Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.0-a-sin-^m-b-trg-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [538]. This is test number [65].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (538)	% 0. (0)
Mathematica	% 100. (538)	% 0. (0)
Maple	% 82.16 (442)	% 17.84 (96)
Maxima	% 37.17 (200)	% 62.83 (338)
Fricas	% 50.93 (274)	% 49.07 (264)
Sympy	% 18.22 (98)	% 81.78 (440)
Giac	% 39.41 (212)	% 60.59 (326)

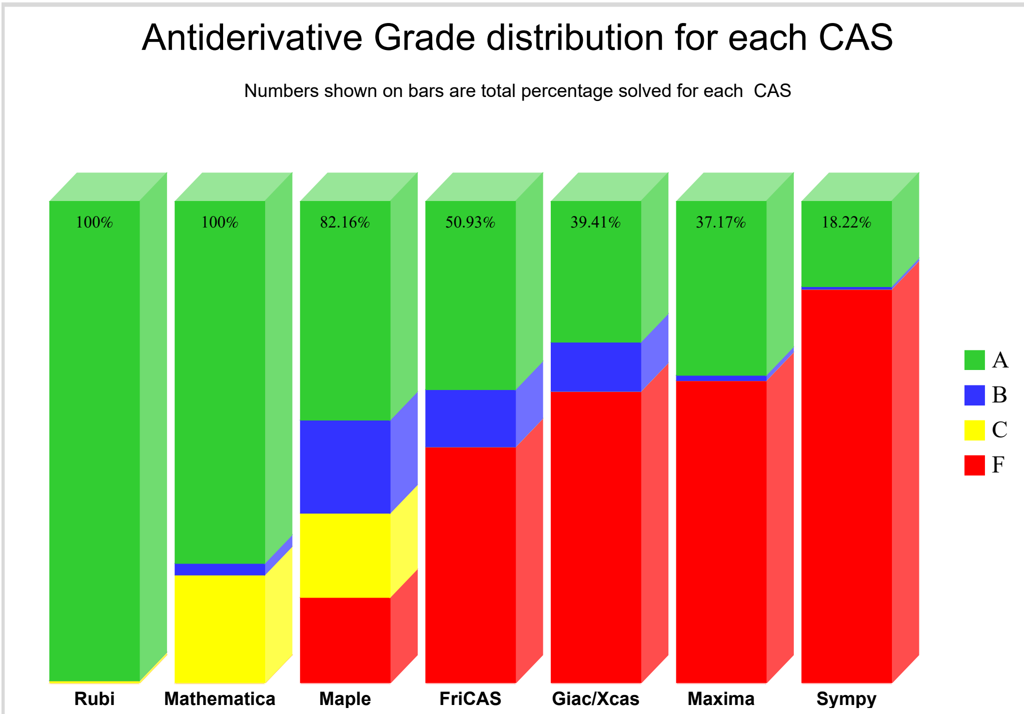
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

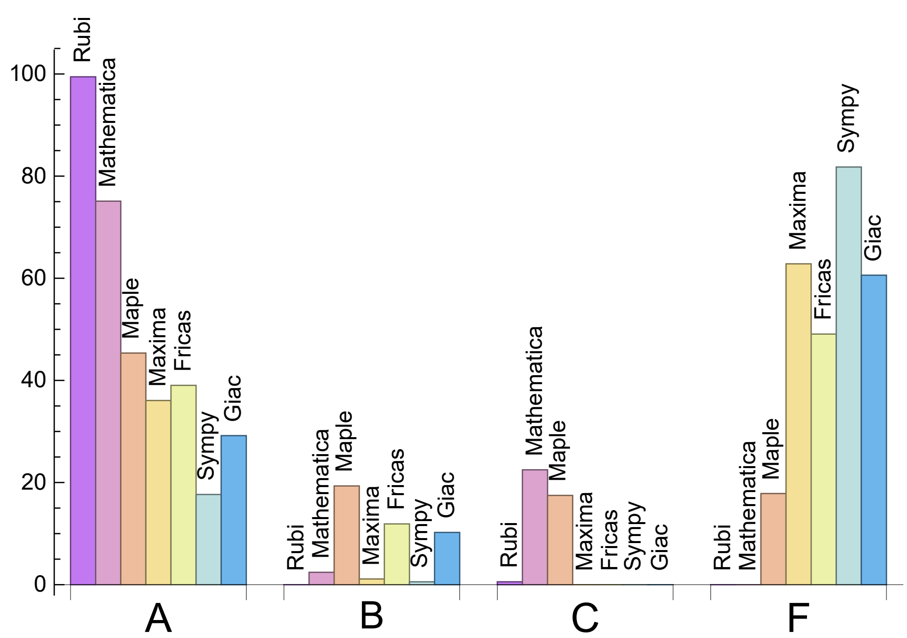
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.44	0.	0.56	0.
Mathematica	75.09	2.42	22.49	0.
Maple	45.35	19.33	17.47	17.84
Maxima	36.06	1.12	0.	62.83
Fricas	39.03	11.9	0.	49.07
Sympy	17.66	0.56	0.	81.78
Giac	29.18	10.22	0.	60.59

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.07	78.71	1.	68.	1.
Mathematica	0.45	109.48	1.63	56.	0.88
Maple	0.09	235.29	2.84	109.	1.7
Maxima	1.02	54.62	1.37	49.	1.2
Fricas	2.26	260.68	4.13	128.	3.04
Sympy	14.01	214.19	4.35	63.	1.68
Giac	1.6	98.06	2.16	68.	1.47

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {486, 488, 489, 490, 491, 496, 497, 498, 499, 500, 501, 502}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

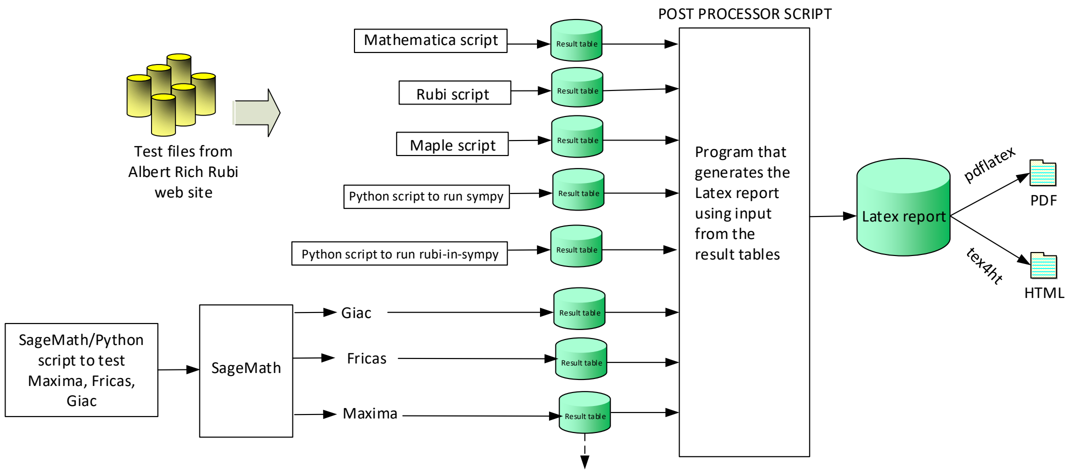
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

B grade: { }

C grade: { 35, 36, 37 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 141, 143, 145, 146, 147, 148, 149, 151, 152, 154, 156, 157, 158, 159, 160, 161, 163, 164, 165, 167, 169, 171, 172, 173, 174, 175, 177, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 204, 205, 206, 207, 208, 209, 222, 223, 224, 225, 226, 227, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 252, 253, 254, 255, 256, 264, 265, 266, 273, 274, 275, 284, 285, 286, 288, 297, 298, 299, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 453, 454, 455, 466, 467, 468, 469, 470, 471, 476, 483, 484, 485, 487, 492, 493, 494, 495, 500, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

B grade: { 44, 87, 88, 126, 150, 153, 155, 176, 178, 183, 210, 221, 366 }

C grade: { 35, 36, 37, 138, 140, 142, 144, 162, 166, 168, 170, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 228, 229, 230, 231, 243, 245, 246, 247, 248, 249, 250, 251, 257, 258, 259, 260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 276, 277, 278, 279, 280, 281, 282, 283, 287, 289, 290, 291, 292, 293, 294, 295, 296, 300, 301, 302, 303, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 450, 451, 452, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 486, 488, 489, 490, 491, 496, 497, 498, 499, 501, 502 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 77, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 113, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 200, 204, 206, 211, 212, 213, 214, 215, 216, 217, 232, 234, 239, 240, 252, 253, 254, 255, 256, 264, 265, 266, 267, 268, 269, 270, 273, 274, 275, 284, 285, 286, 291, 292, 294, 295, 297, 298, 299, 342, 357, 374, 388, 400, 410, 411, 412, 413, 424, 425, 426, 427, 437, 438, 439, 440, 453, 454, 455, 456, 457, 469, 470, 471, 476, 477, 478, 479 }

B grade: { 10, 74, 76, 78, 79, 82, 110, 112, 114, 116, 163, 196, 198, 199, 201, 202, 203, 205, 207, 208, 209, 210, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 235, 236, 237, 238, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 257, 258, 259, 260, 261, 276, 277, 278, 279, 280, 281, 288, 293, 371, 372, 373, 375, 376, 377, 385, 386, 387, 389, 390, 397, 398, 399, 401, 402, 403, 414, 415, 416, 428, 429, 430, 441, 442, 443, 458, 459, 460, 461, 462, 463, 464, 465, 468, 480, 481, 482, 494 }

C grade: { 262, 263, 271, 272, 282, 283, 287, 289, 290, 296, 300, 301, 302, 303, 356, 378, 379, 380, 381, 382, 383, 384, 391, 392, 393, 394, 395, 396, 404, 405, 406, 407, 408, 409, 417, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 466, 467, 472, 473, 474, 475, 493, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537 }

F grade: { 33, 34, 35, 36, 37, 38, 39, 40, 41, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 538 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 204, 205, 206, 207, 208, 209, 210, 221, 252, 253, 254, 255, 256, 371, 372, 373, 374, 385, 386, 387, 388, 397, 398, 399, 400, 410, 411, 412, 413, 424, 425, 426, 427, 437, 438, 439, 440, 492, 493, 494

B grade: { 75, 79, 111, 113, 117, 126 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 389, 390, 391, 392, 393, 394, 395, 396, 401, 402, 403, 404, 405, 406, 407, 408, 409, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 428, 429, 430, 431, 432, 433, 434, 435, 436, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 155, 157, 158, 159, 160, 161, 163, 165, 167, 169, 170, 171, 172, 173, 174, 175, 179, 183, 187, 188, 189, 190, 191, 192, 193, 204, 205, 206, 207, 208, 209, 210, 221, 222, 223, 225, 231, 242, 251, 252, 253, 254, 255, 256, 264, 265, 266, 273, 274, 275, 284, 285, 286, 288, 297, 298, 299, 324, 327, 328, 329, 332, 333, 334, 335, 340, 341, 342, 355, 356, 357, 371, 372, 373, 374, 385, 386, 387, 388, 397, 398, 399, 400, 410, 411, 412, 413, 424, 425, 426, 427, 437, 438, 439, 440, 453, 454, 455, 469, 470, 471, 492, 493, 494 }

B grade: { 53, 54, 63, 83, 86, 104, 111, 126, 127, 128, 140, 150, 152, 153, 154, 156, 162, 164, 166, 168, 176, 177, 178, 180, 181, 182, 184, 185, 186, 224, 226, 227, 228, 229, 230, 243, 244, 245, 246, 247, 248, 249, 250, 289, 290, 375, 376, 377, 389, 390, 401, 402, 403, 414, 415, 416, 428, 429, 430, 441, 442, 443, 468, 476 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 257, 258, 259, 260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 276, 277, 278, 279, 280, 281, 282, 283, 287, 291, 292, 293, 294, 295, 296, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 330, 331, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 378, 379, 380, 381, 382, 383, 384, 391, 392, 393, 394, 395, 396, 404, 405, 406, 407, 408, 409, 417, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 42, 43, 44, 49, 50, 51, 52, 58, 59, 60, 61, 66, 67, 68, 69, 70, 80, 81, 82, 83, 89, 90, 91, 92, 98, 99, 100, 101, 102, 103, 104, 120, 121, 122, 123, 124, 125, 133, 134, 135, 136, 137, 138, 139, 145, 146, 147, 148, 149, 150, 151, 157, 158, 159, 160, 161, 162, 163, 164, 165, 171, 172, 173, 174, 175, 176, 177, 178, 179, 186, 187, 188, 189, 190, 191, 204, 205, 206, 207, 252, 254, 340, 341, 342, 355, 356, 357 }

B grade: { 185, 253, 256 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 45, 46, 47, 48, 53, 54, 55, 56, 57, 62, 63, 64, 65, 71, 72, 73, 74, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 93, 94, 95, 96, 97, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 126, 127, 128, 129, 130, 131, 132, 140, 141, 142, 143, 144, 152, 153, 154, 155, 156, 166, 167, 168, 169, 170, 180, 181, 182, 183, 184, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 110, 118, 123, 125, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 149, 151, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 177, 178, 179, 185, 186, 188, 189, 190, 191, 192, 193, 204, 205, 206, 207, 208, 209, 210, 227, 228, 229, 230, 231, 248, 249, 250, 251, 252, 253, 254, 255, 256, 342, 357, 371, 372, 373, 374, 375, 376, 377, 385, 386, 387, 388, 389, 390, 397, 398, 399, 401, 402, 403, 412, 426, 427, 439, 440 }

B grade: { 98, 99, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 124, 126, 127, 128, 129, 130, 131, 132, 138, 145, 146, 147, 148, 150, 152, 153, 154, 155, 156, 162, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 340, 341, 355, 356, 400, 413 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 187, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 378, 379, 380, 381, 382, 383, 384, 391, 392, 393, 394, 395, 396, 404, 405, 406, 407, 408, 409, 410, 411, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	22	12	15	23	14	15
normalized size	1	1.	2.	1.09	1.36	2.09	1.27	1.36
time (sec)	N/A	0.004	0.009	0.029	0.947	2.499	0.132	1.091

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	32	55	46	24
normalized size	1	1.	0.92	1.08	1.28	2.2	1.84	0.96
time (sec)	N/A	0.009	0.028	0.025	0.995	2.523	0.209	1.135

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	29	22	30	55	37	34
normalized size	1	1.	1.07	0.81	1.11	2.04	1.37	1.26
time (sec)	N/A	0.01	0.01	0.072	1.022	2.145	0.441	1.14

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	33	38	45	89	95	43
normalized size	1	1.	0.72	0.83	0.98	1.93	2.07	0.93
time (sec)	N/A	0.02	0.04	0.044	1.009	2.315	0.979	1.102

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	44	32	46	89	60	51
normalized size	1	1.	1.05	0.76	1.1	2.12	1.43	1.21
time (sec)	N/A	0.013	0.012	0.037	0.995	2.185	1.838	1.122

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	45	48	65	120	139	62
normalized size	1	1.	0.67	0.72	0.97	1.79	2.07	0.93
time (sec)	N/A	0.033	0.042	0.036	1.007	2.202	3.546	1.105

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	42	59	115	80	68
normalized size	1	1.	1.09	0.78	1.09	2.13	1.48	1.26
time (sec)	N/A	0.016	0.009	0.038	0.979	2.275	6.821	1.133

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	55	58	80	155	184	81
normalized size	1	1.	0.62	0.66	0.91	1.76	2.09	0.92
time (sec)	N/A	0.048	0.056	0.034	1.023	2.275	12.31	1.132

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	45	84	0	0	0	0
normalized size	1	1.	0.75	1.4	0.	0.	0.	0.
time (sec)	N/A	0.027	0.069	0.093	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	35	118	0	0	0	0
normalized size	1	1.	0.85	2.88	0.	0.	0.	0.
time (sec)	N/A	0.015	0.038	0.039	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	33	72	0	0	0	0
normalized size	1	1.	0.8	1.76	0.	0.	0.	0.
time (sec)	N/A	0.015	0.036	0.033	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	21	77	0	0	0	0
normalized size	1	1.	1.11	4.05	0.	0.	0.	0.
time (sec)	N/A	0.008	0.025	0.032	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	21	57	0	0	0	0
normalized size	1	1.	1.11	3.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.025	0.034	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	32	110	0	0	0	0
normalized size	1	1.	0.86	2.97	0.	0.	0.	0.
time (sec)	N/A	0.013	0.053	0.039	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	33	72	0	0	0	0
normalized size	1	1.	0.8	1.76	0.	0.	0.	0.
time (sec)	N/A	0.015	0.049	0.036	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	132	0	0	0	0
normalized size	1	1.	0.85	2.2	0.	0.	0.	0.
time (sec)	N/A	0.024	0.05	0.037	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	55	104	0	0	0	0
normalized size	1	1.	0.79	1.49	0.	0.	0.	0.
time (sec)	N/A	0.029	0.119	0.029	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	142	0	0	0	0
normalized size	1	1.	0.94	3.02	0.	0.	0.	0.
time (sec)	N/A	0.017	0.087	0.032	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	40	88	0	0	0	0
normalized size	1	1.	0.85	1.87	0.	0.	0.	0.
time (sec)	N/A	0.016	0.038	0.026	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	24	91	0	0	0	0
normalized size	1	1.	1.14	4.33	0.	0.	0.	0.
time (sec)	N/A	0.008	0.017	0.026	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	24	69	0	0	0	0
normalized size	1	1.	1.14	3.29	0.	0.	0.	0.
time (sec)	N/A	0.008	0.019	0.023	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	39	132	0	0	0	0
normalized size	1	1.	0.91	3.07	0.	0.	0.	0.
time (sec)	N/A	0.015	0.075	0.027	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	88	0	0	0	0
normalized size	1	1.	0.91	1.87	0.	0.	0.	0.
time (sec)	N/A	0.016	0.104	0.026	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	55	160	0	0	0	0
normalized size	1	1.	0.79	2.29	0.	0.	0.	0.
time (sec)	N/A	0.027	0.284	0.031	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	80	108	0	0	0	0
normalized size	1	1.	0.78	1.05	0.	0.	0.	0.
time (sec)	N/A	0.052	0.153	0.042	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	66	152	0	0	0	0
normalized size	1	1.	0.88	2.03	0.	0.	0.	0.
time (sec)	N/A	0.032	0.104	0.036	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	62	97	0	0	0	0
normalized size	1	1.	0.83	1.29	0.	0.	0.	0.
time (sec)	N/A	0.033	0.052	0.034	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	98	0	0	0	0
normalized size	1	1.	0.98	2.28	0.	0.	0.	0.
time (sec)	N/A	0.018	0.021	0.036	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	74	0	0	0	0
normalized size	1	1.	0.98	1.72	0.	0.	0.	0.
time (sec)	N/A	0.018	0.027	0.032	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	54	141	0	0	0	0
normalized size	1	1.	0.74	1.93	0.	0.	0.	0.
time (sec)	N/A	0.032	0.047	0.043	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	55	105	0	0	0	0
normalized size	1	1.	0.71	1.36	0.	0.	0.	0.
time (sec)	N/A	0.033	0.075	0.04	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	68	168	0	0	0	0
normalized size	1	1.	0.65	1.6	0.	0.	0.	0.
time (sec)	N/A	0.051	0.158	0.042	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.055	0.128	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.035	0.128	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	517	58	55	0	0	0	0	0
normalized size	1	0.11	0.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.033	0.069	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	58	55	0	0	0	0	0
normalized size	1	0.23	0.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.045	0.058	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	56	53	0	0	0	0	0
normalized size	1	0.21	0.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.041	0.048	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.042	0.036	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.043	0.47	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.04	0.433	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	76	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.079	0.861	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	31	41	32
normalized size	1	1.	1.	0.93	1.2	2.07	2.73	2.13
time (sec)	N/A	0.019	0.005	0.006	1.009	1.814	1.118	1.148

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	31	22	18
normalized size	1	1.	1.	0.93	1.2	2.07	1.47	1.2
time (sec)	N/A	0.021	0.005	0.004	0.993	1.803	0.505	1.126

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	37	14	18	31	19	18
normalized size	1	1.	2.47	0.93	1.2	2.07	1.27	1.2
time (sec)	N/A	0.011	0.013	0.001	1.013	1.676	0.231	1.149

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	24	31	0	24
normalized size	1	1.	1.	1.	2.	2.58	0.	2.
time (sec)	N/A	0.004	0.006	0.01	0.988	1.926	0.	1.202

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	16	27	0	16
normalized size	1	1.	1.	1.1	1.6	2.7	0.	1.6
time (sec)	N/A	0.011	0.007	0.009	0.974	1.877	0.	1.131

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	23	32	0	18
normalized size	1	1.	1.	0.93	1.53	2.13	0.	1.2
time (sec)	N/A	0.019	0.01	0.008	0.99	1.675	0.	1.158

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	32	0	18
normalized size	1	1.	1.	0.93	1.2	2.13	0.	1.2
time (sec)	N/A	0.019	0.008	0.008	1.022	1.823	0.	1.182

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	60	62	142	88	73
normalized size	1	1.	0.77	0.98	1.02	2.33	1.44	1.2
time (sec)	N/A	0.043	0.156	0.039	0.999	1.981	22.218	1.231

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	50	49	115	66	73
normalized size	1	1.	0.8	1.09	1.07	2.5	1.43	1.59
time (sec)	N/A	0.038	0.092	0.034	0.985	1.915	7.791	1.149

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	40	35	84	44	35
normalized size	1	1.	0.87	1.29	1.13	2.71	1.42	1.13
time (sec)	N/A	0.035	0.059	0.036	0.987	1.973	2.1	1.234

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	57	20	18
normalized size	1	1.	1.	0.93	1.2	3.8	1.33	1.2
time (sec)	N/A	0.018	0.004	0.003	0.977	1.858	0.515	1.223

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	23	19	24	72	0	24
normalized size	1	1.	1.64	1.36	1.71	5.14	0.	1.71
time (sec)	N/A	0.008	0.007	0.016	1.49	1.824	0.	1.132

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	18	80	0	18
normalized size	1	1.	1.	1.47	1.2	5.33	0.	1.2
time (sec)	N/A	0.03	0.007	0.019	1.023	1.735	0.	1.163

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	56	42	35	107	0	35
normalized size	1	1.	1.81	1.35	1.13	3.45	0.	1.13
time (sec)	N/A	0.034	0.041	0.02	0.992	1.792	0.	1.158

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	77	60	49	138	0	49
normalized size	1	1.	1.67	1.3	1.07	3.	0.	1.07
time (sec)	N/A	0.039	0.043	0.022	0.989	1.845	0.	1.174

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	98	78	62	165	0	62
normalized size	1	1.	1.61	1.28	1.02	2.7	0.	1.02
time (sec)	N/A	0.04	0.035	0.023	0.979	1.595	0.	1.204

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	52	64	65	149	189	81
normalized size	1	1.	0.59	0.73	0.74	1.69	2.15	0.92
time (sec)	N/A	0.066	0.144	0.039	0.998	1.622	12.097	1.175

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	40	54	50	116	136	62
normalized size	1	1.	0.6	0.81	0.75	1.73	2.03	0.93
time (sec)	N/A	0.051	0.088	0.037	1.012	1.538	3.694	1.202

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	23	43	32	84	92	24
normalized size	1	1.	0.5	0.93	0.7	1.83	2.	0.52
time (sec)	N/A	0.04	0.033	0.012	0.974	1.575	1.017	1.159

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	32	55	46	24
normalized size	1	1.	0.92	1.08	1.28	2.2	1.84	0.96
time (sec)	N/A	0.009	0.024	0.	0.96	1.561	0.216	1.137

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	31	46	99	0	49
normalized size	1	1.	1.	1.35	2.	4.3	0.	2.13
time (sec)	N/A	0.015	0.011	0.017	0.991	1.685	0.	1.241

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	53	62	163	0	65
normalized size	1	1.	1.	1.56	1.82	4.79	0.	1.91
time (sec)	N/A	0.023	0.014	0.019	0.979	1.634	0.	1.293

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	74	88	193	0	111
normalized size	1	1.	1.	1.35	1.6	3.51	0.	2.02
time (sec)	N/A	0.045	0.045	0.021	1.043	1.707	0.	1.258

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	95	123	227	0	99
normalized size	1	1.	1.	1.25	1.62	2.99	0.	1.3
time (sec)	N/A	0.059	0.065	0.023	1.	1.739	0.	1.226

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	48	34	49	62	63	36
normalized size	1	1.	1.55	1.1	1.58	2.	2.03	1.16
time (sec)	N/A	0.033	0.129	0.01	0.97	1.695	12.261	1.169

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	34	35	62	46	36
normalized size	1	1.	0.87	1.1	1.13	2.	1.48	1.16
time (sec)	N/A	0.034	0.091	0.013	0.981	1.666	7.054	1.166

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	35	34	35	62	42	35
normalized size	1	1.	1.13	1.1	1.13	2.	1.35	1.13
time (sec)	N/A	0.033	0.015	0.01	0.985	1.615	3.582	1.236

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	34	35	62	46	36
normalized size	1	1.	0.87	1.1	1.13	2.	1.48	1.16
time (sec)	N/A	0.032	0.062	0.012	0.979	1.638	2.135	1.246

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	58	20	18
normalized size	1	1.	1.	0.93	1.2	3.87	1.33	1.2
time (sec)	N/A	0.018	0.003	0.003	0.976	1.552	0.98	1.21

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	25	27	34	63	0	39
normalized size	1	1.	0.89	0.96	1.21	2.25	0.	1.39
time (sec)	N/A	0.021	0.017	0.016	0.975	1.657	0.	1.206

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	40	26	53	0	31
normalized size	1	1.	1.	1.9	1.24	2.52	0.	1.48
time (sec)	N/A	0.021	0.022	0.014	0.988	1.572	0.	1.192

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	42	89	0	57
normalized size	1	1.	0.93	0.96	1.56	3.3	0.	2.11
time (sec)	N/A	0.011	0.024	0.019	1.001	1.637	0.	1.186

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	60	34	65	0	34
normalized size	1	1.	1.	2.22	1.26	2.41	0.	1.26
time (sec)	N/A	0.02	0.023	0.02	1.011	1.554	0.	1.168

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	53	65	0	34
normalized size	1	1.	1.	1.47	3.53	4.33	0.	2.27
time (sec)	N/A	0.028	0.005	0.019	0.979	1.599	0.	1.194

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	78	34	66	0	34
normalized size	1	1.	1.	2.52	1.1	2.13	0.	1.1
time (sec)	N/A	0.032	0.048	0.019	0.976	1.608	0.	1.186

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	28	42	66	66	0	34
normalized size	1	1.	0.9	1.35	2.13	2.13	0.	1.1
time (sec)	N/A	0.033	0.035	0.02	0.995	1.575	0.	1.19

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	96	34	66	0	34
normalized size	1	1.	1.	3.1	1.1	2.13	0.	1.1
time (sec)	N/A	0.032	0.028	0.021	1.017	1.675	0.	1.176

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	28	60	80	66	0	34
normalized size	1	1.	0.9	1.94	2.58	2.13	0.	1.1
time (sec)	N/A	0.032	0.042	0.02	1.031	1.632	0.	1.201

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	78	62	173	88	111
normalized size	1	1.	0.77	1.28	1.02	2.84	1.44	1.82
time (sec)	N/A	0.04	0.201	0.011	0.982	1.719	59.681	1.168

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	68	49	140	66	92
normalized size	1	1.	0.8	1.48	1.07	3.04	1.43	2.
time (sec)	N/A	0.036	0.111	0.013	0.99	1.683	21.526	1.15

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	58	35	108	44	35
normalized size	1	1.	0.87	1.87	1.13	3.48	1.42	1.13
time (sec)	N/A	0.032	0.07	0.013	0.978	1.605	7.484	1.156

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	81	20	18
normalized size	1	1.	1.	0.93	1.2	5.4	1.33	1.2
time (sec)	N/A	0.018	0.003	0.003	0.987	1.565	2.027	1.15

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	31	54	55	108	0	55
normalized size	1	1.	0.78	1.35	1.38	2.7	0.	1.38
time (sec)	N/A	0.039	0.113	0.017	1.486	1.607	0.	1.237

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	38	28	39	115	0	39
normalized size	1	1.	1.36	1.	1.39	4.11	0.	1.39
time (sec)	N/A	0.016	0.009	0.02	1.502	1.648	0.	1.697

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	18	104	0	18
normalized size	1	1.	1.	1.47	1.2	6.93	0.	1.2
time (sec)	N/A	0.028	0.007	0.019	0.994	1.538	0.	1.143

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	77	42	35	131	0	35
normalized size	1	1.	2.48	1.35	1.13	4.23	0.	1.13
time (sec)	N/A	0.032	0.035	0.023	1.007	1.623	0.	1.186

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	98	60	49	163	0	49
normalized size	1	1.	2.13	1.3	1.07	3.54	0.	1.07
time (sec)	N/A	0.036	0.039	0.023	0.988	1.661	0.	1.169

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	62	82	65	180	231	100
normalized size	1	1.	0.56	0.74	0.59	1.62	2.08	0.9
time (sec)	N/A	0.097	0.196	0.011	1.008	1.719	32.538	1.146

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	33	72	45	146	189	43
normalized size	1	1.	0.37	0.8	0.5	1.62	2.1	0.48
time (sec)	N/A	0.083	0.044	0.012	0.997	1.605	12.39	1.161

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	40	61	50	117	136	62
normalized size	1	1.	0.58	0.88	0.72	1.7	1.97	0.9
time (sec)	N/A	0.068	0.067	0.01	1.11	1.665	3.845	1.175

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	33	38	45	89	95	43
normalized size	1	1.	0.72	0.83	0.98	1.93	2.07	0.93
time (sec)	N/A	0.02	0.017	0.	1.149	1.617	1.01	1.133

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	44	62	132	0	65
normalized size	1	1.	1.	1.16	1.63	3.47	0.	1.71
time (sec)	N/A	0.027	0.015	0.016	1.031	1.705	0.	1.173

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	40	66	76	200	0	78
normalized size	1	1.	0.82	1.35	1.55	4.08	0.	1.59
time (sec)	N/A	0.029	0.091	0.02	1.012	1.73	0.	1.199

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	45	87	96	200	0	85
normalized size	1	1.	0.82	1.58	1.75	3.64	0.	1.55
time (sec)	N/A	0.042	0.121	0.019	1.061	1.682	0.	1.214

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	99	108	123	227	0	99
normalized size	1	1.	1.27	1.38	1.58	2.91	0.	1.27
time (sec)	N/A	0.075	0.028	0.021	1.148	1.752	0.	1.187

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	64	129	150	255	0	144
normalized size	1	1.	0.65	1.3	1.52	2.58	0.	1.45
time (sec)	N/A	0.089	0.313	0.022	1.083	1.769	0.	1.22

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	68	52	62	97	83	115
normalized size	1	1.	1.48	1.13	1.35	2.11	1.8	2.5
time (sec)	N/A	0.041	0.379	0.013	1.027	1.701	81.059	1.175

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	52	49	97	68	111
normalized size	1	1.	0.8	1.13	1.07	2.11	1.48	2.41
time (sec)	N/A	0.036	0.273	0.013	0.989	1.741	54.047	1.14

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	50	52	49	93	63	58
normalized size	1	1.	1.09	1.13	1.07	2.02	1.37	1.26
time (sec)	N/A	0.04	0.027	0.013	0.997	1.677	31.496	1.119

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	52	49	95	68	92
normalized size	1	1.	0.8	1.13	1.07	2.07	1.48	2.
time (sec)	N/A	0.036	0.143	0.015	0.996	1.71	19.904	1.133

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	48	52	35	89	42	35
normalized size	1	1.	1.55	1.68	1.13	2.87	1.35	1.13
time (sec)	N/A	0.032	0.12	0.011	1.001	1.695	11.609	1.148

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	52	49	95	68	73
normalized size	1	1.	0.8	1.13	1.07	2.07	1.48	1.59
time (sec)	N/A	0.035	0.09	0.012	0.982	1.598	6.948	1.167

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	85	20	18
normalized size	1	1.	1.	0.93	1.2	5.67	1.33	1.2
time (sec)	N/A	0.017	0.004	0.004	0.964	1.508	3.411	1.145

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	35	40	50	90	0	305
normalized size	1	1.	0.88	1.	1.25	2.25	0.	7.62
time (sec)	N/A	0.026	0.033	0.014	0.957	1.705	0.	1.189

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	39	50	43	85	0	134
normalized size	1	1.	1.05	1.35	1.16	2.3	0.	3.62
time (sec)	N/A	0.034	0.033	0.017	0.966	1.571	0.	1.14

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	60	55	139	0	246
normalized size	1	1.	0.77	1.4	1.28	3.23	0.	5.72
time (sec)	N/A	0.038	0.04	0.021	0.958	1.626	0.	1.2

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	70	47	90	0	135
normalized size	1	1.	1.	1.84	1.24	2.37	0.	3.55
time (sec)	N/A	0.025	0.025	0.021	0.97	1.638	0.	1.178

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	37	40	73	116	0	305
normalized size	1	1.	0.86	0.93	1.7	2.7	0.	7.09
time (sec)	N/A	0.021	0.052	0.026	0.965	1.674	0.	1.226

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	88	47	93	0	97
normalized size	1	1.	1.	2.15	1.15	2.27	0.	2.37
time (sec)	N/A	0.023	0.028	0.023	1.009	1.582	0.	1.16

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	80	89	0	65
normalized size	1	1.	1.	1.47	5.33	5.93	0.	4.33
time (sec)	N/A	0.027	0.009	0.03	0.965	1.523	0.	1.181

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	106	47	96	0	157
normalized size	1	1.	1.	2.3	1.02	2.09	0.	3.41
time (sec)	N/A	0.036	0.032	0.023	0.958	1.513	0.	1.169

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	38	42	93	90	0	126
normalized size	1	1.	1.23	1.35	3.	2.9	0.	4.06
time (sec)	N/A	0.031	0.055	0.023	0.988	1.537	0.	1.199

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	124	47	96	0	216
normalized size	1	1.	1.	2.7	1.02	2.09	0.	4.7
time (sec)	N/A	0.035	0.033	0.023	0.987	1.576	0.	1.179

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	38	60	107	95	0	188
normalized size	1	1.	0.83	1.3	2.33	2.07	0.	4.09
time (sec)	N/A	0.039	0.059	0.023	0.973	1.593	0.	1.191

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	142	47	99	0	275
normalized size	1	1.	1.	3.09	1.02	2.15	0.	5.98
time (sec)	N/A	0.035	0.033	0.024	0.974	1.61	0.	1.193

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	38	78	120	97	0	247
normalized size	1	1.	0.83	1.7	2.61	2.11	0.	5.37
time (sec)	N/A	0.04	0.129	0.026	0.974	1.642	0.	1.203

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	52	79	89	231	0	92
normalized size	1	1.	0.79	1.2	1.35	3.5	0.	1.39
time (sec)	N/A	0.047	0.186	0.022	0.96	1.764	0.	1.227

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	96	61	116	0	194
normalized size	1	1.	1.	1.92	1.22	2.32	0.	3.88
time (sec)	N/A	0.028	0.04	0.026	0.975	1.585	0.	1.217

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	75	58	76	178	1085	196
normalized size	1	1.	1.42	1.09	1.43	3.36	20.47	3.7
time (sec)	N/A	0.029	0.032	0.016	1.032	1.735	11.017	1.177

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	47	93	1086	230
normalized size	1	1.	1.	0.98	1.18	2.32	27.15	5.75
time (sec)	N/A	0.027	0.015	0.016	0.973	1.722	6.079	1.165

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	60	45	62	146	410	136
normalized size	1	1.	1.58	1.18	1.63	3.84	10.79	3.58
time (sec)	N/A	0.026	0.025	0.011	0.958	1.67	3.415	1.163

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	34	68	369	34
normalized size	1	1.	1.	0.96	1.26	2.52	13.67	1.26
time (sec)	N/A	0.021	0.014	0.016	0.96	1.679	1.935	1.171

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	42	32	46	115	92	77
normalized size	1	1.	1.83	1.39	2.	5.	4.	3.35
time (sec)	N/A	0.015	0.015	0.013	0.977	1.603	1.072	1.216

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	19	12	15	34	17	16
normalized size	1	1.	1.73	1.09	1.36	3.09	1.55	1.45
time (sec)	N/A	0.004	0.011	0.004	0.971	1.655	0.412	1.184

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	31	12	38	85	0	76
normalized size	1	1.	2.82	1.09	3.45	7.73	0.	6.91
time (sec)	N/A	0.01	0.021	0.017	0.977	1.894	0.	1.199

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	42	34	49	154	0	74
normalized size	1	1.	1.83	1.48	2.13	6.7	0.	3.22
time (sec)	N/A	0.022	0.026	0.017	0.986	1.609	0.	1.198

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	36	26	54	154	0	167
normalized size	1	1.	1.33	0.96	2.	5.7	0.	6.19
time (sec)	N/A	0.02	0.031	0.022	1.049	1.688	0.	1.22

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	57	47	68	193	0	136
normalized size	1	1.	1.5	1.24	1.79	5.08	0.	3.58
time (sec)	N/A	0.026	0.023	0.022	1.014	1.623	0.	1.235

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	46	39	88	185	0	230
normalized size	1	1.	1.18	1.	2.26	4.74	0.	5.9
time (sec)	N/A	0.026	0.091	0.023	0.974	1.743	0.	1.19

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	72	60	81	225	0	196
normalized size	1	1.	1.36	1.13	1.53	4.25	0.	3.7
time (sec)	N/A	0.028	0.023	0.026	0.964	1.697	0.	1.196

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	56	52	115	212	0	289
normalized size	1	1.	0.98	0.91	2.02	3.72	0.	5.07
time (sec)	N/A	0.031	0.146	0.025	1.008	1.717	0.	1.225

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	62	57	111	82	57
normalized size	1	1.	1.	1.24	1.14	2.22	1.64	1.14
time (sec)	N/A	0.038	0.026	0.013	0.986	1.959	8.109	1.19

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	41	66	85	135	119	74
normalized size	1	1.	0.67	1.08	1.39	2.21	1.95	1.21
time (sec)	N/A	0.046	0.138	0.011	1.485	1.842	4.808	1.245

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	52	43	84	61	43
normalized size	1	1.	1.	1.37	1.13	2.21	1.61	1.13
time (sec)	N/A	0.034	0.017	0.012	0.99	1.782	2.675	1.153

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	31	56	58	104	75	58
normalized size	1	1.	0.78	1.4	1.45	2.6	1.88	1.45
time (sec)	N/A	0.037	0.134	0.012	1.483	1.825	1.636	1.16

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	42	27	53	39	27
normalized size	1	1.	1.	1.83	1.17	2.3	1.7	1.17
time (sec)	N/A	0.02	0.012	0.011	0.992	1.834	0.983	1.132

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	29	21	24	72	29	47
normalized size	1	1.	1.93	1.4	1.6	4.8	1.93	3.13
time (sec)	N/A	0.008	0.013	0.01	1.479	1.937	0.694	1.169

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	18	28	20	18
normalized size	1	1.	1.	1.27	1.64	2.55	1.82	1.64
time (sec)	N/A	0.009	0.008	0.002	0.995	1.811	0.685	1.152

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	27	33	49	136	0	51
normalized size	1	1.	1.17	1.43	2.13	5.91	0.	2.22
time (sec)	N/A	0.022	0.015	0.017	0.99	1.989	0.	1.198

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	13	31	30	74	0	22
normalized size	1	1.	0.59	1.41	1.36	3.36	0.	1.
time (sec)	N/A	0.03	0.017	0.044	0.986	1.795	0.	1.143

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	27	55	82	228	0	85
normalized size	1	1.	0.55	1.12	1.67	4.65	0.	1.73
time (sec)	N/A	0.043	0.013	0.023	1.017	1.982	0.	1.22

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	46	50	43	108	0	43
normalized size	1	1.	1.21	1.32	1.13	2.84	0.	1.13
time (sec)	N/A	0.037	0.033	0.022	0.986	1.785	0.	1.199

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	27	76	107	261	0	99
normalized size	1	1.	0.39	1.09	1.53	3.73	0.	1.41
time (sec)	N/A	0.047	0.014	0.029	1.	2.099	0.	1.22

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	45	74	61	190	1484	312
normalized size	1	1.	0.78	1.28	1.05	3.28	25.59	5.38
time (sec)	N/A	0.044	0.102	0.014	0.972	2.024	22.474	1.245

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	103	81	89	265	719	220
normalized size	1	1.	1.56	1.23	1.35	4.02	10.89	3.33
time (sec)	N/A	0.045	0.039	0.013	0.977	2.046	11.723	1.168

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	35	61	47	159	614	252
normalized size	1	1.	0.81	1.42	1.09	3.7	14.28	5.86
time (sec)	N/A	0.039	0.062	0.013	0.974	1.942	6.341	1.211

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	86	68	76	232	241	189
normalized size	1	1.	1.76	1.39	1.55	4.73	4.92	3.86
time (sec)	N/A	0.029	0.027	0.013	1.018	2.035	3.358	1.185

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	34	27	31	108	42	49
normalized size	1	1.	1.21	0.96	1.11	3.86	1.5	1.75
time (sec)	N/A	0.013	0.095	0.011	0.978	2.395	1.005	1.137

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	75	55	62	200	58	126
normalized size	1	1.	2.21	1.62	1.82	5.88	1.71	3.71
time (sec)	N/A	0.023	0.027	0.01	0.982	2.575	1.641	1.178

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	38	24	18
normalized size	1	1.	1.	0.93	1.2	2.53	1.6	1.2
time (sec)	N/A	0.018	0.012	0.004	0.961	2.49	0.977	1.131

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	34	26	49	176	0	161
normalized size	1	1.	1.26	0.96	1.81	6.52	0.	5.96
time (sec)	N/A	0.022	0.042	0.018	0.999	2.382	0.	1.167

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	143	57	82	261	0	189
normalized size	1	1.	2.92	1.16	1.67	5.33	0.	3.86
time (sec)	N/A	0.043	0.248	0.017	0.97	2.256	0.	1.206

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	61	48	86	261	0	254
normalized size	1	1.	1.42	1.12	2.	6.07	0.	5.91
time (sec)	N/A	0.038	0.018	0.02	1.002	2.211	0.	1.24

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	205	78	99	301	0	220
normalized size	1	1.	3.11	1.18	1.5	4.56	0.	3.33
time (sec)	N/A	0.043	0.422	0.023	0.998	2.33	0.	1.179

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	69	111	286	0	313
normalized size	1	1.	0.97	1.19	1.91	4.93	0.	5.4
time (sec)	N/A	0.043	0.225	0.024	1.006	2.378	0.	1.23

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	90	76	176	105	76
normalized size	1	1.	1.	1.32	1.12	2.59	1.54	1.12
time (sec)	N/A	0.044	0.038	0.043	0.99	2.272	24.745	1.19

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	53	94	101	230	141	92
normalized size	1	1.	0.66	1.18	1.26	2.88	1.76	1.15
time (sec)	N/A	0.048	0.281	0.042	1.492	2.326	15.645	1.186

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	80	59	142	82	55
normalized size	1	1.	1.	1.51	1.11	2.68	1.55	1.04
time (sec)	N/A	0.04	0.026	0.012	0.984	2.347	8.077	1.166

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	43	84	74	197	97	74
normalized size	1	1.	0.75	1.47	1.3	3.46	1.7	1.3
time (sec)	N/A	0.041	0.177	0.012	1.485	2.234	4.735	1.155

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	68	47	117	63	47
normalized size	1	1.	1.	1.84	1.27	3.16	1.7	1.27
time (sec)	N/A	0.025	0.019	0.012	0.99	2.172	2.756	1.163

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	33	26	46	166	48	84
normalized size	1	1.	1.22	0.96	1.7	6.15	1.78	3.11
time (sec)	N/A	0.017	0.009	0.014	1.48	2.09	1.673	1.167

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	60	34	89	42	34
normalized size	1	1.	1.	2.31	1.31	3.42	1.62	1.31
time (sec)	N/A	0.021	0.014	0.013	0.993	2.095	1.629	1.129

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	18	78	71	18
normalized size	1	1.	1.	1.47	1.2	5.2	4.73	1.2
time (sec)	N/A	0.029	0.01	0.012	0.993	2.113	2.841	1.178

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	58	24	18
normalized size	1	1.	1.	0.93	1.2	3.87	1.6	1.2
time (sec)	N/A	0.018	0.01	0.003	0.973	2.074	1.477	1.116

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	31	46	68	252	0	70
normalized size	1	1.	0.82	1.21	1.79	6.63	0.	1.84
time (sec)	N/A	0.026	0.013	0.021	0.98	2.242	0.	1.175

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	45	50	47	135	0	47
normalized size	1	1.	1.22	1.35	1.27	3.65	0.	1.27
time (sec)	N/A	0.035	0.035	0.021	0.969	2.097	0.	1.16

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	31	76	99	342	0	97
normalized size	1	1.	0.47	1.15	1.5	5.18	0.	1.47
time (sec)	N/A	0.042	0.013	0.026	1.009	2.285	0.	1.204

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	68	59	165	0	42
normalized size	1	1.	0.81	1.28	1.11	3.11	0.	0.79
time (sec)	N/A	0.038	0.019	0.026	1.016	2.149	0.	1.146

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	31	97	123	375	0	115
normalized size	1	1.	0.35	1.09	1.38	4.21	0.	1.29
time (sec)	N/A	0.049	0.014	0.026	0.996	2.337	0.	1.213

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	55	107	76	274	1664	374
normalized size	1	1.	0.8	1.55	1.1	3.97	24.12	5.42
time (sec)	N/A	0.048	0.109	0.012	0.968	2.428	59.755	1.214

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	141	115	120	378	869	282
normalized size	1	1.	1.58	1.29	1.35	4.25	9.76	3.17
time (sec)	N/A	0.05	0.041	0.011	0.967	2.444	33.469	1.209

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	47	95	66	239	733	313
normalized size	1	1.	0.81	1.64	1.14	4.12	12.64	5.4
time (sec)	N/A	0.043	0.154	0.013	0.961	2.249	18.965	1.205

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	123	102	107	344	330	221
normalized size	1	1.	1.76	1.46	1.53	4.91	4.71	3.16
time (sec)	N/A	0.036	0.031	0.012	0.978	2.301	9.934	1.191

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	46	39	51	188	61	221
normalized size	1	1.	1.1	0.93	1.21	4.48	1.45	5.26
time (sec)	N/A	0.021	0.105	0.013	0.97	2.344	2.703	1.214

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	113	89	96	315	92	188
normalized size	1	1.	2.05	1.62	1.75	5.73	1.67	3.42
time (sec)	N/A	0.042	0.031	0.012	0.982	2.141	4.94	1.187

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	34	99	44	34
normalized size	1	1.	1.	1.47	2.27	6.6	2.93	2.27
time (sec)	N/A	0.028	0.007	0.01	0.989	1.729	2.67	1.129

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	113	76	88	308	58	132
normalized size	1	1.	2.05	1.38	1.6	5.6	1.05	2.4
time (sec)	N/A	0.045	0.034	0.011	0.971	1.951	4.732	1.162

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	68	24	18
normalized size	1	1.	1.	0.93	1.2	4.53	1.6	1.2
time (sec)	N/A	0.018	0.009	0.003	0.983	1.698	2.461	1.141

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	44	39	69	285	0	223
normalized size	1	1.	1.1	0.98	1.72	7.12	0.	5.58
time (sec)	N/A	0.027	0.111	0.02	0.971	1.866	0.	1.336

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	129	78	107	374	0	220
normalized size	1	1.	1.84	1.11	1.53	5.34	0.	3.14
time (sec)	N/A	0.043	4.559	0.022	0.969	1.989	0.	1.182

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	54	69	100	366	0	313
normalized size	1	1.	0.93	1.19	1.72	6.31	0.	5.4
time (sec)	N/A	0.04	0.341	0.026	0.985	1.928	0.	1.246

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	268	99	123	416	0	282
normalized size	1	1.	3.01	1.11	1.38	4.67	0.	3.17
time (sec)	N/A	0.046	0.504	0.025	0.973	1.897	0.	1.22

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	91	90	124	397	0	375
normalized size	1	1.	1.32	1.3	1.8	5.75	0.	5.43
time (sec)	N/A	0.046	0.028	0.026	0.973	1.947	0.	1.242

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	27	22	19	93	29	19
normalized size	1	1.	1.59	1.29	1.12	5.47	1.71	1.12
time (sec)	N/A	0.026	0.022	0.007	0.959	1.771	0.061	1.127

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	19	86	14	24
normalized size	1	1.	1.	1.29	1.12	5.06	0.82	1.41
time (sec)	N/A	0.026	0.008	0.007	0.96	1.742	0.102	1.137

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	62	34	0
normalized size	1	1.	1.	0.86	1.09	2.82	1.55	0.
time (sec)	N/A	0.025	0.033	0.007	0.975	1.956	63.5	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	57	34	28
normalized size	1	1.	1.	0.86	1.09	2.59	1.55	1.27
time (sec)	N/A	0.022	0.017	0.006	0.958	1.91	1.587	1.125

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	24	42	32	24
normalized size	1	1.	1.	0.95	1.2	2.1	1.6	1.2
time (sec)	N/A	0.023	0.015	0.008	0.983	1.909	1.236	1.209

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	24	61	31	24
normalized size	1	1.	1.	0.95	1.2	3.05	1.55	1.2
time (sec)	N/A	0.026	0.024	0.004	0.979	1.842	6.079	1.17

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	66	32	35
normalized size	1	1.	1.	0.86	1.09	3.	1.45	1.59
time (sec)	N/A	0.027	0.028	0.006	0.977	1.827	66.857	1.164

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	66	0	35
normalized size	1	1.	1.	0.86	1.09	3.	0.	1.59
time (sec)	N/A	0.026	0.039	0.004	0.961	1.765	0.	1.155

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	66	0	35
normalized size	1	1.	1.	0.86	1.09	3.	0.	1.59
time (sec)	N/A	0.027	0.063	0.004	0.976	1.721	0.	1.183

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	60	249	0	0	0	0
normalized size	1	1.	0.48	1.98	0.	0.	0.	0.
time (sec)	N/A	0.101	0.136	0.095	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	60	236	0	0	0	0
normalized size	1	1.	0.48	1.87	0.	0.	0.	0.
time (sec)	N/A	0.098	0.134	0.059	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	57	223	0	0	0	0
normalized size	1	1.	0.58	2.28	0.	0.	0.	0.
time (sec)	N/A	0.078	0.05	0.063	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	57	208	0	0	0	0
normalized size	1	1.	0.58	2.12	0.	0.	0.	0.
time (sec)	N/A	0.079	0.071	0.052	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	58	194	0	0	0	0
normalized size	1	1.	0.84	2.81	0.	0.	0.	0.
time (sec)	N/A	0.056	0.086	0.052	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	58	188	0	0	0	0
normalized size	1	1.	0.84	2.72	0.	0.	0.	0.
time (sec)	N/A	0.058	0.105	0.053	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	60	168	0	0	0	0
normalized size	1	1.	0.88	2.47	0.	0.	0.	0.
time (sec)	N/A	0.064	0.082	0.065	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	242	0	0	0	0
normalized size	1	1.	0.83	3.36	0.	0.	0.	0.
time (sec)	N/A	0.064	0.081	0.063	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	59	365	0	0	0	0
normalized size	1	1.	0.59	3.65	0.	0.	0.	0.
time (sec)	N/A	0.082	0.078	0.091	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	59	396	0	0	0	0
normalized size	1	1.	0.59	3.96	0.	0.	0.	0.
time (sec)	N/A	0.082	0.064	0.068	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	57	63	49	88	65	49
normalized size	1	1.	1.27	1.4	1.09	1.96	1.44	1.09
time (sec)	N/A	0.042	0.323	0.043	0.974	1.895	11.453	87.946

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	57	92	49	72	63	62
normalized size	1	1.	1.33	2.14	1.14	1.67	1.47	1.44
time (sec)	N/A	0.046	0.179	0.033	0.984	1.875	5.357	1.186

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	70	47	92	61	57
normalized size	1	1.	1.07	1.63	1.09	2.14	1.42	1.33
time (sec)	N/A	0.051	0.075	0.1	0.983	1.872	6.164	1.201

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	48	85	46	97	63	55
normalized size	1	1.	1.12	1.98	1.07	2.26	1.47	1.28
time (sec)	N/A	0.052	0.111	0.089	0.971	2.295	67.283	1.182

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	70	98	50	99	0	61
normalized size	1	1.	1.63	2.28	1.16	2.3	0.	1.42
time (sec)	N/A	0.051	0.246	0.103	0.962	2.122	0.	1.18

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	70	111	50	100	0	61
normalized size	1	1.	1.56	2.47	1.11	2.22	0.	1.36
time (sec)	N/A	0.05	0.279	0.149	0.973	2.068	0.	1.188

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	94	124	50	100	0	61
normalized size	1	1.	2.09	2.76	1.11	2.22	0.	1.36
time (sec)	N/A	0.051	0.548	0.178	0.977	2.178	0.	1.213

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	57	275	0	0	0	0
normalized size	1	1.	0.37	1.76	0.	0.	0.	0.
time (sec)	N/A	0.149	0.131	0.069	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	57	262	0	0	0	0
normalized size	1	1.	0.37	1.68	0.	0.	0.	0.
time (sec)	N/A	0.147	0.097	0.062	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	65	249	0	0	0	0
normalized size	1	1.	0.51	1.95	0.	0.	0.	0.
time (sec)	N/A	0.126	0.074	0.06	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	65	255	0	0	0	0
normalized size	1	1.	0.51	1.99	0.	0.	0.	0.
time (sec)	N/A	0.133	0.108	0.068	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	58	221	0	0	0	0
normalized size	1	1.	0.59	2.23	0.	0.	0.	0.
time (sec)	N/A	0.096	0.065	0.059	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	58	208	0	0	0	0
normalized size	1	1.	0.59	2.1	0.	0.	0.	0.
time (sec)	N/A	0.098	0.07	0.063	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	60	213	0	0	0	0
normalized size	1	1.	0.6	2.13	0.	0.	0.	0.
time (sec)	N/A	0.105	0.059	0.068	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	60	286	0	0	0	0
normalized size	1	1.	0.59	2.8	0.	0.	0.	0.
time (sec)	N/A	0.106	0.069	0.064	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	65	366	0	0	0	0
normalized size	1	1.	0.64	3.59	0.	0.	0.	0.
time (sec)	N/A	0.113	0.071	0.096	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	65	398	0	0	0	0
normalized size	1	1.	0.64	3.9	0.	0.	0.	0.
time (sec)	N/A	0.111	0.069	0.066	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	111	103	49	123	0	0
normalized size	1	1.	2.13	1.98	0.94	2.37	0.	0.
time (sec)	N/A	0.035	0.279	0.088	0.972	2.239	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	83	318	0	836	0	0
normalized size	1	1.	0.83	3.18	0.	8.36	0.	0.
time (sec)	N/A	0.077	0.191	0.112	0.	3.802	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	80	280	0	791	0	0
normalized size	1	1.	0.81	2.83	0.	7.99	0.	0.
time (sec)	N/A	0.071	0.181	0.109	0.	3.822	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	68	244	0	757	0	0
normalized size	1	1.	0.87	3.13	0.	9.71	0.	0.
time (sec)	N/A	0.065	0.111	0.128	0.	3.719	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	65	204	0	702	0	0
normalized size	1	1.	0.84	2.65	0.	9.12	0.	0.
time (sec)	N/A	0.063	0.065	0.141	0.	3.799	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	51	179	0	664	0	0
normalized size	1	1.	0.88	3.09	0.	11.45	0.	0.
time (sec)	N/A	0.051	0.036	0.112	0.	2.64	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	50	177	0	676	0	70
normalized size	1	1.	0.85	3.	0.	11.46	0.	1.19
time (sec)	N/A	0.051	0.04	0.104	0.	2.539	0.	1.122

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	36	422	0	853	0	89
normalized size	1	1.	0.46	5.41	0.	10.94	0.	1.14
time (sec)	N/A	0.067	0.055	0.297	0.	2.727	0.	1.129

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	38	624	0	879	0	104
normalized size	1	1.	0.47	7.7	0.	10.85	0.	1.28
time (sec)	N/A	0.064	0.058	0.226	0.	2.641	0.	1.14

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	38	882	0	942	0	127
normalized size	1	1.	0.38	8.82	0.	9.42	0.	1.27
time (sec)	N/A	0.078	0.068	0.22	0.	2.705	0.	1.138

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	38	1082	0	946	0	130
normalized size	1	1.	0.37	10.5	0.	9.18	0.	1.26
time (sec)	N/A	0.075	0.083	0.223	0.	2.693	0.	1.128

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	89	242	0	0	0	0
normalized size	1	1.	0.72	1.95	0.	0.	0.	0.
time (sec)	N/A	0.101	0.359	0.204	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	74	229	0	0	0	0
normalized size	1	1.	0.77	2.39	0.	0.	0.	0.
time (sec)	N/A	0.082	0.24	0.203	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	73	216	0	0	0	0
normalized size	1	1.	0.76	2.25	0.	0.	0.	0.
time (sec)	N/A	0.081	0.222	0.197	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	58	203	0	0	0	0
normalized size	1	1.	0.88	3.08	0.	0.	0.	0.
time (sec)	N/A	0.063	0.145	0.396	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	56	190	0	0	0	0
normalized size	1	1.	0.85	2.88	0.	0.	0.	0.
time (sec)	N/A	0.064	0.098	0.266	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	56	203	0	0	0	0
normalized size	1	1.	0.86	3.12	0.	0.	0.	0.
time (sec)	N/A	0.057	0.081	0.323	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	47	188	0	0	0	0
normalized size	1	1.	0.73	2.94	0.	0.	0.	0.
time (sec)	N/A	0.058	0.084	0.27	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	65	209	0	0	0	0
normalized size	1	1.	0.69	2.22	0.	0.	0.	0.
time (sec)	N/A	0.08	0.172	0.319	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	62	190	0	0	0	0
normalized size	1	1.	0.63	1.94	0.	0.	0.	0.
time (sec)	N/A	0.083	0.141	0.235	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	82	408	0	0	0	0
normalized size	1	1.	0.65	3.24	0.	0.	0.	0.
time (sec)	N/A	0.102	0.168	0.299	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	137	433	0	1061	0	0
normalized size	1	1.	1.01	3.21	0.	7.86	0.	0.
time (sec)	N/A	0.089	2.076	0.221	0.	3.569	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	78	394	0	1027	0	0
normalized size	1	1.	0.69	3.49	0.	9.09	0.	0.
time (sec)	N/A	0.082	0.635	0.204	0.	3.493	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	118	327	0	991	0	0
normalized size	1	1.	1.04	2.89	0.	8.77	0.	0.
time (sec)	N/A	0.08	1.204	0.207	0.	3.51	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	65	300	0	988	0	0
normalized size	1	1.	0.71	3.3	0.	10.86	0.	0.
time (sec)	N/A	0.073	0.291	0.291	0.	2.45	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	76	294	0	922	0	0
normalized size	1	1.	0.84	3.23	0.	10.13	0.	0.
time (sec)	N/A	0.071	0.185	0.2	0.	2.662	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	62	289	0	940	0	0
normalized size	1	1.	0.67	3.11	0.	10.11	0.	0.
time (sec)	N/A	0.066	0.25	0.256	0.	2.761	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	69	297	0	922	0	123
normalized size	1	1.	0.74	3.19	0.	9.91	0.	1.32
time (sec)	N/A	0.065	0.228	0.312	0.	2.662	0.	1.138

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	91	705	0	1091	0	162
normalized size	1	1.	0.79	6.13	0.	9.49	0.	1.41
time (sec)	N/A	0.083	0.246	0.338	0.	2.714	0.	1.141

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	92	909	0	1111	0	154
normalized size	1	1.	0.8	7.9	0.	9.66	0.	1.34
time (sec)	N/A	0.083	0.367	0.345	0.	2.809	0.	1.142

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	102	1165	0	1166	0	185
normalized size	1	1.	0.74	8.5	0.	8.51	0.	1.35
time (sec)	N/A	0.092	0.455	0.381	0.	2.872	0.	1.163

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	59	34	28
normalized size	1	1.	1.	0.86	1.09	2.68	1.55	1.27
time (sec)	N/A	0.022	0.02	0.004	0.977	2.208	33.716	1.235

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	18	49	167	18
normalized size	1	1.	0.86	0.67	0.86	2.33	7.95	0.86
time (sec)	N/A	0.024	0.013	0.042	0.978	2.143	45.799	1.101

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	18	65	24	18
normalized size	1	1.	0.86	0.67	0.86	3.1	1.14	0.86
time (sec)	N/A	0.024	0.013	0.04	0.964	2.228	68.453	1.106

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	18	68	0	18
normalized size	1	1.	0.86	0.67	0.86	3.24	0.	0.86
time (sec)	N/A	0.025	0.012	0.036	0.963	2.344	0.	1.098

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	14	18	45	323	18
normalized size	1	1.	0.84	0.74	0.95	2.37	17.	0.95
time (sec)	N/A	0.023	0.009	0.039	1.004	2.178	51.113	1.114

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	70	540	0	0	0	0
normalized size	1	1.	0.53	4.09	0.	0.	0.	0.
time (sec)	N/A	0.16	0.103	0.224	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	70	518	0	0	0	0
normalized size	1	1.	0.74	5.45	0.	0.	0.	0.
time (sec)	N/A	0.103	0.089	0.147	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	67	505	0	0	0	0
normalized size	1	1.	1.26	9.53	0.	0.	0.	0.
time (sec)	N/A	0.049	0.065	0.107	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	70	493	0	0	0	0
normalized size	1	1.	0.75	5.3	0.	0.	0.	0.
time (sec)	N/A	0.107	0.107	0.134	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	70	528	0	0	0	0
normalized size	1	1.	0.52	3.94	0.	0.	0.	0.
time (sec)	N/A	0.163	0.15	0.144	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	70	514	0	0	0	0
normalized size	1	1.	0.22	1.61	0.	0.	0.	0.
time (sec)	N/A	0.295	0.119	0.079	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	67	271	0	0	0	0
normalized size	1	1.	0.24	0.97	0.	0.	0.	0.
time (sec)	N/A	0.189	0.062	0.083	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	38	0	112	0	0
normalized size	1	1.	1.	1.03	0.	3.03	0.	0.
time (sec)	N/A	0.053	0.083	0.108	0.	3.296	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	57	50	0	144	0	0
normalized size	1	1.	0.76	0.67	0.	1.92	0.	0.
time (sec)	N/A	0.113	0.242	0.107	0.	3.702	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	67	60	0	176	0	0
normalized size	1	1.	0.6	0.54	0.	1.57	0.	0.
time (sec)	N/A	0.171	0.235	0.144	0.	4.005	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	71	216	0	0	0	0
normalized size	1	1.	0.54	1.65	0.	0.	0.	0.
time (sec)	N/A	0.179	0.12	0.132	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	67	182	0	0	0	0
normalized size	1	1.	0.72	1.96	0.	0.	0.	0.
time (sec)	N/A	0.117	0.081	0.118	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	67	186	0	0	0	0
normalized size	1	1.	0.68	1.9	0.	0.	0.	0.
time (sec)	N/A	0.123	0.168	0.093	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	70	215	0	0	0	0
normalized size	1	1.	0.53	1.62	0.	0.	0.	0.
time (sec)	N/A	0.186	0.156	0.103	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	67	654	0	0	0	0
normalized size	1	1.	0.21	2.04	0.	0.	0.	0.
time (sec)	N/A	0.281	0.07	0.078	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	67	642	0	0	0	0
normalized size	1	1.	0.21	2.05	0.	0.	0.	0.
time (sec)	N/A	0.276	0.164	0.092	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	40	38	0	127	0	0
normalized size	1	1.	1.08	1.03	0.	3.43	0.	0.
time (sec)	N/A	0.06	0.123	0.066	0.	3.549	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	57	50	0	159	0	0
normalized size	1	1.	0.54	0.47	0.	1.5	0.	0.
time (sec)	N/A	0.184	0.298	0.072	0.	4.182	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	67	60	0	194	0	0
normalized size	1	1.	0.48	0.43	0.	1.38	0.	0.
time (sec)	N/A	0.24	0.31	0.109	0.	5.362	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	72	545	0	0	0	0
normalized size	1	1.	0.43	3.28	0.	0.	0.	0.
time (sec)	N/A	0.235	0.173	0.148	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	70	532	0	0	0	0
normalized size	1	1.	0.53	4.06	0.	0.	0.	0.
time (sec)	N/A	0.176	0.178	0.08	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	67	519	0	0	0	0
normalized size	1	1.	0.71	5.46	0.	0.	0.	0.
time (sec)	N/A	0.108	0.095	0.121	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	67	508	0	0	0	0
normalized size	1	1.	0.71	5.4	0.	0.	0.	0.
time (sec)	N/A	0.12	0.12	0.093	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	70	531	0	0	0	0
normalized size	1	1.	0.53	3.99	0.	0.	0.	0.
time (sec)	N/A	0.174	0.182	0.105	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	72	544	0	0	0	0
normalized size	1	1.	0.43	3.24	0.	0.	0.	0.
time (sec)	N/A	0.24	0.182	0.086	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	67	510	0	0	0	0
normalized size	1	1.	0.21	1.59	0.	0.	0.	0.
time (sec)	N/A	0.258	0.126	0.118	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	67	532	0	0	0	0
normalized size	1	1.	0.21	1.69	0.	0.	0.	0.
time (sec)	N/A	0.266	0.126	0.093	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	40	38	0	150	0	0
normalized size	1	1.	1.08	1.03	0.	4.05	0.	0.
time (sec)	N/A	0.06	0.166	0.066	0.	3.213	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	57	50	0	188	0	0
normalized size	1	1.	0.54	0.47	0.	1.77	0.	0.
time (sec)	N/A	0.175	0.305	0.069	0.	4.387	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	67	60	0	227	0	0
normalized size	1	1.	0.48	0.43	0.	1.61	0.	0.
time (sec)	N/A	0.235	0.477	0.116	0.	5.816	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	57	702	0	0	0	0
normalized size	1	1.	0.25	3.11	0.	0.	0.	0.
time (sec)	N/A	0.149	0.056	0.178	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	33	0	63	0	0
normalized size	1	1.	1.	2.06	0.	3.94	0.	0.
time (sec)	N/A	0.023	0.018	0.048	0.	2.695	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	38	171	0	1613	0	0
normalized size	1	1.	0.31	1.4	0.	13.22	0.	0.
time (sec)	N/A	0.082	0.012	0.055	0.	5.776	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	38	2667	0	1666	0	0
normalized size	1	1.	0.27	18.65	0.	11.65	0.	0.
time (sec)	N/A	0.114	0.011	0.171	0.	5.432	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	70	216	0	0	0	0
normalized size	1	1.	0.53	1.64	0.	0.	0.	0.
time (sec)	N/A	0.178	0.112	0.112	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	68	188	0	0	0	0
normalized size	1	1.	0.74	2.04	0.	0.	0.	0.
time (sec)	N/A	0.113	0.107	0.099	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	65	151	0	0	0	0
normalized size	1	1.	1.23	2.85	0.	0.	0.	0.
time (sec)	N/A	0.055	0.057	0.089	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	65	184	0	0	0	0
normalized size	1	1.	0.67	1.9	0.	0.	0.	0.
time (sec)	N/A	0.114	0.106	0.112	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	70	212	0	0	0	0
normalized size	1	1.	0.52	1.58	0.	0.	0.	0.
time (sec)	N/A	0.172	0.133	0.125	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	65	312	0	0	0	0
normalized size	1	1.	0.23	1.11	0.	0.	0.	0.
time (sec)	N/A	0.177	0.061	0.096	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	36	38	0	92	0	0
normalized size	1	1.	1.03	1.09	0.	2.63	0.	0.
time (sec)	N/A	0.054	0.062	0.069	0.	2.619	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	52	50	0	128	0	0
normalized size	1	1.	0.69	0.67	0.	1.71	0.	0.
time (sec)	N/A	0.111	0.173	0.079	0.	2.687	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	67	60	0	157	0	0
normalized size	1	1.	0.6	0.54	0.	1.4	0.	0.
time (sec)	N/A	0.169	0.244	0.088	0.	3.366	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	55	298	0	0	0	0
normalized size	1	1.	0.32	1.71	0.	0.	0.	0.
time (sec)	N/A	0.089	0.026	0.086	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	55	953	0	0	0	0
normalized size	1	1.	0.28	4.79	0.	0.	0.	0.
time (sec)	N/A	0.114	0.035	0.079	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	57	1281	0	0	0	0
normalized size	1	1.	0.28	6.37	0.	0.	0.	0.
time (sec)	N/A	0.115	0.034	0.095	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	57	1966	0	0	0	0
normalized size	1	1.	0.25	8.7	0.	0.	0.	0.
time (sec)	N/A	0.139	0.039	0.113	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.055	0.13	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.044	0.07	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.036	0.068	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.04	0.063	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.038	0.071	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.055	0.115	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.051	0.075	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.041	0.02	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.051	0.066	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.047	0.073	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.052	0.086	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.043	0.056	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.035	0.065	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.042	0.059	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.039	0.066	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.047	0.087	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.045	0.059	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.034	0.022	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.042	0.069	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.037	0.07	0.	0.	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	57	0	0	424	0	0
normalized size	1	1.	0.45	0.	0.	3.31	0.	0.
time (sec)	N/A	0.151	0.041	0.096	0.	2.246	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	57	0	0	0	0	0
normalized size	1	1.	0.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.329	0.041	0.07	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	57	0	0	0	0	0
normalized size	1	1.	0.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.348	0.052	0.046	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	57	0	0	572	0	0
normalized size	1	1.	0.37	0.	0.	3.69	0.	0.
time (sec)	N/A	0.175	0.056	0.052	0.	3.238	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	57	0	0	566	0	0
normalized size	1	1.	0.37	0.	0.	3.65	0.	0.
time (sec)	N/A	0.11	0.053	0.051	0.	2.914	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	57	0	0	444	0	0
normalized size	1	1.	0.45	0.	0.	3.47	0.	0.
time (sec)	N/A	0.082	0.026	0.06	0.	2.889	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	55	0	0	0	0	0
normalized size	1	1.	0.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.3	0.028	0.06	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	55	0	0	0	0	0
normalized size	1	1.	0.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.326	0.034	0.046	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	57	0	0	555	0	0
normalized size	1	1.	0.37	0.	0.	3.58	0.	0.
time (sec)	N/A	0.111	0.035	0.049	0.	3.194	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	57	0	0	621	0	0
normalized size	1	1.	0.37	0.	0.	4.01	0.	0.
time (sec)	N/A	0.134	0.034	0.049	0.	2.949	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	0	62	0	0
normalized size	1	1.	1.	0.	0.	3.88	0.	0.
time (sec)	N/A	0.022	0.011	0.044	0.	2.455	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	0	42	0	0
normalized size	1	1.	1.	0.	0.	2.62	0.	0.
time (sec)	N/A	0.023	0.015	0.046	0.	2.382	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	79	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.108	0.562	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	82	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.117	0.573	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	85	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.076	0.399	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.095	0.432	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	55	0	0	177	2050	335
normalized size	1	1.	0.74	0.	0.	2.39	27.7	4.53
time (sec)	N/A	0.07	0.316	1.115	0.	2.608	79.342	1.125

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	0	0	113	530	159
normalized size	1	1.	0.96	0.	0.	2.26	10.6	3.18
time (sec)	N/A	0.051	0.085	0.801	0.	2.452	13.193	1.131

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	25	25	0	58	58	32
normalized size	1	1.	1.04	1.04	0.	2.42	2.42	1.33
time (sec)	N/A	0.025	0.011	0.	0.	2.458	1.675	1.136

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.022	0.26	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.024	0.259	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.053	0.885	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.047	0.783	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.039	0.457	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.047	0.349	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.044	0.3	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	78	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.14	0.089	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.069	0.087	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.071	0.071	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	78	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.093	0.069	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	78	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.084	0.069	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	83	0	0	209	2462	336
normalized size	1	1.	1.09	0.	0.	2.75	32.39	4.42
time (sec)	N/A	0.066	0.3	0.893	0.	2.79	86.708	1.16

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	1076	0	122	694	158
normalized size	1	1.	1.	21.52	0.	2.44	13.88	3.16
time (sec)	N/A	0.05	0.117	1.467	0.	2.262	14.66	1.16

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	26	26	0	59	61	34
normalized size	1	1.	1.04	1.04	0.	2.36	2.44	1.36
time (sec)	N/A	0.022	0.01	0.003	0.	2.241	1.809	1.156

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	52	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.033	0.385	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	86	0	0	0	0	0
normalized size	1	1.	1.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.217	0.391	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	86	0	0	0	0	0
normalized size	1	1.	1.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.193	0.219	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.112	0.655	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.083	0.992	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	64	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.043	0.254	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	80	0	0	0	0	0
normalized size	1	1.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.177	0.359	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	82	0	0	0	0	0
normalized size	1	1.	1.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.213	0.2	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	158	0	0	0	0	0
normalized size	1	1.	2.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.415	0.078	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.166	0.067	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	82	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.107	0.071	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	82	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.118	0.06	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	79	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.137	0.059	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	58	517	85	149	0	146
normalized size	1	1.	0.68	6.08	1.	1.75	0.	1.72
time (sec)	N/A	0.059	0.34	0.283	1.055	2.24	0.	1.172

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	48	507	68	117	0	112
normalized size	1	1.	0.76	8.05	1.08	1.86	0.	1.78
time (sec)	N/A	0.05	0.205	0.137	1.038	2.215	0.	1.145

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	36	497	47	84	0	77
normalized size	1	1.	0.88	12.12	1.15	2.05	0.	1.88
time (sec)	N/A	0.043	0.16	0.148	1.007	2.168	0.	1.111

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	31	54	0	32
normalized size	1	1.	1.	0.94	1.72	3.	0.	1.78
time (sec)	N/A	0.03	0.034	0.019	1.015	2.12	0.	1.158

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	73	169	0	660	0	86
normalized size	1	1.	1.26	2.91	0.	11.38	0.	1.48
time (sec)	N/A	0.046	0.343	0.122	0.	2.654	0.	1.154

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	95	603	0	941	0	139
normalized size	1	1.	1.02	6.48	0.	10.12	0.	1.49
time (sec)	N/A	0.076	0.612	0.148	0.	2.666	0.	1.129

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	107	1089	0	1173	0	181
normalized size	1	1.	0.87	8.85	0.	9.54	0.	1.47
time (sec)	N/A	0.086	0.902	0.158	0.	2.686	0.	1.156

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	73	165	0	0	0	0
normalized size	1	1.	0.59	1.34	0.	0.	0.	0.
time (sec)	N/A	0.143	0.15	0.253	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	61	143	0	0	0	0
normalized size	1	1.	0.64	1.51	0.	0.	0.	0.
time (sec)	N/A	0.098	0.105	0.158	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	51	123	0	0	0	0
normalized size	1	1.	0.76	1.84	0.	0.	0.	0.
time (sec)	N/A	0.058	0.094	0.143	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	98	0	0	0	0
normalized size	1	1.	1.	2.58	0.	0.	0.	0.
time (sec)	N/A	0.021	0.034	0.119	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	47	184	0	0	0	0
normalized size	1	1.	0.76	2.97	0.	0.	0.	0.
time (sec)	N/A	0.057	0.106	0.139	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	63	335	0	0	0	0
normalized size	1	1.	0.66	3.53	0.	0.	0.	0.
time (sec)	N/A	0.096	0.22	0.165	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	73	485	0	0	0	0
normalized size	1	1.	0.59	3.94	0.	0.	0.	0.
time (sec)	N/A	0.138	0.442	0.181	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	52	969	97	136	0	143
normalized size	1	1.	0.63	11.67	1.17	1.64	0.	1.72
time (sec)	N/A	0.063	0.142	0.237	0.995	2.232	0.	1.14

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	42	959	74	108	0	109
normalized size	1	1.	0.67	15.22	1.17	1.71	0.	1.73
time (sec)	N/A	0.056	0.091	0.172	1.033	2.245	0.	1.128

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	30	949	50	72	0	68
normalized size	1	1.	0.73	23.15	1.22	1.76	0.	1.66
time (sec)	N/A	0.05	0.063	0.139	0.993	2.202	0.	1.188

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	31	38	0	36
normalized size	1	1.	1.	0.94	1.72	2.11	0.	2.
time (sec)	N/A	0.034	0.027	0.013	0.995	2.197	0.	1.326

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	85	235	0	737	0	107
normalized size	1	1.	1.1	3.05	0.	9.57	0.	1.39
time (sec)	N/A	0.053	0.914	0.148	0.	3.05	0.	1.369

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	97	644	0	991	0	181
normalized size	1	1.	0.86	5.7	0.	8.77	0.	1.6
time (sec)	N/A	0.084	2.279	0.129	0.	2.894	0.	1.403

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	70	330	0	0	0	0
normalized size	1	1.	0.55	2.58	0.	0.	0.	0.
time (sec)	N/A	0.149	0.151	0.233	0.	0.	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	60	320	0	0	0	0
normalized size	1	1.	0.61	3.27	0.	0.	0.	0.
time (sec)	N/A	0.108	0.115	0.179	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	48	312	0	0	0	0
normalized size	1	1.	0.73	4.73	0.	0.	0.	0.
time (sec)	N/A	0.066	0.079	0.142	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	48	322	0	0	0	0
normalized size	1	1.	0.73	4.88	0.	0.	0.	0.
time (sec)	N/A	0.037	0.053	0.174	0.	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	57	322	0	0	0	0
normalized size	1	1.	0.63	3.58	0.	0.	0.	0.
time (sec)	N/A	0.082	0.147	0.146	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	77	622	0	0	0	0
normalized size	1	1.	0.62	5.02	0.	0.	0.	0.
time (sec)	N/A	0.129	0.206	0.158	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	52	532	89	169	0	146
normalized size	1	1.	0.61	6.26	1.05	1.99	0.	1.72
time (sec)	N/A	0.062	0.426	0.152	1.054	2.339	0.	1.209

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	42	522	70	135	0	108
normalized size	1	1.	0.67	8.29	1.11	2.14	0.	1.71
time (sec)	N/A	0.056	0.34	0.148	1.042	2.235	0.	1.17

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	32	357	49	97	0	72
normalized size	1	1.	0.78	8.71	1.2	2.37	0.	1.76
time (sec)	N/A	0.049	0.198	0.151	1.033	2.088	0.	1.157

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	31	63	0	49
normalized size	1	1.	1.	0.85	1.55	3.15	0.	2.45
time (sec)	N/A	0.036	0.042	0.013	1.045	2.163	0.	1.15

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	87	237	0	868	0	123
normalized size	1	1.	1.12	3.04	0.	11.13	0.	1.58
time (sec)	N/A	0.054	0.191	0.119	0.	2.69	0.	1.17

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	109	699	0	1133	0	173
normalized size	1	1.	0.96	6.19	0.	10.03	0.	1.53
time (sec)	N/A	0.084	1.955	0.16	0.	2.709	0.	1.155

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	119	1161	0	1395	0	215
normalized size	1	1.	0.83	8.12	0.	9.76	0.	1.5
time (sec)	N/A	0.098	1.199	0.126	0.	3.009	0.	1.159

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	74	168	0	0	0	0
normalized size	1	1.	0.57	1.29	0.	0.	0.	0.
time (sec)	N/A	0.147	0.206	0.177	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	64	144	0	0	0	0
normalized size	1	1.	0.64	1.44	0.	0.	0.	0.
time (sec)	N/A	0.103	0.134	0.14	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	52	126	0	0	0	0
normalized size	1	1.	0.74	1.8	0.	0.	0.	0.
time (sec)	N/A	0.065	0.124	0.129	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	51	128	0	0	0	0
normalized size	1	1.	0.73	1.83	0.	0.	0.	0.
time (sec)	N/A	0.035	0.075	0.123	0.	0.	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	67	202	0	0	0	0
normalized size	1	1.	0.68	2.06	0.	0.	0.	0.
time (sec)	N/A	0.102	0.181	0.145	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	79	352	0	0	0	0
normalized size	1	1.	0.64	2.86	0.	0.	0.	0.
time (sec)	N/A	0.15	0.406	0.175	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	52	56	85	159	0	0
normalized size	1	1.	0.6	0.64	0.98	1.83	0.	0.
time (sec)	N/A	0.057	0.218	0.155	1.002	2.719	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	42	46	68	128	0	0
normalized size	1	1.	0.65	0.71	1.05	1.97	0.	0.
time (sec)	N/A	0.05	0.163	0.128	1.033	2.586	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	36	50	96	0	68
normalized size	1	1.	0.74	0.84	1.16	2.23	0.	1.58
time (sec)	N/A	0.045	0.105	0.115	1.009	2.416	0.	1.489

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	31	65	0	51
normalized size	1	1.	1.	0.85	1.55	3.25	0.	2.55
time (sec)	N/A	0.031	0.045	0.019	1.003	2.355	0.	1.439

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	73	161	0	670	0	0
normalized size	1	1.	1.24	2.73	0.	11.36	0.	0.
time (sec)	N/A	0.042	0.106	0.117	0.	2.961	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	425	0	950	0	0
normalized size	1	1.	1.	4.57	0.	10.22	0.	0.
time (sec)	N/A	0.067	1.165	0.141	0.	2.911	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	107	729	0	1188	0	0
normalized size	1	1.	0.87	5.93	0.	9.66	0.	0.
time (sec)	N/A	0.079	2.236	0.146	0.	2.995	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	73	338	0	0	0	0
normalized size	1	1.	0.59	2.75	0.	0.	0.	0.
time (sec)	N/A	0.143	0.481	0.185	0.	0.	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	63	328	0	0	0	0
normalized size	1	1.	0.66	3.45	0.	0.	0.	0.
time (sec)	N/A	0.1	0.338	0.15	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	60	316	0	0	0	0
normalized size	1	1.	0.9	4.72	0.	0.	0.	0.
time (sec)	N/A	0.059	0.134	0.185	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	306	0	0	0	0
normalized size	1	1.	1.	8.05	0.	0.	0.	0.
time (sec)	N/A	0.02	0.037	0.127	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	48	316	0	0	0	0
normalized size	1	1.	0.76	5.02	0.	0.	0.	0.
time (sec)	N/A	0.058	0.152	0.144	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	74	618	0	0	0	0
normalized size	1	1.	0.78	6.51	0.	0.	0.	0.
time (sec)	N/A	0.101	0.209	0.177	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	86	918	0	0	0	0
normalized size	1	1.	0.7	7.46	0.	0.	0.	0.
time (sec)	N/A	0.143	0.141	0.187	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	52	56	85	165	0	0
normalized size	1	1.	0.6	0.64	0.98	1.9	0.	0.
time (sec)	N/A	0.063	0.436	0.146	1.002	2.708	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	42	46	68	134	0	0
normalized size	1	1.	0.65	0.71	1.05	2.06	0.	0.
time (sec)	N/A	0.058	0.252	0.114	1.	2.64	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	36	50	99	0	68
normalized size	1	1.	0.74	0.84	1.16	2.3	0.	1.58
time (sec)	N/A	0.05	0.172	0.102	1.001	2.676	0.	1.478

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	31	68	0	41
normalized size	1	1.	1.	0.85	1.55	3.4	0.	2.05
time (sec)	N/A	0.036	0.054	0.012	0.985	2.488	0.	1.466

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	89	221	0	813	0	0
normalized size	1	1.	1.14	2.83	0.	10.42	0.	0.
time (sec)	N/A	0.054	1.963	0.108	0.	4.071	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	98	426	0	957	0	0
normalized size	1	1.	1.05	4.58	0.	10.29	0.	0.
time (sec)	N/A	0.074	0.466	0.125	0.	3.021	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	109	729	0	1192	0	0
normalized size	1	1.	0.89	5.93	0.	9.69	0.	0.
time (sec)	N/A	0.086	0.654	0.133	0.	3.059	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	81	173	0	0	0	0
normalized size	1	1.	0.64	1.37	0.	0.	0.	0.
time (sec)	N/A	0.134	0.163	0.167	0.	0.	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	71	153	0	0	0	0
normalized size	1	1.	0.72	1.56	0.	0.	0.	0.
time (sec)	N/A	0.082	0.13	0.133	0.	0.	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	59	131	0	0	0	0
normalized size	1	1.	0.82	1.82	0.	0.	0.	0.
time (sec)	N/A	0.036	0.06	0.115	0.	0.	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	58	191	0	0	0	0
normalized size	1	1.	0.85	2.81	0.	0.	0.	0.
time (sec)	N/A	0.066	0.102	0.124	0.	0.	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	62	343	0	0	0	0
normalized size	1	1.	0.61	3.36	0.	0.	0.	0.
time (sec)	N/A	0.104	0.222	0.142	0.	0.	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	74	493	0	0	0	0
normalized size	1	1.	0.56	3.73	0.	0.	0.	0.
time (sec)	N/A	0.144	0.311	0.178	0.	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	62	56	85	169	0	0
normalized size	1	1.	0.71	0.64	0.98	1.94	0.	0.
time (sec)	N/A	0.063	0.361	0.19	1.019	2.924	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	52	46	68	135	0	0
normalized size	1	1.	0.8	0.71	1.05	2.08	0.	0.
time (sec)	N/A	0.059	0.219	0.165	1.006	2.671	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	36	50	100	0	68
normalized size	1	1.	0.98	0.84	1.16	2.33	0.	1.58
time (sec)	N/A	0.053	0.149	0.113	1.012	2.472	0.	1.567

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	31	68	0	41
normalized size	1	1.	1.	0.85	1.55	3.4	0.	2.05
time (sec)	N/A	0.037	0.074	0.011	1.003	2.476	0.	1.462

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	90	377	0	826	0	0
normalized size	1	1.	1.11	4.65	0.	10.2	0.	0.
time (sec)	N/A	0.055	0.206	0.121	0.	3.739	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	98	437	0	969	0	0
normalized size	1	1.	1.05	4.7	0.	10.42	0.	0.
time (sec)	N/A	0.073	2.347	0.127	0.	2.75	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	110	737	0	1199	0	0
normalized size	1	1.	0.89	5.99	0.	9.75	0.	0.
time (sec)	N/A	0.084	2.533	0.141	0.	2.787	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	83	343	0	0	0	0
normalized size	1	1.	0.66	2.72	0.	0.	0.	0.
time (sec)	N/A	0.132	0.498	0.158	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	66	333	0	0	0	0
normalized size	1	1.	0.67	3.4	0.	0.	0.	0.
time (sec)	N/A	0.082	0.392	0.129	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	323	0	0	0	0
normalized size	1	1.	0.83	4.49	0.	0.	0.	0.
time (sec)	N/A	0.035	0.085	0.15	0.	0.	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	51	313	0	0	0	0
normalized size	1	1.	0.75	4.6	0.	0.	0.	0.
time (sec)	N/A	0.066	0.164	0.137	0.	0.	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	79	623	0	0	0	0
normalized size	1	1.	0.77	6.11	0.	0.	0.	0.
time (sec)	N/A	0.108	0.252	0.162	0.	0.	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	87	923	0	0	0	0
normalized size	1	1.	0.66	6.99	0.	0.	0.	0.
time (sec)	N/A	0.149	0.178	0.181	0.	0.	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	80	546	0	0	0	0
normalized size	1	1.	0.18	1.22	0.	0.	0.	0.
time (sec)	N/A	0.476	0.367	0.162	0.	0.	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	65	512	0	0	0	0
normalized size	1	1.	0.16	1.24	0.	0.	0.	0.
time (sec)	N/A	0.348	0.252	0.107	0.	0.	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	55	273	0	0	0	0
normalized size	1	1.	0.15	0.73	0.	0.	0.	0.
time (sec)	N/A	0.265	0.129	0.105	0.	0.	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	37	40	0	108	0	0
normalized size	1	1.	1.12	1.21	0.	3.27	0.	0.
time (sec)	N/A	0.052	0.074	0.123	0.	3.855	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	52	52	0	176	0	0
normalized size	1	1.	0.73	0.73	0.	2.48	0.	0.
time (sec)	N/A	0.109	0.198	0.122	0.	3.616	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	65	62	0	240	0	0
normalized size	1	1.	0.61	0.58	0.	2.26	0.	0.
time (sec)	N/A	0.165	0.214	0.133	0.	4.389	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	90	212	0	0	0	0
normalized size	1	1.	0.7	1.66	0.	0.	0.	0.
time (sec)	N/A	0.215	0.872	0.158	0.	0.	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	66	184	0	0	0	0
normalized size	1	1.	0.73	2.02	0.	0.	0.	0.
time (sec)	N/A	0.151	1.249	0.125	0.	0.	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	66	153	0	0	0	0
normalized size	1	1.	1.25	2.89	0.	0.	0.	0.
time (sec)	N/A	0.099	0.372	0.13	0.	0.	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	75	279	0	0	0	0
normalized size	1	1.	0.79	2.94	0.	0.	0.	0.
time (sec)	N/A	0.154	0.432	0.135	0.	0.	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	111	532	0	0	0	0
normalized size	1	1.	0.85	4.09	0.	0.	0.	0.
time (sec)	N/A	0.214	1.016	0.154	0.	0.	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	86	532	0	0	0	0
normalized size	1	1.	0.75	4.63	0.	0.	0.	0.
time (sec)	N/A	0.164	0.537	0.118	0.	0.	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	74	511	0	0	0	0
normalized size	1	1.	0.87	6.01	0.	0.	0.	0.
time (sec)	N/A	0.119	0.293	0.136	0.	0.	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	60	497	0	0	0	0
normalized size	1	1.	1.18	9.75	0.	0.	0.	0.
time (sec)	N/A	0.081	1.084	0.118	0.	0.	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	63	484	0	0	0	0
normalized size	1	1.	0.78	5.98	0.	0.	0.	0.
time (sec)	N/A	0.12	0.245	0.109	0.	0.	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	82	1030	0	0	0	0
normalized size	1	1.	0.71	8.96	0.	0.	0.	0.
time (sec)	N/A	0.162	0.462	0.159	0.	0.	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	218	648	0	0	0	0
normalized size	1	1.	0.6	1.79	0.	0.	0.	0.
time (sec)	N/A	0.268	1.549	0.129	0.	0.	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	166	304	0	0	0	0
normalized size	1	1.	0.51	0.93	0.	0.	0.	0.
time (sec)	N/A	0.192	0.939	0.11	0.	0.	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	70	0	117	0	0
normalized size	1	1.	1.	2.33	0.	3.9	0.	0.
time (sec)	N/A	0.038	0.102	0.096	0.	2.322	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	42	82	0	181	0	0
normalized size	1	1.	0.69	1.34	0.	2.97	0.	0.
time (sec)	N/A	0.078	0.139	0.107	0.	2.476	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	52	92	0	243	0	0
normalized size	1	1.	0.57	1.01	0.	2.67	0.	0.
time (sec)	N/A	0.122	0.201	0.129	0.	2.683	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	62	102	0	308	0	0
normalized size	1	1.	0.51	0.84	0.	2.55	0.	0.
time (sec)	N/A	0.162	0.254	0.15	0.	3.01	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	97	572	0	0	0	0
normalized size	1	1.	0.2	1.17	0.	0.	0.	0.
time (sec)	N/A	0.553	0.444	0.145	0.	0.	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	453	82	546	0	0	0	0
normalized size	1	1.	0.18	1.21	0.	0.	0.	0.
time (sec)	N/A	0.447	0.332	0.123	0.	0.	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	76	516	0	0	0	0
normalized size	1	1.	0.18	1.23	0.	0.	0.	0.
time (sec)	N/A	0.356	0.231	0.144	0.	0.	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	66	957	0	0	0	0
normalized size	1	1.	0.16	2.33	0.	0.	0.	0.
time (sec)	N/A	0.36	0.207	0.111	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	45	40	0	157	0	0
normalized size	1	1.	1.29	1.14	0.	4.49	0.	0.
time (sec)	N/A	0.058	0.11	0.085	0.	2.461	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	103	246	0	0	0	0
normalized size	1	1.	0.6	1.43	0.	0.	0.	0.
time (sec)	N/A	0.275	0.862	0.138	0.	0.	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	87	218	0	0	0	0
normalized size	1	1.	0.64	1.61	0.	0.	0.	0.
time (sec)	N/A	0.216	0.432	0.123	0.	0.	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	84	190	0	0	0	0
normalized size	1	1.	0.89	2.02	0.	0.	0.	0.
time (sec)	N/A	0.152	0.538	0.118	0.	0.	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	78	284	0	0	0	0
normalized size	1	1.	0.78	2.84	0.	0.	0.	0.
time (sec)	N/A	0.156	0.496	0.102	0.	0.	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	119	540	0	0	0	0
normalized size	1	1.	0.87	3.94	0.	0.	0.	0.
time (sec)	N/A	0.219	0.868	0.122	0.	0.	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	131	793	0	0	0	0
normalized size	1	1.	0.75	4.56	0.	0.	0.	0.
time (sec)	N/A	0.282	1.203	0.168	0.	0.	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	96	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	8.716	0.116	0.	0.	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	96	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	1.357	0.092	0.	0.	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	106	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	1.374	0.102	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	289	0	0	0	0	0
normalized size	1	1.	3.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	1.763	0.098	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	116	0	0	0	0	0
normalized size	1	1.	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	3.821	0.095	0.	0.	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	285	0	0	0	0	0
normalized size	1	1.	3.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	1.387	0.516	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	287	0	0	0	0	0
normalized size	1	1.	3.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.17	0.49	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	287	0	0	0	0	0
normalized size	1	1.	3.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.134	0.432	0.	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	289	0	0	0	0	0
normalized size	1	1.	3.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.128	0.455	0.	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	115	208	0	0
normalized size	1	1.	1.	0.	1.44	2.6	0.	0.
time (sec)	N/A	0.073	0.367	1.006	1.039	1.8	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	47	1732	80	123	0	0
normalized size	1	1.	0.9	33.31	1.54	2.37	0.	0.
time (sec)	N/A	0.053	0.145	1.523	1.034	1.767	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	120	38	58	0	0
normalized size	1	1.	0.88	4.8	1.52	2.32	0.	0.
time (sec)	N/A	0.033	0.021	0.029	1.008	1.649	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	92	0	0	0	0	0
normalized size	1	1.	1.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.332	0.403	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	48	48	2113	0	0	0	0	0
normalized size	1	1.	44.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	16.831	0.397	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	8327	0	0	0	0	0
normalized size	1	1.	114.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	25.619	0.859	0.	0.	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	6192	0	0	0	0	0
normalized size	1	1.	84.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	24.741	0.701	0.	0.	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	4143	0	0	0	0	0
normalized size	1	1.	56.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	18.737	0.917	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	61	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.047	0.169	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	2638	0	0	0	0	0
normalized size	1	1.	36.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	14.708	0.295	0.	0.	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	3833	0	0	0	0	0
normalized size	1	1.	52.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	17.415	0.208	0.	0.	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	104	0	0	0	0	0
normalized size	1	1.	1.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.758	0.111	0.	0.	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	75	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.123	0.105	0.	0.	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	72	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.134	0.092	0.	0.	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	73	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.149	0.089	0.	0.	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	67	214	0	0	0	0
normalized size	1	1.	0.67	2.14	0.	0.	0.	0.
time (sec)	N/A	0.078	0.176	0.226	0.	0.	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	62	538	0	0	0	0
normalized size	1	1.	0.83	7.17	0.	0.	0.	0.
time (sec)	N/A	0.054	0.115	0.198	0.	0.	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	55	187	0	0	0	0
normalized size	1	1.	0.76	2.6	0.	0.	0.	0.
time (sec)	N/A	0.052	0.074	0.142	0.	0.	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	525	0	0	0	0
normalized size	1	1.	0.98	11.93	0.	0.	0.	0.
time (sec)	N/A	0.028	0.046	0.13	0.	0.	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	165	0	0	0	0
normalized size	1	1.	0.98	3.84	0.	0.	0.	0.
time (sec)	N/A	0.019	0.035	0.093	0.	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	57	514	0	0	0	0
normalized size	1	1.	0.84	7.56	0.	0.	0.	0.
time (sec)	N/A	0.039	0.128	0.121	0.	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	55	319	0	0	0	0
normalized size	1	1.	0.74	4.31	0.	0.	0.	0.
time (sec)	N/A	0.04	0.09	0.127	0.	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	68	1054	0	0	0	0
normalized size	1	1.	0.68	10.54	0.	0.	0.	0.
time (sec)	N/A	0.058	0.14	0.158	0.	0.	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	68	216	0	0	0	0
normalized size	1	1.	0.66	2.1	0.	0.	0.	0.
time (sec)	N/A	0.076	0.123	0.113	0.	0.	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	62	545	0	0	0	0
normalized size	1	1.	0.81	7.08	0.	0.	0.	0.
time (sec)	N/A	0.056	0.189	0.151	0.	0.	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	56	189	0	0	0	0
normalized size	1	1.	0.75	2.52	0.	0.	0.	0.
time (sec)	N/A	0.053	0.069	0.118	0.	0.	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	531	0	0	0	0
normalized size	1	1.	0.98	11.54	0.	0.	0.	0.
time (sec)	N/A	0.038	0.027	0.131	0.	0.	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	165	0	0	0	0
normalized size	1	1.	0.98	3.75	0.	0.	0.	0.
time (sec)	N/A	0.028	0.016	0.125	0.	0.	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	54	520	0	0	0	0
normalized size	1	1.	0.76	7.32	0.	0.	0.	0.
time (sec)	N/A	0.033	0.078	0.105	0.	0.	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	58	319	0	0	0	0
normalized size	1	1.	0.81	4.43	0.	0.	0.	0.
time (sec)	N/A	0.039	0.176	0.118	0.	0.	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	68	1054	0	0	0	0
normalized size	1	1.	0.66	10.23	0.	0.	0.	0.
time (sec)	N/A	0.058	0.244	0.135	0.	0.	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	70	208	0	0	0	0
normalized size	1	1.	0.69	2.04	0.	0.	0.	0.
time (sec)	N/A	0.071	0.137	0.154	0.	0.	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	57	547	0	0	0	0
normalized size	1	1.	0.79	7.6	0.	0.	0.	0.
time (sec)	N/A	0.051	0.169	0.148	0.	0.	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	64	181	0	0	0	0
normalized size	1	1.	0.86	2.45	0.	0.	0.	0.
time (sec)	N/A	0.045	0.08	0.128	0.	0.	0.	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	521	0	0	0	0
normalized size	1	1.	0.98	12.12	0.	0.	0.	0.
time (sec)	N/A	0.019	0.015	0.124	0.	0.	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	165	0	0	0	0
normalized size	1	1.	0.98	3.59	0.	0.	0.	0.
time (sec)	N/A	0.021	0.017	0.109	0.	0.	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	52	522	0	0	0	0
normalized size	1	1.	0.74	7.46	0.	0.	0.	0.
time (sec)	N/A	0.039	0.084	0.117	0.	0.	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	60	319	0	0	0	0
normalized size	1	1.	0.78	4.14	0.	0.	0.	0.
time (sec)	N/A	0.039	0.071	0.129	0.	0.	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	70	216	0	0	0	0
normalized size	1	1.	0.68	2.1	0.	0.	0.	0.
time (sec)	N/A	0.074	0.114	0.137	0.	0.	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	60	546	0	0	0	0
normalized size	1	1.	0.81	7.38	0.	0.	0.	0.
time (sec)	N/A	0.046	0.032	0.128	0.	0.	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	63	189	0	0	0	0
normalized size	1	1.	0.82	2.45	0.	0.	0.	0.
time (sec)	N/A	0.033	0.018	0.108	0.	0.	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	533	0	0	0	0
normalized size	1	1.	0.98	11.59	0.	0.	0.	0.
time (sec)	N/A	0.021	0.031	0.106	0.	0.	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	165	0	0	0	0
normalized size	1	1.	0.98	3.59	0.	0.	0.	0.
time (sec)	N/A	0.022	0.017	0.107	0.	0.	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	55	522	0	0	0	0
normalized size	1	1.	0.75	7.15	0.	0.	0.	0.
time (sec)	N/A	0.039	0.051	0.115	0.	0.	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	60	319	0	0	0	0
normalized size	1	1.	0.78	4.14	0.	0.	0.	0.
time (sec)	N/A	0.039	0.067	0.138	0.	0.	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	73	1054	0	0	0	0
normalized size	1	1.	0.7	10.04	0.	0.	0.	0.
time (sec)	N/A	0.059	0.113	0.148	0.	0.	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	102	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	9.795	0.781	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [290] had the largest ratio of [0.6154]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	6	0.167
2	A	2	2	1.	8	0.25
3	A	2	1	1.	8	0.125
4	A	3	2	1.	8	0.25
5	A	2	1	1.	8	0.125
6	A	4	2	1.	8	0.25
7	A	2	1	1.	8	0.125
8	A	5	2	1.	8	0.25
9	A	3	2	1.	8	0.25
10	A	2	2	1.	8	0.25
11	A	2	2	1.	8	0.25
12	A	1	1	1.	8	0.125
13	A	1	1	1.	8	0.125
14	A	2	2	1.	8	0.25
15	A	2	2	1.	8	0.25
16	A	3	2	1.	8	0.25
17	A	3	2	1.	10	0.2
18	A	2	2	1.	10	0.2
19	A	2	2	1.	10	0.2
20	A	1	1	1.	10	0.1

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	1	1	1.	10	0.1
22	A	2	2	1.	10	0.2
23	A	2	2	1.	10	0.2
24	A	3	2	1.	10	0.2
25	A	4	3	1.	12	0.25
26	A	3	3	1.	12	0.25
27	A	3	3	1.	12	0.25
28	A	2	2	1.	12	0.167
29	A	2	2	1.	12	0.167
30	A	3	3	1.	12	0.25
31	A	3	3	1.	12	0.25
32	A	4	3	1.	12	0.25
33	A	1	1	1.	12	0.083
34	A	1	1	1.	12	0.083
35	C	1	1	0.11	12	0.083
36	C	1	1	0.23	12	0.083
37	C	1	1	0.21	12	0.083
38	A	1	1	1.	12	0.083
39	A	1	1	1.	8	0.125
40	A	1	1	1.	10	0.1
41	A	2	2	1.	21	0.095
42	A	2	2	1.	15	0.133
43	A	2	2	1.	15	0.133
44	A	2	2	1.	13	0.154
45	A	1	1	1.	6	0.167
46	A	2	2	1.	13	0.154
47	A	2	2	1.	15	0.133
48	A	2	2	1.	15	0.133
49	A	3	2	1.	17	0.118
50	A	3	2	1.	17	0.118
51	A	3	2	1.	17	0.118
52	A	2	2	1.	15	0.133
53	A	2	2	1.	8	0.25
54	A	2	2	1.	17	0.118
55	A	3	2	1.	17	0.118
56	A	3	2	1.	17	0.118
57	A	3	2	1.	17	0.118
58	A	5	3	1.	17	0.176
59	A	4	3	1.	17	0.176
60	A	3	3	1.	17	0.176
61	A	2	2	1.	8	0.25
62	A	3	3	1.	13	0.231
63	A	2	2	1.	15	0.133
64	A	3	3	1.	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
65	A	4	3	1.	17	0.176
66	A	3	2	1.	17	0.118
67	A	3	2	1.	17	0.118
68	A	3	2	1.	17	0.118
69	A	3	2	1.	17	0.118
70	A	2	2	1.	15	0.133
71	A	3	2	1.	15	0.133
72	A	3	2	1.	15	0.133
73	A	2	2	1.	8	0.25
74	A	2	1	1.	15	0.067
75	A	2	2	1.	17	0.118
76	A	3	2	1.	17	0.118
77	A	3	2	1.	17	0.118
78	A	3	2	1.	17	0.118
79	A	3	2	1.	17	0.118
80	A	3	2	1.	17	0.118
81	A	3	2	1.	17	0.118
82	A	3	2	1.	17	0.118
83	A	2	2	1.	15	0.133
84	A	4	4	1.	17	0.235
85	A	3	2	1.	8	0.25
86	A	2	2	1.	17	0.118
87	A	3	2	1.	17	0.118
88	A	3	2	1.	17	0.118
89	A	6	3	1.	17	0.176
90	A	5	3	1.	17	0.176
91	A	4	3	1.	17	0.176
92	A	3	2	1.	8	0.25
93	A	4	3	1.	15	0.2
94	A	4	4	1.	15	0.267
95	A	3	2	1.	15	0.133
96	A	4	3	1.	17	0.176
97	A	5	3	1.	17	0.176
98	A	4	3	1.	17	0.176
99	A	3	2	1.	17	0.118
100	A	4	3	1.	17	0.176
101	A	3	2	1.	17	0.118
102	A	3	2	1.	17	0.118
103	A	3	2	1.	17	0.118
104	A	2	2	1.	15	0.133
105	A	4	3	1.	15	0.2
106	A	3	2	1.	17	0.118
107	A	4	3	1.	17	0.176
108	A	3	2	1.	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
109	A	3	2	1.	8	0.25
110	A	3	2	1.	15	0.133
111	A	2	2	1.	17	0.118
112	A	3	2	1.	17	0.118
113	A	3	2	1.	17	0.118
114	A	3	2	1.	17	0.118
115	A	4	3	1.	17	0.176
116	A	3	2	1.	17	0.118
117	A	4	3	1.	17	0.176
118	A	5	4	1.	17	0.235
119	A	3	2	1.	15	0.133
120	A	4	3	1.	15	0.2
121	A	4	3	1.	15	0.2
122	A	4	3	1.	15	0.2
123	A	3	2	1.	15	0.133
124	A	3	3	1.	13	0.231
125	A	1	1	1.	6	0.167
126	A	2	2	1.	13	0.154
127	A	3	3	1.	15	0.2
128	A	3	2	1.	15	0.133
129	A	4	3	1.	15	0.2
130	A	4	3	1.	15	0.2
131	A	4	3	1.	15	0.2
132	A	4	3	1.	15	0.2
133	A	3	2	1.	17	0.118
134	A	5	4	1.	17	0.235
135	A	3	2	1.	17	0.118
136	A	4	4	1.	17	0.235
137	A	3	2	1.	15	0.133
138	A	2	2	1.	8	0.25
139	A	2	2	1.	13	0.154
140	A	3	3	1.	15	0.2
141	A	3	2	1.	17	0.118
142	A	4	4	1.	17	0.235
143	A	3	2	1.	17	0.118
144	A	5	4	1.	17	0.235
145	A	4	3	1.	17	0.176
146	A	5	4	1.	17	0.235
147	A	4	3	1.	17	0.176
148	A	4	4	1.	15	0.267
149	A	2	2	1.	8	0.25
150	A	2	2	1.	15	0.133
151	A	2	2	1.	15	0.133
152	A	3	2	1.	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
153	A	4	4	1.	17	0.235
154	A	4	3	1.	17	0.176
155	A	5	4	1.	17	0.235
156	A	4	3	1.	17	0.176
157	A	3	2	1.	17	0.118
158	A	6	4	1.	17	0.235
159	A	3	2	1.	17	0.118
160	A	5	4	1.	17	0.235
161	A	3	2	1.	15	0.133
162	A	3	2	1.	8	0.25
163	A	2	1	1.	15	0.067
164	A	2	2	1.	17	0.118
165	A	2	2	1.	15	0.133
166	A	4	3	1.	15	0.2
167	A	3	2	1.	17	0.118
168	A	5	4	1.	17	0.235
169	A	3	2	1.	17	0.118
170	A	6	4	1.	17	0.235
171	A	4	3	1.	17	0.176
172	A	6	4	1.	17	0.235
173	A	4	3	1.	17	0.176
174	A	5	4	1.	15	0.267
175	A	3	2	1.	8	0.25
176	A	3	2	1.	15	0.133
177	A	2	2	1.	17	0.118
178	A	3	3	1.	17	0.176
179	A	2	2	1.	15	0.133
180	A	4	3	1.	15	0.2
181	A	5	4	1.	17	0.235
182	A	4	3	1.	17	0.176
183	A	6	4	1.	17	0.235
184	A	4	3	1.	17	0.176
185	A	3	2	1.	9	0.222
186	A	3	2	1.	9	0.222
187	A	2	2	1.	19	0.105
188	A	2	2	1.	19	0.105
189	A	2	2	1.	19	0.105
190	A	2	2	1.	19	0.105
191	A	2	2	1.	19	0.105
192	A	2	2	1.	19	0.105
193	A	2	2	1.	19	0.105
194	A	5	4	1.	21	0.19
195	A	5	4	1.	21	0.19
196	A	4	4	1.	21	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	4	4	1.	21	0.19
198	A	3	3	1.	21	0.143
199	A	3	3	1.	21	0.143
200	A	3	3	1.	21	0.143
201	A	3	3	1.	21	0.143
202	A	4	4	1.	21	0.19
203	A	4	4	1.	21	0.19
204	A	3	2	1.	21	0.095
205	A	3	2	1.	21	0.095
206	A	3	2	1.	21	0.095
207	A	3	2	1.	21	0.095
208	A	3	2	1.	21	0.095
209	A	3	2	1.	21	0.095
210	A	3	2	1.	21	0.095
211	A	6	4	1.	21	0.19
212	A	6	4	1.	21	0.19
213	A	5	4	1.	21	0.19
214	A	5	4	1.	21	0.19
215	A	4	3	1.	21	0.143
216	A	4	3	1.	21	0.143
217	A	4	4	1.	21	0.19
218	A	4	4	1.	21	0.19
219	A	4	3	1.	21	0.143
220	A	4	3	1.	21	0.143
221	A	3	2	1.	19	0.105
222	A	7	6	1.	19	0.316
223	A	7	6	1.	19	0.316
224	A	6	6	1.	19	0.316
225	A	6	6	1.	19	0.316
226	A	5	5	1.	19	0.263
227	A	5	5	1.	19	0.263
228	A	6	6	1.	19	0.316
229	A	6	6	1.	19	0.316
230	A	7	6	1.	19	0.316
231	A	7	6	1.	19	0.316
232	A	5	4	1.	21	0.19
233	A	4	4	1.	21	0.19
234	A	4	4	1.	21	0.19
235	A	3	3	1.	21	0.143
236	A	3	3	1.	21	0.143
237	A	3	3	1.	21	0.143
238	A	3	3	1.	21	0.143
239	A	4	4	1.	21	0.19
240	A	4	4	1.	21	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
241	A	5	4	1.	21	0.19
242	A	8	7	1.	21	0.333
243	A	7	7	1.	21	0.333
244	A	7	7	1.	21	0.333
245	A	6	6	1.	21	0.286
246	A	6	6	1.	21	0.286
247	A	6	6	1.	21	0.286
248	A	6	6	1.	21	0.286
249	A	7	7	1.	21	0.333
250	A	7	7	1.	21	0.333
251	A	8	7	1.	21	0.333
252	A	2	2	1.	19	0.105
253	A	3	2	1.	11	0.182
254	A	3	2	1.	11	0.182
255	A	3	2	1.	11	0.182
256	A	3	2	1.	11	0.182
257	A	4	3	1.	25	0.12
258	A	3	3	1.	25	0.12
259	A	2	2	1.	25	0.08
260	A	3	3	1.	25	0.12
261	A	4	3	1.	25	0.12
262	A	11	8	1.	25	0.32
263	A	10	7	1.	25	0.28
264	A	1	1	1.	25	0.04
265	A	2	2	1.	25	0.08
266	A	3	2	1.	25	0.08
267	A	4	4	1.	25	0.16
268	A	3	3	1.	25	0.12
269	A	3	3	1.	25	0.12
270	A	4	4	1.	25	0.16
271	A	11	8	1.	25	0.32
272	A	11	8	1.	25	0.32
273	A	1	1	1.	25	0.04
274	A	3	3	1.	25	0.12
275	A	4	3	1.	25	0.12
276	A	5	4	1.	25	0.16
277	A	4	4	1.	25	0.16
278	A	3	3	1.	25	0.12
279	A	3	3	1.	25	0.12
280	A	4	4	1.	25	0.16
281	A	5	4	1.	25	0.16
282	A	11	8	1.	25	0.32
283	A	11	8	1.	25	0.32
284	A	1	1	1.	25	0.04

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
285	A	3	3	1.	25	0.12
286	A	4	3	1.	25	0.12
287	A	12	8	1.	21	0.381
288	A	1	1	1.	13	0.077
289	A	10	7	1.	13	0.538
290	A	11	8	1.	13	0.615
291	A	4	3	1.	25	0.12
292	A	3	3	1.	25	0.12
293	A	2	2	1.	25	0.08
294	A	3	3	1.	25	0.12
295	A	4	3	1.	25	0.12
296	A	10	7	1.	25	0.28
297	A	1	1	1.	25	0.04
298	A	2	2	1.	25	0.08
299	A	3	2	1.	25	0.08
300	A	10	7	1.	21	0.333
301	A	11	8	1.	21	0.381
302	A	11	8	1.	21	0.381
303	A	12	8	1.	21	0.381
304	A	1	1	1.	21	0.048
305	A	1	1	1.	21	0.048
306	A	1	1	1.	12	0.083
307	A	1	1	1.	21	0.048
308	A	1	1	1.	21	0.048
309	A	1	1	1.	21	0.048
310	A	1	1	1.	21	0.048
311	A	1	1	1.	12	0.083
312	A	1	1	1.	21	0.048
313	A	1	1	1.	21	0.048
314	A	1	1	1.	21	0.048
315	A	1	1	1.	21	0.048
316	A	1	1	1.	12	0.083
317	A	1	1	1.	21	0.048
318	A	1	1	1.	21	0.048
319	A	1	1	1.	21	0.048
320	A	1	1	1.	21	0.048
321	A	1	1	1.	12	0.083
322	A	1	1	1.	21	0.048
323	A	1	1	1.	21	0.048
324	A	8	8	1.	21	0.381
325	A	11	7	1.	21	0.333
326	A	12	8	1.	21	0.381
327	A	9	9	1.	21	0.429
328	A	9	9	1.	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
329	A	8	8	1.	21	0.381
330	A	11	7	1.	21	0.333
331	A	12	8	1.	21	0.381
332	A	9	9	1.	21	0.429
333	A	9	9	1.	21	0.429
334	A	1	1	1.	13	0.077
335	A	1	1	1.	13	0.077
336	A	1	1	1.	17	0.059
337	A	1	1	1.	19	0.053
338	A	1	1	1.	19	0.053
339	A	1	1	1.	21	0.048
340	A	3	2	1.	19	0.105
341	A	3	2	1.	19	0.105
342	A	2	2	1.	17	0.118
343	A	2	2	1.	17	0.118
344	A	2	2	1.	19	0.105
345	A	1	1	1.	19	0.053
346	A	1	1	1.	19	0.053
347	A	1	1	1.	10	0.1
348	A	1	1	1.	19	0.053
349	A	1	1	1.	19	0.053
350	A	1	1	1.	23	0.043
351	A	1	1	1.	23	0.043
352	A	1	1	1.	23	0.043
353	A	1	1	1.	23	0.043
354	A	1	1	1.	23	0.043
355	A	3	2	1.	19	0.105
356	A	3	2	1.	19	0.105
357	A	2	2	1.	17	0.118
358	A	2	2	1.	17	0.118
359	A	2	2	1.	19	0.105
360	A	2	2	1.	19	0.105
361	A	1	1	1.	19	0.053
362	A	1	1	1.	19	0.053
363	A	1	1	1.	10	0.1
364	A	1	1	1.	19	0.053
365	A	1	1	1.	19	0.053
366	A	1	1	1.	23	0.043
367	A	1	1	1.	23	0.043
368	A	1	1	1.	23	0.043
369	A	1	1	1.	23	0.043
370	A	1	1	1.	23	0.043
371	A	3	2	1.	21	0.095
372	A	3	2	1.	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
373	A	3	2	1.	21	0.095
374	A	2	2	1.	19	0.105
375	A	5	5	1.	19	0.263
376	A	6	6	1.	21	0.286
377	A	7	6	1.	21	0.286
378	A	5	3	1.	21	0.143
379	A	4	3	1.	21	0.143
380	A	3	3	1.	21	0.143
381	A	2	2	1.	12	0.167
382	A	3	3	1.	21	0.143
383	A	4	3	1.	21	0.143
384	A	5	3	1.	21	0.143
385	A	3	2	1.	21	0.095
386	A	3	2	1.	21	0.095
387	A	3	2	1.	21	0.095
388	A	2	2	1.	19	0.105
389	A	6	6	1.	19	0.316
390	A	7	7	1.	21	0.333
391	A	5	4	1.	21	0.19
392	A	4	4	1.	21	0.19
393	A	3	3	1.	21	0.143
394	A	3	3	1.	12	0.25
395	A	4	4	1.	21	0.19
396	A	5	4	1.	21	0.19
397	A	3	2	1.	21	0.095
398	A	3	2	1.	21	0.095
399	A	3	2	1.	21	0.095
400	A	2	2	1.	19	0.105
401	A	6	6	1.	19	0.316
402	A	7	7	1.	21	0.333
403	A	8	7	1.	21	0.333
404	A	5	4	1.	21	0.19
405	A	4	4	1.	21	0.19
406	A	3	3	1.	21	0.143
407	A	3	3	1.	12	0.25
408	A	4	4	1.	21	0.19
409	A	5	4	1.	21	0.19
410	A	3	2	1.	21	0.095
411	A	3	2	1.	21	0.095
412	A	3	2	1.	21	0.095
413	A	2	2	1.	19	0.105
414	A	5	5	1.	19	0.263
415	A	6	6	1.	21	0.286
416	A	7	6	1.	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
417	A	5	3	1.	21	0.143
418	A	4	3	1.	21	0.143
419	A	3	3	1.	21	0.143
420	A	2	2	1.	12	0.167
421	A	3	3	1.	21	0.143
422	A	4	3	1.	21	0.143
423	A	5	3	1.	21	0.143
424	A	3	2	1.	21	0.095
425	A	3	2	1.	21	0.095
426	A	3	2	1.	21	0.095
427	A	2	2	1.	19	0.105
428	A	6	6	1.	19	0.316
429	A	6	6	1.	21	0.286
430	A	7	7	1.	21	0.333
431	A	5	4	1.	21	0.19
432	A	4	4	1.	21	0.19
433	A	3	3	1.	12	0.25
434	A	3	3	1.	21	0.143
435	A	4	4	1.	21	0.19
436	A	5	4	1.	21	0.19
437	A	3	2	1.	21	0.095
438	A	3	2	1.	21	0.095
439	A	3	2	1.	21	0.095
440	A	2	2	1.	19	0.105
441	A	6	6	1.	19	0.316
442	A	6	6	1.	21	0.286
443	A	7	7	1.	21	0.333
444	A	5	4	1.	21	0.19
445	A	4	4	1.	21	0.19
446	A	3	3	1.	12	0.25
447	A	3	3	1.	21	0.143
448	A	4	4	1.	21	0.19
449	A	5	4	1.	21	0.19
450	A	13	9	1.	25	0.36
451	A	12	9	1.	25	0.36
452	A	11	8	1.	25	0.32
453	A	1	1	1.	25	0.04
454	A	2	2	1.	25	0.08
455	A	3	2	1.	25	0.08
456	A	5	4	1.	25	0.16
457	A	4	4	1.	25	0.16
458	A	3	3	1.	25	0.12
459	A	4	4	1.	25	0.16
460	A	5	4	1.	25	0.16

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
461	A	5	4	1.	23	0.174
462	A	4	4	1.	23	0.174
463	A	3	3	1.	23	0.13
464	A	4	4	1.	23	0.174
465	A	5	4	1.	23	0.174
466	A	12	9	1.	23	0.391
467	A	11	8	1.	23	0.348
468	A	1	1	1.	23	0.043
469	A	2	2	1.	23	0.087
470	A	3	2	1.	23	0.087
471	A	4	2	1.	23	0.087
472	A	14	10	1.	25	0.4
473	A	13	10	1.	25	0.4
474	A	12	9	1.	25	0.36
475	A	12	9	1.	25	0.36
476	A	1	1	1.	25	0.04
477	A	6	5	1.	25	0.2
478	A	5	5	1.	25	0.2
479	A	4	4	1.	25	0.16
480	A	4	4	1.	25	0.16
481	A	5	5	1.	25	0.2
482	A	6	5	1.	25	0.2
483	A	2	2	1.	23	0.087
484	A	2	2	1.	23	0.087
485	A	2	2	1.	23	0.087
486	A	2	2	1.	23	0.087
487	A	2	2	1.	23	0.087
488	A	2	2	1.	17	0.118
489	A	2	2	1.	19	0.105
490	A	2	2	1.	19	0.105
491	A	2	2	1.	21	0.095
492	A	3	2	1.	19	0.105
493	A	3	2	1.	19	0.105
494	A	2	2	1.	17	0.118
495	A	2	2	1.	17	0.118
496	A	2	2	1.	19	0.105
497	A	2	2	1.	19	0.105
498	A	2	2	1.	19	0.105
499	A	2	2	1.	19	0.105
500	A	2	2	1.	10	0.2
501	A	2	2	1.	19	0.105
502	A	2	2	1.	19	0.105
503	A	2	2	1.	23	0.087
504	A	2	2	1.	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
505	A	2	2	1.	23	0.087
506	A	2	2	1.	23	0.087
507	A	5	4	1.	21	0.19
508	A	4	4	1.	21	0.19
509	A	4	4	1.	21	0.19
510	A	3	3	1.	19	0.158
511	A	2	2	1.	12	0.167
512	A	4	4	1.	19	0.21
513	A	4	4	1.	21	0.19
514	A	5	4	1.	21	0.19
515	A	5	4	1.	21	0.19
516	A	4	4	1.	21	0.19
517	A	4	4	1.	21	0.19
518	A	3	3	1.	21	0.143
519	A	3	3	1.	19	0.158
520	A	3	3	1.	12	0.25
521	A	4	4	1.	19	0.21
522	A	5	4	1.	21	0.19
523	A	5	4	1.	21	0.19
524	A	4	4	1.	21	0.19
525	A	4	4	1.	19	0.21
526	A	2	2	1.	12	0.167
527	A	3	3	1.	19	0.158
528	A	4	4	1.	21	0.19
529	A	4	4	1.	21	0.19
530	A	5	4	1.	21	0.19
531	A	4	4	1.	19	0.21
532	A	3	3	1.	12	0.25
533	A	3	3	1.	19	0.158
534	A	3	3	1.	21	0.143
535	A	4	4	1.	21	0.19
536	A	4	4	1.	21	0.19
537	A	5	4	1.	21	0.19
538	A	2	2	1.	21	0.095

Chapter 3

Listing of integrals

3.1 $\int \sin(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\cos(a+bx)}{b}$$

```
[Out] -(Cos[a + b*x]/b)
```

Rubi [A] time = 0.0040155, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2638}

$$-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x],x]
```

```
[Out] -(Cos[a + b*x]/b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(a+bx)\,dx = -\frac{\cos(a+bx)}{b}$$

Mathematica [A] time = 0.0085557, size = 22, normalized size = 2.

$$\frac{\sin(a)\sin(bx)}{b} - \frac{\cos(a)\cos(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x],x]
```

```
[Out] -((Cos[a]*Cos[b*x])/b) + (Sin[a]*Sin[b*x])/b
```

Maple [A] time = 0.029, size = 12, normalized size = 1.1

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a),x)
```

```
[Out] -cos(b*x+a)/b
```

Maxima [A] time = 0.946618, size = 15, normalized size = 1.36

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -cos(b*x + a)/b
```

Fricas [A] time = 2.49887, size = 23, normalized size = 2.09

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -cos(b*x + a)/b
```

Sympy [A] time = 0.132351, size = 14, normalized size = 1.27

$$\begin{cases} -\frac{\cos(a+bx)}{b} & \text{for } b \neq 0 \\ x\sin(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x)
```

```
[Out] Piecewise((-cos(a + b*x)/b, Ne(b, 0)), (x*sin(a), True))
```

Giac [A] time = 1.09055, size = 15, normalized size = 1.36

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x, algorithm="giac")
```

```
[Out] -cos(b*x + a)/b
```

3.2 $\int \sin^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

```
[Out] x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rubi [A] time = 0.008774, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2,x]

[Out] x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\,dx &= -\frac{\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0283056, size = 23, normalized size = 0.92

$$-\frac{\sin(2(a+bx))-2(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2,x]

[Out] -(-2*(a + b*x) + Sin[2*(a + b*x)])/(4*b)
```

Maple [A] time = 0.025, size = 27, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

Maxima [A] time = 0.995329, size = 32, normalized size = 1.28

$$\frac{2\,bx+2\,a-\sin(2\,bx+2\,a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))/b
```

Fricas [A] time = 2.52304, size = 55, normalized size = 2.2

$$\frac{bx-\cos(bx+a)\sin(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x - cos(b*x + a)*sin(b*x + a))/b
```

Sympy [A] time = 0.209137, size = 46, normalized size = 1.84

$$\begin{cases} \frac{x\sin^2(a+bx)}{2}+\frac{x\cos^2(a+bx)}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b\neq 0\\ x\sin^2(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a +
 b*x)/(2*b), Ne(b, 0)), (x*sin(a)**2, True))
```

Giac [A] time = 1.13473, size = 24, normalized size = 0.96

$$\frac{1}{2}x-\frac{\sin(2\,bx+2\,a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*b*x + 2*a)/b
```

3.3 $\int \sin^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

```
[Out] -(Cos[a + b*x]/b) + Cos[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.010362, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3,x]
```

```
[Out] -(Cos[a + b*x]/b) + Cos[a + b*x]^3/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0102068, size = 29, normalized size = 1.07

$$\frac{\cos(3(a+bx))}{12b}-\frac{3\cos(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3,x]
```

```
[Out] (-3*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b)
```

Maple [A] time = 0.072, size = 22, normalized size = 0.8

$$-\frac{\left(2+(\sin(bx+a))^2\right)\cos(bx+a)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3,x)
```

```
[Out] -1/3/b*(2+sin(b*x+a)^2)*cos(b*x+a)
```

Maxima [A] time = 1.02229, size = 30, normalized size = 1.11

$$\frac{\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

Fricas [A] time = 2.14492, size = 55, normalized size = 2.04

$$\frac{\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

Sympy [A] time = 0.440955, size = 37, normalized size = 1.37

$$\begin{cases} -\frac{\sin^{2}{\left(a+bx \right)}\cos{\left(a+bx \right)}}{b} - \frac{2\cos^{3}{\left(a+bx \right)}}{3b} & \text{for}\: b \neq 0 \\ x\sin^{3}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)/b - 2*cos(a + b*x)**3/(3*b), Ne(b,
 0)), (x*sin(a)**3, True))
```

Giac [A] time = 1.14002, size = 34, normalized size = 1.26

$$\frac{\cos\left(bx+a\right)^{3}}{3\,b}-\frac{\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*cos(b*x + a)^3/b - cos(b*x + a)/b
```

3.4 $\int \sin^4(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}-\frac{3\sin(a+bx)\cos(a+bx)}{8b}+\frac{3x}{8}$$

```
[Out] (3*x)/8 - (3*Cos[a + b*x]*Sin[a + b*x])/(8*b) - (Cos[a + b*x]*Sin[a + b*x]^
3)/(4*b)
```

Rubi [A] time = 0.0197972, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}-\frac{3\sin(a+bx)\cos(a+bx)}{8b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[a + b*x]*Sin[a + b*x])/(8*b) - (Cos[a + b*x]*Sin[a + b*x]^
3)/(4*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^4(a+bx)\,dx &= -\frac{\cos(a+bx)\sin^3(a+bx)}{4b}+\frac{3}{4}\int \sin^2(a+bx)\,dx\\ &= -\frac{3\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos(a+bx)\sin^3(a+bx)}{4b}+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}-\frac{3\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos(a+bx)\sin^3(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.040468, size = 33, normalized size = 0.72

$$\frac{12(a+bx)-8\sin(2(a+bx))+\sin(4(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4,x]
```

```
[Out] (12*(a + b*x) - 8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)])/(32*b)
```

Maple [A] time = 0.044, size = 38, normalized size = 0.8

$$\frac{1}{b}\left(-\frac{\cos(bx+a)}{4}\left((\sin(bx+a))^3+\frac{3\sin(bx+a)}{2}\right)+\frac{3bx}{8}+\frac{3a}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)
```

Maxima [A] time = 1.00929, size = 45, normalized size = 0.98

$$\frac{12\,bx+12\,a+\sin(4\,bx+4\,a)-8\,\sin(2\,bx+2\,a)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/32*(12*b*x + 12*a + sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))/b
```

Fricas [A] time = 2.31536, size = 89, normalized size = 1.93

$$\frac{3\,bx+\left(2\,\cos(bx+a)^3-5\,\cos(bx+a)\right)\sin(bx+a)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(3*b*x + (2*cos(b*x + a)^3 - 5*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [A] time = 0.979039, size = 95, normalized size = 2.07

$$\begin{cases}\frac{3x\sin^4(a+bx)}{8}+\frac{3x\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{3x\cos^4(a+bx)}{8}-\frac{5\sin^3(a+bx)\cos(a+bx)}{8b}-\frac{3\sin(a+bx)\cos^3(a+bx)}{8b} & \text{for } b\neq 0\\ x\sin^4(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**4/8 + 3*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
3*x*cos(a + b*x)**4/8 - 5*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*sin(a + b*
x)*cos(a + b*x)**3/(8*b), Ne(b, 0)), (x*sin(a)**4, True))
```

Giac [A] time = 1.10223, size = 43, normalized size = 0.93

$$\frac{3}{8}x + \frac{\sin(4bx + 4a)}{32b} - \frac{\sin(2bx + 2a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*b*x + 4*a)/b - 1/4*sin(2*b*x + 2*a)/b
```

3.5 $\int \sin^5(a+bx)\,dx$

Optimal. Leaf size=42

$$-\frac{\cos^5(a+bx)}{5b}+\frac{2\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

```
[Out] -(Cos[a + b*x]/b) + (2*Cos[a + b*x]^3)/(3*b) - Cos[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0130112, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$-\frac{\cos^5(a+bx)}{5b}+\frac{2\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^5,x]
```

```
[Out] -(Cos[a + b*x]/b) + (2*Cos[a + b*x]^3)/(3*b) - Cos[a + b*x]^5/(5*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^5(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos(a+bx)}{b}+\frac{2\cos^3(a+bx)}{3b}-\frac{\cos^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0121988, size = 44, normalized size = 1.05

$$-\frac{5\cos(a+bx)}{8b}+\frac{5\cos(3(a+bx))}{48b}-\frac{\cos(5(a+bx))}{80b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^5,x]
```

```
[Out] (-5*Cos[a + b*x])/(8*b) + (5*Cos[3*(a + b*x)])/(48*b) - Cos[5*(a + b*x)]/(8
0*b)
```

Maple [A] time = 0.037, size = 32, normalized size = 0.8

$$-\frac{\cos(bx+a)}{5b}\left(\frac{8}{3}+\left(\sin(bx+a)\right)^4+\frac{4\left(\sin(bx+a)\right)^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^5,x)
```

```
[Out] -1/5/b*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)
```

Maxima [A] time = 0.995149, size = 46, normalized size = 1.1

$$-\frac{3\cos\left(bx+a\right)^{5}-10\cos\left(bx+a\right)^{3}+15\cos\left(bx+a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/15*(3*cos(b*x + a)^5 - 10*cos(b*x + a)^3 + 15*cos(b*x + a))/b
```

Fricas [A] time = 2.1845, size = 89, normalized size = 2.12

$$-\frac{3\cos\left(bx+a\right)^{5}-10\cos\left(bx+a\right)^{3}+15\cos\left(bx+a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/15*(3*cos(b*x + a)^5 - 10*cos(b*x + a)^3 + 15*cos(b*x + a))/b
```

Sympy [A] time = 1.83787, size = 60, normalized size = 1.43

$$\begin{cases} -\frac{\sin^{4}{\left(a+bx\right)}\cos{\left(a+bx\right)}}{b}-\frac{4\sin^{2}{\left(a+bx\right)}\cos^{3}{\left(a+bx\right)}}{3b}-\frac{8\cos^{5}{\left(a+bx\right)}}{15b} & \text{for}\: b\neq 0\\ x\sin^{5}{\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)/b - 4*sin(a + b*x)**2*cos(a + b*x)
**3/(3*b) - 8*cos(a + b*x)**5/(15*b), Ne(b, 0)), (x*sin(a)**5, True))
```

Giac [A] time = 1.12195, size = 51, normalized size = 1.21

$$-\frac{\cos\left(bx+a\right)^{5}}{5\,b}+\frac{2\cos\left(bx+a\right)^{3}}{3\,b}-\frac{\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/5*cos(b*x + a)^5/b + 2/3*cos(b*x + a)^3/b - cos(b*x + a)/b
```

3.6 $\int \sin^6(a+bx)\,dx$

Optimal. Leaf size=67

$$-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}-\frac{5\sin^3(a+bx)\cos(a+bx)}{24b}-\frac{5\sin(a+bx)\cos(a+bx)}{16b}+\frac{5x}{16}$$

```
[Out] (5*x)/16 - (5*Cos[a + b*x]*Sin[a + b*x])/(16*b) - (5*Cos[a + b*x]*Sin[a + b
*x]^3)/(24*b) - (Cos[a + b*x]*Sin[a + b*x]^5)/(6*b)
```

Rubi [A] time = 0.0327676, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}-\frac{5\sin^3(a+bx)\cos(a+bx)}{24b}-\frac{5\sin(a+bx)\cos(a+bx)}{16b}+\frac{5x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^6,x]
```

```
[Out] (5*x)/16 - (5*Cos[a + b*x]*Sin[a + b*x])/(16*b) - (5*Cos[a + b*x]*Sin[a + b
*x]^3)/(24*b) - (Cos[a + b*x]*Sin[a + b*x]^5)/(6*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \sin^6(a+bx)\,dx &= -\frac{\cos(a+bx)\sin^5(a+bx)}{6b}+\frac{5}{6}\int \sin^4(a+bx)\,dx\\
&= -\frac{5\cos(a+bx)\sin^3(a+bx)}{24b}-\frac{\cos(a+bx)\sin^5(a+bx)}{6b}+\frac{5}{8}\int \sin^2(a+bx)\,dx\\
&= -\frac{5\cos(a+bx)\sin(a+bx)}{16b}-\frac{5\cos(a+bx)\sin^3(a+bx)}{24b}-\frac{\cos(a+bx)\sin^5(a+bx)}{6b}+\frac{5\int 1\,dx}{16}\\
&= \frac{5x}{16}-\frac{5\cos(a+bx)\sin(a+bx)}{16b}-\frac{5\cos(a+bx)\sin^3(a+bx)}{24b}-\frac{\cos(a+bx)\sin^5(a+bx)}{6b}
\end{aligned}$$

Mathematica [A] time = 0.0419155, size = 45, normalized size = 0.67

$$\frac{-45\sin(2(a+bx))+9\sin(4(a+bx))-\sin(6(a+bx))+60a+60bx}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^6,x]
```

```
[Out] (60*a + 60*b*x - 45*Sin[2*(a + b*x)] + 9*Sin[4*(a + b*x)] - Sin[6*(a + b*x)
])/(192*b)
```

Maple [A] time = 0.036, size = 48, normalized size = 0.7

$$\frac{1}{b}\left(-\frac{\cos(bx+a)}{6}\left((\sin(bx+a))^5+\frac{5(\sin(bx+a))^3}{4}+\frac{15\sin(bx+a)}{8}\right)+\frac{5bx}{16}+\frac{5a}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^6,x)
```

```
[Out] 1/b*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b
*x+5/16*a)
```

Maxima [A] time = 1.00742, size = 65, normalized size = 0.97

$$\frac{4\sin(2bx+2a)^3+60bx+60a+9\sin(4bx+4a)-48\sin(2bx+2a)}{192b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] 1/192*(4*sin(2*b*x + 2*a)^3 + 60*b*x + 60*a + 9*sin(4*b*x + 4*a) - 48*sin(2
*b*x + 2*a))/b
```

Fricas [A] time = 2.20246, size = 120, normalized size = 1.79

$$\frac{15bx-\left(8\cos(bx+a)^5-26\cos(bx+a)^3+33\cos(bx+a)\right)\sin(bx+a)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(15*b*x - (8*cos(b*x + a)^5 - 26*cos(b*x + a)^3 + 33*cos(b*x + a))*sin
(b*x + a))/b
```

Sympy [A] time = 3.54554, size = 139, normalized size = 2.07

$$\begin{cases}\frac{5x\sin^6(a+bx)}{16}+\frac{15x\sin^4(a+bx)\cos^2(a+bx)}{16}+\frac{15x\sin^2(a+bx)\cos^4(a+bx)}{16}+\frac{5x\cos^6(a+bx)}{16}-\frac{11\sin^5(a+bx)\cos(a+bx)}{16b}-\frac{5\sin^3(a+bx)\cos^3(a+b}{6b} \\ x\sin^6(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**6,x)
```

```
[Out] Piecewise((5*x*sin(a + b*x)**6/16 + 15*x*sin(a + b*x)**4*cos(a + b*x)**2/16
 + 15*x*sin(a + b*x)**2*cos(a + b*x)**4/16 + 5*x*cos(a + b*x)**6/16 - 11*si
n(a + b*x)**5*cos(a + b*x)/(16*b) - 5*sin(a + b*x)**3*cos(a + b*x)**3/(6*b)
 - 5*sin(a + b*x)*cos(a + b*x)**5/(16*b), Ne(b, 0)), (x*sin(a)**6, True))
```

Giac [A] time = 1.10528, size = 62, normalized size = 0.93

$$\frac{5}{16}x - \frac{\sin(6bx + 6a)}{192b} + \frac{3\sin(4bx + 4a)}{64b} - \frac{15\sin(2bx + 2a)}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 5/16*x - 1/192*sin(6*b*x + 6*a)/b + 3/64*sin(4*b*x + 4*a)/b - 15/64*sin(2*b
*x + 2*a)/b
```

3.7 $\int \sin^7(a+bx)\,dx$

Optimal. Leaf size=54

$$\frac{\cos^7(a+bx)}{7b}-\frac{3\cos^5(a+bx)}{5b}+\frac{\cos^3(a+bx)}{b}-\frac{\cos(a+bx)}{b}$$

```
[Out] -(Cos[a + b*x]/b) + Cos[a + b*x]^3/b - (3*Cos[a + b*x]^5)/(5*b) + Cos[a + b
*x]^7/(7*b)
```

Rubi [A] time = 0.0157233, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\cos^7(a+bx)}{7b}-\frac{3\cos^5(a+bx)}{5b}+\frac{\cos^3(a+bx)}{b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^7,x]
```

```
[Out] -(Cos[a + b*x]/b) + Cos[a + b*x]^3/b - (3*Cos[a + b*x]^5)/(5*b) + Cos[a + b
*x]^7/(7*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^7(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{b}-\frac{3\cos^5(a+bx)}{5b}+\frac{\cos^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.0092732, size = 59, normalized size = 1.09

$$-\frac{35\cos(a+bx)}{64b}+\frac{7\cos(3(a+bx))}{64b}-\frac{7\cos(5(a+bx))}{320b}+\frac{\cos(7(a+bx))}{448b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^7,x]
```

```
[Out] (-35*Cos[a + b*x])/(64*b) + (7*Cos[3*(a + b*x)])/(64*b) - (7*Cos[5*(a + b*x
)])/(320*b) + Cos[7*(a + b*x)]/(448*b)
```

Maple [A] time = 0.038, size = 42, normalized size = 0.8

$$-\frac{\cos(bx+a)}{7b}\left(\frac{16}{5}+(\sin(bx+a))^6+\frac{6(\sin(bx+a))^4}{5}+\frac{8(\sin(bx+a))^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^7,x)
```

```
[Out] -1/7/b*(16/5+sin(b*x+a)^6+6/5*sin(b*x+a)^4+8/5*sin(b*x+a)^2)*cos(b*x+a)
```

Maxima [A] time = 0.979021, size = 59, normalized size = 1.09

$$\frac{5\cos(bx+a)^7 - 21\cos(bx+a)^5 + 35\cos(bx+a)^3 - 35\cos(bx+a)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/35*(5*cos(b*x + a)^7 - 21*cos(b*x + a)^5 + 35*cos(b*x + a)^3 - 35*cos(b*x
 + a))/b
```

Fricas [A] time = 2.27506, size = 115, normalized size = 2.13

$$\frac{5\cos(bx+a)^7 - 21\cos(bx+a)^5 + 35\cos(bx+a)^3 - 35\cos(bx+a)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(b*x + a)^7 - 21*cos(b*x + a)^5 + 35*cos(b*x + a)^3 - 35*cos(b*x
 + a))/b
```

Sympy [A] time = 6.82139, size = 80, normalized size = 1.48

$$\begin{cases} -\frac{\sin^6(a+bx)\cos(a+bx)}{b} - \frac{2\sin^4(a+bx)\cos^3(a+bx)}{b} - \frac{8\sin^2(a+bx)\cos^5(a+bx)}{5b} - \frac{16\cos^7(a+bx)}{35b} & \text{for } b \neq 0 \\ x\sin^7(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**7,x)
```

```
[Out] Piecewise((-sin(a + b*x)**6*cos(a + b*x)/b - 2*sin(a + b*x)**4*cos(a + b*x)
**3/b - 8*sin(a + b*x)**2*cos(a + b*x)**5/(5*b) - 16*cos(a + b*x)**7/(35*b)
, Ne(b, 0)), (x*sin(a)**7, True))
```

Giac [A] time = 1.13322, size = 68, normalized size = 1.26

$$\frac{\cos(bx+a)^7}{7\,b} - \frac{3\cos(bx+a)^5}{5\,b} + \frac{\cos(bx+a)^3}{b} - \frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/7*cos(b*x + a)^7/b - 3/5*cos(b*x + a)^5/b + cos(b*x + a)^3/b - cos(b*x +
a)/b
```

3.8 $\int \sin^8(a+bx)\,dx$

Optimal. Leaf size=88

$$-\frac{\sin^7(a+bx)\cos(a+bx)}{8b}-\frac{7\sin^5(a+bx)\cos(a+bx)}{48b}-\frac{35\sin^3(a+bx)\cos(a+bx)}{192b}-\frac{35\sin(a+bx)\cos(a+bx)}{128b}+\frac{35x}{128}$$

```
[Out] (35*x)/128 - (35*Cos[a + b*x]*Sin[a + b*x])/(128*b) - (35*Cos[a + b*x]*Sin[
a + b*x]^3)/(192*b) - (7*Cos[a + b*x]*Sin[a + b*x]^5)/(48*b) - (Cos[a + b*x
]*Sin[a + b*x]^7)/(8*b)
```

Rubi [A] time = 0.0484121, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$-\frac{\sin^7(a+bx)\cos(a+bx)}{8b}-\frac{7\sin^5(a+bx)\cos(a+bx)}{48b}-\frac{35\sin^3(a+bx)\cos(a+bx)}{192b}-\frac{35\sin(a+bx)\cos(a+bx)}{128b}+\frac{35x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^8,x]
```

```
[Out] (35*x)/128 - (35*Cos[a + b*x]*Sin[a + b*x])/(128*b) - (35*Cos[a + b*x]*Sin[
a + b*x]^3)/(192*b) - (7*Cos[a + b*x]*Sin[a + b*x]^5)/(48*b) - (Cos[a + b*x
]*Sin[a + b*x]^7)/(8*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^8(a+bx)\,dx &= -\frac{\cos(a+bx)\sin^7(a+bx)}{8b}+\frac{7}{8}\int \sin^6(a+bx)\,dx\\ &= -\frac{7\cos(a+bx)\sin^5(a+bx)}{48b}-\frac{\cos(a+bx)\sin^7(a+bx)}{8b}+\frac{35}{48}\int \sin^4(a+bx)\,dx\\ &= -\frac{35\cos(a+bx)\sin^3(a+bx)}{192b}-\frac{7\cos(a+bx)\sin^5(a+bx)}{48b}-\frac{\cos(a+bx)\sin^7(a+bx)}{8b}+\frac{35}{64}\int \sin^2(\\ &= -\frac{35\cos(a+bx)\sin(a+bx)}{128b}-\frac{35\cos(a+bx)\sin^3(a+bx)}{192b}-\frac{7\cos(a+bx)\sin^5(a+bx)}{48b}-\cos(a+b\\ &= \frac{35x}{128}-\frac{35\cos(a+bx)\sin(a+bx)}{128b}-\frac{35\cos(a+bx)\sin^3(a+bx)}{192b}-\frac{7\cos(a+bx)\sin^5(a+bx)}{48b}-\cos(\end{aligned}$$

Mathematica [A] time = 0.0557637, size = 55, normalized size = 0.62

$$\frac{-672\sin(2(a+bx))+168\sin(4(a+bx))-32\sin(6(a+bx))+3\sin(8(a+bx))+840a+840bx}{3072b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^8,x]
```

```
[Out] (840*a + 840*b*x - 672*Sin[2*(a + b*x)] + 168*Sin[4*(a + b*x)] - 32*Sin[6*(
a + b*x)] + 3*Sin[8*(a + b*x)])/(3072*b)
```

Maple [A] time = 0.034, size = 58, normalized size = 0.7

$$\frac{1}{b}\left(-\frac{\cos(bx+a)}{8}\left((\sin(bx+a))^7+\frac{7(\sin(bx+a))^5}{6}+\frac{35(\sin(bx+a))^3}{24}+\frac{35\sin(bx+a)}{16}\right)+\frac{35bx}{128}+\frac{35a}{128}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^8,x)
```

```
[Out] 1/b*(-1/8*(sin(b*x+a)^7+7/6*sin(b*x+a)^5+35/24*sin(b*x+a)^3+35/16*sin(b*x+a
))*cos(b*x+a)+35/128*b*x+35/128*a)
```

Maxima [A] time = 1.02275, size = 80, normalized size = 0.91

$$\frac{128\sin(2bx+2a)^3+840bx+840a+3\sin(8bx+8a)+168\sin(4bx+4a)-768\sin(2bx+2a)}{3072b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] 1/3072*(128*sin(2*b*x + 2*a)^3 + 840*b*x + 840*a + 3*sin(8*b*x + 8*a) + 168
*sin(4*b*x + 4*a) - 768*sin(2*b*x + 2*a))/b
```

Fricas [A] time = 2.27506, size = 155, normalized size = 1.76

$$\frac{105bx+\left(48\cos(bx+a)^7-200\cos(bx+a)^5+326\cos(bx+a)^3-279\cos(bx+a)\right)\sin(bx+a)}{384b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] 1/384*(105*b*x + (48*cos(b*x + a)^7 - 200*cos(b*x + a)^5 + 326*cos(b*x + a)
^3 - 279*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [A] time = 12.3102, size = 184, normalized size = 2.09

$$\begin{cases}\frac{35x\sin^8(a+bx)}{128}+\frac{35x\sin^6(a+bx)\cos^2(a+bx)}{32}+\frac{105x\sin^4(a+bx)\cos^4(a+bx)}{64}+\frac{35x\sin^2(a+bx)\cos^6(a+bx)}{32}+\frac{35x\cos^8(a+bx)}{128}-\frac{93\sin^7(a+bx)\cos}{128b} \\ x\sin^8(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**8,x)
```

```
[Out] Piecewise((35*x*sin(a + b*x)**8/128 + 35*x*sin(a + b*x)**6*cos(a + b*x)**2/
32 + 105*x*sin(a + b*x)**4*cos(a + b*x)**4/64 + 35*x*sin(a + b*x)**2*cos(a
+ b*x)**6/32 + 35*x*cos(a + b*x)**8/128 - 93*sin(a + b*x)**7*cos(a + b*x)/(
128*b) - 511*sin(a + b*x)**5*cos(a + b*x)**3/(384*b) - 385*sin(a + b*x)**3*
cos(a + b*x)**5/(384*b) - 35*sin(a + b*x)*cos(a + b*x)**7/(128*b), Ne(b, 0)
), (x*sin(a)**8, True))
```

Giac [A] time = 1.13179, size = 81, normalized size = 0.92

$$\frac{35}{128}\,x + \frac{\sin\left(8\,bx + 8\,a\right)}{1024\,b} - \frac{\sin\left(6\,bx + 6\,a\right)}{96\,b} + \frac{7\,\sin\left(4\,bx + 4\,a\right)}{128\,b} - \frac{7\,\sin\left(2\,bx + 2\,a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^8,x, algorithm="giac")
```

```
[Out] 35/128*x + 1/1024*sin(8*b*x + 8*a)/b - 1/96*sin(6*b*x + 6*a)/b + 7/128*sin(
4*b*x + 4*a)/b - 7/32*sin(2*b*x + 2*a)/b
```

3.9 $\int \sin^{\frac{7}{2}}(bx)\,dx$

Optimal. Leaf size=60

$$-\frac{10F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{21b}-\frac{2\sin^{\frac{5}{2}}(bx)\cos(bx)}{7b}-\frac{10\sqrt{\sin(bx)}\cos(bx)}{21b}$$

```
[Out] (-10*EllipticF[Pi/4 - (b*x)/2, 2])/(21*b) - (10*Cos[b*x]*Sqrt[Sin[b*x]])/(2
1*b) - (2*Cos[b*x]*Sin[b*x]^(5/2))/(7*b)
```

Rubi [A] time = 0.026722, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 2641}

$$-\frac{10F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{21b}-\frac{2\sin^{\frac{5}{2}}(bx)\cos(bx)}{7b}-\frac{10\sqrt{\sin(bx)}\cos(bx)}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[b*x]^(7/2),x]
```

```
[Out] (-10*EllipticF[Pi/4 - (b*x)/2, 2])/(21*b) - (10*Cos[b*x]*Sqrt[Sin[b*x]])/(2
1*b) - (2*Cos[b*x]*Sin[b*x]^(5/2))/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sin^{\frac{7}{2}}(bx)\,dx &= -\frac{2\cos(bx)\sin^{\frac{5}{2}}(bx)}{7b}+\frac{5}{7}\int \sin^{\frac{3}{2}}(bx)\,dx\\
&= -\frac{10\cos(bx)\sqrt{\sin(bx)}}{21b}-\frac{2\cos(bx)\sin^{\frac{5}{2}}(bx)}{7b}+\frac{5}{21}\int \frac{1}{\sqrt{\sin(bx)}}\,dx\\
&= -\frac{10F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{21b}-\frac{10\cos(bx)\sqrt{\sin(bx)}}{21b}-\frac{2\cos(bx)\sin^{\frac{5}{2}}(bx)}{7b}
\end{aligned}$$

Mathematica [A] time = 0.0686002, size = 45, normalized size = 0.75

$$\frac{\sqrt{\sin(bx)}(3\cos(3bx)-23\cos(bx))-20F\left(\frac{1}{4}(\pi-2bx)\middle|2\right)}{42b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[b*x]^(7/2),x]
```

```
[Out] (-20*EllipticF[(Pi - 2*b*x)/4, 2] + (-23*Cos[b*x] + 3*Cos[3*b*x])*Sqrt[Sin[
b*x]])/(42*b)
```

Maple [A] time = 0.093, size = 84, normalized size = 1.4

$$\frac{1}{b\cos(bx)}\left(\frac{2\sin(bx)(\cos(bx))^4}{7}+\frac{5}{21}\sqrt{\sin(bx)+1}\sqrt{-2\sin(bx)+2}\sqrt{-\sin(bx)}\text{EllipticF}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)-\frac{16(\text{co}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x)^(7/2),x)
```

```
[Out] (2/7*sin(b*x)*cos(b*x)^4+5/21*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-si
n(b*x))^(1/2)*EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2))-16/21*cos(b*x)^2*si
n(b*x))/cos(b*x)/sin(b*x)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos(bx)^2-1\right)\sin(bx)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x)^2 - 1)*sin(b*x)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x)^(7/2), x)
```

3.10 $\int \sin^{\frac{5}{2}}(bx)\,dx$

Optimal. Leaf size=41

$$-\frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\sin^{\frac{3}{2}}(bx)\cos(bx)}{5b}$$

```
[Out] (-6*EllipticE[Pi/4 - (b*x)/2, 2])/(5*b) - (2*Cos[b*x]*Sin[b*x]^(3/2))/(5*b)
```

Rubi [A] time = 0.015399, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 2639}

$$-\frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\sin^{\frac{3}{2}}(bx)\cos(bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[b*x]^(5/2),x]
```

```
[Out] (-6*EllipticE[Pi/4 - (b*x)/2, 2])/(5*b) - (2*Cos[b*x]*Sin[b*x]^(3/2))/(5*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^{\frac{5}{2}}(bx)\,dx &= -\frac{2\cos(bx)\sin^{\frac{3}{2}}(bx)}{5b}+\frac{3}{5}\int \sqrt{\sin(bx)}\,dx\\ &= -\frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\cos(bx)\sin^{\frac{3}{2}}(bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0384482, size = 35, normalized size = 0.85

$$-\frac{2\left(3E\left(\frac{1}{4}(\pi-2bx)\middle|2\right)+\sin^{\frac{3}{2}}(bx)\cos(bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[b*x]^(5/2),x]
```

```
[Out] (-2*(3*EllipticE[(Pi - 2*b*x)/4, 2] + Cos[b*x]*Sin[b*x]^(3/2)))/(5*b)
```

Maple [B] time = 0.039, size = 118, normalized size = 2.9

$$\frac{1}{b\cos(bx)}\left(\frac{2(\sin(bx))^4}{5}-\frac{2(\sin(bx))^2}{5}-\frac{6}{5}\sqrt{\sin(bx)+1}\sqrt{-2\sin(bx)+2}\sqrt{-\sin(bx)}\mathrm{EllipticE}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x)^(5/2),x)
```

```
[Out] (2/5*sin(b*x)^4-2/5*sin(b*x)^2-6/5*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)
*(-sin(b*x))^(1/2)*EllipticE((sin(b*x)+1)^(1/2),1/2*2^(1/2))+3/5*(sin(b*x)+
1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticF((sin(b*x)+1)^(1/
2),1/2*2^(1/2)))/cos(b*x)/sin(b*x)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos(bx)^2-1\right)\sqrt{\sin(bx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x)^2 - 1)*sqrt(sin(b*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin (bx)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x)^(5/2), x)
```

3.11 $\int \sin^{\frac{3}{2}}(bx)\,dx$

Optimal. Leaf size=41

$$-\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{3b}-\frac{2\sqrt{\sin(bx)}\cos(bx)}{3b}$$

```
[Out] (-2*EllipticF[Pi/4 - (b*x)/2, 2])/(3*b) - (2*Cos[b*x]*Sqrt[Sin[b*x]])/(3*b)
```

Rubi [A] time = 0.0154634, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 2641}

$$-\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{3b}-\frac{2\sqrt{\sin(bx)}\cos(bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[b*x]^(3/2),x]
```

```
[Out] (-2*EllipticF[Pi/4 - (b*x)/2, 2])/(3*b) - (2*Cos[b*x]*Sqrt[Sin[b*x]])/(3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^{\frac{3}{2}}(bx)\,dx &= -\frac{2\cos(bx)\sqrt{\sin(bx)}}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\sin(bx)}}\,dx\\ &= -\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{3b}-\frac{2\cos(bx)\sqrt{\sin(bx)}}{3b}\end{aligned}$$

Mathematica [A] time = 0.0361527, size = 33, normalized size = 0.8

$$-\frac{2\left(F\left(\frac{1}{4}(\pi-2bx)\middle|2\right)+\sqrt{\sin(bx)}\cos(bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[b*x]^(3/2),x]
```

```
[Out] (-2*(EllipticF[(Pi - 2*b*x)/4, 2] + Cos[b*x]*Sqrt[Sin[b*x]]))/(3*b)
```

Maple [A] time = 0.033, size = 72, normalized size = 1.8

$$\frac{1}{b\cos(bx)}\left(\frac{1}{3}\sqrt{\sin(bx)+1}\sqrt{-2\sin(bx)+2}\sqrt{-\sin(bx)}\mathrm{EllipticF}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)-\frac{2(\cos(bx))^2\sin(bx)}{3}\right)\frac{1}{\sqrt{\sin(bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x)^(3/2),x)
```

```
[Out] (1/3*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticF((
sin(b*x)+1)^(1/2),1/2*2^(1/2))-2/3*cos(b*x)^2*sin(b*x))/cos(b*x)/sin(b*x)^(
1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sin(bx)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sin(b*x)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^{\frac{3}{2}}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)**(3/2),x)
```

```
[Out] Integral(sin(b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x)^(3/2), x)
```

3.12 $\int \sqrt{\sin(bx)}\,dx$

Optimal. Leaf size=19

$$-\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

```
[Out] (-2*EllipticE[Pi/4 - (b*x)/2, 2])/b
```

Rubi [A] time = 0.0076162, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2639}

$$-\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[b*x]],x]

[Out] (-2*EllipticE[Pi/4 - (b*x)/2, 2])/b
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sin(bx)}\,dx = -\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

Mathematica [A] time = 0.0247763, size = 21, normalized size = 1.11

$$-\frac{2E\left(\frac{1}{2}\left(\frac{\pi}{2}-bx\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sin[b*x]],x]

[Out] (-2*EllipticE[(Pi/2 - b*x)/2, 2])/b
```

Maple [A] time = 0.032, size = 77, normalized size = 4.1

$$-\frac{1}{b\cos(bx)}\sqrt{\sin(bx)+1}\sqrt{-2\sin(bx)+2}\sqrt{-\sin(bx)}\left(2\,\mathrm{EllipticE}\left(\sqrt{\sin(bx)+1},1/2\sqrt{2}\right)-\mathrm{EllipticF}\left(\sqrt{\sin(bx)+1},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x)^(1/2),x)
```

```
[Out] -(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*(2*EllipticE((s
in(b*x)+1)^(1/2),1/2*2^(1/2))-EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2)))/co
s(b*x)/sin(b*x)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(b*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{\sin(bx)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sin(b*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)**(1/2),x)
```

```
[Out] Integral(sqrt(sin(b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(b*x)), x)
```

3.13 $\int \frac{1}{\sqrt{\sin(bx)}}\,dx$

Optimal. Leaf size=19

$$-\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

```
[Out] (-2*EllipticF[Pi/4 - (b*x)/2, 2])/b
```

Rubi [A] time = 0.008046, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2641}

$$-\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sin[b*x]],x]
```

```
[Out] (-2*EllipticF[Pi/4 - (b*x)/2, 2])/b
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = -\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

Mathematica [A] time = 0.024718, size = 21, normalized size = 1.11

$$-\frac{2F\left(\frac{1}{2}\left(\frac{\pi}{2}-bx\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sin[b*x]],x]
```

```
[Out] (-2*EllipticF[(Pi/2 - b*x)/2, 2])/b
```

Maple [A] time = 0.034, size = 57, normalized size = 3.

$$\frac{1}{b\cos(bx)}\sqrt{\sin(bx)+1}\sqrt{-2\sin(bx)+2}\sqrt{-\sin(bx)}\text{EllipticF}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)\frac{1}{\sqrt{\sin(bx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x)^(1/2),x)
```

```
[Out] (sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticF((sin(b
*x)+1)^(1/2),1/2*2^(1/2))/cos(b*x)/sin(b*x)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(sin(b*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{\sin(bx)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(sin(b*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sin(b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sin(b*x)), x)
```

3.14 $\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx$

Optimal. Leaf size=37

$$\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}-\frac{2\cos(bx)}{b\sqrt{\sin(bx)}}$$

```
[Out] (2*EllipticE[Pi/4 - (b*x)/2, 2])/b - (2*Cos[b*x])/(b*Sqrt[Sin[b*x]])
```

Rubi [A] time = 0.0131666, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2636, 2639}

$$\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}-\frac{2\cos(bx)}{b\sqrt{\sin(bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[b*x]^(-3/2),x]
```

```
[Out] (2*EllipticE[Pi/4 - (b*x)/2, 2])/b - (2*Cos[b*x])/(b*Sqrt[Sin[b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx &= -\frac{2\cos(bx)}{b\sqrt{\sin(bx)}}-\int \sqrt{\sin(bx)}\,dx\\ &= \frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}-\frac{2\cos(bx)}{b\sqrt{\sin(bx)}}\end{aligned}$$

Mathematica [A] time = 0.0533583, size = 32, normalized size = 0.86

$$\frac{2\left(E\left(\frac{1}{4}(\pi-2bx)\middle|2\right)-\frac{\cos(bx)}{\sqrt{\sin(bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[b*x]^(-3/2),x]
```

```
[Out] (2*(EllipticE[(Pi - 2*b*x)/4, 2] - Cos[b*x]/Sqrt[Sin[b*x]]))/b
```

Maple [A] time = 0.039, size = 110, normalized size = 3.

$$\frac{1}{b\cos(bx)}\left(2\sqrt{\sin(bx)+1}\sqrt{-2\sin(bx)+2}\sqrt{-\sin(bx)}\mathrm{EllipticE}\left(\sqrt{\sin(bx)+1},1/2\sqrt{2}\right)-\sqrt{\sin(bx)+1}\sqrt{-2\sin(bx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x)^(3/2),x)
```

```
[Out] (2*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticE((si
n(b*x)+1)^(1/2),1/2*2^(1/2))-(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin
(b*x))^(1/2)*EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2))-2*cos(b*x)^2)/cos(b*
x)/sin(b*x)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{\sin(bx)}}{\cos(bx)^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(sin(b*x))/(cos(b*x)^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)**(3/2),x)
```

```
[Out] Integral(sin(b*x)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x)^(-3/2), x)
```

3.15 $\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx$

Optimal. Leaf size=41

$$-\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{3b}-\frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)}$$

```
[Out] (-2*EllipticF[Pi/4 - (b*x)/2, 2])/(3*b) - (2*Cos[b*x])/(3*b*Sin[b*x]^(3/2))
```

Rubi [A] time = 0.0149834, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2636, 2641}

$$-\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{3b}-\frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[b*x]^(-5/2),x]
```

```
[Out] (-2*EllipticF[Pi/4 - (b*x)/2, 2])/(3*b) - (2*Cos[b*x])/(3*b*Sin[b*x]^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx &= -\frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)}+\frac{1}{3}\int \frac{1}{\sqrt{\sin(bx)}}\,dx\\ &= -\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{3b}-\frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)}\end{aligned}$$

Mathematica [A] time = 0.0493103, size = 33, normalized size = 0.8

$$-\frac{2\left(F\left(\frac{1}{4}(\pi-2bx)\middle|2\right)+\frac{\cos(bx)}{\sin^{\frac{3}{2}}(bx)}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[b*x]^(-5/2),x]
```

```
[Out] (-2*(EllipticF[(Pi - 2*b*x)/4, 2] + Cos[b*x]/Sin[b*x]^(3/2)))/(3*b)
```

Maple [A] time = 0.036, size = 72, normalized size = 1.8

$$\frac{1}{3\,b\cos\left(bx\right)}\left(\sqrt{\sin\left(bx\right)+1}\sqrt{-2\,\sin\left(bx\right)+2}\sqrt{-\sin\left(bx\right)}\mathrm{EllipticF}\left(\sqrt{\sin\left(bx\right)+1},\frac{\sqrt{2}}{2}\right)\sin\left(bx\right)-2\,\left(\cos\left(bx\right)\right)^{2}\right)\left(\sin\left(bx\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x)^(5/2),x)
```

```
[Out] 1/3/sin(b*x)^(3/2)*((sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1
/2)*EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2))*sin(b*x)-2*cos(b*x)^2)/cos(b*
x)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin\left(bx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{\left(\cos\left(bx\right)^{2}-1\right)\sqrt{\sin\left(bx\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-1/((cos(b*x)^2 - 1)*sqrt(sin(b*x))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin^{\frac{5}{2}}\left(bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)**(5/2),x)
```

```
[Out] Integral(sin(b*x)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x)^(-5/2), x)
```

3.16 $\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx$

Optimal. Leaf size=60

$$\frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)}-\frac{6\cos(bx)}{5b\sqrt{\sin(bx)}}$$

```
[Out] (6*EllipticE[Pi/4 - (b*x)/2, 2])/(5*b) - (2*Cos[b*x])/(5*b*Sin[b*x]^(5/2))
- (6*Cos[b*x])/(5*b*Sqrt[Sin[b*x]])
```

Rubi [A] time = 0.0243357, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2636, 2639}

$$\frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)}-\frac{6\cos(bx)}{5b\sqrt{\sin(bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[b*x]^(-7/2),x]
```

```
[Out] (6*EllipticE[Pi/4 - (b*x)/2, 2])/(5*b) - (2*Cos[b*x])/(5*b*Sin[b*x]^(5/2))
- (6*Cos[b*x])/(5*b*Sqrt[Sin[b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx &= -\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)}+\frac{3}{5}\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx\\ &= -\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)}-\frac{6\cos(bx)}{5b\sqrt{\sin(bx)}}-\frac{3}{5}\int \sqrt{\sin(bx)}\,dx\\ &= \frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)}-\frac{6\cos(bx)}{5b\sqrt{\sin(bx)}}\end{aligned}$$

Mathematica [A] time = 0.049694, size = 51, normalized size = 0.85

$$\frac{-7\cos(bx)+3\cos(3bx)+12\sin^{\frac{5}{2}}(bx)E\left(\frac{1}{4}(\pi-2bx)\middle|2\right)}{10b\sin^{\frac{5}{2}}(bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[b*x]^(-7/2),x]
```

```
[Out] (-7*Cos[b*x] + 3*Cos[3*b*x] + 12*EllipticE[(Pi - 2*b*x)/4, 2]*Sin[b*x]^(5/2
))/(10*b*Sin[b*x]^(5/2))
```

Maple [A] time = 0.037, size = 132, normalized size = 2.2

$$\frac{1}{5\,b\cos(bx)}\left(6\sqrt{\sin(bx)+1}\sqrt{-2\sin(bx)+2}\sqrt{-\sin(bx)}\,(\sin(bx))^2\,\mathrm{EllipticE}\left(\sqrt{\sin(bx)+1},1/2\sqrt{2}\right)-3\sqrt{\sin(bx)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x)^(7/2),x)
```

```
[Out] 1/5/sin(b*x)^(5/2)*(6*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^
(1/2)*sin(b*x)^2*EllipticE((sin(b*x)+1)^(1/2),1/2*2^(1/2))-3*(sin(b*x)+1)^(
1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*sin(b*x)^2*EllipticF((sin(b*x)
+1)^(1/2),1/2*2^(1/2))+6*sin(b*x)^4-4*sin(b*x)^2-2)/cos(b*x)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(bx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x)^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sin(bx)}}{\cos(bx)^4-2\cos(bx)^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sin(b*x))/(cos(b*x)^4 - 2*cos(b*x)^2 + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(bx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x)^(-7/2), x)
```

3.17 $\int \sin^{\frac{7}{2}}(a+bx)\,dx$

Optimal. Leaf size=70

$$\frac{10F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{21b}-\frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}-\frac{10\sqrt{\sin(a+bx)}\cos(a+bx)}{21b}$$

```
[Out] (10*EllipticF[(a - Pi/2 + b*x)/2, 2])/(21*b) - (10*Cos[a + b*x]*Sqrt[Sin[a
+ b*x]])/(21*b) - (2*Cos[a + b*x]*Sin[a + b*x]^(5/2))/(7*b)
```

Rubi [A] time = 0.0287382, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2635, 2641}

$$\frac{10F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{21b}-\frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}-\frac{10\sqrt{\sin(a+bx)}\cos(a+bx)}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(7/2),x]
```

```
[Out] (10*EllipticF[(a - Pi/2 + b*x)/2, 2])/(21*b) - (10*Cos[a + b*x]*Sqrt[Sin[a
+ b*x]])/(21*b) - (2*Cos[a + b*x]*Sin[a + b*x]^(5/2))/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^{\frac{7}{2}}(a+bx)\,dx &= -\frac{2\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{7}\int \sin^{\frac{3}{2}}(a+bx)\,dx\\ &= -\frac{10\cos(a+bx)\sqrt{\sin(a+bx)}}{21b}-\frac{2\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{21}\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx\\ &= \frac{10F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{21b}-\frac{10\cos(a+bx)\sqrt{\sin(a+bx)}}{21b}-\frac{2\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.118821, size = 55, normalized size = 0.79

$$\frac{\sqrt{\sin(a+bx)}(3\cos(3(a+bx))-23\cos(a+bx))-20F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)}{42b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(7/2),x]
```

```
[Out] (-20*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + (-23*Cos[a + b*x] + 3*Cos[3*(a +
 b*x)])*Sqrt[Sin[a + b*x]])/(42*b)
```

Maple [A] time = 0.029, size = 104, normalized size = 1.5

$$\frac{1}{b\cos(bx+a)}\left(\frac{2\sin(bx+a)(\cos(bx+a))^4}{7}+\frac{5}{21}\sqrt{\sin(bx+a)+1}\sqrt{-2\sin(bx+a)+2}\sqrt{-\sin(bx+a)}\text{EllipticF}\left(\sqrt{\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(7/2),x)
```

```
[Out] (2/7*sin(b*x+a)*cos(b*x+a)^4+5/21*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1
/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-16/21*c
os(b*x+a)^2*sin(b*x+a))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos(bx+a)^2-1\right)\sin(bx+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(b*x + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx+a)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(7/2), x)
```

3.18 $\int \sin^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=47

$$\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{5b}-\frac{2\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{5b}$$

```
[Out] (6*EllipticE[(a - Pi/2 + b*x)/2, 2])/(5*b) - (2*Cos[a + b*x]*Sin[a + b*x]^(
3/2))/(5*b)
```

Rubi [A] time = 0.0169634, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2635, 2639}

$$\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{5b}-\frac{2\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(5/2),x]
```

```
[Out] (6*EllipticE[(a - Pi/2 + b*x)/2, 2])/(5*b) - (2*Cos[a + b*x]*Sin[a + b*x]^(
3/2))/(5*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^{\frac{5}{2}}(a+bx)\,dx &= -\frac{2\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{5b}+\frac{3}{5}\int\sqrt{\sin(a+bx)}\,dx\\ &= \frac{6E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{5b}-\frac{2\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0867625, size = 44, normalized size = 0.94

$$-\frac{\sqrt{\sin(a+bx)}\sin(2(a+bx))+6E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(5/2),x]
```

```
[Out] -(6*EllipticE[(-2*a + Pi - 2*b*x)/4, 2] + Sqrt[Sin[a + b*x]]*Sin[2*(a + b*x
)])/(5*b)
```

Maple [A] time = 0.032, size = 142, normalized size = 3.

$$\frac{1}{b\cos(bx+a)}\left(\frac{2(\sin(bx+a))^4}{5}-\frac{2(\sin(bx+a))^2}{5}-\frac{6}{5}\sqrt{\sin(bx+a)+1}\sqrt{-2\sin(bx+a)+2}\sqrt{-\sin(bx+a)}\mathrm{Ellipti}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(5/2),x)
```

```
[Out] (2/5*sin(b*x+a)^4-2/5*sin(b*x+a)^2-6/5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+
2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+3/
5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Elliptic
F((sin(b*x+a)+1)^(1/2),1/2*2^(1/2)))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos(bx+a)^2-1\right)\sqrt{\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(sin(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(5/2), x)
```

3.19 $\int \sin^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=47

$$\frac{2F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b}-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}$$

```
[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2])/(3*b) - (2*Cos[a + b*x]*Sqrt[Sin[a + b
*x]])/(3*b)
```

Rubi [A] time = 0.0164747, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2635, 2641}

$$\frac{2F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b}-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(3/2),x]
```

```
[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2])/(3*b) - (2*Cos[a + b*x]*Sqrt[Sin[a + b
*x]])/(3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^{\frac{3}{2}}(a+bx)\,dx &= -\frac{2\cos(a+bx)\sqrt{\sin(a+bx)}}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx\\ &= \frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{3b}-\frac{2\cos(a+bx)\sqrt{\sin(a+bx)}}{3b}\end{aligned}$$

Mathematica [A] time = 0.0378158, size = 40, normalized size = 0.85

$$-\frac{2\left(F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+\sqrt{\sin(a+bx)}\cos(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(3/2),x]
```

```
[Out] (-2*(EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + Cos[a + b*x]*Sqrt[Sin[a + b*x]])
)/(3*b)
```

Maple [A] time = 0.026, size = 88, normalized size = 1.9

$$\frac{1}{b\cos\left(bx+a\right)}\left(\frac{1}{3}\sqrt{\sin\left(bx+a\right)+1}\sqrt{-2\sin\left(bx+a\right)+2}\sqrt{-\sin\left(bx+a\right)}\mathrm{EllipticF}\left(\sqrt{\sin\left(bx+a\right)+1},\frac{\sqrt{2}}{2}\right)-\frac{2\left(\cos\left(bx+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(3/2),x)
```

```
[Out] (1/3*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Ellip
ticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2/3*cos(b*x+a)^2*sin(b*x+a))/cos(b*x
+a)/sin(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(bx+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sin\left(bx+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sin(b*x + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^{\frac{3}{2}}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(3/2), x)
```

3.20 $\int \sqrt{\sin(a+bx)}\,dx$

Optimal. Leaf size=21

$$\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

```
[Out] (2*EllipticE[(a - Pi/2 + b*x)/2, 2])/b
```

Rubi [A] time = 0.0077866, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2639}

$$\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a - Pi/2 + b*x)/2, 2])/b
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sin(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b}$$

Mathematica [A] time = 0.0168481, size = 24, normalized size = 1.14

$$-\frac{2E\left(\frac{1}{2}\left(-a-bx+\frac{\pi}{2}\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sin[a + b*x]],x]
```

```
[Out] (-2*EllipticE[(-a + Pi/2 - b*x)/2, 2])/b
```

Maple [A] time = 0.026, size = 91, normalized size = 4.3

$$-\frac{1}{b\cos(bx+a)}\sqrt{\sin(bx+a)+1}\sqrt{-2\sin(bx+a)+2}\sqrt{-\sin(bx+a)}\left(2\,\mathrm{EllipticE}\left(\sqrt{\sin(bx+a)+1},1/2\sqrt{2}\right)-\mathrm{Ellipti}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(1/2),x)
```

```
[Out] -(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*(2*Ellipt
icE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^
(1/2)))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\sin(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sin(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(sin(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(b*x + a)), x)
```

3.21 $\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx$

Optimal. Leaf size=21

$$\frac{2F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

```
[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2])/b
```

Rubi [A] time = 0.0077145, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2641}

$$\frac{2F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sin[a + b*x]],x]

[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2])/b
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx=\frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b}$$

Mathematica [A] time = 0.0185693, size = 24, normalized size = 1.14

$$-\frac{2F\left(\frac{1}{2}\left(-a-bx+\frac{\pi}{2}\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sin[a + b*x]],x]

[Out] (-2*EllipticF[(-a + Pi/2 - b*x)/2, 2])/b
```

Maple [A] time = 0.023, size = 69, normalized size = 3.3

$$\frac{1}{b\cos(bx+a)}\sqrt{\sin(bx+a)+1}\sqrt{-2\sin(bx+a)+2}\sqrt{-\sin(bx+a)}\,\mathrm{EllipticF}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)\frac{1}{\sqrt{\sin(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x+a)^(1/2),x)
```

```
[Out] (sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF(
(sin(b*x+a)+1)^(1/2),1/2*2^(1/2))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(sin(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{\sin(bx+a)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(sin(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sin(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sin(b*x + a)), x)
```

3.22 $\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=43

$$-\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}$$

```
[Out] (-2*EllipticE[(a - Pi/2 + b*x)/2, 2])/b - (2*Cos[a + b*x])/(b*Sqrt[Sin[a +
b*x]])
```

Rubi [A] time = 0.0146858, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2636, 2639}

$$-\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(-3/2),x]
```

```
[Out] (-2*EllipticE[(a - Pi/2 + b*x)/2, 2])/b - (2*Cos[a + b*x])/(b*Sqrt[Sin[a +
b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx &= -\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}-\int \sqrt{\sin(a+bx)}\,dx\\ &= -\frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b}-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0753019, size = 39, normalized size = 0.91

$$\frac{2\left(E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)-\frac{\cos(a+bx)}{\sqrt{\sin(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(-3/2),x]
```

```
[Out] (2*(EllipticE[(-2*a + Pi - 2*b*x)/4, 2] - Cos[a + b*x]/Sqrt[Sin[a + b*x]]))
/b
```

Maple [A] time = 0.027, size = 132, normalized size = 3.1

$$\frac{1}{b\cos(bx+a)}\left(2\sqrt{\sin(bx+a)+1}\sqrt{-2\sin(bx+a)+2}\sqrt{-\sin(bx+a)}\text{EllipticE}\left(\sqrt{\sin(bx+a)+1},1/2\sqrt{2}\right)-\sqrt{\sin(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x+a)^(3/2),x)
```

```
[Out] (2*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Ellipti
cE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)
^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*co
s(b*x+a)^2)/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{\sin(bx+a)}}{\cos(bx+a)^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(sin(b*x + a))/(cos(b*x + a)^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(-3/2), x)
```

3.23 $\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=47

$$\frac{2F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b}-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

```
[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2])/(3*b) - (2*Cos[a + b*x])/(3*b*Sin[a +
b*x]^(3/2))
```

Rubi [A] time = 0.0163912, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2636, 2641}

$$\frac{2F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b}-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(-5/2),x]
```

```
[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2])/(3*b) - (2*Cos[a + b*x])/(3*b*Sin[a +
b*x]^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx&=-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}+\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx\\&=\frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{3b}-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.103562, size = 43, normalized size = 0.91

$$\frac{2\left(F\left(\frac{1}{4}(2a+2bx-\pi)\middle|2\right)-\frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(-5/2),x]
```

```
[Out] (2*(EllipticF[(2*a - Pi + 2*b*x)/4, 2] - Cos[a + b*x]/Sin[a + b*x]^(3/2)))/
(3*b)
```

Maple [A] time = 0.026, size = 88, normalized size = 1.9

$$\frac{1}{3\,b\cos\left(bx+a\right)}\left(\sqrt{\sin\left(bx+a\right)+1}\sqrt{-2\,\sin\left(bx+a\right)+2}\sqrt{-\sin\left(bx+a\right)}\mathrm{EllipticF}\left(\sqrt{\sin\left(bx+a\right)+1},\frac{\sqrt{2}}{2}\right)\sin\left(bx+a\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x+a)^(5/2),x)
```

```
[Out] 1/3/sin(b*x+a)^(3/2)*((sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*
x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))*sin(b*x+a)-2*cos(b*
x+a)^2)/cos(b*x+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{\left(\cos\left(bx+a\right)^{2}-1\right)\sqrt{\sin\left(bx+a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-1/((cos(b*x + a)^2 - 1)*sqrt(sin(b*x + a))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin^{\frac{5}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)**(5/2),x)
```

```
[Out] Integral(sin(a + b*x)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(-5/2), x)
```

3.24 $\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=70

$$-\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{5b}-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}-\frac{6\cos(a+bx)}{5b\sqrt{\sin(a+bx)}}$$

```
[Out] (-6*EllipticE[(a - Pi/2 + b*x)/2, 2])/(5*b) - (2*Cos[a + b*x])/(5*b*Sin[a +
 b*x]^(5/2)) - (6*Cos[a + b*x])/(5*b*Sqrt[Sin[a + b*x]])
```

Rubi [A] time = 0.0268507, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2636, 2639}

$$-\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{5b}-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}-\frac{6\cos(a+bx)}{5b\sqrt{\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(-7/2),x]
```

```
[Out] (-6*EllipticE[(a - Pi/2 + b*x)/2, 2])/(5*b) - (2*Cos[a + b*x])/(5*b*Sin[a +
 b*x]^(5/2)) - (6*Cos[a + b*x])/(5*b*Sqrt[Sin[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx &= -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}+\frac{3}{5}\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx\\ &= -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}-\frac{6\cos(a+bx)}{5b\sqrt{\sin(a+bx)}}-\frac{3}{5}\int \sqrt{\sin(a+bx)}\,dx\\ &= -\frac{6E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{5b}-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}-\frac{6\cos(a+bx)}{5b\sqrt{\sin(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.284052, size = 55, normalized size = 0.79

$$\frac{2\left(3E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)-\frac{\left(3\sin^2(a+bx)+1\right)\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(-7/2),x]
```

```
[Out] (2*(3*EllipticE[(-2*a + Pi - 2*b*x)/4, 2] - (Cos[a + b*x]*(1 + 3*Sin[a + b*
x]^2))/Sin[a + b*x]^(5/2)))/(5*b)
```

Maple [A] time = 0.031, size = 160, normalized size = 2.3

$$\frac{1}{5\,b\cos\left(bx+a\right)}\left(6\,\sqrt{\sin\left(bx+a\right)+1}\sqrt{-2\,\sin\left(bx+a\right)+2}\sqrt{-\sin\left(bx+a\right)}\left(\sin\left(bx+a\right)\right)^{2}\mathrm{EllipticE}\left(\sqrt{\sin\left(bx+a\right)+1},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x+a)^(7/2),x)
```

```
[Out] 1/5/sin(b*x+a)^(5/2)*(6*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(
b*x+a))^(1/2)*sin(b*x+a)^2*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-3*(s
in(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*sin(b*x+a)^2
*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+6*sin(b*x+a)^4-4*sin(b*x+a)^2-
2)/cos(b*x+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sin\left(bx+a\right)}}{\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sin(b*x + a))/(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(-7/2), x)
```

3.25 $\int (c\sin(a+bx))^{7/2}\,dx$

Optimal. Leaf size=103

$$-\frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b}+\frac{10c^4\sqrt{\sin(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{21b\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}$$

```
[Out] (10*c^4*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(21*b*Sqrt[c*S
in[a + b*x]]) - (10*c^3*Cos[a + b*x]*Sqrt[c*Sin[a + b*x]])/(21*b) - (2*c*Co
s[a + b*x]*(c*Sin[a + b*x])^(5/2))/(7*b)
```

Rubi [A] time = 0.0523121, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 2642, 2641}

$$-\frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b}+\frac{10c^4\sqrt{\sin(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{21b\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(7/2),x]
```

```
[Out] (10*c^4*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(21*b*Sqrt[c*S
in[a + b*x]]) - (10*c^3*Cos[a + b*x]*Sqrt[c*Sin[a + b*x]])/(21*b) - (2*c*Co
s[a + b*x]*(c*Sin[a + b*x])^(5/2))/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c\sin(a+bx))^{7/2}\,dx = -\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b} + \frac{1}{7}\left(5c^2\right)\int (c\sin(a+bx))^{3/2}\,dx$$
$$= -\frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b} - \frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b} + \frac{1}{21}\left(5c^4\right)\int \frac{1}{\sqrt{c\sin(a+bx}}$$
$$= -\frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b} - \frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b} + \frac{\left(5c^4\sqrt{\sin(a+bx)}\right)\int \frac{}{\sqrt{\sin(}}}{21\sqrt{c\sin(a+bx)}}$$
$$= \frac{10c^4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{21b\sqrt{c\sin(a+bx)}} - \frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b} - \frac{2c\cos(a+bx)(c\sin}{7b}$$

Mathematica [A] time = 0.153076, size = 80, normalized size = 0.78

$$\frac{c^3\sqrt{c\sin(a+bx)}\left(\sqrt{\sin(a+bx)}(3\cos(3(a+bx))-23\cos(a+bx))-20F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{42b\sqrt{\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(7/2),x]
```

```
[Out] (c^3*(-20*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + (-23*Cos[a + b*x] + 3*Cos[3
*(a + b*x)])*Sqrt[Sin[a + b*x]])*Sqrt[c*Sin[a + b*x]])/(42*b*Sqrt[Sin[a + b
*x]])
```

Maple [A] time = 0.042, size = 108, normalized size = 1.1

$$-\frac{c^4}{21\,b\cos(bx+a)}\left(-6\,(\sin(bx+a))^5+5\,\sqrt{-\sin(bx+a)+1}\sqrt{2\,\sin(bx+a)+2}\sqrt{\sin(bx+a)}\mathrm{EllipticF}\left(\sqrt{-\sin(bx+a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(7/2),x)
```

```
[Out] -1/21*c^4*(-6*sin(b*x+a)^5+5*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*s
in(b*x+a)^(1/2)*EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-4*sin(b*x+a)^3
+10*sin(b*x+a))/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(c^3\cos\left(bx+a\right)^2-c^3\right)\sqrt{c\sin\left(bx+a\right)}\sin\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^3*cos(b*x + a)^2 - c^3)*sqrt(c*sin(b*x + a))*sin(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(bx+a\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(7/2), x)
```

3.26 $\int (c\sin(a+bx))^{5/2}\,dx$

Optimal. Leaf size=75

$$\frac{6c^2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5b\sqrt{\sin(a+bx)}}-\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}$$

```
[Out] (6*c^2*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[c*Sin[a + b*x]])/(5*b*Sqrt[Sin
[a + b*x]]) - (2*c*Cos[a + b*x]*(c*Sin[a + b*x])^(3/2))/(5*b)
```

Rubi [A] time = 0.0316081, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 2640, 2639}

$$\frac{6c^2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5b\sqrt{\sin(a+bx)}}-\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2),x]
```


```
[Out] (6*c^2*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[c*Sin[a + b*x]])/(5*b*Sqrt[Sin
[a + b*x]]) - (2*c*Cos[a + b*x]*(c*Sin[a + b*x])^(3/2))/(5*b)
```


Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c\sin(a+bx))^{5/2}\,dx &= -\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}+\frac{1}{5}\left(3c^2\right)\int\sqrt{c\sin(a+bx)}\,dx\\
&= -\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}+\frac{\left(3c^2\sqrt{c\sin(a+bx)}\right)\int\sqrt{\sin(a+bx)}\,dx}{5\sqrt{\sin(a+bx)}}\\
&= \frac{6c^2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5b\sqrt{\sin(a+bx)}}-\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}
\end{aligned}$$

Mathematica [A] time = 0.104112, size = 66, normalized size = 0.88

$$-\frac{(c\sin(a+bx))^{5/2}\left(\sqrt{\sin(a+bx)}\sin(2(a+bx))+6E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{5b\sin^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] -((c*Sin[a + b*x])^(5/2)*(6*EllipticE[(-2*a + Pi - 2*b*x)/4, 2] + Sqrt[Sin[
a + b*x]]*Sin[2*(a + b*x)]))/(5*b*Sin[a + b*x]^(5/2))
```

Maple [A] time = 0.036, size = 152, normalized size = 2.

$$-\frac{c^3}{5\,b\cos(bx+a)}\left(6\sqrt{-\sin(bx+a)+1}\sqrt{2\sin(bx+a)+2}\sqrt{\sin(bx+a)}\mathrm{EllipticE}\left(\sqrt{-\sin(bx+a)+1},1/2\sqrt{2}\right)-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2),x)
```

```
[Out] -1/5*c^3*(6*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*E
llipticE((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-3*(-sin(b*x+a)+1)^(1/2)*(2*sin(
b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2
))-2*sin(b*x+a)^4+2*sin(b*x+a)^2)/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(c^2\cos(bx+a)^2-c^2\right)\sqrt{c\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(c*sin(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2), x)
```

3.27 $\int (c\sin(a+bx))^{3/2}\,dx$

Optimal. Leaf size=75

$$\frac{2c^2\sqrt{\sin(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}$$

```
[Out] (2*c^2*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b*Sqrt[c*Sin
[a + b*x]]) - (2*c*Cos[a + b*x]*Sqrt[c*Sin[a + b*x]])/(3*b)
```

Rubi [A] time = 0.0326279, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 2642, 2641}

$$\frac{2c^2\sqrt{\sin(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (2*c^2*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b*Sqrt[c*Sin
[a + b*x]]) - (2*c*Cos[a + b*x]*Sqrt[c*Sin[a + b*x]])/(3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c\sin(a+bx))^{3/2}\,dx &= -\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}+\frac{1}{3}c^2\int\frac{1}{\sqrt{c\sin(a+bx)}}\,dx\\
&= -\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}+\frac{\left(c^2\sqrt{\sin(a+bx)}\right)\int\frac{1}{\sqrt{\sin(a+bx)}}\,dx}{3\sqrt{c\sin(a+bx)}}\\
&= \frac{2c^2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{3b\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}
\end{aligned}$$

Mathematica [A] time = 0.0516922, size = 62, normalized size = 0.83

$$-\frac{2(c\sin(a+bx))^{3/2}\left(F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+\sqrt{\sin(a+bx)}\cos(a+bx)\right)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (-2*(EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + Cos[a + b*x]*Sqrt[Sin[a + b*x]])
*(c*Sin[a + b*x])^(3/2))/(3*b*Sin[a + b*x]^(3/2))
```

Maple [A] time = 0.034, size = 97, normalized size = 1.3

$$-\frac{c^2}{3\,b\cos(bx+a)}\left(\sqrt{-\sin(bx+a)+1}\sqrt{2\sin(bx+a)+2}\sqrt{\sin(bx+a)}\mathrm{EllipticF}\left(\sqrt{-\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-2\,(\sin(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2),x)
```

```
[Out] -1/3*c^2*((-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*Ell
ipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*sin(b*x+a)^3+2*sin(b*x+a))/cos(
b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(c\sin(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c\sin(bx+a)}c\sin(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))*c*sin(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(c\sin(a+bx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2),x)
```

```
[Out] Integral((c*sin(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2), x)
```

3.28 $\int \sqrt{c \sin(a+bx)}\,dx$

Optimal. Leaf size=43

$$\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

```
[Out] (2*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[c*Sin[a + b*x]])/(b*Sqrt[Sin[a + b
*x]])
```

Rubi [A] time = 0.0183664, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2640, 2639}

$$\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[c*Sin[a + b*x]])/(b*Sqrt[Sin[a + b
*x]])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c\sin(a+bx)}\,dx &= \frac{\sqrt{c\sin(a+bx)}\int \sqrt{\sin(a+bx)}\,dx}{\sqrt{\sin(a+bx)}}\\ &= \frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0211787, size = 42, normalized size = 0.98

$$-\frac{2E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (-2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[c*Sin[a + b*x]])/(b*Sqrt[Sin[a
 + b*x]])
```

Maple [A] time = 0.036, size = 98, normalized size = 2.3

$$-{\frac{c}{b\cos \left(bx+a \right) }\sqrt{-\sin \left(bx+a \right) +1}\sqrt{2\,\sin \left(bx+a \right) +2}\sqrt{\sin \left(bx+a \right) } \left(2\,{\it EllipticE} \left(\sqrt{-\sin \left(bx+a \right) +1},1/2\,\sqrt{2} \right) -{\it Elli}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/2),x)
```

```
[Out] -c*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*(2*Ellipti
cE((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2
^(1/2)))/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\sin \left(bx+a \right) }\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\sqrt{c\sin \left(bx+a \right) }, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\sin \left(a+bx \right) }\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(c*sin(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a)), x)
```

3.29 $\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{\sin(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{c\sin(a+bx)}}$$

```
[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt[c*Sin[a + b
*x]])
```

Rubi [A] time = 0.0179524, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2642, 2641}

$$\frac{2\sqrt{\sin(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt[c*Sin[a + b
*x]])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx &= \frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{\sqrt{c\sin(a+bx)}}\\ &= \frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b\sqrt{c\sin(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0265183, size = 42, normalized size = 0.98

$$-\frac{2\sqrt{\sin(a+bx)}F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)}{b\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (-2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt[c*Sin[a
 + b*x]])
```

Maple [A] time = 0.032, size = 74, normalized size = 1.7

$$-\frac{1}{b\cos(bx+a)}\sqrt{-\sin(bx+a)+1}\sqrt{2\sin(bx+a)+2}\sqrt{\sin(bx+a)}\mathrm{EllipticF}\left(\sqrt{-\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)\frac{1}{\sqrt{c\sin(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*EllipticF((-
sin(b*x+a)+1)^(1/2),1/2*2^(1/2))/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*sin(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c\sin(bx+a)}}{c\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))/(c*sin(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*sin(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c \sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*sin(b*x + a)), x)
```

3.30 $\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{bc^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}$$

```
[Out] (-2*Cos[a + b*x])/(b*c*Sqrt[c*Sin[a + b*x]]) - (2*EllipticE[(a - Pi/2 + b*x
)/2, 2]*Sqrt[c*Sin[a + b*x]])/(b*c^2*Sqrt[Sin[a + b*x]])
```

Rubi [A] time = 0.0323439, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2636, 2640, 2639}

$$-\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{bc^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(-3/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(b*c*Sqrt[c*Sin[a + b*x]]) - (2*EllipticE[(a - Pi/2 + b*x
)/2, 2]*Sqrt[c*Sin[a + b*x]])/(b*c^2*Sqrt[Sin[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx &= -\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}-\frac{\int \sqrt{c\sin(a+bx)}\,dx}{c^2}\\ &= -\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}-\frac{\sqrt{c\sin(a+bx)}\int \sqrt{\sin(a+bx)}\,dx}{c^2\sqrt{\sin(a+bx)}}\\ &= -\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}-\frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{bc^2\sqrt{\sin(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0467522, size = 54, normalized size = 0.74

$$-\frac{2\left(\cos(a+bx)-\sqrt{\sin(a+bx)}E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{bc\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(-3/2),x]
```

```
[Out] (-2*(Cos[a + b*x] - EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]])
)/(b*c*Sqrt[c*Sin[a + b*x]])
```

Maple [A] time = 0.043, size = 141, normalized size = 1.9

$$\frac{1}{cb\cos(bx+a)}\left(2\sqrt{-\sin(bx+a)+1}\sqrt{2\sin(bx+a)+2}\sqrt{\sin(bx+a)}\mathrm{EllipticE}\left(\sqrt{-\sin(bx+a)+1},1/2\sqrt{2}\right)-\sqrt{-s\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a))^(3/2),x)
```

```
[Out] 1/c*(2*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*Ellipt
icE((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+
2)^(1/2)*sin(b*x+a)^(1/2)*EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*co
s(b*x+a)^2)/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{c\sin(bx+a)}}{c^2\cos(bx+a)^2-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*sin(b*x + a))/(c^2*cos(b*x + a)^2 - c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sin(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))**(3/2),x)
```

```
[Out] Integral((c*sin(a + b*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(-3/2), x)
```

3.31 $\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{\sin(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3bc^2\sqrt{c\sin(a+bx)}}-\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}$$

```
[Out] (-2*Cos[a + b*x])/(3*b*c*(c*Sin[a + b*x])^(3/2)) + (2*EllipticF[(a - Pi/2 +
 b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b*c^2*Sqrt[c*Sin[a + b*x]])
```

Rubi [A] time = 0.0325813, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2636, 2642, 2641}

$$\frac{2\sqrt{\sin(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3bc^2\sqrt{c\sin(a+bx)}}-\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(-5/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(3*b*c*(c*Sin[a + b*x])^(3/2)) + (2*EllipticF[(a - Pi/2 +
 b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b*c^2*Sqrt[c*Sin[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx &= -\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}+\frac{\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx}{3c^2}\\
&= -\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}+\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{3c^2\sqrt{c\sin(a+bx)}}\\
&= -\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}+\frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{3bc^2\sqrt{c\sin(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.0752905, size = 55, normalized size = 0.71

$$-\frac{2\left(\cos(a+bx)+\sin^{\frac{3}{2}}(a+bx)F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{3bc(c\sin(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(-5/2),x]
```

```
[Out] (-2*(Cos[a + b*x] + EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/2))
)/(3*b*c*(c*Sin[a + b*x])^(3/2))
```

Maple [A] time = 0.04, size = 105, normalized size = 1.4

$$-\frac{1}{3\,c^2\left(\sin\left(bx+a\right)\right)^2\cos\left(bx+a\right)b}\left(\sqrt{-\sin\left(bx+a\right)+1}\sqrt{2\,\sin\left(bx+a\right)+2}\left(\sin\left(bx+a\right)\right)^{\frac{5}{2}}\text{EllipticF}\left(\sqrt{-\sin\left(bx+a\right)+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a))^(5/2),x)
```

```
[Out] -1/3/c^2*((-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(5/2)*Ell
ipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*sin(b*x+a)^3+2*sin(b*x+a))/sin(
b*x+a)^2/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\sin\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{c\sin\left(bx+a\right)}}{\left(c^3\cos\left(bx+a\right)^2-c^3\right)\sin\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*sin(b*x + a))/((c^3*cos(b*x + a)^2 - c^3)*sin(b*x + a)), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sin(a+bx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))**(5/2),x)
```

```
[Out] Integral((c*sin(a + b*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sin(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(-5/2), x)
```

3.32 $\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}}-\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5bc^4\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}$$

```
[Out] (-2*Cos[a + b*x])/(5*b*c*(c*Sin[a + b*x])^(5/2)) - (6*Cos[a + b*x])/(5*b*c^
3*Sqrt[c*Sin[a + b*x]]) - (6*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[c*Sin[a
+ b*x]])/(5*b*c^4*Sqrt[Sin[a + b*x]])
```

Rubi [A] time = 0.0510645, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2636, 2640, 2639}

$$-\frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}}-\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5bc^4\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(-7/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(5*b*c*(c*Sin[a + b*x])^(5/2)) - (6*Cos[a + b*x])/(5*b*c^
3*Sqrt[c*Sin[a + b*x]]) - (6*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[c*Sin[a
+ b*x]])/(5*b*c^4*Sqrt[Sin[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = -\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}} + \frac{3\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx}{5c^2}$$
$$= -\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}} - \frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}} - \frac{3\int \sqrt{c\sin(a+bx)}\,dx}{5c^4}$$
$$= -\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}} - \frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}} - \frac{\left(3\sqrt{c\sin(a+bx)}\right)\int \sqrt{\sin(a+bx)}\,dx}{5c^4\sqrt{\sin(a+bx)}}$$
$$= -\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}} - \frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}} - \frac{6E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5bc^4\sqrt{\sin(a+bx)}}$$

Mathematica [A] time = 0.157936, size = 68, normalized size = 0.65

$$-\frac{2\left(\frac{3}{2}\sin(2(a+bx))+\cot(a+bx)-3\sin^{\frac{3}{2}}(a+bx)E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{5bc^2(c\sin(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(-7/2),x]
```

```
[Out] (-2*(Cot[a + b*x] - 3*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/2
) + (3*Sin[2*(a + b*x)])/2))/(5*b*c^2*(c*Sin[a + b*x])^(3/2))
```

Maple [A] time = 0.042, size = 168, normalized size = 1.6

$$\frac{1}{5\,c^3\left(\sin\left(bx+a\right)\right)^3\cos\left(bx+a\right)b}\left(6\sqrt{-\sin\left(bx+a\right)+1}\sqrt{2\,\sin\left(bx+a\right)+2}\left(\sin\left(bx+a\right)\right)^{7/2}\mathrm{EllipticE}\left(\sqrt{-\sin\left(bx+a\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a))^(7/2),x)
```

```
[Out] 1/5/c^3*(6*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(7/2)*El
lipticE((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-3*(-sin(b*x+a)+1)^(1/2)*(2*sin(b
*x+a)+2)^(1/2)*sin(b*x+a)^(7/2)*EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2)
)+6*sin(b*x+a)^5-4*sin(b*x+a)^3-2*sin(b*x+a))/sin(b*x+a)^3/cos(b*x+a)/(c*si
n(b*x+a))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\sin\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c\sin\left(bx+a\right)}}{c^{4}\cos\left(bx+a\right)^{4}-2\,c^{4}\cos\left(bx+a\right)^{2}+c^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))/(c^4*cos(b*x + a)^4 - 2*c^4*cos(b*x + a)^2 +
c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(c\sin\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(-7/2), x)
```

3.33 $\int (c\sin(a+bx))^{4/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(a+bx)(c\sin(a+bx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\sin^2(a+bx)\right)}{7bc\sqrt{\cos^2(a+bx)}}$$

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Sin[a + b*x]^2]*(c*Sin[a
+ b*x])^(7/3))/(7*b*c*Sqrt[Cos[a + b*x]^2])
```

Rubi [A] time = 0.0143208, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cos(a+bx)(c\sin(a+bx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\sin^2(a+bx)\right)}{7bc\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(4/3),x]
```

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Sin[a + b*x]^2]*(c*Sin[a
+ b*x])^(7/3))/(7*b*c*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (c\sin(a+bx))^{4/3}\,dx=\frac{3\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\sin^2(a+bx)\right)(c\sin(a+bx))^{7/3}}{7bc\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.0549374, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(a+bx)}\tan(a+bx)(c\sin(a+bx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\sin^2(a+bx)\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(4/3),x]
```

```
[Out] (3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 7/6, 13/6, Sin[a + b*x]^2]*(
c*Sin[a + b*x])^(4/3)*Tan[a + b*x])/(7*b)
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(4/3),x)
```

```
[Out] int((c*sin(b*x+a))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((c\sin(bx+a))^{\frac{1}{3}}c\sin(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^(1/3)*c*sin(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(4/3), x)
```

3.34 $\int (c\sin(a+bx))^{2/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(a+bx)(c\sin(a+bx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sin^2(a+bx)\right)}{5bc\sqrt{\cos^2(a+bx)}}$$

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[a + b*x]^2]*(c*Sin[a
+ b*x])^(5/3))/(5*b*c*Sqrt[Cos[a + b*x]^2])
```

Rubi [A] time = 0.0138778, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cos(a+bx)(c\sin(a+bx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sin^2(a+bx)\right)}{5bc\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(2/3),x]
```

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[a + b*x]^2]*(c*Sin[a
+ b*x])^(5/3))/(5*b*c*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (c\sin(a+bx))^{2/3}\,dx = \frac{3\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sin^2(a+bx)\right)(c\sin(a+bx))^{5/3}}{5bc\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.0349906, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(a+bx)}\tan(a+bx)(c\sin(a+bx))^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sin^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(2/3),x]
```

```
[Out] (3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[a + b*x]^2]*(
c*Sin[a + b*x])^(2/3)*Tan[a + b*x])/(5*b)
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int (c \sin (bx + a))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(2/3),x)
```

```
[Out] int((c*sin(b*x+a))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c \sin (bx + a))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((c \sin (bx + a))^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c \sin (a + bx))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(2/3),x)
```

```
[Out] Integral((c*sin(a + b*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c \sin (bx + a))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(2/3), x)
```

3.35 $\int \sqrt[3]{c\sin(a+bx)}\,dx$

Optimal. Leaf size=517

$$\frac{3\left(1-i\sqrt{3}\right)\sqrt{3-i\sqrt{3}}\sqrt[3]{c}\sec(a+bx)\sqrt{1-\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}}\sqrt{\frac{2(c\sin(a+bx))^{2/3}}{\left(3-i\sqrt{3}\right)c^{2/3}}+\frac{\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2(c\sin(a+bx))^{2/3}}{\left(3+i\sqrt{3}\right)c^{2/3}}+\frac{-\sqrt{3}+i}{-\sqrt{3}+3i}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{}}{}\right.\right.}{2\sqrt{2}b}$$

```
[Out] (-3*Sqrt[(3*(3 - I*Sqrt[3]))/2]*c^(1/3)*EllipticE[ArcSin[(Sqrt[2]*Sqrt[1 -
(c*Sin[a + b*x])^(2/3)/c^(2/3)])/Sqrt[3 + I*Sqrt[3]]], (3*I - Sqrt[3])/(3*I
 + Sqrt[3])]*Sec[a + b*x]*Sqrt[1 - (c*Sin[a + b*x])^(2/3)/c^(2/3)]*Sqrt[(I
+ Sqrt[3])/(3*I + Sqrt[3]) + (2*(c*Sin[a + b*x])^(2/3))/((3 - I*Sqrt[3])*c^
(2/3))]*Sqrt[(I - Sqrt[3])/(3*I - Sqrt[3]) + (2*(c*Sin[a + b*x])^(2/3))/((3
 + I*Sqrt[3])*c^(2/3))])/b + (3*(1 - I*Sqrt[3])*Sqrt[3 - I*Sqrt[3]]*c^(1/3)
*EllipticF[ArcSin[(Sqrt[2]*Sqrt[1 - (c*Sin[a + b*x])^(2/3)/c^(2/3)])/Sqrt[3
 - I*Sqrt[3]]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]*Sec[a + b*x]*Sqrt[1 - (c*S
in[a + b*x])^(2/3)/c^(2/3)]*Sqrt[(I + Sqrt[3])/(3*I + Sqrt[3]) + (2*(c*Sin[
a + b*x])^(2/3))/((3 - I*Sqrt[3])*c^(2/3))]*Sqrt[(I - Sqrt[3])/(3*I - Sqrt[
3]) + (2*(c*Sin[a + b*x])^(2/3))/((3 + I*Sqrt[3])*c^(2/3))])/(2*Sqrt[2]*b)
```

Rubi [C] time = 0.0135465, antiderivative size = 58, normalized size of antiderivative = 0.11, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cos(a+bx)(c\sin(a+bx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(a+bx)\right)}{4bc\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(1/3),x]
```

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[a + b*x]^2]*(c*Sin[a +
 b*x])^(4/3))/(4*b*c*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{c\sin(a+bx)}\,dx=\frac{3\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(a+bx)\right)(c\sin(a+bx))^{4/3}}{4bc\sqrt{\cos^2(a+bx)}}$$

Mathematica [C] time = 0.0333308, size = 55, normalized size = 0.11

$$\frac{3\sqrt{\cos^2(a+bx)}\tan(a+bx)\sqrt[3]{c\sin(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[a + b*x]^2]*(c
*Sin[a + b*x])^(1/3)*Tan[a + b*x])/(4*b)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \sqrt[3]{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/3),x)
```

```
[Out] int((c*sin(b*x+a))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((c\sin(bx+a))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{c\sin(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/3),x)
```

```
[Out] Integral((c*sin(a + b*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin (bx+a))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(1/3), x)
```

3.36 $\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=252

$$-\frac{3\sqrt{3-i\sqrt{3}}\sec(a+bx)\sqrt{1-\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}}\sqrt{\frac{2(c\sin(a+bx))^{2/3}}{(3-i\sqrt{3})c^{2/3}}+\frac{\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2(c\sin(a+bx))^{2/3}}{(3+i\sqrt{3})c^{2/3}}+\frac{-\sqrt{3}+i}{-\sqrt{3}+3i}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{1-\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}}}{\sqrt{3-i\sqrt{3}}}\right)|\right.}{\sqrt{2}b\sqrt[3]{c}}$$

```
[Out] (-3*Sqrt[3 - I*Sqrt[3]]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[1 - (c*Sin[a + b*x])
^(2/3)/c^(2/3)])/Sqrt[3 - I*Sqrt[3]]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]*Sec
[a + b*x]*Sqrt[1 - (c*Sin[a + b*x])^(2/3)/c^(2/3)]*Sqrt[(I + Sqrt[3])/(3*I
+ Sqrt[3]) + (2*(c*Sin[a + b*x])^(2/3))/((3 - I*Sqrt[3])*c^(2/3))]*Sqrt[(I
- Sqrt[3])/(3*I - Sqrt[3]) + (2*(c*Sin[a + b*x])^(2/3))/((3 + I*Sqrt[3])*c^
(2/3))])/(Sqrt[2]*b*c^(1/3))
```

Rubi [C] time = 0.0165006, antiderivative size = 58, normalized size of antiderivative = 0.23, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cos(a+bx)(c\sin(a+bx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sin^2(a+bx)\right)}{2bc\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(-1/3),x]
```

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[a + b*x]^2]*(c*Sin[a +
 b*x])^(2/3))/(2*b*c*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx=\frac{3\cos(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sin^2(a+bx)\right)(c\sin(a+bx))^{2/3}}{2bc\sqrt{\cos^2(a+bx)}}$$

Mathematica [C] time = 0.0452701, size = 55, normalized size = 0.22

$$\frac{3\sqrt{\cos^2(a+bx)}\tan(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sin^2(a+bx)\right)}{2b\sqrt[3]{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(-1/3),x]
```

```
[Out] (3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[a + b*x]^2]*Ta
n[a + b*x])/(2*b*(c*Sin[a + b*x])^(1/3))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{c \sin \left(b x+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a))^(1/3),x)
```

```
[Out] int(1/(c*sin(b*x+a))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c \sin \left(b x+a\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c \sin \left(b x+a\right)\right)^{\frac{2}{3}}}{c \sin \left(b x+a\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^(2/3)/(c*sin(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{c \sin \left(a+b x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))**(1/3),x)
```

```
[Out] Integral((c*sin(a + b*x))**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\sin\left(bx+a\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(-1/3), x)
```

3.37 $\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx$

Optimal. Leaf size=271

$$\frac{3^{3/4}\sec(a+bx)\sqrt[3]{c\sin(a+bx)}\left(c^{2/3}-(c\sin(a+bx))^{2/3}\right)\sqrt{\frac{c^{4/3}\left(\frac{(c\sin(a+bx))^{4/3}}{c^{4/3}}+\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}+1\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)(c\sin(a+bx))^{2/3}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)(c\sin(a+bx)}{c^{2/3}-\left(1+\sqrt{3}\right)(c\sin(a+bx)}\right.\right.}{2bc^{5/3}\sqrt{-\frac{(c\sin(a+bx))^{2/3}\left(c^{2/3}-(c\sin(a+bx))^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)(c\sin(a+bx))^{2/3}\right)^2}}}$$

```
[Out] (3^(3/4)*EllipticF[ArcCos[(c^(2/3) - (1 - Sqrt[3])*(c*Sin[a + b*x])^(2/3))/
(c^(2/3) - (1 + Sqrt[3])*(c*Sin[a + b*x])^(2/3))], (2 + Sqrt[3])/4]*Sec[a +
 b*x]*(c*Sin[a + b*x])^(1/3)*(c^(2/3) - (c*Sin[a + b*x])^(2/3))*Sqrt[(c^(4/
3)*(1 + (c*Sin[a + b*x])^(2/3)/c^(2/3) + (c*Sin[a + b*x])^(4/3)/c^(4/3)))/(
c^(2/3) - (1 + Sqrt[3])*(c*Sin[a + b*x])^(2/3))^2])/(2*b*c^(5/3)*Sqrt[-(((c
*Sin[a + b*x])^(2/3)*(c^(2/3) - (c*Sin[a + b*x])^(2/3)))/(c^(2/3) - (1 + Sq
rt[3])*(c*Sin[a + b*x])^(2/3))^2)])
```

Rubi [C] time = 0.0155328, antiderivative size = 56, normalized size of antiderivative = 0.21, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cos(a+bx)\sqrt[3]{c\sin(a+bx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(a+bx)\right)}{bc\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(-2/3),x]
```

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[a + b*x]^2]*(c*Sin[a +
 b*x])^(1/3))/(b*c*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx=\frac{3\cos(a+bx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(a+bx)\right)\sqrt[3]{c\sin(a+bx)}}{bc\sqrt{\cos^2(a+bx)}}$$

Mathematica [C] time = 0.0407623, size = 53, normalized size = 0.2

$$\frac{3\sqrt{\cos^2(a+bx)}\tan(a+bx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(a+bx)\right)}{b(c\sin(a+bx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(-2/3),x]
```

```
[Out] (3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[a + b*x]^2]*Ta
n[a + b*x])/(b*(c*Sin[a + b*x])^(2/3))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a))^(2/3),x)
```

```
[Out] int(1/(c*sin(b*x+a))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sin(bx+a))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(-2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(c\sin(bx+a))^{\frac{1}{3}}}{c\sin(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^(1/3)/(c*sin(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sin(a+bx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))**(2/3),x)
```

```
[Out] Integral((c*sin(a + b*x))**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sin(bx+a))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(-2/3), x)
```

3.38 $\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3\cos(a+bx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sin^2(a+bx)\right)}{bc\sqrt{\cos^2(a+bx)}\sqrt[3]{c\sin(a+bx)}}$$

```
[Out] (-3*Cos[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sin[a + b*x]^2])/(b*c*Sq
rt[Cos[a + b*x]^2]*(c*Sin[a + b*x])^(1/3))
```

Rubi [A] time = 0.0167059, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\cos(a+bx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sin^2(a+bx)\right)}{bc\sqrt{\cos^2(a+bx)}\sqrt[3]{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(-4/3),x]
```

```
[Out] (-3*Cos[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sin[a + b*x]^2])/(b*c*Sq
rt[Cos[a + b*x]^2]*(c*Sin[a + b*x])^(1/3))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = -\frac{3\cos(a+bx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sin^2(a+bx)\right)}{bc\sqrt{\cos^2(a+bx)}\sqrt[3]{c\sin(a+bx)}}$$

Mathematica [A] time = 0.0415464, size = 53, normalized size = 0.95

$$-\frac{3\sqrt{\cos^2(a+bx)}\tan(a+bx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sin^2(a+bx)\right)}{b(c\sin(a+bx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(-4/3),x]
```

```
[Out] (-3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sin[a + b*x]^2]*
Tan[a + b*x])/(b*(c*Sin[a + b*x])^(4/3))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a))^(4/3),x)
```

```
[Out] int(1/(c*sin(b*x+a))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sin(bx+a))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(-4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(c\sin(bx+a))^{\frac{2}{3}}}{c^2\cos(bx+a)^2-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(c*sin(b*x + a))^(2/3)/(c^2*cos(b*x + a)^2 - c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sin(a+bx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))**(4/3),x)
```

```
[Out] Integral((c*sin(a + b*x))**(-4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sin(bx+a))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(-4/3), x)
```

3.39 $\int \sin^n(a+bx)\,dx$

Optimal. Leaf size=63

$$\frac{\cos(a+bx)\sin^{n+1}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{b(n+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]*
Sin[a + b*x]^(1 + n))/(b*(1 + n)*Sqrt[Cos[a + b*x]^2])
```

Rubi [A] time = 0.0159425, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2643}

$$\frac{\cos(a+bx)\sin^{n+1}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{b(n+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^n,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]*
Sin[a + b*x]^(1 + n))/(b*(1 + n)*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sin^n(a+bx)\,dx=\frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)\sin^{1+n}(a+bx)}{b(1+n)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.0429127, size = 63, normalized size = 1.

$$\frac{\sqrt{\cos^2(a+bx)}\sec(a+bx)\sin^{n+1}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^n,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a +
b*x]^2]*Sec[a + b*x]*Sin[a + b*x]^(1 + n))/(b*(1 + n))
```

Maple [F] time = 0.47, size = 0, normalized size = 0.

$$\int (\sin (bx + a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^n,x)
```

```
[Out] int(sin(b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin (bx + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sin (bx + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(sin(b*x + a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^n (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**n,x)
```

```
[Out] Integral(sin(a + b*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin (bx + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^n, x)
```

3.40 $\int (c\sin(a+bx))^n\,dx$

Optimal. Leaf size=68

$$\frac{\cos(a+bx)(c\sin(a+bx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bc(n+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(1 + n))/(b*c*(1 + n)*Sqrt[Cos[a + b*x]^2])
```

Rubi [A] time = 0.0179258, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2643}

$$\frac{\cos(a+bx)(c\sin(a+bx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bc(n+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^n,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(1 + n))/(b*c*(1 + n)*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (c\sin(a+bx))^n\,dx=\frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+n}}{bc(1+n)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.0396145, size = 63, normalized size = 0.93

$$\frac{\sqrt{\cos^2(a+bx)}\tan(a+bx)(c\sin(a+bx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^n,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a +
b*x]^2]*(c*Sin[a + b*x])^n*Tan[a + b*x])/(b*(1 + n))
```

Maple [F] time = 0.433, size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^n,x)
```

```
[Out] int((c*sin(b*x+a))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((c\sin(bx+a))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(a+bx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**n,x)
```

```
[Out] Integral((c*sin(a + b*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^n, x)
```

3.41 $\int (a\sin(e+fx))^m (b\sin(e+fx))^n\,dx$

Optimal. Leaf size=81

$$\frac{\cos(e+fx)(a\sin(e+fx))^{m+1}(b\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\sin^2(e+fx)\right)}{af(m+n+1)\sqrt{\cos^2(e+fx)}}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Sin[e +
f*x]^2]*(a*Sin[e + f*x])^(1 + m)*(b*Sin[e + f*x])^n)/(a*f*(1 + m + n)*Sqrt[
Cos[e + f*x]^2])
```

Rubi [A] time = 0.0297226, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$\frac{\cos(e+fx)(a\sin(e+fx))^{m+1}(b\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\sin^2(e+fx)\right)}{af(m+n+1)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*(b*Sin[e + f*x])^n,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Sin[e +
f*x]^2]*(a*Sin[e + f*x])^(1 + m)*(b*Sin[e + f*x])^n)/(a*f*(1 + m + n)*Sqrt[
Cos[e + f*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx &= \left((a\sin(e+fx))^{-n}(b\sin(e+fx))^n\right)\int (a\sin(e+fx))^{m+n}\,dx\\ &= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);\sin^2(e+fx)\right)(a\sin(e+fx))^{1}}{af(1+m+n)\sqrt{\cos^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0785823, size = 76, normalized size = 0.94

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)(a\sin(e+fx))^m(b\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\sin^2(e+fx)\right)}{f(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m*(b*Sin[e + f*x])^n,x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2,
Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Sin[e + f*x])^n*Tan[e + f*x])/(f*(1 +
 m + n))
```

Maple [F] time = 0.861, size = 0, normalized size = 0.

$$\int \left(a \sin \left(fx+e\right)\right)^{m} \left(b \sin \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x)
```

```
[Out] int((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(fx+e\right)\right)^{m} \left(b \sin \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*(b*sin(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a \sin \left(fx+e\right)\right)^{m} \left(b \sin \left(fx+e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*(b*sin(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(e+fx\right)\right)^{m} \left(b \sin \left(e+fx\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*(b*sin(f*x+e))**n,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*(b*sin(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e))^m (b\sin(fx+e))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*(b*sin(f*x + e))^n, x)
```

3.42 $\int \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\cos^4(a+bx)}{4b}$$

```
[Out] -Cos[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0193339, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2565, 30}

$$-\frac{\cos^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -Cos[a + b*x]^4/(4*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sin(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^3\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0045508, size = 15, normalized size = 1.

$$-\frac{\cos^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -Cos[a + b*x]^4/(4*b)
```

Maple [A] time = 0.006, size = 14, normalized size = 0.9

$$-\frac{(\cos(bx+a))^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] -1/4*cos(b*x+a)^4/b
```

Maxima [A] time = 1.00887, size = 18, normalized size = 1.2

$$-\frac{\cos(bx+a)^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*cos(b*x + a)^4/b
```

Fricas [A] time = 1.81433, size = 31, normalized size = 2.07

$$-\frac{\cos(bx+a)^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*cos(b*x + a)^4/b
```

Sympy [A] time = 1.11814, size = 41, normalized size = 2.73

$$\begin{cases} \frac{\sin^4(a+bx)}{4b} + \frac{\sin^2(a+bx)\cos^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sin(a)\cos^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((sin(a + b*x)**4/(4*b) + sin(a + b*x)**2*cos(a + b*x)**2/(2*b), N
e(b, 0)), (x*sin(a)*cos(a)**3, True))
```

Giac [A] time = 1.14808, size = 32, normalized size = 2.13

$$-\frac{\sin(bx+a)^4 - 2\sin(bx+a)^2}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(sin(b*x + a)^4 - 2*sin(b*x + a)^2)/b
```

3.43 $\int \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\cos^3(a+bx)}{3b}$$

```
[Out] -Cos[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0206154, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2565, 30}

$$-\frac{\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -Cos[a + b*x]^3/(3*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^2\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0045087, size = 15, normalized size = 1.

$$-\frac{\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -Cos[a + b*x]^3/(3*b)
```

Maple [A] time = 0.004, size = 14, normalized size = 0.9

$$-\frac{(\cos(bx+a))^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] -1/3*cos(b*x+a)^3/b
```

Maxima [A] time = 0.993427, size = 18, normalized size = 1.2

$$-\frac{\cos(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/3*cos(b*x + a)^3/b
```

Fricas [A] time = 1.80328, size = 31, normalized size = 2.07

$$-\frac{\cos(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/3*cos(b*x + a)^3/b
```

Sympy [A] time = 0.505304, size = 22, normalized size = 1.47

$$\begin{cases} -\frac{\cos^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sin(a)\cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-cos(a + b*x)**3/(3*b), Ne(b, 0)), (x*sin(a)*cos(a)**2, True))
```

Giac [A] time = 1.12636, size = 18, normalized size = 1.2

$$-\frac{\cos(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/3*cos(b*x + a)^3/b
```

3.44 $\int \cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sin^2(a+bx)}{2b}$$

```
[Out] Sin[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0108554, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2564, 30}

$$\frac{\sin^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] Sin[a + b*x]^2/(2*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin(a+bx)\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^2(a+bx)}{2b}\end{aligned}$$

Mathematica [B] time = 0.0126988, size = 37, normalized size = 2.47

$$\frac{1}{2}\left(\frac{\sin(2a)\sin(2bx)}{2b}-\frac{\cos(2a)\cos(2bx)}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-(Cos[2*a]*Cos[2*b*x])/(2*b) + (Sin[2*a]*Sin[2*b*x])/(2*b))/2
```

Maple [A] time = 0.001, size = 14, normalized size = 0.9

$$\frac{(\sin(bx+a))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/2*sin(b*x+a)^2/b
```

Maxima [A] time = 1.01269, size = 18, normalized size = 1.2

$$-\frac{\cos(bx+a)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*cos(b*x + a)^2/b
```

Fricas [A] time = 1.67621, size = 31, normalized size = 2.07

$$-\frac{\cos(bx+a)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*cos(b*x + a)^2/b
```

Sympy [A] time = 0.23076, size = 19, normalized size = 1.27

$$\begin{cases} \frac{\sin^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sin(a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((sin(a + b*x)**2/(2*b), Ne(b, 0)), (x*sin(a)*cos(a), True))
```

Giac [A] time = 1.14891, size = 18, normalized size = 1.2

$$\frac{\sin(bx+a)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*sin(b*x + a)^2/b
```

3.45 $\int \tan(a+bx)\,dx$

Optimal. Leaf size=12

$$-\frac{\log(\cos(a+bx))}{b}$$

```
[Out] -(Log[Cos[a + b*x]]/b)
```

Rubi [A] time = 0.0044883, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$-\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*x],x]
```

```
[Out] -(Log[Cos[a + b*x]]/b)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tan(a+bx)\,dx = -\frac{\log(\cos(a+bx))}{b}$$

Mathematica [A] time = 0.006467, size = 12, normalized size = 1.

$$-\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*x],x]
```

```
[Out] -(Log[Cos[a + b*x]]/b)
```

Maple [A] time = 0.01, size = 12, normalized size = 1.

$$\frac{\ln(\sec(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/b*ln(sec(b*x+a))
```

Maxima [A] time = 0.988365, size = 24, normalized size = 2.

$$-\frac{\log\left(-\sin\left(bx+a\right)^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*log(-sin(b*x + a)^2 + 1)/b
```

Fricas [A] time = 1.92592, size = 31, normalized size = 2.58

$$-\frac{\log\left(-\cos\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -log(-cos(b*x + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a+bx\right)\sec\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*sec(a + b*x), x)
```

Giac [A] time = 1.20248, size = 24, normalized size = 2.

$$-\frac{\log\left(\frac{|\cos(bx+a)|}{|b|}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -log(abs(cos(b*x + a))/abs(b))/b
```

3.46 $\int \sec(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\sec(a+bx)}{b}$$

```
[Out] Sec[a + b*x]/b
```

Rubi [A] time = 0.0107023, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2606, 8}

$$\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] Sec[a + b*x]/b
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec(a+bx)\tan(a+bx)\,dx &= \frac{\text{Subst}\left(\int 1\,dx, x, \sec(a+bx)\right)}{b}\\ &= \frac{\sec(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.006631, size = 10, normalized size = 1.

$$\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] Sec[a + b*x]/b
```

Maple [A] time = 0.009, size = 11, normalized size = 1.1

$$\frac{\sec(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] sec(b*x+a)/b
```

Maxima [A] time = 0.974452, size = 16, normalized size = 1.6

$$\frac{1}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/(b*cos(b*x + a))
```

Fricas [A] time = 1.87666, size = 27, normalized size = 2.7

$$\frac{1}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/(b*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*sec(a + b*x)**2, x)
```

Giac [A] time = 1.1312, size = 16, normalized size = 1.6

$$\frac{1}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/(b*cos(b*x + a))
```

3.47 $\int \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sec^2(a+bx)}{2b}$$

```
[Out] Sec[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0194794, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$\frac{\sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] Sec[a + b*x]^2/(2*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)\tan(a+bx)\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \sec(a+bx)\right)}{b}\\ &= \frac{\sec^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.010365, size = 15, normalized size = 1.

$$\frac{\sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] Sec[a + b*x]^2/(2*b)
```

Maple [A] time = 0.008, size = 14, normalized size = 0.9

$$\frac{(\sec(bx+a))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 1/2*sec(b*x+a)^2/b
```

Maxima [A] time = 0.989576, size = 23, normalized size = 1.53

$$-\frac{1}{2\left(\sin(bx+a)^2-1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2/((sin(b*x + a)^2 - 1)*b)
```

Fricas [A] time = 1.67507, size = 32, normalized size = 2.13

$$\frac{1}{2\,b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2/(b*cos(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(a+bx)\sec^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*sec(a + b*x)**3, x)
```

Giac [A] time = 1.15784, size = 18, normalized size = 1.2

$$\frac{1}{2\,b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2/(b*cos(b*x + a)^2)
```

3.48 $\int \sec^3(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sec^3(a+bx)}{3b}$$

```
[Out] Sec[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.018593, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$\frac{\sec^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*Tan[a + b*x],x]
```

```
[Out] Sec[a + b*x]^3/(3*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^3(a+bx)\tan(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, \sec(a+bx)\right)}{b}\\ &= \frac{\sec^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0075239, size = 15, normalized size = 1.

$$\frac{\sec^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*Tan[a + b*x],x]
```

```
[Out] Sec[a + b*x]^3/(3*b)
```

Maple [A] time = 0.008, size = 14, normalized size = 0.9

$$\frac{(\sec(bx + a))^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*sin(b*x+a),x)
```

```
[Out] 1/3*sec(b*x+a)^3/b
```

Maxima [A] time = 1.0225, size = 18, normalized size = 1.2

$$\frac{1}{3b\cos(bx + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3/(b*cos(b*x + a)^3)
```

Fricas [A] time = 1.82284, size = 32, normalized size = 2.13

$$\frac{1}{3b\cos(bx + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3/(b*cos(b*x + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(a + bx)\sec^4(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*sec(a + b*x)**4, x)
```

Giac [A] time = 1.18209, size = 18, normalized size = 1.2

$$\frac{1}{3b\cos(bx + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a),x, algorithm="giac")

[Out] 1/3/(b*cos(b*x + a)^3)
```

3.49 $\int \cos^7(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=61

$$-\frac{\sin^9(a+bx)}{9b}+\frac{3\sin^7(a+bx)}{7b}-\frac{3\sin^5(a+bx)}{5b}+\frac{\sin^3(a+bx)}{3b}$$

```
[Out] Sin[a + b*x]^3/(3*b) - (3*Sin[a + b*x]^5)/(5*b) + (3*Sin[a + b*x]^7)/(7*b)
- Sin[a + b*x]^9/(9*b)
```

Rubi [A] time = 0.0430188, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 270}

$$-\frac{\sin^9(a+bx)}{9b}+\frac{3\sin^7(a+bx)}{7b}-\frac{3\sin^5(a+bx)}{5b}+\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^7*Sin[a + b*x]^2,x]
```

```
[Out] Sin[a + b*x]^3/(3*b) - (3*Sin[a + b*x]^5)/(5*b) + (3*Sin[a + b*x]^7)/(7*b)
- Sin[a + b*x]^9/(9*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^7(a+bx)\sin^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)^3\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^2-3x^4+3x^6-x^8\right)dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\sin^3(a+bx)}{3b}-\frac{3\sin^5(a+bx)}{5b}+\frac{3\sin^7(a+bx)}{7b}-\frac{\sin^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] time = 0.15612, size = 47, normalized size = 0.77

$$\frac{\sin^3(a+bx)(1389\cos(2(a+bx))+330\cos(4(a+bx))+35\cos(6(a+bx))+1606)}{10080b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^7*Sin[a + b*x]^2,x]
```

```
[Out] ((1606 + 1389*Cos[2*(a + b*x)] + 330*Cos[4*(a + b*x)] + 35*Cos[6*(a + b*x)]
)*Sin[a + b*x]^3)/(10080*b)
```

Maple [A] time = 0.039, size = 60, normalized size = 1.

$$\frac{1}{b}\left(-\frac{\sin(bx+a)(\cos(bx+a))^8}{9}+\frac{\sin(bx+a)}{63}\left(\frac{16}{5}+(\cos(bx+a))^6+\frac{6(\cos(bx+a))^4}{5}+\frac{8(\cos(bx+a))^2}{5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/9*sin(b*x+a)*cos(b*x+a)^8+1/63*(16/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+
8/5*cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [A] time = 0.99892, size = 62, normalized size = 1.02

$$-\frac{35\sin(bx+a)^9-135\sin(bx+a)^7+189\sin(bx+a)^5-105\sin(bx+a)^3}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/315*(35*sin(b*x + a)^9 - 135*sin(b*x + a)^7 + 189*sin(b*x + a)^5 - 105*s
in(b*x + a)^3)/b
```

Fricas [A] time = 1.98078, size = 142, normalized size = 2.33

$$-\frac{\left(35\cos(bx+a)^8-5\cos(bx+a)^6-6\cos(bx+a)^4-8\cos(bx+a)^2-16\right)\sin(bx+a)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/315*(35*cos(b*x + a)^8 - 5*cos(b*x + a)^6 - 6*cos(b*x + a)^4 - 8*cos(b*x
 + a)^2 - 16)*sin(b*x + a)/b
```

Sympy [A] time = 22.2179, size = 88, normalized size = 1.44

$$\begin{cases}\frac{16\sin^9(a+bx)}{315b}+\frac{8\sin^7(a+bx)\cos^2(a+bx)}{35b}+\frac{2\sin^5(a+bx)\cos^4(a+bx)}{5b}+\frac{\sin^3(a+bx)\cos^6(a+bx)}{3b} & \text{for } b\neq 0\\ x\sin^2(a)\cos^7(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((16*sin(a + b*x)**9/(315*b) + 8*sin(a + b*x)**7*cos(a + b*x)**2/(
35*b) + 2*sin(a + b*x)**5*cos(a + b*x)**4/(5*b) + sin(a + b*x)**3*cos(a + b
*x)**6/(3*b), Ne(b, 0)), (x*sin(a)**2*cos(a)**7, True))
```

Giac [A] time = 1.23135, size = 73, normalized size = 1.2

$$-\frac{\sin(9\,bx+9\,a)}{2304\,b}-\frac{5\,\sin(7\,bx+7\,a)}{1792\,b}-\frac{\sin(5\,bx+5\,a)}{160\,b}+\frac{7\,\sin(bx+a)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2304*sin(9*b*x + 9*a)/b - 5/1792*sin(7*b*x + 7*a)/b - 1/160*sin(5*b*x +
5*a)/b + 7/128*sin(b*x + a)/b
```

3.50 $\int \cos^5(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sin^7(a+bx)}{7b}-\frac{2\sin^5(a+bx)}{5b}+\frac{\sin^3(a+bx)}{3b}$$

```
[Out] Sin[a + b*x]^3/(3*b) - (2*Sin[a + b*x]^5)/(5*b) + Sin[a + b*x]^7/(7*b)
```

Rubi [A] time = 0.0382698, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 270}

$$\frac{\sin^7(a+bx)}{7b}-\frac{2\sin^5(a+bx)}{5b}+\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*Sin[a + b*x]^2,x]
```

```
[Out] Sin[a + b*x]^3/(3*b) - (2*Sin[a + b*x]^5)/(5*b) + Sin[a + b*x]^7/(7*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(a+bx)\sin^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\sin^3(a+bx)}{3b}-\frac{2\sin^5(a+bx)}{5b}+\frac{\sin^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.0922559, size = 37, normalized size = 0.8

$$\frac{\sin^3(a+bx)(108\cos(2(a+bx))+15\cos(4(a+bx))+157)}{840b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*Sin[a + b*x]^2,x]
```

```
[Out] ((157 + 108*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)])*Sin[a + b*x]^3)/(840*b)
```

Maple [A] time = 0.034, size = 50, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{\sin(bx+a)(\cos(bx+a))^6}{7}+\frac{\sin(bx+a)}{35}\left(\frac{8}{3}+(\cos(bx+a))^4+\frac{4(\cos(bx+a))^2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/7*sin(b*x+a)*cos(b*x+a)^6+1/35*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*
sin(b*x+a))
```

Maxima [A] time = 0.985084, size = 49, normalized size = 1.07

$$\frac{15\sin(bx+a)^7-42\sin(bx+a)^5+35\sin(bx+a)^3}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*sin(b*x + a)^7 - 42*sin(b*x + a)^5 + 35*sin(b*x + a)^3)/b
```

Fricas [A] time = 1.91543, size = 115, normalized size = 2.5

$$-\frac{\left(15\cos(bx+a)^6-3\cos(bx+a)^4-4\cos(bx+a)^2-8\right)\sin(bx+a)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/105*(15*cos(b*x + a)^6 - 3*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b*
x + a)/b
```

Sympy [A] time = 7.79142, size = 66, normalized size = 1.43

$$\begin{cases}\frac{8\sin^7(a+bx)}{105b}+\frac{4\sin^5(a+bx)\cos^2(a+bx)}{15b}+\frac{\sin^3(a+bx)\cos^4(a+bx)}{3b} & \text{for } b\neq 0\\ x\sin^2(a)\cos^5(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((8*sin(a + b*x)**7/(105*b) + 4*sin(a + b*x)**5*cos(a + b*x)**2/(1
5*b) + sin(a + b*x)**3*cos(a + b*x)**4/(3*b), Ne(b, 0)), (x*sin(a)**2*cos(a
```

```
)**5, True))
```

Giac [A] time = 1.14921, size = 73, normalized size = 1.59

$$-\frac{\sin\left(7\,bx+7\,a\right)}{448\,b}-\frac{3\,\sin\left(5\,bx+5\,a\right)}{320\,b}-\frac{\sin\left(3\,bx+3\,a\right)}{192\,b}+\frac{5\,\sin\left(bx+a\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/448*sin(7*b*x + 7*a)/b - 3/320*sin(5*b*x + 5*a)/b - 1/192*sin(3*b*x + 3*
a)/b + 5/64*sin(b*x + a)/b
```

3.51 $\int \cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sin^3(a+bx)}{3b}-\frac{\sin^5(a+bx)}{5b}$$

```
[Out] Sin[a + b*x]^3/(3*b) - Sin[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0348122, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 14}

$$\frac{\sin^3(a+bx)}{3b}-\frac{\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] Sin[a + b*x]^3/(3*b) - Sin[a + b*x]^5/(5*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sin^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\sin^3(a+bx)}{3b}-\frac{\sin^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0585984, size = 27, normalized size = 0.87

$$\frac{\sin^3(a+bx)(3\cos(2(a+bx))+7)}{30b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((7 + 3*Cos[2*(a + b*x)])*Sin[a + b*x]^3)/(30*b)
```

Maple [A] time = 0.036, size = 40, normalized size = 1.3

$$\frac{1}{b}\left(-\frac{\sin(bx+a)(\cos(bx+a))^4}{5}+\frac{\left(2+(\cos(bx+a))^2\right)\sin(bx+a)}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [A] time = 0.98687, size = 35, normalized size = 1.13

$$-\frac{3\sin(bx+a)^5-5\sin(bx+a)^3}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/15*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)/b
```

Fricas [A] time = 1.97271, size = 84, normalized size = 2.71

$$-\frac{\left(3\cos(bx+a)^4-\cos(bx+a)^2-2\right)\sin(bx+a)}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)*sin(b*x + a)/b
```

Sympy [A] time = 2.1003, size = 44, normalized size = 1.42

$$\begin{cases}\frac{2\sin^5(a+bx)}{15b}+\frac{\sin^3(a+bx)\cos^2(a+bx)}{3b} & \text{for } b\neq 0\\ x\sin^2(a)\cos^3(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*sin(a + b*x)**5/(15*b) + sin(a + b*x)**3*cos(a + b*x)**2/(3*b)
, Ne(b, 0)), (x*sin(a)**2*cos(a)**3, True))
```

Giac [A] time = 1.2339, size = 35, normalized size = 1.13

$$-\frac{3 \sin\left(bx + a\right)^{5} - 5 \sin\left(bx + a\right)^{3}}{15 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/15*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)/b
```

3.52 $\int \cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sin^3(a+bx)}{3b}$$

```
[Out] Sin[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0183016, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2564, 30}

$$\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] Sin[a + b*x]^3/(3*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0035197, size = 15, normalized size = 1.

$$\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] Sin[a + b*x]^3/(3*b)
```

Maple [A] time = 0.003, size = 14, normalized size = 0.9

$$\frac{(\sin(bx+a))^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/3*sin(b*x+a)^3/b
```

Maxima [A] time = 0.976664, size = 18, normalized size = 1.2

$$\frac{\sin(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*sin(b*x + a)^3/b
```

Fricas [A] time = 1.85827, size = 57, normalized size = 3.8

$$-\frac{\left(\cos(bx+a)^2-1\right)\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(b*x + a)^2 - 1)*sin(b*x + a)/b
```

Sympy [A] time = 0.514821, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sin^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sin^2(a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((sin(a + b*x)**3/(3*b), Ne(b, 0)), (x*sin(a)**2*cos(a), True))
```

Giac [A] time = 1.22347, size = 18, normalized size = 1.2

$$\frac{\sin(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*sin(b*x + a)^3/b
```

3.53 $\int \tan^2(a+bx)\,dx$

Optimal. Leaf size=14

$$\frac{\tan(a+bx)}{b}-x$$

```
[Out] -x + Tan[a + b*x]/b
```

Rubi [A] time = 0.0081899, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$\frac{\tan(a+bx)}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*x]^2,x]

[Out] -x + Tan[a + b*x]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tan^2(a+bx)\,dx &= \frac{\tan(a+bx)}{b}-\int 1\,dx\\ &= -x+\frac{\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0071568, size = 23, normalized size = 1.64

$$\frac{\tan(a+bx)}{b}-\frac{\tan^{-1}(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*x]^2,x]

[Out] -(ArcTan[Tan[a + b*x]]/b) + Tan[a + b*x]/b
```

Maple [A] time = 0.016, size = 19, normalized size = 1.4

$$\frac{\tan(bx+a)-bx-a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(tan(b*x+a)-b*x-a)
```

Maxima [A] time = 1.48968, size = 24, normalized size = 1.71

$$-\frac{bx + a - \tan\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b*x + a - tan(b*x + a))/b
```

Fricas [B] time = 1.82419, size = 72, normalized size = 5.14

$$-\frac{bx\cos\left(bx + a\right) - \sin\left(bx + a\right)}{b\cos\left(bx + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b*x*cos(b*x + a) - sin(b*x + a))/(b*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^{2}\left(a + bx\right)\sec^{2}\left(a + bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)**2, x)
```

Giac [A] time = 1.1317, size = 24, normalized size = 1.71

$$-\frac{bx + a - \tan\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(b*x + a - tan(b*x + a))/b
```

3.54 $\int \sec^2(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\tan^3(a+bx)}{3b}$$

```
[Out] Tan[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0295692, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$\frac{\tan^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2*Tan[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]^3/(3*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)\tan^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\tan^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0068882, size = 15, normalized size = 1.

$$\frac{\tan^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2*Tan[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]^3/(3*b)
```

Maple [A] time = 0.019, size = 22, normalized size = 1.5

$$\frac{(\sin(bx+a))^3}{3b(\cos(bx+a))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*sin(b*x+a)^2,x)
```

```
[Out] 1/3/b*sin(b*x+a)^3/cos(b*x+a)^3
```

Maxima [A] time = 1.02254, size = 18, normalized size = 1.2

$$\frac{\tan(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*tan(b*x + a)^3/b
```

Fricas [B] time = 1.73518, size = 80, normalized size = 5.33

$$-\frac{\left(\cos(bx+a)^2-1\right)\sin(bx+a)}{3b\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(b*x + a)^2 - 1)*sin(b*x + a)/(b*cos(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16347, size = 18, normalized size = 1.2

$$\frac{\tan(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*tan(b*x + a)^3/b
```

3.55 $\int \sec^4(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{3b}$$

```
[Out] Tan[a + b*x]^3/(3*b) + Tan[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0340102, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 14}

$$\frac{\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4*Tan[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]^3/(3*b) + Tan[a + b*x]^5/(5*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sec^4(a+bx)\tan^2(a+bx)\,dx&=\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,\tan(a+bx)\right)}{b}\\&=\frac{\text{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,\tan(a+bx)\right)}{b}\\&=\frac{\tan^3(a+bx)}{3b}+\frac{\tan^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0411832, size = 56, normalized size = 1.81

$$-\frac{2\tan(a+bx)}{15b}+\frac{\tan(a+bx)\sec^4(a+bx)}{5b}-\frac{\tan(a+bx)\sec^2(a+bx)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4*Tan[a + b*x]^2,x]
```

```
[Out] (-2*Tan[a + b*x])/(15*b) - (Sec[a + b*x]^2*Tan[a + b*x])/(15*b) + (Sec[a +
b*x]^4*Tan[a + b*x])/(5*b)
```

Maple [A] time = 0.02, size = 42, normalized size = 1.4

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^3}{5(\cos(bx+a))^5}+\frac{2(\sin(bx+a))^3}{15(\cos(bx+a))^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/5*sin(b*x+a)^3/cos(b*x+a)^5+2/15*sin(b*x+a)^3/cos(b*x+a)^3)
```

Maxima [A] time = 0.991552, size = 35, normalized size = 1.13

$$\frac{3\tan(bx+a)^5+5\tan(bx+a)^3}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(b*x + a)^5 + 5*tan(b*x + a)^3)/b
```

Fricas [A] time = 1.79198, size = 107, normalized size = 3.45

$$-\frac{\left(2\cos(bx+a)^4+\cos(bx+a)^2-3\right)\sin(bx+a)}{15\,b\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(b*x + a)^4 + cos(b*x + a)^2 - 3)*sin(b*x + a)/(b*cos(b*x + a)^
5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15766, size = 35, normalized size = 1.13

$$\frac{3 \tan\left(bx + a\right)^{5} + 5 \tan\left(bx + a\right)^{3}}{15 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(b*x + a)^5 + 5*tan(b*x + a)^3)/b
```

3.56 $\int \sec^6(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\tan^7(a+bx)}{7b}+\frac{2\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{3b}$$

```
[Out] Tan[a + b*x]^3/(3*b) + (2*Tan[a + b*x]^5)/(5*b) + Tan[a + b*x]^7/(7*b)
```

Rubi [A] time = 0.038507, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 270}

$$\frac{\tan^7(a+bx)}{7b}+\frac{2\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6*Tan[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]^3/(3*b) + (2*Tan[a + b*x]^5)/(5*b) + Tan[a + b*x]^7/(7*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(a+bx)\tan^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^2\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^2+2x^4+x^6\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\tan^3(a+bx)}{3b}+\frac{2\tan^5(a+bx)}{5b}+\frac{\tan^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.0426543, size = 77, normalized size = 1.67

$$-\frac{8\tan(a+bx)}{105b}+\frac{\tan(a+bx)\sec^6(a+bx)}{7b}-\frac{\tan(a+bx)\sec^4(a+bx)}{35b}-\frac{4\tan(a+bx)\sec^2(a+bx)}{105b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6*Tan[a + b*x]^2,x]
```

```
[Out] (-8*Tan[a + b*x])/(105*b) - (4*Sec[a + b*x]^2*Tan[a + b*x])/(105*b) - (Sec[
a + b*x]^4*Tan[a + b*x])/(35*b) + (Sec[a + b*x]^6*Tan[a + b*x])/(7*b)
```

Maple [A] time = 0.022, size = 60, normalized size = 1.3

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^3}{7(\cos(bx+a))^7}+\frac{4(\sin(bx+a))^3}{35(\cos(bx+a))^5}+\frac{8(\sin(bx+a))^3}{105(\cos(bx+a))^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^8*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/7*sin(b*x+a)^3/cos(b*x+a)^7+4/35*sin(b*x+a)^3/cos(b*x+a)^5+8/105*sin
(b*x+a)^3/cos(b*x+a)^3)
```

Maxima [A] time = 0.989448, size = 49, normalized size = 1.07

$$\frac{15\tan(bx+a)^7+42\tan(bx+a)^5+35\tan(bx+a)^3}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*tan(b*x + a)^7 + 42*tan(b*x + a)^5 + 35*tan(b*x + a)^3)/b
```

Fricas [A] time = 1.84528, size = 138, normalized size = 3.

$$-\frac{\left(8\cos(bx+a)^6+4\cos(bx+a)^4+3\cos(bx+a)^2-15\right)\sin(bx+a)}{105\,b\cos(bx+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/105*(8*cos(b*x + a)^6 + 4*cos(b*x + a)^4 + 3*cos(b*x + a)^2 - 15)*sin(b*
x + a)/(b*cos(b*x + a)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**8*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17382, size = 49, normalized size = 1.07

$$\frac{15 \tan\left(bx+a\right)^{7}+42 \tan\left(bx+a\right)^{5}+35 \tan\left(bx+a\right)^{3}}{105 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*tan(b*x + a)^7 + 42*tan(b*x + a)^5 + 35*tan(b*x + a)^3)/b
```

3.57 $\int \sec^8(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=61

$$\frac{\tan^9(a+bx)}{9b}+\frac{3\tan^7(a+bx)}{7b}+\frac{3\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{3b}$$

```
[Out] Tan[a + b*x]^3/(3*b) + (3*Tan[a + b*x]^5)/(5*b) + (3*Tan[a + b*x]^7)/(7*b)
+ Tan[a + b*x]^9/(9*b)
```

Rubi [A] time = 0.039758, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 270}

$$\frac{\tan^9(a+bx)}{9b}+\frac{3\tan^7(a+bx)}{7b}+\frac{3\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^8*Tan[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]^3/(3*b) + (3*Tan[a + b*x]^5)/(5*b) + (3*Tan[a + b*x]^7)/(7*b)
+ Tan[a + b*x]^9/(9*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^8(a+bx)\tan^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^3\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^2+3x^4+3x^6+x^8\right)dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\tan^3(a+bx)}{3b}+\frac{3\tan^5(a+bx)}{5b}+\frac{3\tan^7(a+bx)}{7b}+\frac{\tan^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] time = 0.0346972, size = 98, normalized size = 1.61

$$-\frac{16\tan(a+bx)}{315b}+\frac{\tan(a+bx)\sec^8(a+bx)}{9b}-\frac{\tan(a+bx)\sec^6(a+bx)}{63b}-\frac{2\tan(a+bx)\sec^4(a+bx)}{105b}-\frac{8\tan(a+bx)}{31}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^8*Tan[a + b*x]^2,x]
```

```
[Out] (-16*Tan[a + b*x])/(315*b) - (8*Sec[a + b*x]^2*Tan[a + b*x])/(315*b) - (2*S
ec[a + b*x]^4*Tan[a + b*x])/(105*b) - (Sec[a + b*x]^6*Tan[a + b*x])/(63*b)
+ (Sec[a + b*x]^8*Tan[a + b*x])/(9*b)
```

Maple [A] time = 0.023, size = 78, normalized size = 1.3

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^3}{9\,(\cos(bx+a))^9}+\frac{2\,(\sin(bx+a))^3}{21\,(\cos(bx+a))^7}+\frac{8\,(\sin(bx+a))^3}{105\,(\cos(bx+a))^5}+\frac{16\,(\sin(bx+a))^3}{315\,(\cos(bx+a))^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^10*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/9*sin(b*x+a)^3/cos(b*x+a)^9+2/21*sin(b*x+a)^3/cos(b*x+a)^7+8/105*sin
(b*x+a)^3/cos(b*x+a)^5+16/315*sin(b*x+a)^3/cos(b*x+a)^3)
```

Maxima [A] time = 0.979227, size = 62, normalized size = 1.02

$$\frac{35\,\tan(bx+a)^9+135\,\tan(bx+a)^7+189\,\tan(bx+a)^5+105\,\tan(bx+a)^3}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*tan(b*x + a)^9 + 135*tan(b*x + a)^7 + 189*tan(b*x + a)^5 + 105*ta
n(b*x + a)^3)/b
```

Fricas [A] time = 1.59504, size = 165, normalized size = 2.7

$$-\frac{\left(16\,\cos(bx+a)^8+8\,\cos(bx+a)^6+6\,\cos(bx+a)^4+5\,\cos(bx+a)^2-35\right)\sin(bx+a)}{315\,b\cos(bx+a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/315*(16*cos(b*x + a)^8 + 8*cos(b*x + a)^6 + 6*cos(b*x + a)^4 + 5*cos(b*x
 + a)^2 - 35)*sin(b*x + a)/(b*cos(b*x + a)^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**10*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20449, size = 62, normalized size = 1.02

$$\frac{35 \tan(bx+a)^9 + 135 \tan(bx+a)^7 + 189 \tan(bx+a)^5 + 105 \tan(bx+a)^3}{315 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/315*(35*tan(b*x + a)^9 + 135*tan(b*x + a)^7 + 189*tan(b*x + a)^5 + 105*ta
n(b*x + a)^3)/b
```

3.58 $\int \cos^6(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=88

$$-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}+\frac{\sin(a+bx)\cos^5(a+bx)}{48b}+\frac{5\sin(a+bx)\cos^3(a+bx)}{192b}+\frac{5\sin(a+bx)\cos(a+bx)}{128b}+\frac{5x}{128}$$

```
[Out] (5*x)/128 + (5*Cos[a + b*x]*Sin[a + b*x])/(128*b) + (5*Cos[a + b*x]^3*Sin[a
 + b*x])/(192*b) + (Cos[a + b*x]^5*Sin[a + b*x])/(48*b) - (Cos[a + b*x]^7*S
in[a + b*x])/(8*b)
```

Rubi [A] time = 0.0658742, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}+\frac{\sin(a+bx)\cos^5(a+bx)}{48b}+\frac{5\sin(a+bx)\cos^3(a+bx)}{192b}+\frac{5\sin(a+bx)\cos(a+bx)}{128b}+\frac{5x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^6*Sin[a + b*x]^2,x]
```

```
[Out] (5*x)/128 + (5*Cos[a + b*x]*Sin[a + b*x])/(128*b) + (5*Cos[a + b*x]^3*Sin[a
 + b*x])/(192*b) + (Cos[a + b*x]^5*Sin[a + b*x])/(48*b) - (Cos[a + b*x]^7*S
in[a + b*x])/(8*b)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx = -\frac{\cos^7(a+bx)\sin(a+bx)}{8b} + \frac{1}{8}\int \cos^6(a+bx)\,dx$$
$$= \frac{\cos^5(a+bx)\sin(a+bx)}{48b} - \frac{\cos^7(a+bx)\sin(a+bx)}{8b} + \frac{5}{48}\int \cos^4(a+bx)\,dx$$
$$= \frac{5\cos^3(a+bx)\sin(a+bx)}{192b} + \frac{\cos^5(a+bx)\sin(a+bx)}{48b} - \frac{\cos^7(a+bx)\sin(a+bx)}{8b} +$$
$$= \frac{5\cos(a+bx)\sin(a+bx)}{128b} + \frac{5\cos^3(a+bx)\sin(a+bx)}{192b} + \frac{\cos^5(a+bx)\sin(a+bx)}{48b} -$$
$$= \frac{5x}{128} + \frac{5\cos(a+bx)\sin(a+bx)}{128b} + \frac{5\cos^3(a+bx)\sin(a+bx)}{192b} + \frac{\cos^5(a+bx)\sin(a}{48b}$$

Mathematica [A] time = 0.144025, size = 52, normalized size = 0.59

$$\frac{48\sin(2(a+bx)) - 24\sin(4(a+bx)) - 16\sin(6(a+bx)) - 3\sin(8(a+bx)) + 120bx}{3072b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^6*Sin[a + b*x]^2,x]
```

```
[Out] (120*b*x + 48*Sin[2*(a + b*x)] - 24*Sin[4*(a + b*x)] - 16*Sin[6*(a + b*x)]
- 3*Sin[8*(a + b*x)])/(3072*b)
```

Maple [A] time = 0.039, size = 64, normalized size = 0.7

$$\frac{1}{b}\left(-\frac{\sin(bx+a)(\cos(bx+a))^7}{8} + \frac{\sin(bx+a)}{48}\left((\cos(bx+a))^5 + \frac{5(\cos(bx+a))^3}{4} + \frac{15\cos(bx+a)}{8}\right) + \frac{5bx}{128} + \frac{5a}{128}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/8*sin(b*x+a)*cos(b*x+a)^7+1/48*(cos(b*x+a)^5+5/4*cos(b*x+a)^3+15/8*
cos(b*x+a))*sin(b*x+a)+5/128*b*x+5/128*a)
```

Maxima [A] time = 0.997626, size = 65, normalized size = 0.74

$$\frac{64\sin(2bx+2a)^3 + 120bx + 120a - 3\sin(8bx+8a) - 24\sin(4bx+4a)}{3072b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3072*(64*sin(2*b*x + 2*a)^3 + 120*b*x + 120*a - 3*sin(8*b*x + 8*a) - 24*s
in(4*b*x + 4*a))/b
```

Fricas [A] time = 1.62223, size = 149, normalized size = 1.69

$$\frac{15bx - \left(48\cos(bx+a)^7 - 8\cos(bx+a)^5 - 10\cos(bx+a)^3 - 15\cos(bx+a)\right)\sin(bx+a)}{384b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(15*b*x - (48*cos(b*x + a)^7 - 8*cos(b*x + a)^5 - 10*cos(b*x + a)^3 -
 15*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [A] time = 12.0965, size = 189, normalized size = 2.15

$$\begin{cases} \frac{5x\sin^8(a+bx)}{128} + \frac{5x\sin^6(a+bx)\cos^2(a+bx)}{32} + \frac{15x\sin^4(a+bx)\cos^4(a+bx)}{64} + \frac{5x\sin^2(a+bx)\cos^6(a+bx)}{32} + \frac{5x\cos^8(a+bx)}{128} + \frac{5\sin^7(a+bx)\cos(a+bx)}{128b} + \\ x\sin^2(a)\cos^6(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((5*x*sin(a + b*x)**8/128 + 5*x*sin(a + b*x)**6*cos(a + b*x)**2/32
 + 15*x*sin(a + b*x)**4*cos(a + b*x)**4/64 + 5*x*sin(a + b*x)**2*cos(a + b*
x)**6/32 + 5*x*cos(a + b*x)**8/128 + 5*sin(a + b*x)**7*cos(a + b*x)/(128*b)
 + 55*sin(a + b*x)**5*cos(a + b*x)**3/(384*b) + 73*sin(a + b*x)**3*cos(a +
b*x)**5/(384*b) - 5*sin(a + b*x)*cos(a + b*x)**7/(128*b), Ne(b, 0)), (x*sin
(a)**2*cos(a)**6, True))
```

Giac [A] time = 1.17535, size = 81, normalized size = 0.92

$$\frac{5}{128}\,x - \frac{\sin(8\,bx + 8\,a)}{1024\,b} - \frac{\sin(6\,bx + 6\,a)}{192\,b} - \frac{\sin(4\,bx + 4\,a)}{128\,b} + \frac{\sin(2\,bx + 2\,a)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5/128*x - 1/1024*sin(8*b*x + 8*a)/b - 1/192*sin(6*b*x + 6*a)/b - 1/128*sin(
4*b*x + 4*a)/b + 1/64*sin(2*b*x + 2*a)/b
```

3.59 $\int \cos^4(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=67

$$-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{\sin(a+bx)\cos^3(a+bx)}{24b}+\frac{\sin(a+bx)\cos(a+bx)}{16b}+\frac{x}{16}$$

```
[Out] x/16 + (Cos[a + b*x]*Sin[a + b*x])/(16*b) + (Cos[a + b*x]^3*Sin[a + b*x])/(
24*b) - (Cos[a + b*x]^5*Sin[a + b*x])/(6*b)
```

Rubi [A] time = 0.051307, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{\sin(a+bx)\cos^3(a+bx)}{24b}+\frac{\sin(a+bx)\cos(a+bx)}{16b}+\frac{x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Sin[a + b*x]^2,x]
```

```
[Out] x/16 + (Cos[a + b*x]*Sin[a + b*x])/(16*b) + (Cos[a + b*x]^3*Sin[a + b*x])/(
24*b) - (Cos[a + b*x]^5*Sin[a + b*x])/(6*b)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(a+bx)\sin^2(a+bx)\,dx &= -\frac{\cos^5(a+bx)\sin(a+bx)}{6b}+\frac{1}{6}\int \cos^4(a+bx)\,dx\\
&= \frac{\cos^3(a+bx)\sin(a+bx)}{24b}-\frac{\cos^5(a+bx)\sin(a+bx)}{6b}+\frac{1}{8}\int \cos^2(a+bx)\,dx\\
&= \frac{\cos(a+bx)\sin(a+bx)}{16b}+\frac{\cos^3(a+bx)\sin(a+bx)}{24b}-\frac{\cos^5(a+bx)\sin(a+bx)}{6b}+\int\\
&= \frac{x}{16}+\frac{\cos(a+bx)\sin(a+bx)}{16b}+\frac{\cos^3(a+bx)\sin(a+bx)}{24b}-\frac{\cos^5(a+bx)\sin(a+bx)}{6b}
\end{aligned}$$

Mathematica [A] time = 0.0880186, size = 40, normalized size = 0.6

$$-\frac{-3\sin(2(a+bx))+3\sin(4(a+bx))+\sin(6(a+bx))-12bx}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Sin[a + b*x]^2,x]
```

```
[Out] -(-12*b*x - 3*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)])/(19
2*b)
```

Maple [A] time = 0.037, size = 54, normalized size = 0.8

$$\frac{1}{b}\left(-\frac{\sin(bx+a)(\cos(bx+a))^5}{6}+\frac{\sin(bx+a)}{24}\left((\cos(bx+a))^3+\frac{3\cos(bx+a)}{2}\right)+\frac{bx}{16}+\frac{a}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/6*sin(b*x+a)*cos(b*x+a)^5+1/24*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*
x+a)+1/16*b*x+1/16*a)
```

Maxima [A] time = 1.01179, size = 50, normalized size = 0.75

$$\frac{4\sin(2bx+2a)^3+12bx+12a-3\sin(4bx+4a)}{192b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/192*(4*sin(2*b*x + 2*a)^3 + 12*b*x + 12*a - 3*sin(4*b*x + 4*a))/b
```

Fricas [A] time = 1.53828, size = 116, normalized size = 1.73

$$\frac{3bx-\left(8\cos(bx+a)^5-2\cos(bx+a)^3-3\cos(bx+a)\right)\sin(bx+a)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*b*x - (8*cos(b*x + a)^5 - 2*cos(b*x + a)^3 - 3*cos(b*x + a))*sin(b*
x + a))/b
```

Sympy [A] time = 3.69407, size = 136, normalized size = 2.03

$$\begin{cases}\frac{x\sin^6(a+bx)}{16}+\frac{3x\sin^4(a+bx)\cos^2(a+bx)}{16}+\frac{3x\sin^2(a+bx)\cos^4(a+bx)}{16}+\frac{x\cos^6(a+bx)}{16}+\frac{\sin^5(a+bx)\cos(a+bx)}{16b}+\frac{\sin^3(a+bx)\cos^3(a+bx)}{6b}-\frac{\sin(a+}{} \\ x\sin^2(a)\cos^4(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**6/16 + 3*x*sin(a + b*x)**4*cos(a + b*x)**2/16 +
3*x*sin(a + b*x)**2*cos(a + b*x)**4/16 + x*cos(a + b*x)**6/16 + sin(a + b*x
)**5*cos(a + b*x)/(16*b) + sin(a + b*x)**3*cos(a + b*x)**3/(6*b) - sin(a +
b*x)*cos(a + b*x)**5/(16*b), Ne(b, 0)), (x*sin(a)**2*cos(a)**4, True))
```

Giac [A] time = 1.20238, size = 62, normalized size = 0.93

$$\frac{1}{16}x - \frac{\sin(6bx + 6a)}{192b} - \frac{\sin(4bx + 4a)}{64b} + \frac{\sin(2bx + 2a)}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*x - 1/192*sin(6*b*x + 6*a)/b - 1/64*sin(4*b*x + 4*a)/b + 1/64*sin(2*b*
x + 2*a)/b
```

3.60 $\int \cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{\sin(a+bx)\cos(a+bx)}{8b}+\frac{x}{8}$$

```
[Out] x/8 + (Cos[a + b*x]*Sin[a + b*x])/(8*b) - (Cos[a + b*x]^3*Sin[a + b*x])/(4*
b)
```

Rubi [A] time = 0.0396221, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$-\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{\sin(a+bx)\cos(a+bx)}{8b}+\frac{x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] x/8 + (Cos[a + b*x]*Sin[a + b*x])/(8*b) - (Cos[a + b*x]^3*Sin[a + b*x])/(4*
b)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin^2(a+bx)\,dx &= -\frac{\cos^3(a+bx)\sin(a+bx)}{4b}+\frac{1}{4}\int \cos^2(a+bx)\,dx\\ &= \frac{\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos^3(a+bx)\sin(a+bx)}{4b}+\frac{\int 1\,dx}{8}\\ &= \frac{x}{8}+\frac{\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos^3(a+bx)\sin(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0325703, size = 23, normalized size = 0.5

$$-\frac{\sin(4(a+bx))-4(a+bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] -(-4*(a + b*x) + Sin[4*(a + b*x)])/(32*b)
```

Maple [A] time = 0.012, size = 43, normalized size = 0.9

$$\frac{1}{b}\left(-\frac{\left(\cos\left(bx+a\right)\right)^{3}\sin\left(bx+a\right)}{4}+\frac{\cos\left(bx+a\right)\sin\left(bx+a\right)}{8}+\frac{bx}{8}+\frac{a}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)
```

Maxima [A] time = 0.97436, size = 32, normalized size = 0.7

$$\frac{4\,bx+4\,a-\sin\left(4\,bx+4\,a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(4*b*x + 4*a - sin(4*b*x + 4*a))/b
```

Fricas [A] time = 1.57523, size = 84, normalized size = 1.83

$$\frac{bx-\left(2\,\cos\left(bx+a\right)^{3}-\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b*x - (2*cos(b*x + a)^3 - cos(b*x + a))*sin(b*x + a))/b
```

Sympy [A] time = 1.01716, size = 92, normalized size = 2.

$$\begin{cases} \frac{x\sin^{4}\left(a+bx\right)}{8}+\frac{x\sin^{2}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{4}+\frac{x\cos^{4}\left(a+bx\right)}{8}+\frac{\sin^{3}\left(a+bx\right)\cos\left(a+bx\right)}{8b}-\frac{\sin\left(a+bx\right)\cos^{3}\left(a+bx\right)}{8b} & \text{for}\ b\neq 0 \\ x\sin^{2}\left(a\right)\cos^{2}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**4/8 + x*sin(a + b*x)**2*cos(a + b*x)**2/4 + x*co
s(a + b*x)**4/8 + sin(a + b*x)**3*cos(a + b*x)/(8*b) - sin(a + b*x)*cos(a +
 b*x)**3/(8*b), Ne(b, 0)), (x*sin(a)**2*cos(a)**2, True))
```

Giac [A] time = 1.15917, size = 24, normalized size = 0.52

$$\frac{1}{8}x - \frac{\sin(4bx + 4a)}{32b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*x - 1/32*sin(4*b*x + 4*a)/b
```

3.61 $\int \sin^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

```
[Out] x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rubi [A] time = 0.0092417, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2,x]
```

```
[Out] x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\,dx &= -\frac{\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0238493, size = 23, normalized size = 0.92

$$-\frac{\sin(2(a+bx))-2(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2,x]
```

```
[Out] -(-2*(a + b*x) + Sin[2*(a + b*x)])/(4*b)
```

Maple [A] time = 0., size = 27, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

Maxima [A] time = 0.959948, size = 32, normalized size = 1.28

$$\frac{2\,bx+2\,a-\sin(2\,bx+2\,a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))/b
```

Fricas [A] time = 1.56115, size = 55, normalized size = 2.2

$$\frac{bx-\cos(bx+a)\sin(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x - cos(b*x + a)*sin(b*x + a))/b
```

Sympy [A] time = 0.216223, size = 46, normalized size = 1.84

$$\begin{cases}\frac{x\sin^2(a+bx)}{2}+\frac{x\cos^2(a+bx)}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b\neq 0\\ x\sin^2(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a +
 b*x)/(2*b), Ne(b, 0)), (x*sin(a)**2, True))
```

Giac [A] time = 1.13709, size = 24, normalized size = 0.96

$$\frac{1}{2}\,x-\frac{\sin(2\,bx+2\,a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*b*x + 2*a)/b
```

3.62 $\int \sin(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}-\frac{\sin(a+bx)}{b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b
```

Rubi [A] time = 0.0147116, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2592, 321, 206}

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sin(a+bx)\tan(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(a+bx)\right)}{b}\\
&= -\frac{\sin(a+bx)}{b}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(a+bx)\right)}{b}\\
&= \frac{\tanh^{-1}(\sin(a+bx))}{b}-\frac{\sin(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0107391, size = 23, normalized size = 1.

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b
```

Maple [A] time = 0.017, size = 31, normalized size = 1.4

$$-\frac{\sin(bx+a)}{b}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] -sin(b*x+a)/b+1/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 0.990652, size = 46, normalized size = 2.

$$\frac{\log(\sin(bx+a)+1)-\log(\sin(bx+a)-1)-2\sin(bx+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))/b
```

Fricas [A] time = 1.68546, size = 99, normalized size = 4.3

$$\frac{\log(\sin(bx+a)+1)-\log(-\sin(bx+a)+1)-2\sin(bx+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1) - 2*sin(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24147, size = 49, normalized size = 2.13

$$\frac{\log\left(\left|\sin\left(bx+a\right)+1\right|\right)-\log\left(\left|\sin\left(bx+a\right)-1\right|\right)-2\,\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(sin(b*x + a) + 1)) - log(abs(sin(b*x + a) - 1)) - 2*sin(b*x +
a))/b
```

3.63 $\int \sec(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tan(a+bx)\sec(a+bx)}{2b}-\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

```
[Out] -ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rubi [A] time = 0.0230989, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$\frac{\tan(a+bx)\sec(a+bx)}{2b}-\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(a+bx)\tan^2(a+bx)\,dx &= \frac{\sec(a+bx)\tan(a+bx)}{2b}-\frac{1}{2}\int \sec(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\sec(a+bx)\tan(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0141463, size = 34, normalized size = 1.

$$\frac{\tan(a+bx)\sec(a+bx)}{2b}-\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Maple [A] time = 0.019, size = 53, normalized size = 1.6

$$\frac{(\sin(bx+a))^3}{2b(\cos(bx+a))^2}+\frac{\sin(bx+a)}{2b}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/2/b*sin(b*x+a)^3/cos(b*x+a)^2+1/2*sin(b*x+a)/b-1/2/b*ln(sec(b*x+a)+tan(b*
x+a))
```

Maxima [A] time = 0.979486, size = 62, normalized size = 1.82

$$-\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1}+\log(\sin(bx+a)+1)-\log(\sin(bx+a)-1)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin
(b*x + a) - 1))/b
```

Fricas [B] time = 1.63399, size = 163, normalized size = 4.79

$$-\frac{\cos(bx+a)^2\log(\sin(bx+a)+1)-\cos(bx+a)^2\log(-\sin(bx+a)+1)-2\sin(bx+a)}{4b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(cos(b*x + a)^2*log(sin(b*x + a) + 1) - cos(b*x + a)^2*log(-sin(b*x +
a) + 1) - 2*sin(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sin^2(a+bx)\sec^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)**3, x)
```

Giac [A] time = 1.29343, size = 65, normalized size = 1.91

$$-\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1}+\log\left(|\sin\left(bx+a\right)+1|\right)-\log\left(|\sin\left(bx+a\right)-1|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(abs(sin(b*x + a) + 1)) - lo
g(abs(sin(b*x + a) - 1)))/b
```

3.64 $\int \sec^3(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=55

$$-\frac{\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}-\frac{\tan(a+bx)\sec(a+bx)}{8b}$$

```
[Out] -ArcTanh[Sin[a + b*x]]/(8*b) - (Sec[a + b*x]*Tan[a + b*x])/(8*b) + (Sec[a +
 b*x]^3*Tan[a + b*x])/(4*b)
```

Rubi [A] time = 0.0448754, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2611, 3768, 3770}

$$-\frac{\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}-\frac{\tan(a+bx)\sec(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*Tan[a + b*x]^2,x]
```

```
[Out] -ArcTanh[Sin[a + b*x]]/(8*b) - (Sec[a + b*x]*Tan[a + b*x])/(8*b) + (Sec[a +
 b*x]^3*Tan[a + b*x])/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^3(a+bx)\tan^2(a+bx)\,dx &= \frac{\sec^3(a+bx)\tan(a+bx)}{4b}-\frac{1}{4}\int \sec^3(a+bx)\,dx\\ &= -\frac{\sec(a+bx)\tan(a+bx)}{8b}+\frac{\sec^3(a+bx)\tan(a+bx)}{4b}-\frac{1}{8}\int \sec(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\sin(a+bx))}{8b}-\frac{\sec(a+bx)\tan(a+bx)}{8b}+\frac{\sec^3(a+bx)\tan(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0447106, size = 55, normalized size = 1.

$$-\frac{\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}-\frac{\tan(a+bx)\sec(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*Tan[a + b*x]^2,x]
```

```
[Out] -ArcTanh[Sin[a + b*x]]/(8*b) - (Sec[a + b*x]*Tan[a + b*x])/(8*b) + (Sec[a +
 b*x]^3*Tan[a + b*x])/(4*b)
```

Maple [A] time = 0.021, size = 74, normalized size = 1.4

$$\frac{(\sin(bx+a))^3}{4b(\cos(bx+a))^4}+\frac{(\sin(bx+a))^3}{8b(\cos(bx+a))^2}+\frac{\sin(bx+a)}{8b}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5*sin(b*x+a)^2,x)
```

```
[Out] 1/4/b*sin(b*x+a)^3/cos(b*x+a)^4+1/8/b*sin(b*x+a)^3/cos(b*x+a)^2+1/8*sin(b*x
+a)/b-1/8/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 1.04326, size = 88, normalized size = 1.6

$$\frac{\frac{2\left(\sin(bx+a)^3+\sin(bx+a)\right)}{\sin(bx+a)^4-2\sin(bx+a)^2+1}-\log(\sin(bx+a)+1)+\log(\sin(bx+a)-1)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(sin(b*x + a)^3 + sin(b*x + a))/(sin(b*x + a)^4 - 2*sin(b*x + a)^2
+ 1) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b
```

Fricas [A] time = 1.70726, size = 193, normalized size = 3.51

$$-\frac{\cos(bx+a)^4\log(\sin(bx+a)+1)-\cos(bx+a)^4\log(-\sin(bx+a)+1)+2\left(\cos(bx+a)^2-2\right)\sin(bx+a)}{16b\cos(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(cos(b*x + a)^4*log(sin(b*x + a) + 1) - cos(b*x + a)^4*log(-sin(b*x +
 a) + 1) + 2*(cos(b*x + a)^2 - 2)*sin(b*x + a))/(b*cos(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25782, size = 111, normalized size = 2.02

$$\frac{\frac{4\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)}{\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)^2-4} - \log\left(\left|\frac{1}{\sin(bx+a)} + \sin(bx+a) + 2\right|\right) + \log\left(\left|\frac{1}{\sin(bx+a)} + \sin(bx+a) - 2\right|\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/32*(4*(1/sin(b*x + a) + sin(b*x + a))/((1/sin(b*x + a) + sin(b*x + a))^2
- 4) - log(abs(1/sin(b*x + a) + sin(b*x + a) + 2)) + log(abs(1/sin(b*x + a)
 + sin(b*x + a) - 2)))/b
```

3.65 $\int \sec^5(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=76

$$-\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}-\frac{\tan(a+bx)\sec^3(a+bx)}{24b}-\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

```
[Out] -ArcTanh[Sin[a + b*x]]/(16*b) - (Sec[a + b*x]*Tan[a + b*x])/(16*b) - (Sec[a
 + b*x]^3*Tan[a + b*x])/(24*b) + (Sec[a + b*x]^5*Tan[a + b*x])/(6*b)
```

Rubi [A] time = 0.0594764, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2611, 3768, 3770}

$$-\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}-\frac{\tan(a+bx)\sec^3(a+bx)}{24b}-\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5*Tan[a + b*x]^2,x]
```

```
[Out] -ArcTanh[Sin[a + b*x]]/(16*b) - (Sec[a + b*x]*Tan[a + b*x])/(16*b) - (Sec[a
 + b*x]^3*Tan[a + b*x])/(24*b) + (Sec[a + b*x]^5*Tan[a + b*x])/(6*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^5(a+bx)\tan^2(a+bx)\,dx &= \frac{\sec^5(a+bx)\tan(a+bx)}{6b}-\frac{1}{6}\int \sec^5(a+bx)\,dx\\ &= -\frac{\sec^3(a+bx)\tan(a+bx)}{24b}+\frac{\sec^5(a+bx)\tan(a+bx)}{6b}-\frac{1}{8}\int \sec^3(a+bx)\,dx\\ &= -\frac{\sec(a+bx)\tan(a+bx)}{16b}-\frac{\sec^3(a+bx)\tan(a+bx)}{24b}+\frac{\sec^5(a+bx)\tan(a+bx)}{6b}-\\ &= -\frac{\tanh^{-1}(\sin(a+bx))}{16b}-\frac{\sec(a+bx)\tan(a+bx)}{16b}-\frac{\sec^3(a+bx)\tan(a+bx)}{24b}+\frac{\sec}{}\end{aligned}$$

Mathematica [A] time = 0.064702, size = 76, normalized size = 1.

$$-\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}-\frac{\tan(a+bx)\sec^3(a+bx)}{24b}-\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5*Tan[a + b*x]^2,x]
```

```
[Out] -ArcTanh[Sin[a + b*x]]/(16*b) - (Sec[a + b*x]*Tan[a + b*x])/(16*b) - (Sec[a
 + b*x]^3*Tan[a + b*x])/(24*b) + (Sec[a + b*x]^5*Tan[a + b*x])/(6*b)
```

Maple [A] time = 0.023, size = 95, normalized size = 1.3

$$\frac{(\sin(bx+a))^3}{6b(\cos(bx+a))^6}+\frac{(\sin(bx+a))^3}{8b(\cos(bx+a))^4}+\frac{(\sin(bx+a))^3}{16b(\cos(bx+a))^2}+\frac{\sin(bx+a)}{16b}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^7*sin(b*x+a)^2,x)
```

```
[Out] 1/6/b*sin(b*x+a)^3/cos(b*x+a)^6+1/8/b*sin(b*x+a)^3/cos(b*x+a)^4+1/16/b*sin(
b*x+a)^3/cos(b*x+a)^2+1/16*sin(b*x+a)/b-1/16/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 1.00015, size = 123, normalized size = 1.62

$$\frac{\frac{2\left(3\sin(bx+a)^5-8\sin(bx+a)^3-3\sin(bx+a)\right)}{\sin(bx+a)^6-3\sin(bx+a)^4+3\sin(bx+a)^2-1}-3\log(\sin(bx+a)+1)+3\log(\sin(bx+a)-1)}{96b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(2*(3*sin(b*x + a)^5 - 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x + a
)^6 - 3*sin(b*x + a)^4 + 3*sin(b*x + a)^2 - 1) - 3*log(sin(b*x + a) + 1) +
3*log(sin(b*x + a) - 1))/b
```

Fricas [A] time = 1.73883, size = 227, normalized size = 2.99

$$-\frac{3\cos(bx+a)^6\log(\sin(bx+a)+1)-3\cos(bx+a)^6\log(-\sin(bx+a)+1)+2\left(3\cos(bx+a)^4+2\cos(bx+a)^2-8\right.}{96b\cos(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/96*(3*cos(b*x + a)^6*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^6*log(-sin(b
*x + a) + 1) + 2*(3*cos(b*x + a)^4 + 2*cos(b*x + a)^2 - 8)*sin(b*x + a))/(b
*cos(b*x + a)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**7*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2256, size = 99, normalized size = 1.3

$$\frac{\frac{2\left(3\sin(bx+a)^5-8\sin(bx+a)^3-3\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^3}-3\log\left(|\sin(bx+a)+1|\right)+3\log\left(|\sin(bx+a)-1|\right)}{96b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/96*(2*(3*sin(b*x + a)^5 - 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x + a
)^2 - 1)^3 - 3*log(abs(sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1)))/b
```

3.66 $\int \cos^5(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\cos^8(a+bx)}{8b}-\frac{\cos^6(a+bx)}{6b}$$

```
[Out] -Cos[a + b*x]^6/(6*b) + Cos[a + b*x]^8/(8*b)
```

Rubi [A] time = 0.0334502, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 14}

$$\frac{\cos^8(a+bx)}{8b}-\frac{\cos^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*Sin[a + b*x]^3,x]

[Out] -Cos[a + b*x]^6/(6*b) + Cos[a + b*x]^8/(8*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^5(a+bx)\sin^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^5\left(1-x^2\right)dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(x^5-x^7\right)dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos^6(a+bx)}{6b}+\frac{\cos^8(a+bx)}{8b}\end{aligned}$$

Mathematica [A] time = 0.129495, size = 48, normalized size = 1.55

$$\frac{-72\cos(2(a+bx))-12\cos(4(a+bx))+8\cos(6(a+bx))+3\cos(8(a+bx))}{3072b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*Sin[a + b*x]^3,x]
```

```
[Out] (-72*Cos[2*(a + b*x)] - 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] + 3*Cos[8*
(a + b*x)])/(3072*b)
```

Maple [A] time = 0.01, size = 34, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^6(\sin(bx+a))^2}{8}-\frac{(\cos(bx+a))^6}{24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/8*cos(b*x+a)^6*sin(b*x+a)^2-1/24*cos(b*x+a)^6)
```

Maxima [A] time = 0.970098, size = 49, normalized size = 1.58

$$\frac{3\sin(bx+a)^8-8\sin(bx+a)^6+6\sin(bx+a)^4}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*sin(b*x + a)^8 - 8*sin(b*x + a)^6 + 6*sin(b*x + a)^4)/b
```

Fricas [A] time = 1.69465, size = 62, normalized size = 2.

$$\frac{3\cos(bx+a)^8-4\cos(bx+a)^6}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

Sympy [A] time = 12.2605, size = 63, normalized size = 2.03

$$\begin{cases}\frac{\sin^8(a+bx)}{24b}+\frac{\sin^6(a+bx)\cos^2(a+bx)}{6b}+\frac{\sin^4(a+bx)\cos^4(a+bx)}{4b} & \text{for } b\neq 0\\ x\sin^3(a)\cos^5(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((sin(a + b*x)**8/(24*b) + sin(a + b*x)**6*cos(a + b*x)**2/(6*b) +
 sin(a + b*x)**4*cos(a + b*x)**4/(4*b), Ne(b, 0)), (x*sin(a)**3*cos(a)**5,
True))
```

Giac [A] time = 1.16947, size = 36, normalized size = 1.16

$$\frac{\cos(bx+a)^8}{8\,b} - \frac{\cos(bx+a)^6}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*cos(b*x + a)^8/b - 1/6*cos(b*x + a)^6/b
```

3.67 $\int \cos^4(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\cos^7(a+bx)}{7b}-\frac{\cos^5(a+bx)}{5b}$$

```
[Out] -Cos[a + b*x]^5/(5*b) + Cos[a + b*x]^7/(7*b)
```

Rubi [A] time = 0.034393, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 14}

$$\frac{\cos^7(a+bx)}{7b}-\frac{\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Sin[a + b*x]^3,x]
```

```
[Out] -Cos[a + b*x]^5/(5*b) + Cos[a + b*x]^7/(7*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^4(a+bx)\sin^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^4\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(x^4-x^6\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\cos^5(a+bx)}{5b}+\frac{\cos^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.0906027, size = 27, normalized size = 0.87

$$\frac{\cos^5(a+bx)(5\cos(2(a+bx))-9)}{70b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Sin[a + b*x]^3,x]
```

```
[Out] (Cos[a + b*x]^5*(-9 + 5*Cos[2*(a + b*x)]))/(70*b)
```

Maple [A] time = 0.013, size = 34, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^5(\sin(bx+a))^2}{7}-\frac{2(\cos(bx+a))^5}{35}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/7*cos(b*x+a)^5*sin(b*x+a)^2-2/35*cos(b*x+a)^5)
```

Maxima [A] time = 0.980987, size = 35, normalized size = 1.13

$$\frac{5\cos(bx+a)^7-7\cos(bx+a)^5}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

Fricas [A] time = 1.6658, size = 62, normalized size = 2.

$$\frac{5\cos(bx+a)^7-7\cos(bx+a)^5}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

Sympy [A] time = 7.05412, size = 46, normalized size = 1.48

$$\begin{cases} -\frac{\sin^2(a+bx)\cos^5(a+bx)}{5b}-\frac{2\cos^7(a+bx)}{35b} & \text{for } b\neq 0 \\ x\sin^3(a)\cos^4(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)**5/(5*b) - 2*cos(a + b*x)**7/(35*b
), Ne(b, 0)), (x*sin(a)**3*cos(a)**4, True))
```

Giac [A] time = 1.16608, size = 36, normalized size = 1.16

$$\frac{\cos(bx+a)^7}{7\,b}-\frac{\cos(bx+a)^5}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/7*cos(b*x + a)^7/b - 1/5*cos(b*x + a)^5/b
```

3.68 $\int \cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sin^4(a+bx)}{4b}-\frac{\sin^6(a+bx)}{6b}$$

```
[Out] Sin[a + b*x]^4/(4*b) - Sin[a + b*x]^6/(6*b)
```

Rubi [A] time = 0.0333805, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 14}

$$\frac{\sin^4(a+bx)}{4b}-\frac{\sin^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] Sin[a + b*x]^4/(4*b) - Sin[a + b*x]^6/(6*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sin^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^3\left(1-x^2\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^3-x^5\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\sin^4(a+bx)}{4b}-\frac{\sin^6(a+bx)}{6b}\end{aligned}$$

Mathematica [A] time = 0.0145893, size = 35, normalized size = 1.13

$$\frac{1}{8}\left(\frac{\cos(6(a+bx))}{24b}-\frac{3\cos(2(a+bx))}{8b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] ((-3*Cos[2*(a + b*x)])/(8*b) + Cos[6*(a + b*x)]/(24*b))/8
```

Maple [A] time = 0.01, size = 34, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^4(\sin(bx+a))^2}{6}-\frac{(\cos(bx+a))^4}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/6*cos(b*x+a)^4*sin(b*x+a)^2-1/12*cos(b*x+a)^4)
```

Maxima [A] time = 0.98458, size = 35, normalized size = 1.13

$$-\frac{2\sin(bx+a)^6-3\sin(bx+a)^4}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)/b
```

Fricas [A] time = 1.61483, size = 62, normalized size = 2.

$$\frac{2\cos(bx+a)^6-3\cos(bx+a)^4}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*cos(b*x + a)^6 - 3*cos(b*x + a)^4)/b
```

Sympy [A] time = 3.582, size = 42, normalized size = 1.35

$$\begin{cases}\frac{\sin^6(a+bx)}{12b}+\frac{\sin^4(a+bx)\cos^2(a+bx)}{4b} & \text{for } b\neq 0\\ x\sin^3(a)\cos^3(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((sin(a + b*x)**6/(12*b) + sin(a + b*x)**4*cos(a + b*x)**2/(4*b),
Ne(b, 0)), (x*sin(a)**3*cos(a)**3, True))
```

Giac [A] time = 1.23579, size = 35, normalized size = 1.13

$$-\frac{2\sin\left(bx+a\right)^{6}-3\sin\left(bx+a\right)^{4}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/12*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)/b
```

3.69 $\int \cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\cos^5(a+bx)}{5b}-\frac{\cos^3(a+bx)}{3b}$$

```
[Out] -Cos[a + b*x]^3/(3*b) + Cos[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0323095, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 14}

$$\frac{\cos^5(a+bx)}{5b}-\frac{\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -Cos[a + b*x]^3/(3*b) + Cos[a + b*x]^5/(5*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\cos^3(a+bx)}{3b}+\frac{\cos^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.061844, size = 27, normalized size = 0.87

$$\frac{\cos^3(a+bx)(3\cos(2(a+bx))-7)}{30b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (Cos[a + b*x]^3*(-7 + 3*Cos[2*(a + b*x)]))/(30*b)
```

Maple [A] time = 0.012, size = 34, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^3(\sin(bx+a))^2}{5}-\frac{2(\cos(bx+a))^3}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/5*cos(b*x+a)^3*sin(b*x+a)^2-2/15*cos(b*x+a)^3)
```

Maxima [A] time = 0.979482, size = 35, normalized size = 1.13

$$\frac{3\cos(bx+a)^5-5\cos(bx+a)^3}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/15*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)/b
```

Fricas [A] time = 1.638, size = 62, normalized size = 2.

$$\frac{3\cos(bx+a)^5-5\cos(bx+a)^3}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)/b
```

Sympy [A] time = 2.13542, size = 46, normalized size = 1.48

$$\begin{cases} -\frac{\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2\cos^5(a+bx)}{15b} & \text{for } b\neq 0 \\ x\sin^3(a)\cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*cos(a + b*x)**5/(15*b
), Ne(b, 0)), (x*sin(a)**3*cos(a)**2, True))
```

Giac [A] time = 1.24596, size = 36, normalized size = 1.16

$$\frac{\cos(bx+a)^5}{5\,b} - \frac{\cos(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/5*cos(b*x + a)^5/b - 1/3*cos(b*x + a)^3/b
```

3.70 $\int \cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sin^4(a+bx)}{4b}$$

```
[Out] Sin[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0176114, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2564, 30}

$$\frac{\sin^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] Sin[a + b*x]^4/(4*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^3\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0026198, size = 15, normalized size = 1.

$$\frac{\sin^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] Sin[a + b*x]^4/(4*b)
```

Maple [A] time = 0.003, size = 14, normalized size = 0.9

$$\frac{(\sin(bx+a))^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/4*sin(b*x+a)^4/b
```

Maxima [A] time = 0.976335, size = 18, normalized size = 1.2

$$\frac{\sin(bx+a)^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*sin(b*x + a)^4/b
```

Fricas [A] time = 1.55172, size = 58, normalized size = 3.87

$$\frac{\cos(bx+a)^4 - 2\cos(bx+a)^2}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(cos(b*x + a)^4 - 2*cos(b*x + a)^2)/b
```

Sympy [A] time = 0.979507, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sin^4(a+bx)}{4b} & \text{for } b \neq 0 \\ x\sin^3(a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((sin(a + b*x)**4/(4*b), Ne(b, 0)), (x*sin(a)**3*cos(a), True))
```

Giac [A] time = 1.21047, size = 18, normalized size = 1.2

$$\frac{\sin(bx+a)^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*sin(b*x + a)^4/b
```

3.71 $\int \sin^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{\cos^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}$$

```
[Out] Cos[a + b*x]^2/(2*b) - Log[Cos[a + b*x]]/b
```

Rubi [A] time = 0.0206091, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\cos^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] Cos[a + b*x]^2/(2*b) - Log[Cos[a + b*x]]/b
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\tan(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)dx, x, \cos(a+bx)\right)}{b}\\ &= \frac{\cos^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.0174835, size = 25, normalized size = 0.89

$$-\frac{\log(\cos(a+bx))-\frac{1}{2}\cos^2(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] -((-Cos[a + b*x]^2/2 + Log[Cos[a + b*x]])/b)
```

Maple [A] time = 0.016, size = 27, normalized size = 1.

$$-\frac{(\sin(bx+a))^2}{2b}-\frac{\ln(\cos(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] -1/2*sin(b*x+a)^2/b-ln(cos(b*x+a))/b
```

Maxima [A] time = 0.975397, size = 34, normalized size = 1.21

$$-\frac{\sin(bx+a)^2+\log\left(\sin(bx+a)^2-1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))/b
```

Fricas [A] time = 1.65688, size = 63, normalized size = 2.25

$$\frac{\cos(bx+a)^2-2\log(-\cos(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(b*x + a)^2 - 2*log(-cos(b*x + a)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20588, size = 39, normalized size = 1.39

$$\frac{\cos(bx+a)^2-\log\left(\frac{\cos(bx+a)^2}{b^2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(cos(b*x + a)^2 - log(cos(b*x + a)^2/b^2))/b
```

3.72 $\int \sin(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}$$

```
[Out] Cos[a + b*x]/b + Sec[a + b*x]/b
```

Rubi [A] time = 0.0209529, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] Cos[a + b*x]/b + Sec[a + b*x]/b
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\tan^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx,x,\cos(a+bx)\right)}{b}\\ &= \frac{\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0221629, size = 21, normalized size = 1.

$$\frac{\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] Cos[a + b*x]/b + Sec[a + b*x]/b
```

Maple [A] time = 0.014, size = 40, normalized size = 1.9

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^4}{\cos(bx+a)}+\left(2+(\sin(bx+a))^2\right)\cos(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(sin(b*x+a)^4/cos(b*x+a)+(2+sin(b*x+a)^2)*cos(b*x+a))
```

Maxima [A] time = 0.988198, size = 26, normalized size = 1.24

$$\frac{\frac{1}{\cos(bx+a)}+\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] (1/cos(b*x + a) + cos(b*x + a))/b
```

Fricas [A] time = 1.57206, size = 53, normalized size = 2.52

$$\frac{\cos(bx+a)^2+1}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (cos(b*x + a)^2 + 1)/(b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1924, size = 31, normalized size = 1.48

$$\frac{\cos(bx+a)}{b} + \frac{1}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] cos(b*x + a)/b + 1/(b*cos(b*x + a))
```

3.73 $\int \tan^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\tan^2(a+bx)}{2b}+\frac{\log(\cos(a+bx))}{b}$$

```
[Out] Log[Cos[a + b*x]]/b + Tan[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0111813, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$\frac{\tan^2(a+bx)}{2b}+\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*x]^3,x]
```

```
[Out] Log[Cos[a + b*x]]/b + Tan[a + b*x]^2/(2*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^3(a+bx)\,dx &= \frac{\tan^2(a+bx)}{2b}-\int \tan(a+bx)\,dx\\ &= \frac{\log(\cos(a+bx))}{b}+\frac{\tan^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.024339, size = 25, normalized size = 0.93

$$\frac{\tan^2(a+bx)+2\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*x]^3,x]
```

```
[Out] (2*Log[Cos[a + b*x]] + Tan[a + b*x]^2)/(2*b)
```

Maple [A] time = 0.019, size = 26, normalized size = 1.

$$\frac{\ln\left(\cos\left(bx+a\right)\right)}{b}+\frac{\left(\tan\left(bx+a\right)\right)^{2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] ln(cos(b*x+a))/b+1/2*tan(b*x+a)^2/b
```

Maxima [A] time = 1.00059, size = 42, normalized size = 1.56

$$-\frac{\frac{1}{\sin\left(bx+a\right)^{2}-1}-\log\left(\sin\left(bx+a\right)^{2}-1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b
```

Fricas [A] time = 1.63733, size = 89, normalized size = 3.3

$$\frac{2\cos\left(bx+a\right)^{2}\log\left(-\cos\left(bx+a\right)\right)+1}{2b\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cos(b*x + a)^2*log(-cos(b*x + a)) + 1)/(b*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18609, size = 57, normalized size = 2.11

$$\frac{\log\left(\frac{\cos\left(bx+a\right)^{2}}{b^{2}}\right)}{2b}-\frac{\cos\left(bx+a\right)^{2}-1}{2b\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(cos(b*x + a)^2/b^2)/b - 1/2*(cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^2)
```

3.74 $\int \sec(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sec^3(a+bx)}{3b}-\frac{\sec(a+bx)}{b}$$

```
[Out] -(Sec[a + b*x]/b) + Sec[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0200087, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2606}

$$\frac{\sec^3(a+bx)}{3b}-\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Tan[a + b*x]^3,x]
```

```
[Out] -(Sec[a + b*x]/b) + Sec[a + b*x]^3/(3*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec(a+bx)\tan^3(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= -\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0233389, size = 27, normalized size = 1.

$$\frac{\sec^3(a+bx)}{3b}-\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Tan[a + b*x]^3,x]
```

```
[Out] -(Sec[a + b*x]/b) + Sec[a + b*x]^3/(3*b)
```

Maple [B] time = 0.02, size = 60, normalized size = 2.2

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^4}{3\,(\cos(bx+a))^3}-\frac{(\sin(bx+a))^4}{3\,\cos(bx+a)}-\frac{\left(2+(\sin(bx+a))^2\right)\cos(bx+a)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/3*sin(b*x+a)^4/cos(b*x+a)^3-1/3*sin(b*x+a)^4/cos(b*x+a)-1/3*(2+sin(b
*x+a)^2)*cos(b*x+a))
```

Maxima [A] time = 1.01126, size = 34, normalized size = 1.26

$$-\frac{3\cos\left(bx+a\right)^{2}-1}{3\,b\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3*(3*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

Fricas [A] time = 1.55351, size = 65, normalized size = 2.41

$$-\frac{3\cos\left(bx+a\right)^{2}-1}{3\,b\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16811, size = 34, normalized size = 1.26

$$-\frac{3\cos\left(bx+a\right)^{2}-1}{3\,b\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

3.75 $\int \sec^2(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\tan^4(a+bx)}{4b}$$

```
[Out] Tan[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0278485, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$\frac{\tan^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2*Tan[a + b*x]^3,x]
```

```
[Out] Tan[a + b*x]^4/(4*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)\tan^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^3\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\tan^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0053825, size = 15, normalized size = 1.

$$\frac{\tan^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2*Tan[a + b*x]^3,x]
```

```
[Out] Tan[a + b*x]^4/(4*b)
```

Maple [A] time = 0.019, size = 22, normalized size = 1.5

$$\frac{(\sin(bx+a))^4}{4\,b\,(\cos(bx+a))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5*sin(b*x+a)^3,x)
```

```
[Out] 1/4/b*sin(b*x+a)^4/cos(b*x+a)^4
```

Maxima [B] time = 0.978591, size = 53, normalized size = 3.53

$$\frac{2\,\sin(bx+a)^2-1}{4\left(\sin(bx+a)^4-2\,\sin(bx+a)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sin(b*x + a)^2 - 1)/((sin(b*x + a)^4 - 2*sin(b*x + a)^2 + 1)*b)
```

Fricas [A] time = 1.59917, size = 65, normalized size = 4.33

$$-\frac{2\,\cos(bx+a)^2-1}{4\,b\cos(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19368, size = 34, normalized size = 2.27

$$-\frac{2\,\cos(bx+a)^2-1}{4\,b\cos(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4)
```

3.76 $\int \sec^3(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sec^5(a+bx)}{5b}-\frac{\sec^3(a+bx)}{3b}$$

```
[Out] -Sec[a + b*x]^3/(3*b) + Sec[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.032495, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 14}

$$\frac{\sec^5(a+bx)}{5b}-\frac{\sec^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*Tan[a + b*x]^3,x]
```

```
[Out] -Sec[a + b*x]^3/(3*b) + Sec[a + b*x]^5/(5*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sec^3(a+bx)\tan^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= -\frac{\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0479105, size = 31, normalized size = 1.

$$\frac{\sec^5(a+bx)}{5b}-\frac{\sec^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*Tan[a + b*x]^3,x]
```

```
[Out] -Sec[a + b*x]^3/(3*b) + Sec[a + b*x]^5/(5*b)
```

Maple [B] time = 0.019, size = 78, normalized size = 2.5

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^4}{5(\cos(bx+a))^5}+\frac{(\sin(bx+a))^4}{15(\cos(bx+a))^3}-\frac{(\sin(bx+a))^4}{15\cos(bx+a)}-\frac{\left(2+(\sin(bx+a))^2\right)\cos(bx+a)}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/5*sin(b*x+a)^4/cos(b*x+a)^5+1/15*sin(b*x+a)^4/cos(b*x+a)^3-1/15*sin(
b*x+a)^4/cos(b*x+a)-1/15*(2+sin(b*x+a)^2)*cos(b*x+a))
```

Maxima [A] time = 0.976277, size = 34, normalized size = 1.1

$$-\frac{5\cos(bx+a)^2-3}{15\,b\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(5*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^5)
```

Fricas [A] time = 1.60811, size = 66, normalized size = 2.13

$$-\frac{5\cos(bx+a)^2-3}{15\,b\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1863, size = 34, normalized size = 1.1

$$-\frac{5 \cos(bx+a)^2 - 3}{15 b \cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/15*(5*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^5)
```

3.77 $\int \sec^4(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sec^6(a+bx)}{6b}-\frac{\sec^4(a+bx)}{4b}$$

```
[Out] -Sec[a + b*x]^4/(4*b) + Sec[a + b*x]^6/(6*b)
```

Rubi [A] time = 0.0325137, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 14}

$$\frac{\sec^6(a+bx)}{6b}-\frac{\sec^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4*Tan[a + b*x]^3,x]
```

```
[Out] -Sec[a + b*x]^4/(4*b) + Sec[a + b*x]^6/(6*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sec^4(a+bx)\tan^3(a+bx)\,dx&=\frac{\text{Subst}\left(\int x^3\left(-1+x^2\right)\,dx,x,\sec(a+bx)\right)}{b}\\&=\frac{\text{Subst}\left(\int\left(-x^3+x^5\right)\,dx,x,\sec(a+bx)\right)}{b}\\&=-\frac{\sec^4(a+bx)}{4b}+\frac{\sec^6(a+bx)}{6b}\end{aligned}$$

Mathematica [A] time = 0.0354476, size = 28, normalized size = 0.9

$$-\frac{3\sec^4(a+bx)-2\sec^6(a+bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4*Tan[a + b*x]^3,x]
```

```
[Out] -(3*Sec[a + b*x]^4 - 2*Sec[a + b*x]^6)/(12*b)
```

Maple [A] time = 0.02, size = 42, normalized size = 1.4

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^4}{6\,(\cos(bx+a))^6}+\frac{(\sin(bx+a))^4}{12\,(\cos(bx+a))^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^7*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/6*sin(b*x+a)^4/cos(b*x+a)^6+1/12*sin(b*x+a)^4/cos(b*x+a)^4)
```

Maxima [A] time = 0.99536, size = 66, normalized size = 2.13

$$-\frac{3\,\sin(bx+a)^2-1}{12\left(\sin(bx+a)^6-3\,\sin(bx+a)^4+3\,\sin(bx+a)^2-1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*sin(b*x + a)^2 - 1)/((sin(b*x + a)^6 - 3*sin(b*x + a)^4 + 3*sin(b*
x + a)^2 - 1)*b)
```

Fricas [A] time = 1.57463, size = 66, normalized size = 2.13

$$-\frac{3\,\cos(bx+a)^2-2}{12\,b\cos(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**7*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18971, size = 34, normalized size = 1.1

$$-\frac{3\cos(bx+a)^2-2}{12\,b\cos(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^6)
```

3.78 $\int \sec^5(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sec^7(a+bx)}{7b}-\frac{\sec^5(a+bx)}{5b}$$

```
[Out] -Sec[a + b*x]^5/(5*b) + Sec[a + b*x]^7/(7*b)
```

Rubi [A] time = 0.0321572, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 14}

$$\frac{\sec^7(a+bx)}{7b}-\frac{\sec^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5*Tan[a + b*x]^3,x]
```

```
[Out] -Sec[a + b*x]^5/(5*b) + Sec[a + b*x]^7/(7*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sec^5(a+bx)\tan^3(a+bx)\,dx&=\frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)dx,x,\sec(a+bx)\right)}{b}\\&=\frac{\text{Subst}\left(\int\left(-x^4+x^6\right)dx,x,\sec(a+bx)\right)}{b}\\&=-\frac{\sec^5(a+bx)}{5b}+\frac{\sec^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.0282945, size = 31, normalized size = 1.

$$\frac{\sec^7(a+bx)}{7b}-\frac{\sec^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5*Tan[a + b*x]^3,x]
```

```
[Out] -Sec[a + b*x]^5/(5*b) + Sec[a + b*x]^7/(7*b)
```

Maple [B] time = 0.021, size = 96, normalized size = 3.1

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^4}{7(\cos(bx+a))^7}+\frac{3(\sin(bx+a))^4}{35(\cos(bx+a))^5}+\frac{(\sin(bx+a))^4}{35(\cos(bx+a))^3}-\frac{(\sin(bx+a))^4}{35\cos(bx+a)}-\frac{\left(2+(\sin(bx+a))^2\right)\cos(bx+a)}{35}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^8*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/7*sin(b*x+a)^4/cos(b*x+a)^7+3/35*sin(b*x+a)^4/cos(b*x+a)^5+1/35*sin(
b*x+a)^4/cos(b*x+a)^3-1/35*sin(b*x+a)^4/cos(b*x+a)-1/35*(2+sin(b*x+a)^2)*co
s(b*x+a))
```

Maxima [A] time = 1.01664, size = 34, normalized size = 1.1

$$-\frac{7\cos(bx+a)^2-5}{35\,b\cos(bx+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/35*(7*cos(b*x + a)^2 - 5)/(b*cos(b*x + a)^7)
```

Fricas [A] time = 1.67524, size = 66, normalized size = 2.13

$$-\frac{7\cos(bx+a)^2-5}{35\,b\cos(bx+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/35*(7*cos(b*x + a)^2 - 5)/(b*cos(b*x + a)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**8*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17587, size = 34, normalized size = 1.1

$$-\frac{7\cos(bx+a)^2-5}{35\,b\cos(bx+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/35*(7*cos(b*x + a)^2 - 5)/(b*cos(b*x + a)^7)
```

3.79 $\int \sec^6(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sec^8(a+bx)}{8b}-\frac{\sec^6(a+bx)}{6b}$$

```
[Out] -Sec[a + b*x]^6/(6*b) + Sec[a + b*x]^8/(8*b)
```

Rubi [A] time = 0.0321571, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 14}

$$\frac{\sec^8(a+bx)}{8b}-\frac{\sec^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6*Tan[a + b*x]^3,x]
```

```
[Out] -Sec[a + b*x]^6/(6*b) + Sec[a + b*x]^8/(8*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sec^6(a+bx)\tan^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^5\left(-1+x^2\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-x^5+x^7\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= -\frac{\sec^6(a+bx)}{6b}+\frac{\sec^8(a+bx)}{8b}\end{aligned}$$

Mathematica [A] time = 0.0415676, size = 28, normalized size = 0.9

$$-\frac{4\sec^6(a+bx)-3\sec^8(a+bx)}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6*Tan[a + b*x]^3,x]
```

```
[Out] -(4*Sec[a + b*x]^6 - 3*Sec[a + b*x]^8)/(24*b)
```

Maple [B] time = 0.02, size = 60, normalized size = 1.9

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^4}{8(\cos(bx+a))^8}+\frac{(\sin(bx+a))^4}{12(\cos(bx+a))^6}+\frac{(\sin(bx+a))^4}{24(\cos(bx+a))^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^9*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/8*sin(b*x+a)^4/cos(b*x+a)^8+1/12*sin(b*x+a)^4/cos(b*x+a)^6+1/24*sin(
b*x+a)^4/cos(b*x+a)^4)
```

Maxima [B] time = 1.03137, size = 80, normalized size = 2.58

$$\frac{4\sin(bx+a)^2-1}{24\left(\sin(bx+a)^8-4\sin(bx+a)^6+6\sin(bx+a)^4-4\sin(bx+a)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(4*sin(b*x + a)^2 - 1)/((sin(b*x + a)^8 - 4*sin(b*x + a)^6 + 6*sin(b*x
 + a)^4 - 4*sin(b*x + a)^2 + 1)*b)
```

Fricas [A] time = 1.6317, size = 66, normalized size = 2.13

$$-\frac{4\cos(bx+a)^2-3}{24\,b\cos(bx+a)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(4*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**9*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2006, size = 34, normalized size = 1.1

$$-\frac{4\cos\left(bx+a\right)^{2}-3}{24\,b\cos\left(bx+a\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/24*(4*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^8)
```

3.80 $\int \cos^7(a+bx)\sin^4(a+bx)\,dx$

Optimal. Leaf size=61

$$-\frac{\sin^{11}(a+bx)}{11b}+\frac{\sin^9(a+bx)}{3b}-\frac{3\sin^7(a+bx)}{7b}+\frac{\sin^5(a+bx)}{5b}$$

```
[Out] Sin[a + b*x]^5/(5*b) - (3*Sin[a + b*x]^7)/(7*b) + Sin[a + b*x]^9/(3*b) - Si
n[a + b*x]^11/(11*b)
```

Rubi [A] time = 0.039662, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 270}

$$-\frac{\sin^{11}(a+bx)}{11b}+\frac{\sin^9(a+bx)}{3b}-\frac{3\sin^7(a+bx)}{7b}+\frac{\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^7*Sin[a + b*x]^4,x]
```

```
[Out] Sin[a + b*x]^5/(5*b) - (3*Sin[a + b*x]^7)/(7*b) + Sin[a + b*x]^9/(3*b) - Si
n[a + b*x]^11/(11*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^7(a+bx)\sin^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^4\left(1-x^2\right)^3\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^4-3x^6+3x^8-x^{10}\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\sin^5(a+bx)}{5b}-\frac{3\sin^7(a+bx)}{7b}+\frac{\sin^9(a+bx)}{3b}-\frac{\sin^{11}(a+bx)}{11b}\end{aligned}$$

Mathematica [A] time = 0.200549, size = 47, normalized size = 0.77

$$\frac{\sin^5(a+bx)(3335\cos(2(a+bx))+910\cos(4(a+bx))+105\cos(6(a+bx))+3042)}{36960b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^7*Sin[a + b*x]^4,x]
```

```
[Out] ((3042 + 3335*Cos[2*(a + b*x)] + 910*Cos[4*(a + b*x)] + 105*Cos[6*(a + b*x)
])*Sin[a + b*x]^5)/(36960*b)
```

Maple [A] time = 0.011, size = 78, normalized size = 1.3

$$\frac{1}{b}\left(-\frac{(\sin(bx+a))^3(\cos(bx+a))^8}{11}-\frac{\sin(bx+a)(\cos(bx+a))^8}{33}+\frac{\sin(bx+a)}{231}\left(\frac{16}{5}+(\cos(bx+a))^6+\frac{6(\cos(bx+a))^4}{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/11*sin(b*x+a)^3*cos(b*x+a)^8-1/33*sin(b*x+a)*cos(b*x+a)^8+1/231*(16
/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [A] time = 0.982321, size = 62, normalized size = 1.02

$$-\frac{105\sin(bx+a)^{11}-385\sin(bx+a)^9+495\sin(bx+a)^7-231\sin(bx+a)^5}{1155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/1155*(105*sin(b*x + a)^11 - 385*sin(b*x + a)^9 + 495*sin(b*x + a)^7 - 23
1*sin(b*x + a)^5)/b
```

Fricas [A] time = 1.71904, size = 173, normalized size = 2.84

$$\frac{\left(105\cos(bx+a)^{10}-140\cos(bx+a)^8+5\cos(bx+a)^6+6\cos(bx+a)^4+8\cos(bx+a)^2+16\right)\sin(bx+a)}{1155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/1155*(105*cos(b*x + a)^10 - 140*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*cos
(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x + a)/b
```

Sympy [A] time = 59.6811, size = 88, normalized size = 1.44

$$\begin{cases} \frac{16\sin^{11}(a+bx)}{1155b}+\frac{8\sin^9(a+bx)\cos^2(a+bx)}{105b}+\frac{6\sin^7(a+bx)\cos^4(a+bx)}{35b}+\frac{\sin^5(a+bx)\cos^6(a+bx)}{5b} & \text{for } b\neq 0 \\ x\sin^4(a)\cos^7(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((16*sin(a + b*x)**11/(1155*b) + 8*sin(a + b*x)**9*cos(a + b*x)**2
/(105*b) + 6*sin(a + b*x)**7*cos(a + b*x)**4/(35*b) + sin(a + b*x)**5*cos(a
 + b*x)**6/(5*b), Ne(b, 0)), (x*sin(a)**4*cos(a)**7, True))
```

Giac [A] time = 1.16777, size = 111, normalized size = 1.82

$$\frac{\sin(11\,bx+11\,a)}{11264\,b}+\frac{\sin(9\,bx+9\,a)}{3072\,b}-\frac{\sin(7\,bx+7\,a)}{7168\,b}-\frac{11\,\sin(5\,bx+5\,a)}{5120\,b}-\frac{\sin(3\,bx+3\,a)}{512\,b}+\frac{7\,\sin(bx+a)}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/11264*sin(11*b*x + 11*a)/b + 1/3072*sin(9*b*x + 9*a)/b - 1/7168*sin(7*b*x
 + 7*a)/b - 11/5120*sin(5*b*x + 5*a)/b - 1/512*sin(3*b*x + 3*a)/b + 7/512*s
in(b*x + a)/b
```

3.81 $\int \cos^5(a+bx)\sin^4(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sin^9(a+bx)}{9b}-\frac{2\sin^7(a+bx)}{7b}+\frac{\sin^5(a+bx)}{5b}$$

```
[Out] Sin[a + b*x]^5/(5*b) - (2*Sin[a + b*x]^7)/(7*b) + Sin[a + b*x]^9/(9*b)
```

Rubi [A] time = 0.0356168, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 270}

$$\frac{\sin^9(a+bx)}{9b}-\frac{2\sin^7(a+bx)}{7b}+\frac{\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*Sin[a + b*x]^4,x]
```

```
[Out] Sin[a + b*x]^5/(5*b) - (2*Sin[a + b*x]^7)/(7*b) + Sin[a + b*x]^9/(9*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(a+bx)\sin^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^4\left(1-x^2\right)^2\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^5(a+bx)}{5b}-\frac{2\sin^7(a+bx)}{7b}+\frac{\sin^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] time = 0.110989, size = 37, normalized size = 0.8

$$\frac{\sin^5(a+bx)(220\cos(2(a+bx))+35\cos(4(a+bx))+249)}{2520b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*Sin[a + b*x]^4,x]
```

```
[Out] ((249 + 220*Cos[2*(a + b*x)] + 35*Cos[4*(a + b*x)])*Sin[a + b*x]^5)/(2520*b
)
```

Maple [A] time = 0.013, size = 68, normalized size = 1.5

$$\frac{1}{b}\left(-\frac{(\sin(bx+a))^3(\cos(bx+a))^6}{9}-\frac{\sin(bx+a)(\cos(bx+a))^6}{21}+\frac{\sin(bx+a)}{105}\left(\frac{8}{3}+(\cos(bx+a))^4+\frac{4(\cos(bx+a)}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/9*sin(b*x+a)^3*cos(b*x+a)^6-1/21*sin(b*x+a)*cos(b*x+a)^6+1/105*(8/3
+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [A] time = 0.990053, size = 49, normalized size = 1.07

$$\frac{35\sin(bx+a)^9-90\sin(bx+a)^7+63\sin(bx+a)^5}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/315*(35*sin(b*x + a)^9 - 90*sin(b*x + a)^7 + 63*sin(b*x + a)^5)/b
```

Fricas [A] time = 1.68277, size = 140, normalized size = 3.04

$$\frac{\left(35\cos(bx+a)^8-50\cos(bx+a)^6+3\cos(bx+a)^4+4\cos(bx+a)^2+8\right)\sin(bx+a)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(35*cos(b*x + a)^8 - 50*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b*x
 + a)^2 + 8)*sin(b*x + a)/b
```

Sympy [A] time = 21.5265, size = 66, normalized size = 1.43

$$\begin{cases}\frac{8\sin^9(a+bx)}{315b}+\frac{4\sin^7(a+bx)\cos^2(a+bx)}{35b}+\frac{\sin^5(a+bx)\cos^4(a+bx)}{5b} & \text{for } b\neq 0\\ x\sin^4(a)\cos^5(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((8*sin(a + b*x)**9/(315*b) + 4*sin(a + b*x)**7*cos(a + b*x)**2/(3
5*b) + sin(a + b*x)**5*cos(a + b*x)**4/(5*b), Ne(b, 0)), (x*sin(a)**4*cos(a
```

```
)**5, True))
```

Giac [A] time = 1.14985, size = 92, normalized size = 2.

$$\frac{\sin(9\,bx+9\,a)}{2304\,b}+\frac{\sin(7\,bx+7\,a)}{1792\,b}-\frac{\sin(5\,bx+5\,a)}{320\,b}-\frac{\sin(3\,bx+3\,a)}{192\,b}+\frac{3\,\sin(bx+a)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/2304*sin(9*b*x + 9*a)/b + 1/1792*sin(7*b*x + 7*a)/b - 1/320*sin(5*b*x + 5
*a)/b - 1/192*sin(3*b*x + 3*a)/b + 3/128*sin(b*x + a)/b
```

3.82 $\int \cos^3(a+bx)\sin^4(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sin^5(a+bx)}{5b}-\frac{\sin^7(a+bx)}{7b}$$

```
[Out] Sin[a + b*x]^5/(5*b) - Sin[a + b*x]^7/(7*b)
```

Rubi [A] time = 0.0324848, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 14}

$$\frac{\sin^5(a+bx)}{5b}-\frac{\sin^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[a + b*x]^4,x]
```

```
[Out] Sin[a + b*x]^5/(5*b) - Sin[a + b*x]^7/(7*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(a+bx)\sin^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^4\left(1-x^2\right)dx, x, \sin(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(x^4-x^6\right)dx, x, \sin(a+bx)\right)}{b}\\
&= \frac{\sin^5(a+bx)}{5b}-\frac{\sin^7(a+bx)}{7b}
\end{aligned}$$

Mathematica [A] time = 0.0697153, size = 27, normalized size = 0.87

$$\frac{\sin^5(a+bx)(5\cos(2(a+bx))+9)}{70b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[a + b*x]^4,x]
```

```
[Out] ((9 + 5*Cos[2*(a + b*x)])*Sin[a + b*x]^5)/(70*b)
```

Maple [B] time = 0.013, size = 58, normalized size = 1.9

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^4(\sin(bx+a))^3}{7}-\frac{3\sin(bx+a)(\cos(bx+a))^4}{35}+\frac{\left(2+(\cos(bx+a))^2\right)\sin(bx+a)}{35}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/7*cos(b*x+a)^4*sin(b*x+a)^3-3/35*sin(b*x+a)*cos(b*x+a)^4+1/35*(2+co
s(b*x+a)^2)*sin(b*x+a))
```

Maxima [A] time = 0.978355, size = 35, normalized size = 1.13

$$-\frac{5\sin(bx+a)^7-7\sin(bx+a)^5}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/35*(5*sin(b*x + a)^7 - 7*sin(b*x + a)^5)/b
```

Fricas [A] time = 1.6046, size = 108, normalized size = 3.48

$$\frac{\left(5\cos(bx+a)^6-8\cos(bx+a)^4+\cos(bx+a)^2+2\right)\sin(bx+a)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(b*x + a)^6 - 8*cos(b*x + a)^4 + cos(b*x + a)^2 + 2)*sin(b*x + a
)/b
```

Sympy [A] time = 7.48428, size = 44, normalized size = 1.42

$$\begin{cases}\frac{2\sin^7(a+bx)}{35b}+\frac{\sin^5(a+bx)\cos^2(a+bx)}{5b} & \text{for } b\neq 0\\ x\sin^4(a)\cos^3(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((2*sin(a + b*x)**7/(35*b) + sin(a + b*x)**5*cos(a + b*x)**2/(5*b)
, Ne(b, 0)), (x*sin(a)**4*cos(a)**3, True))
```

Giac [A] time = 1.15579, size = 35, normalized size = 1.13

$$-\frac{5 \sin(bx+a)^7 - 7 \sin(bx+a)^5}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/35*(5*sin(b*x + a)^7 - 7*sin(b*x + a)^5)/b
```

3.83 $\int \cos(a+bx)\sin^4(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sin^5(a+bx)}{5b}$$

```
[Out] Sin[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0176357, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2564, 30}

$$\frac{\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[a + b*x]^4,x]
```

```
[Out] Sin[a + b*x]^5/(5*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^4\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0030475, size = 15, normalized size = 1.

$$\frac{\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[a + b*x]^4,x]
```

```
[Out] Sin[a + b*x]^5/(5*b)
```

Maple [A] time = 0.003, size = 14, normalized size = 0.9

$$\frac{(\sin(bx+a))^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^4,x)
```

```
[Out] 1/5*sin(b*x+a)^5/b
```

Maxima [A] time = 0.986536, size = 18, normalized size = 1.2

$$\frac{\sin(bx+a)^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/5*sin(b*x + a)^5/b
```

Fricas [B] time = 1.56526, size = 81, normalized size = 5.4

$$\frac{\left(\cos(bx+a)^4 - 2\cos(bx+a)^2 + 1\right)\sin(bx+a)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sin(b*x + a)/b
```

Sympy [A] time = 2.02707, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sin^5(a+bx)}{5b} & \text{for } b \neq 0 \\ x\sin^4(a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((sin(a + b*x)**5/(5*b), Ne(b, 0)), (x*sin(a)**4*cos(a), True))
```

Giac [A] time = 1.14989, size = 18, normalized size = 1.2

$$\frac{\sin(bx+a)^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/5*sin(b*x + a)^5/b
```

3.84 $\int \sin^2(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{3\tan(a+bx)}{2b}-\frac{\sin^2(a+bx)\tan(a+bx)}{2b}-\frac{3x}{2}$$

```
[Out] (-3*x)/2 + (3*Tan[a + b*x])/(2*b) - (Sin[a + b*x]^2*Tan[a + b*x])/(2*b)
```

Rubi [A] time = 0.0387557, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2591, 288, 321, 203}

$$\frac{3\tan(a+bx)}{2b}-\frac{\sin^2(a+bx)\tan(a+bx)}{2b}-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Tan[a + b*x]^2,x]
```

```
[Out] (-3*x)/2 + (3*Tan[a + b*x])/(2*b) - (Sin[a + b*x]^2*Tan[a + b*x])/(2*b)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= -\frac{\sin^2(a+bx)\tan(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(a+bx)\right)}{2b}$$
$$= \frac{3\tan(a+bx)}{2b} - \frac{\sin^2(a+bx)\tan(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(a+bx)\right)}{2b}$$
$$= -\frac{3x}{2} + \frac{3\tan(a+bx)}{2b} - \frac{\sin^2(a+bx)\tan(a+bx)}{2b}$$

Mathematica [A] time = 0.113434, size = 31, normalized size = 0.78

$$\frac{-6(a+bx)+\sin(2(a+bx))+4\tan(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Tan[a + b*x]^2,x]
```

```
[Out] (-6*(a + b*x) + Sin[2*(a + b*x)] + 4*Tan[a + b*x])/(4*b)
```

Maple [A] time = 0.017, size = 54, normalized size = 1.4

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^5}{\cos(bx+a)} + \left((\sin(bx+a))^3 + \frac{3\sin(bx+a)}{2}\right)\cos(bx+a) - \frac{3bx}{2} - \frac{3a}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(sin(b*x+a)^5/cos(b*x+a)+(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/2*b
*x-3/2*a)
```

Maxima [A] time = 1.48577, size = 55, normalized size = 1.38

$$-\frac{3\,bx + 3\,a - \frac{\tan(bx+a)}{\tan(bx+a)^2+1} - 2\,\tan(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x + 3*a - tan(b*x + a)/(tan(b*x + a)^2 + 1) - 2*tan(b*x + a))/b
```

Fricas [A] time = 1.60722, size = 108, normalized size = 2.7

$$-\frac{3\,bx\cos(bx+a) - \left(\cos(bx+a)^2 + 2\right)\sin(bx+a)}{2\,b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/2*(3*b*x*cos(b*x + a) - (cos(b*x + a)^2 + 2)*sin(b*x + a))/(b*cos(b*x +
a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23741, size = 55, normalized size = 1.38

$$-\frac{3\,bx+3\,a-\frac{\tan(bx+a)}{\tan(bx+a)^2+1}-2\,\tan\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/2*(3*b*x + 3*a - tan(b*x + a)/(tan(b*x + a)^2 + 1) - 2*tan(b*x + a))/b
```

3.85 $\int \tan^4(a + bx)\, dx$

Optimal. Leaf size=28

$$\frac{\tan^3(a + bx)}{3b} - \frac{\tan(a + bx)}{b} + x$$

```
[Out] x - Tan[a + b*x]/b + Tan[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0160507, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$\frac{\tan^3(a + bx)}{3b} - \frac{\tan(a + bx)}{b} + x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*x]^4,x]
```

```
[Out] x - Tan[a + b*x]/b + Tan[a + b*x]^3/(3*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tan^4(a + bx)\, dx &= \frac{\tan^3(a + bx)}{3b} - \int \tan^2(a + bx)\, dx\\ &= -\frac{\tan(a + bx)}{b} + \frac{\tan^3(a + bx)}{3b} + \int 1\, dx\\ &= x - \frac{\tan(a + bx)}{b} + \frac{\tan^3(a + bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0094322, size = 38, normalized size = 1.36

$$\frac{\tan^3(a + bx)}{3b} + \frac{\tan^{-1}(\tan(a + bx))}{b} - \frac{\tan(a + bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*x]^4,x]
```

```
[Out] ArcTan[Tan[a + b*x]]/b - Tan[a + b*x]/b + Tan[a + b*x]^3/(3*b)
```

Maple [A] time = 0.02, size = 28, normalized size = 1.

$$\frac{1}{b}\left(\frac{(\tan (bx+a))^3}{3}-\tan (bx+a)+bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(1/3*tan(b*x+a)^3-tan(b*x+a)+b*x+a)
```

Maxima [A] time = 1.50201, size = 39, normalized size = 1.39

$$\frac{\tan (bx+a)^3+3\,bx+3\,a-3\,\tan (bx+a)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(b*x + a)^3 + 3*b*x + 3*a - 3*tan(b*x + a))/b
```

Fricas [A] time = 1.6484, size = 115, normalized size = 4.11

$$\frac{3\,bx\cos (bx+a)^3-\left(4\,\cos (bx+a)^2-1\right)\sin (bx+a)}{3\,b\cos (bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*x*cos(b*x + a)^3 - (4*cos(b*x + a)^2 - 1)*sin(b*x + a))/(b*cos(b*x
 + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.69654, size = 39, normalized size = 1.39

$$\frac{\tan (bx+a)^3+3\,bx+3\,a-3\,\tan (bx+a)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/3*(tan(b*x + a)^3 + 3*b*x + 3*a - 3*tan(b*x + a))/b
```

3.86 $\int \sec^2(a+bx)\tan^4(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\tan^5(a+bx)}{5b}$$

```
[Out] Tan[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0276559, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$\frac{\tan^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2*Tan[a + b*x]^4,x]
```

```
[Out] Tan[a + b*x]^5/(5*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)\tan^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^4\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\tan^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0073462, size = 15, normalized size = 1.

$$\frac{\tan^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2*Tan[a + b*x]^4,x]
```

```
[Out] Tan[a + b*x]^5/(5*b)
```

Maple [A] time = 0.019, size = 22, normalized size = 1.5

$$\frac{(\sin(bx+a))^5}{5\,b\,(\cos(bx+a))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6*sin(b*x+a)^4,x)
```

```
[Out] 1/5/b*sin(b*x+a)^5/cos(b*x+a)^5
```

Maxima [A] time = 0.993974, size = 18, normalized size = 1.2

$$\frac{\tan(bx+a)^5}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/5*tan(b*x + a)^5/b
```

Fricas [B] time = 1.53812, size = 104, normalized size = 6.93

$$\frac{\left(\cos(bx+a)^4 - 2\,\cos(bx+a)^2 + 1\right)\sin(bx+a)}{5\,b\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sin(b*x + a)/(b*cos(b*x + a)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14274, size = 18, normalized size = 1.2

$$\frac{\tan(bx+a)^5}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/5*tan(b*x + a)^5/b
```

3.87 $\int \sec^4(a+bx)\tan^4(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\tan^7(a+bx)}{7b}+\frac{\tan^5(a+bx)}{5b}$$

```
[Out] Tan[a + b*x]^5/(5*b) + Tan[a + b*x]^7/(7*b)
```

Rubi [A] time = 0.0320576, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 14}

$$\frac{\tan^7(a+bx)}{7b}+\frac{\tan^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4*Tan[a + b*x]^4,x]
```

```
[Out] Tan[a + b*x]^5/(5*b) + Tan[a + b*x]^7/(7*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \sec^4(a+bx)\tan^4(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int x^4\left(1+x^2\right)\,dx,x,\tan(a+bx)\right)}{b}\\
&= \frac{\operatorname{Subst}\left(\int \left(x^4+x^6\right)\,dx,x,\tan(a+bx)\right)}{b}\\
&= \frac{\tan^5(a+bx)}{5b}+\frac{\tan^7(a+bx)}{7b}
\end{aligned}$$

Mathematica [B] time = 0.0345766, size = 77, normalized size = 2.48

$$\frac{2\tan(a+bx)}{35b}+\frac{\tan(a+bx)\sec^6(a+bx)}{7b}-\frac{8\tan(a+bx)\sec^4(a+bx)}{35b}+\frac{\tan(a+bx)\sec^2(a+bx)}{35b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4*Tan[a + b*x]^4,x]
```

```
[Out] (2*Tan[a + b*x])/(35*b) + (Sec[a + b*x]^2*Tan[a + b*x])/(35*b) - (8*Sec[a +
 b*x]^4*Tan[a + b*x])/(35*b) + (Sec[a + b*x]^6*Tan[a + b*x])/(7*b)
```

Maple [A] time = 0.023, size = 42, normalized size = 1.4

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^5}{7(\cos(bx+a))^7}+\frac{2(\sin(bx+a))^5}{35(\cos(bx+a))^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^8*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(1/7*sin(b*x+a)^5/cos(b*x+a)^7+2/35*sin(b*x+a)^5/cos(b*x+a)^5)
```

Maxima [A] time = 1.00731, size = 35, normalized size = 1.13

$$\frac{5\tan(bx+a)^7+7\tan(bx+a)^5}{35b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/35*(5*tan(b*x + a)^7 + 7*tan(b*x + a)^5)/b
```

Fricas [A] time = 1.62264, size = 131, normalized size = 4.23

$$\frac{\left(2\cos(bx+a)^6+\cos(bx+a)^4-8\cos(bx+a)^2+5\right)\sin(bx+a)}{35b\cos(bx+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/35*(2*cos(b*x + a)^6 + cos(b*x + a)^4 - 8*cos(b*x + a)^2 + 5)*sin(b*x + a
)/(b*cos(b*x + a)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**8*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18599, size = 35, normalized size = 1.13

$$\frac{5 \tan (bx + a)^7 + 7 \tan (bx + a)^5}{35 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/35*(5*tan(b*x + a)^7 + 7*tan(b*x + a)^5)/b
```

3.88 $\int \sec^6(a+bx)\tan^4(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\tan^9(a+bx)}{9b}+\frac{2\tan^7(a+bx)}{7b}+\frac{\tan^5(a+bx)}{5b}$$

```
[Out] Tan[a + b*x]^5/(5*b) + (2*Tan[a + b*x]^7)/(7*b) + Tan[a + b*x]^9/(9*b)
```

Rubi [A] time = 0.0357062, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 270}

$$\frac{\tan^9(a+bx)}{9b}+\frac{2\tan^7(a+bx)}{7b}+\frac{\tan^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6*Tan[a + b*x]^4,x]
```

```
[Out] Tan[a + b*x]^5/(5*b) + (2*Tan[a + b*x]^7)/(7*b) + Tan[a + b*x]^9/(9*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(a+bx)\tan^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^4\left(1+x^2\right)^2\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^4+2x^6+x^8\right)dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\tan^5(a+bx)}{5b}+\frac{2\tan^7(a+bx)}{7b}+\frac{\tan^9(a+bx)}{9b}\end{aligned}$$

Mathematica [B] time = 0.0388211, size = 98, normalized size = 2.13

$$\frac{8\tan(a+bx)}{315b}+\frac{\tan(a+bx)\sec^8(a+bx)}{9b}-\frac{10\tan(a+bx)\sec^6(a+bx)}{63b}+\frac{\tan(a+bx)\sec^4(a+bx)}{105b}+\frac{4\tan(a+bx)\,\text{s}}{315}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6*Tan[a + b*x]^4,x]
```

```
[Out] (8*Tan[a + b*x])/(315*b) + (4*Sec[a + b*x]^2*Tan[a + b*x])/(315*b) + (Sec[a
 + b*x]^4*Tan[a + b*x])/(105*b) - (10*Sec[a + b*x]^6*Tan[a + b*x])/(63*b) +
 (Sec[a + b*x]^8*Tan[a + b*x])/(9*b)
```

Maple [A] time = 0.023, size = 60, normalized size = 1.3

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^5}{9(\cos(bx+a))^9}+\frac{4(\sin(bx+a))^5}{63(\cos(bx+a))^7}+\frac{8(\sin(bx+a))^5}{315(\cos(bx+a))^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^10*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(1/9*sin(b*x+a)^5/cos(b*x+a)^9+4/63*sin(b*x+a)^5/cos(b*x+a)^7+8/315*sin
(b*x+a)^5/cos(b*x+a)^5)
```

Maxima [A] time = 0.98759, size = 49, normalized size = 1.07

$$\frac{35\tan(bx+a)^9+90\tan(bx+a)^7+63\tan(bx+a)^5}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/315*(35*tan(b*x + a)^9 + 90*tan(b*x + a)^7 + 63*tan(b*x + a)^5)/b
```

Fricas [A] time = 1.66066, size = 163, normalized size = 3.54

$$\frac{\left(8\cos(bx+a)^8+4\cos(bx+a)^6+3\cos(bx+a)^4-50\cos(bx+a)^2+35\right)\sin(bx+a)}{315\,b\cos(bx+a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(8*cos(b*x + a)^8 + 4*cos(b*x + a)^6 + 3*cos(b*x + a)^4 - 50*cos(b*x
+ a)^2 + 35)*sin(b*x + a)/(b*cos(b*x + a)^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**10*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16883, size = 49, normalized size = 1.07

$$\frac{35\,\tan\left(bx+a\right)^{9}+90\,\tan\left(bx+a\right)^{7}+63\,\tan\left(bx+a\right)^{5}}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/315*(35*tan(b*x + a)^9 + 90*tan(b*x + a)^7 + 63*tan(b*x + a)^5)/b
```

3.89 $\int \cos^6(a+bx)\sin^4(a+bx)\,dx$

Optimal. Leaf size=111

$$-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}-\frac{3\sin(a+bx)\cos^7(a+bx)}{80b}+\frac{\sin(a+bx)\cos^5(a+bx)}{160b}+\frac{\sin(a+bx)\cos^3(a+bx)}{128b}+\frac{3\sin(a}{}$$

```
[Out] (3*x)/256 + (3*Cos[a + b*x]*Sin[a + b*x])/(256*b) + (Cos[a + b*x]^3*Sin[a +
 b*x])/(128*b) + (Cos[a + b*x]^5*Sin[a + b*x])/(160*b) - (3*Cos[a + b*x]^7*
Sin[a + b*x])/(80*b) - (Cos[a + b*x]^7*Sin[a + b*x]^3)/(10*b)
```

Rubi [A] time = 0.096975, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}-\frac{3\sin(a+bx)\cos^7(a+bx)}{80b}+\frac{\sin(a+bx)\cos^5(a+bx)}{160b}+\frac{\sin(a+bx)\cos^3(a+bx)}{128b}+\frac{3\sin(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^6*Sin[a + b*x]^4,x]
```

```
[Out] (3*x)/256 + (3*Cos[a + b*x]*Sin[a + b*x])/(256*b) + (Cos[a + b*x]^3*Sin[a +
 b*x])/(128*b) + (Cos[a + b*x]^5*Sin[a + b*x])/(160*b) - (3*Cos[a + b*x]^7*
Sin[a + b*x])/(80*b) - (Cos[a + b*x]^7*Sin[a + b*x]^3)/(10*b)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^6(a+bx)\sin^4(a+bx)\,dx &= -\frac{\cos^7(a+bx)\sin^3(a+bx)}{10b} + \frac{3}{10}\int \cos^6(a+bx)\sin^2(a+bx)\,dx \\
&= -\frac{3\cos^7(a+bx)\sin(a+bx)}{80b} - \frac{\cos^7(a+bx)\sin^3(a+bx)}{10b} + \frac{3}{80}\int \cos^6(a+bx)\,dx \\
&= \frac{\cos^5(a+bx)\sin(a+bx)}{160b} - \frac{3\cos^7(a+bx)\sin(a+bx)}{80b} - \frac{\cos^7(a+bx)\sin^3(a+bx)}{10b} + \\
&= \frac{\cos^3(a+bx)\sin(a+bx)}{128b} + \frac{\cos^5(a+bx)\sin(a+bx)}{160b} - \frac{3\cos^7(a+bx)\sin(a+bx)}{80b} - \\
&= \frac{3\cos(a+bx)\sin(a+bx)}{256b} + \frac{\cos^3(a+bx)\sin(a+bx)}{128b} + \frac{\cos^5(a+bx)\sin(a+bx)}{160b} - \\
&= \frac{3x}{256} + \frac{3\cos(a+bx)\sin(a+bx)}{256b} + \frac{\cos^3(a+bx)\sin(a+bx)}{128b} + \frac{\cos^5(a+bx)\sin(a+
\end{aligned}
$$

Mathematica [A] time = 0.195666, size = 62, normalized size = 0.56

$$
\frac{20\sin(2(a+bx)) - 40\sin(4(a+bx)) - 10\sin(6(a+bx)) + 5\sin(8(a+bx)) + 2\sin(10(a+bx)) + 120bx}{10240b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^6*Sin[a + b*x]^4,x]
```

```
[Out] (120*b*x + 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] - 10*Sin[6*(a + b*x)]
+ 5*Sin[8*(a + b*x)] + 2*Sin[10*(a + b*x)])/(10240*b)
```

Maple [A] time = 0.011, size = 82, normalized size = 0.7

$$
\frac{1}{b}\left(-\frac{(\sin(bx+a))^3(\cos(bx+a))^7}{10} - \frac{3\sin(bx+a)(\cos(bx+a))^7}{80} + \frac{\sin(bx+a)}{160}\left((\cos(bx+a))^5 + \frac{5(\cos(bx+a))^3}{4}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/10*sin(b*x+a)^3*cos(b*x+a)^7-3/80*sin(b*x+a)*cos(b*x+a)^7+1/160*(co
s(b*x+a)^5+5/4*cos(b*x+a)^3+15/8*cos(b*x+a))*sin(b*x+a)+3/256*b*x+3/256*a)
```

Maxima [A] time = 1.00769, size = 65, normalized size = 0.59

$$
\frac{32\sin(2bx+2a)^5 + 120bx + 120a + 5\sin(8bx+8a) - 40\sin(4bx+4a)}{10240b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/10240*(32*sin(2*b*x + 2*a)^5 + 120*b*x + 120*a + 5*sin(8*b*x + 8*a) - 40*
sin(4*b*x + 4*a))/b
```

Fricas [A] time = 1.71907, size = 180, normalized size = 1.62

$$\frac{15\,bx + \left(128\,\cos\left(bx+a\right)^{9} - 176\,\cos\left(bx+a\right)^{7} + 8\,\cos\left(bx+a\right)^{5} + 10\,\cos\left(bx+a\right)^{3} + 15\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{1280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/1280*(15*b*x + (128*cos(b*x + a)^9 - 176*cos(b*x + a)^7 + 8*cos(b*x + a)^
5 + 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [A] time = 32.5377, size = 231, normalized size = 2.08

$$\begin{cases} \frac{3x\sin^{10}(a+bx)}{256} + \frac{15x\sin^{8}(a+bx)\cos^{2}(a+bx)}{256} + \frac{15x\sin^{6}(a+bx)\cos^{4}(a+bx)}{128} + \frac{15x\sin^{4}(a+bx)\cos^{6}(a+bx)}{128} + \frac{15x\sin^{2}(a+bx)\cos^{8}(a+bx)}{256} + \frac{3x\cos^{10}(a}{256} \\ x\sin^{4}(a)\cos^{6}(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**10/256 + 15*x*sin(a + b*x)**8*cos(a + b*x)**2/
256 + 15*x*sin(a + b*x)**6*cos(a + b*x)**4/128 + 15*x*sin(a + b*x)**4*cos(a
 + b*x)**6/128 + 15*x*sin(a + b*x)**2*cos(a + b*x)**8/256 + 3*x*cos(a + b*x
)**10/256 + 3*sin(a + b*x)**9*cos(a + b*x)/(256*b) + 7*sin(a + b*x)**7*cos(
a + b*x)**3/(128*b) + sin(a + b*x)**5*cos(a + b*x)**5/(10*b) - 7*sin(a + b*
x)**3*cos(a + b*x)**7/(128*b) - 3*sin(a + b*x)*cos(a + b*x)**9/(256*b), Ne(
b, 0)), (x*sin(a)**4*cos(a)**6, True))
```

Giac [A] time = 1.14561, size = 100, normalized size = 0.9

$$\frac{3}{256}\,x + \frac{\sin\left(10\,bx + 10\,a\right)}{5120\,b} + \frac{\sin\left(8\,bx + 8\,a\right)}{2048\,b} - \frac{\sin\left(6\,bx + 6\,a\right)}{1024\,b} - \frac{\sin\left(4\,bx + 4\,a\right)}{256\,b} + \frac{\sin\left(2\,bx + 2\,a\right)}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/256*x + 1/5120*sin(10*b*x + 10*a)/b + 1/2048*sin(8*b*x + 8*a)/b - 1/1024*
sin(6*b*x + 6*a)/b - 1/256*sin(4*b*x + 4*a)/b + 1/512*sin(2*b*x + 2*a)/b
```

3.90 $\int \cos^4(a+bx)\sin^4(a+bx)\,dx$

Optimal. Leaf size=90

$$-\frac{\sin^3(a+bx)\cos^5(a+bx)}{8b}-\frac{\sin(a+bx)\cos^5(a+bx)}{16b}+\frac{\sin(a+bx)\cos^3(a+bx)}{64b}+\frac{3\sin(a+bx)\cos(a+bx)}{128b}+\frac{3x}{128}$$

```
[Out] (3*x)/128 + (3*Cos[a + b*x]*Sin[a + b*x])/(128*b) + (Cos[a + b*x]^3*Sin[a +
 b*x])/(64*b) - (Cos[a + b*x]^5*Sin[a + b*x])/(16*b) - (Cos[a + b*x]^5*Sin[
a + b*x]^3)/(8*b)
```

Rubi [A] time = 0.0832022, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$-\frac{\sin^3(a+bx)\cos^5(a+bx)}{8b}-\frac{\sin(a+bx)\cos^5(a+bx)}{16b}+\frac{\sin(a+bx)\cos^3(a+bx)}{64b}+\frac{3\sin(a+bx)\cos(a+bx)}{128b}+\frac{3x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Sin[a + b*x]^4,x]
```


```
[Out] (3*x)/128 + (3*Cos[a + b*x]*Sin[a + b*x])/(128*b) + (Cos[a + b*x]^3*Sin[a +
 b*x])/(64*b) - (Cos[a + b*x]^5*Sin[a + b*x])/(16*b) - (Cos[a + b*x]^5*Sin[
a + b*x]^3)/(8*b)
```


Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx = -\frac{\cos^5(a+bx)\sin^3(a+bx)}{8b} + \frac{3}{8}\int \cos^4(a+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{\cos^5(a+bx)\sin(a+bx)}{16b} - \frac{\cos^5(a+bx)\sin^3(a+bx)}{8b} + \frac{1}{16}\int \cos^4(a+bx)\,dx$$
$$= \frac{\cos^3(a+bx)\sin(a+bx)}{64b} - \frac{\cos^5(a+bx)\sin(a+bx)}{16b} - \frac{\cos^5(a+bx)\sin^3(a+bx)}{8b} + \frac{3}{64}$$
$$= \frac{3\cos(a+bx)\sin(a+bx)}{128b} + \frac{\cos^3(a+bx)\sin(a+bx)}{64b} - \frac{\cos^5(a+bx)\sin(a+bx)}{16b} - \cos$$
$$= \frac{3x}{128} + \frac{3\cos(a+bx)\sin(a+bx)}{128b} + \frac{\cos^3(a+bx)\sin(a+bx)}{64b} - \frac{\cos^5(a+bx)\sin(a+bx)}{16b}$$

Mathematica [A] time = 0.0437347, size = 33, normalized size = 0.37

$$\frac{24(a+bx) - 8\sin(4(a+bx)) + \sin(8(a+bx))}{1024b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Sin[a + b*x]^4,x]
```

```
[Out] (24*(a + b*x) - 8*Sin[4*(a + b*x)] + Sin[8*(a + b*x)])/(1024*b)
```

Maple [A] time = 0.012, size = 72, normalized size = 0.8

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^5(\sin(bx+a))^3}{8} - \frac{\sin(bx+a)(\cos(bx+a))^5}{16} + \frac{\sin(bx+a)}{64}\left((\cos(bx+a))^3 + \frac{3\cos(bx+a)}{2}\right) + \frac{3bx}{128}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/8*cos(b*x+a)^5*sin(b*x+a)^3-1/16*sin(b*x+a)*cos(b*x+a)^5+1/64*(cos(
b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/128*b*x+3/128*a)
```

Maxima [A] time = 0.997071, size = 45, normalized size = 0.5

$$\frac{24\,bx + 24\,a + \sin(8\,bx + 8\,a) - 8\,\sin(4\,bx + 4\,a)}{1024\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/1024*(24*b*x + 24*a + sin(8*b*x + 8*a) - 8*sin(4*b*x + 4*a))/b
```

Fricas [A] time = 1.60475, size = 146, normalized size = 1.62

$$\frac{3\,bx + \left(16\,\cos(bx+a)^7 - 24\,\cos(bx+a)^5 + 2\,\cos(bx+a)^3 + 3\,\cos(bx+a)\right)\sin(bx+a)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(3*b*x + (16*cos(b*x + a)^7 - 24*cos(b*x + a)^5 + 2*cos(b*x + a)^3 +
3*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [A] time = 12.3905, size = 189, normalized size = 2.1

$$\begin{cases} \frac{3x\sin^8{(a+bx)}}{128} + \frac{3x\sin^6{(a+bx)}\cos^2{(a+bx)}}{32} + \frac{9x\sin^4{(a+bx)}\cos^4{(a+bx)}}{64} + \frac{3x\sin^2{(a+bx)}\cos^6{(a+bx)}}{32} + \frac{3x\cos^8{(a+bx)}}{128} + \frac{3\sin^7{(a+bx)}\cos{(a+bx)}}{128b} \\ x\sin^4{(a)}\cos^4{(a)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**8/128 + 3*x*sin(a + b*x)**6*cos(a + b*x)**2/32
 + 9*x*sin(a + b*x)**4*cos(a + b*x)**4/64 + 3*x*sin(a + b*x)**2*cos(a + b*x
)**6/32 + 3*x*cos(a + b*x)**8/128 + 3*sin(a + b*x)**7*cos(a + b*x)/(128*b)
+ 11*sin(a + b*x)**5*cos(a + b*x)**3/(128*b) - 11*sin(a + b*x)**3*cos(a + b
*x)**5/(128*b) - 3*sin(a + b*x)*cos(a + b*x)**7/(128*b), Ne(b, 0)), (x*sin(
a)**4*cos(a)**4, True))
```

Giac [A] time = 1.16124, size = 43, normalized size = 0.48

$$\frac{3}{128}\,x + \frac{\sin\left(8\,bx + 8\,a\right)}{1024\,b} - \frac{\sin\left(4\,bx + 4\,a\right)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/128*x + 1/1024*sin(8*b*x + 8*a)/b - 1/128*sin(4*b*x + 4*a)/b
```

3.91 $\int \cos^2(a+bx)\sin^4(a+bx)\,dx$

Optimal. Leaf size=69

$$-\frac{\sin^3(a+bx)\cos^3(a+bx)}{6b}-\frac{\sin(a+bx)\cos^3(a+bx)}{8b}+\frac{\sin(a+bx)\cos(a+bx)}{16b}+\frac{x}{16}$$

```
[Out] x/16 + (Cos[a + b*x]*Sin[a + b*x])/(16*b) - (Cos[a + b*x]^3*Sin[a + b*x])/(
8*b) - (Cos[a + b*x]^3*Sin[a + b*x]^3)/(6*b)
```

Rubi [A] time = 0.0683697, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$-\frac{\sin^3(a+bx)\cos^3(a+bx)}{6b}-\frac{\sin(a+bx)\cos^3(a+bx)}{8b}+\frac{\sin(a+bx)\cos(a+bx)}{16b}+\frac{x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x]^4,x]
```

```
[Out] x/16 + (Cos[a + b*x]*Sin[a + b*x])/(16*b) - (Cos[a + b*x]^3*Sin[a + b*x])/(
8*b) - (Cos[a + b*x]^3*Sin[a + b*x]^3)/(6*b)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(a+bx)\sin^4(a+bx)\,dx &= -\frac{\cos^3(a+bx)\sin^3(a+bx)}{6b}+\frac{1}{2}\int \cos^2(a+bx)\sin^2(a+bx)\,dx\\
&= -\frac{\cos^3(a+bx)\sin(a+bx)}{8b}-\frac{\cos^3(a+bx)\sin^3(a+bx)}{6b}+\frac{1}{8}\int \cos^2(a+bx)\,dx\\
&= \frac{\cos(a+bx)\sin(a+bx)}{16b}-\frac{\cos^3(a+bx)\sin(a+bx)}{8b}-\frac{\cos^3(a+bx)\sin^3(a+bx)}{6b}+\frac{\int 1\,dx}{16}\\
&= \frac{x}{16}+\frac{\cos(a+bx)\sin(a+bx)}{16b}-\frac{\cos^3(a+bx)\sin(a+bx)}{8b}-\frac{\cos^3(a+bx)\sin^3(a+bx)}{6b}
\end{aligned}$$

Mathematica [A] time = 0.0672379, size = 40, normalized size = 0.58

$$\frac{-3\sin(2(a+bx))-3\sin(4(a+bx))+\sin(6(a+bx))+12bx}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x]^4,x]
```

```
[Out] (12*b*x - 3*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)])/(192*
b)
```

Maple [A] time = 0.01, size = 61, normalized size = 0.9

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^3(\sin(bx+a))^3}{6}-\frac{(\cos(bx+a))^3\sin(bx+a)}{8}+\frac{\cos(bx+a)\sin(bx+a)}{16}+\frac{bx}{16}+\frac{a}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/6*cos(b*x+a)^3*sin(b*x+a)^3-1/8*cos(b*x+a)^3*sin(b*x+a)+1/16*cos(b*
x+a)*sin(b*x+a)+1/16*b*x+1/16*a)
```

Maxima [A] time = 1.10965, size = 50, normalized size = 0.72

$$-\frac{4\sin(2bx+2a)^3-12bx-12a+3\sin(4bx+4a)}{192b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/192*(4*sin(2*b*x + 2*a)^3 - 12*b*x - 12*a + 3*sin(4*b*x + 4*a))/b
```

Fricas [A] time = 1.66524, size = 117, normalized size = 1.7

$$\frac{3bx+\left(8\cos(bx+a)^5-14\cos(bx+a)^3+3\cos(bx+a)\right)\sin(bx+a)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(3*b*x + (8*cos(b*x + a)^5 - 14*cos(b*x + a)^3 + 3*cos(b*x + a))*sin(b
*x + a))/b
```

Sympy [A] time = 3.8449, size = 136, normalized size = 1.97

$$\begin{cases}\frac{x\sin^6(a+bx)}{16}+\frac{3x\sin^4(a+bx)\cos^2(a+bx)}{16}+\frac{3x\sin^2(a+bx)\cos^4(a+bx)}{16}+\frac{x\cos^6(a+bx)}{16}+\frac{\sin^5(a+bx)\cos(a+bx)}{16b}-\frac{\sin^3(a+bx)\cos^3(a+bx)}{6b}-\text{si}\\ x\sin^4(a)\cos^2(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**6/16 + 3*x*sin(a + b*x)**4*cos(a + b*x)**2/16 +
3*x*sin(a + b*x)**2*cos(a + b*x)**4/16 + x*cos(a + b*x)**6/16 + sin(a + b*x
)**5*cos(a + b*x)/(16*b) - sin(a + b*x)**3*cos(a + b*x)**3/(6*b) - sin(a +
b*x)*cos(a + b*x)**5/(16*b), Ne(b, 0)), (x*sin(a)**4*cos(a)**2, True))
```

Giac [A] time = 1.17517, size = 62, normalized size = 0.9

$$\frac{1}{16}x + \frac{\sin(6bx + 6a)}{192b} - \frac{\sin(4bx + 4a)}{64b} - \frac{\sin(2bx + 2a)}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*x + 1/192*sin(6*b*x + 6*a)/b - 1/64*sin(4*b*x + 4*a)/b - 1/64*sin(2*b*
x + 2*a)/b
```

3.92 $\int \sin^4(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}-\frac{3\sin(a+bx)\cos(a+bx)}{8b}+\frac{3x}{8}$$

```
[Out] (3*x)/8 - (3*Cos[a + b*x]*Sin[a + b*x])/(8*b) - (Cos[a + b*x]*Sin[a + b*x]^
3)/(4*b)
```

Rubi [A] time = 0.0199329, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}-\frac{3\sin(a+bx)\cos(a+bx)}{8b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[a + b*x]*Sin[a + b*x])/(8*b) - (Cos[a + b*x]*Sin[a + b*x]^
3)/(4*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^4(a+bx)\,dx &= -\frac{\cos(a+bx)\sin^3(a+bx)}{4b}+\frac{3}{4}\int \sin^2(a+bx)\,dx\\ &= -\frac{3\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos(a+bx)\sin^3(a+bx)}{4b}+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}-\frac{3\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos(a+bx)\sin^3(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0172524, size = 33, normalized size = 0.72

$$\frac{12(a+bx)-8\sin(2(a+bx))+\sin(4(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4,x]
```

```
[Out] (12*(a + b*x) - 8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)])/(32*b)
```

Maple [A] time = 0., size = 38, normalized size = 0.8

$$\frac{1}{b}\left(-\frac{\cos(bx+a)}{4}\left((\sin(bx+a))^3+\frac{3\sin(bx+a)}{2}\right)+\frac{3bx}{8}+\frac{3a}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)
```

Maxima [A] time = 1.14888, size = 45, normalized size = 0.98

$$\frac{12\,bx+12\,a+\sin(4\,bx+4\,a)-8\,\sin(2\,bx+2\,a)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/32*(12*b*x + 12*a + sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))/b
```

Fricas [A] time = 1.61686, size = 89, normalized size = 1.93

$$\frac{3\,bx+\left(2\,\cos(bx+a)^3-5\,\cos(bx+a)\right)\sin(bx+a)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(3*b*x + (2*cos(b*x + a)^3 - 5*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [A] time = 1.01031, size = 95, normalized size = 2.07

$$\begin{cases} \frac{3x\sin^4(a+bx)}{8}+\frac{3x\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{3x\cos^4(a+bx)}{8}-\frac{5\sin^3(a+bx)\cos(a+bx)}{8b}-\frac{3\sin(a+bx)\cos^3(a+bx)}{8b} & \text{for } b\neq 0 \\ x\sin^4(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**4/8 + 3*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
3*x*cos(a + b*x)**4/8 - 5*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*sin(a + b*
x)*cos(a + b*x)**3/(8*b), Ne(b, 0)), (x*sin(a)**4, True))
```

Giac [A] time = 1.13314, size = 43, normalized size = 0.93

$$\frac{3}{8}\,x + \frac{\sin\left(4\,bx + 4\,a\right)}{32\,b} - \frac{\sin\left(2\,bx + 2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*b*x + 4*a)/b - 1/4*sin(2*b*x + 2*a)/b
```

3.93 $\int \sin^3(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=38

$$-\frac{\sin^3(a+bx)}{3b}-\frac{\sin(a+bx)}{b}+\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0266243, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2592, 302, 206}

$$-\frac{\sin^3(a+bx)}{3b}-\frac{\sin(a+bx)}{b}+\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Tan[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sin^3(a+bx)\tan(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx,x,\sin(a+bx)\right)}{b}\\
&= -\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{\tanh^{-1}(\sin(a+bx))}{b}-\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] time = 0.0146207, size = 38, normalized size = 1.

$$-\frac{\sin^3(a+bx)}{3b}-\frac{\sin(a+bx)}{b}+\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Tan[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Maple [A] time = 0.016, size = 44, normalized size = 1.2

$$-\frac{(\sin(bx+a))^3}{3b}-\frac{\sin(bx+a)}{b}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^4,x)
```

```
[Out] -1/3*sin(b*x+a)^3/b-sin(b*x+a)/b+1/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 1.03131, size = 62, normalized size = 1.63

$$-\frac{2\sin(bx+a)^3-3\log(\sin(bx+a)+1)+3\log(\sin(bx+a)-1)+6\sin(bx+a)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*sin(b*x + a)^3 - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1)
+ 6*sin(b*x + a))/b
```

Fricas [A] time = 1.70495, size = 132, normalized size = 3.47

$$\frac{2\left(\cos(bx+a)^2-4\right)\sin(bx+a)+3\log(\sin(bx+a)+1)-3\log(-\sin(bx+a)+1)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(cos(b*x + a)^2 - 4)*sin(b*x + a) + 3*log(sin(b*x + a) + 1) - 3*log(
-sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17344, size = 65, normalized size = 1.71

$$-\frac{2\sin\left(bx+a\right)^{3}-3\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+3\log\left(\left|\sin\left(bx+a\right)-1\right|\right)+6\sin\left(bx+a\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*sin(b*x + a)^3 - 3*log(abs(sin(b*x + a) + 1)) + 3*log(abs(sin(b*x +
 a) - 1)) + 6*sin(b*x + a))/b
```

3.94 $\int \sin(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=49

$$\frac{3\sin(a+bx)}{2b}+\frac{\sin(a+bx)\tan^2(a+bx)}{2b}-\frac{3\tanh^{-1}(\sin(a+bx))}{2b}$$

```
[Out] (-3*ArcTanh[Sin[a + b*x]])/(2*b) + (3*Sin[a + b*x])/(2*b) + (Sin[a + b*x]*T
an[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0292958, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2592, 288, 321, 206}

$$\frac{3\sin(a+bx)}{2b}+\frac{\sin(a+bx)\tan^2(a+bx)}{2b}-\frac{3\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[a + b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Sin[a + b*x]])/(2*b) + (3*Sin[a + b*x])/(2*b) + (Sin[a + b*x]*T
an[a + b*x]^2)/(2*b)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sin(a+bx)\tan^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{\sin(a+bx)\tan^2(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b}$$
$$= \frac{3\sin(a+bx)}{2b} + \frac{\sin(a+bx)\tan^2(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b}$$
$$= -\frac{3\tanh^{-1}(\sin(a+bx))}{2b} + \frac{3\sin(a+bx)}{2b} + \frac{\sin(a+bx)\tan^2(a+bx)}{2b}$$

Mathematica [A] time = 0.0907542, size = 40, normalized size = 0.82

$$\frac{(\cos(2(a+bx))+2)\tan(a+bx)\sec(a+bx)-3\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[a + b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Sin[a + b*x]] + (2 + Cos[2*(a + b*x)])*Sec[a + b*x]*Tan[a + b*x
])/(2*b)
```

Maple [A] time = 0.02, size = 66, normalized size = 1.4

$$\frac{(\sin(bx+a))^5}{2\,b\,(\cos(bx+a))^2} + \frac{(\sin(bx+a))^3}{2\,b} + \frac{3\,\sin(bx+a)}{2\,b} - \frac{3\,\ln(\sec(bx+a)+\tan(bx+a))}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*sin(b*x+a)^4,x)
```

```
[Out] 1/2/b*sin(b*x+a)^5/cos(b*x+a)^2+1/2*sin(b*x+a)^3/b+3/2*sin(b*x+a)/b-3/2/b*l
n(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 1.01236, size = 76, normalized size = 1.55

$$-\frac{\frac{2\,\sin(bx+a)}{\sin(bx+a)^2-1} + 3\,\log(\sin(bx+a)+1) - 3\,\log(\sin(bx+a)-1) - 4\,\sin(bx+a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 3*log(sin(b*x + a) + 1) - 3*log
(sin(b*x + a) - 1) - 4*sin(b*x + a))/b
```

Fricas [A] time = 1.72964, size = 200, normalized size = 4.08

$$-\frac{3\cos(bx+a)^2\log(\sin(bx+a)+1)-3\cos(bx+a)^2\log(-\sin(bx+a)+1)-2\left(2\cos(bx+a)^2+1\right)\sin(bx+a)}{4\,b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/4*(3*cos(b*x + a)^2*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^2*log(-sin(b*
x + a) + 1) - 2*(2*cos(b*x + a)^2 + 1)*sin(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19916, size = 78, normalized size = 1.59

$$-\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1}+3\log(|\sin(bx+a)+1|)-3\log(|\sin(bx+a)-1|)-4\sin(bx+a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 3*log(abs(sin(b*x + a) + 1)) -
3*log(abs(sin(b*x + a) - 1)) - 4*sin(b*x + a))/b
```

3.95 $\int \sec(a+bx)\tan^4(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{3\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\tan^3(a+bx)\sec(a+bx)}{4b}-\frac{3\tan(a+bx)\sec(a+bx)}{8b}$$

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(8*b) - (3*Sec[a + b*x]*Tan[a + b*x])/(8*b) + (Se
c[a + b*x]*Tan[a + b*x]^3)/(4*b)
```

Rubi [A] time = 0.0423837, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$\frac{3\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\tan^3(a+bx)\sec(a+bx)}{4b}-\frac{3\tan(a+bx)\sec(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Tan[a + b*x]^4,x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(8*b) - (3*Sec[a + b*x]*Tan[a + b*x])/(8*b) + (Se
c[a + b*x]*Tan[a + b*x]^3)/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(a+bx)\tan^4(a+bx)\,dx&=\frac{\sec(a+bx)\tan^3(a+bx)}{4b}-\frac{3}{4}\int \sec(a+bx)\tan^2(a+bx)\,dx\\&=-\frac{3\sec(a+bx)\tan(a+bx)}{8b}+\frac{\sec(a+bx)\tan^3(a+bx)}{4b}+\frac{3}{8}\int \sec(a+bx)\,dx\\&=\frac{3\tanh^{-1}(\sin(a+bx))}{8b}-\frac{3\sec(a+bx)\tan(a+bx)}{8b}+\frac{\sec(a+bx)\tan^3(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.120879, size = 45, normalized size = 0.82

$$\frac{6\tanh^{-1}(\sin(a+bx))-(5\cos(2(a+bx))+1)\tan(a+bx)\sec^3(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Tan[a + b*x]^4,x]
```

```
[Out] (6*ArcTanh[Sin[a + b*x]] - (1 + 5*Cos[2*(a + b*x)])*Sec[a + b*x]^3*Tan[a +
b*x])/(16*b)
```

Maple [A] time = 0.019, size = 87, normalized size = 1.6

$$\frac{(\sin(bx+a))^5}{4b(\cos(bx+a))^4}-\frac{(\sin(bx+a))^5}{8b(\cos(bx+a))^2}-\frac{(\sin(bx+a))^3}{8b}-\frac{3\sin(bx+a)}{8b}+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5*sin(b*x+a)^4,x)
```

```
[Out] 1/4/b*sin(b*x+a)^5/cos(b*x+a)^4-1/8/b*sin(b*x+a)^5/cos(b*x+a)^2-1/8*sin(b*x
+a)^3/b-3/8*sin(b*x+a)/b+3/8/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 1.06134, size = 96, normalized size = 1.75

$$\frac{\frac{2\left(5\sin(bx+a)^3-3\sin(bx+a)\right)}{\sin(bx+a)^4-2\sin(bx+a)^2+1}+3\log(\sin(bx+a)+1)-3\log(\sin(bx+a)-1)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(5*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x + a)^4 - 2*sin(b*x + a
)^2 + 1) + 3*log(sin(b*x + a) + 1) - 3*log(sin(b*x + a) - 1))/b
```

Fricas [A] time = 1.68203, size = 200, normalized size = 3.64

$$\frac{3\cos(bx+a)^4\log(\sin(bx+a)+1)-3\cos(bx+a)^4\log(-\sin(bx+a)+1)-2\left(5\cos(bx+a)^2-2\right)\sin(bx+a)}{16b\cos(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/16*(3*cos(b*x + a)^4*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^4*log(-sin(b*
x + a) + 1) - 2*(5*cos(b*x + a)^2 - 2)*sin(b*x + a))/(b*cos(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21393, size = 85, normalized size = 1.55

$$\frac{\frac{2\left(5\sin(bx+a)^3-3\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^2}+3\log\left(\left|\sin\left(bx+a\right)+1\right|\right)-3\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(2*(5*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 + 3*log(
abs(sin(b*x + a) + 1)) - 3*log(abs(sin(b*x + a) - 1)))/b
```

3.96 $\int \sec^3(a+bx)\tan^4(a+bx)\,dx$

Optimal. Leaf size=78

$$\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan^3(a+bx)\sec^3(a+bx)}{6b}-\frac{\tan(a+bx)\sec^3(a+bx)}{8b}+\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/(16*b) + (Sec[a + b*x]*Tan[a + b*x])/(16*b) - (Sec[a
+ b*x]^3*Tan[a + b*x])/(8*b) + (Sec[a + b*x]^3*Tan[a + b*x]^3)/(6*b)
```

Rubi [A] time = 0.074756, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2611, 3768, 3770}

$$\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan^3(a+bx)\sec^3(a+bx)}{6b}-\frac{\tan(a+bx)\sec^3(a+bx)}{8b}+\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*Tan[a + b*x]^4,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(16*b) + (Sec[a + b*x]*Tan[a + b*x])/(16*b) - (Sec[a
+ b*x]^3*Tan[a + b*x])/(8*b) + (Sec[a + b*x]^3*Tan[a + b*x]^3)/(6*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec^3(a+bx)\tan^4(a+bx)\,dx &= \frac{\sec^3(a+bx)\tan^3(a+bx)}{6b}-\frac{1}{2}\int \sec^3(a+bx)\tan^2(a+bx)\,dx\\
&= -\frac{\sec^3(a+bx)\tan(a+bx)}{8b}+\frac{\sec^3(a+bx)\tan^3(a+bx)}{6b}+\frac{1}{8}\int \sec^3(a+bx)\,dx\\
&= \frac{\sec(a+bx)\tan(a+bx)}{16b}-\frac{\sec^3(a+bx)\tan(a+bx)}{8b}+\frac{\sec^3(a+bx)\tan^3(a+bx)}{6b}+\\
&= \frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\sec(a+bx)\tan(a+bx)}{16b}-\frac{\sec^3(a+bx)\tan(a+bx)}{8b}+
\end{aligned}$$

Mathematica [A] time = 0.028292, size = 99, normalized size = 1.27

$$\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan^3(a+bx)\sec^3(a+bx)}{3b}-\frac{\tan(a+bx)\sec^5(a+bx)}{6b}+\frac{\tan(a+bx)\sec^3(a+bx)}{24b}+\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*Tan[a + b*x]^4,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(16*b) + (Sec[a + b*x]*Tan[a + b*x])/(16*b) + (Sec[a
+ b*x]^3*Tan[a + b*x])/(24*b) - (Sec[a + b*x]^5*Tan[a + b*x])/(6*b) + (Sec[
a + b*x]^3*Tan[a + b*x]^3)/(3*b)
```

Maple [A] time = 0.021, size = 108, normalized size = 1.4

$$\frac{(\sin(bx+a))^5}{6b(\cos(bx+a))^6}+\frac{(\sin(bx+a))^5}{24b(\cos(bx+a))^4}-\frac{(\sin(bx+a))^5}{48b(\cos(bx+a))^2}-\frac{(\sin(bx+a))^3}{48b}-\frac{\sin(bx+a)}{16b}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^7*sin(b*x+a)^4,x)
```

```
[Out] 1/6/b*sin(b*x+a)^5/cos(b*x+a)^6+1/24/b*sin(b*x+a)^5/cos(b*x+a)^4-1/48/b*sin
(b*x+a)^5/cos(b*x+a)^2-1/48*sin(b*x+a)^3/b-1/16*sin(b*x+a)/b+1/16/b*ln(sec(
b*x+a)+tan(b*x+a))
```

Maxima [A] time = 1.14841, size = 123, normalized size = 1.58

$$-\frac{\frac{2\left(3\sin(bx+a)^5+8\sin(bx+a)^3-3\sin(bx+a)\right)}{\sin(bx+a)^6-3\sin(bx+a)^4+3\sin(bx+a)^2-1}-3\log(\sin(bx+a)+1)+3\log(\sin(bx+a)-1)}{96b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(3*sin(b*x + a)^5 + 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x +
a)^6 - 3*sin(b*x + a)^4 + 3*sin(b*x + a)^2 - 1) - 3*log(sin(b*x + a) + 1) +
 3*log(sin(b*x + a) - 1))/b
```

Fricas [A] time = 1.75174, size = 227, normalized size = 2.91

$$\frac{3\cos(bx+a)^6\log(\sin(bx+a)+1)-3\cos(bx+a)^6\log(-\sin(bx+a)+1)+2\left(3\cos(bx+a)^4-14\cos(bx+a)^2+8\right)\sin(bx+a)}{96b\cos(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(3*cos(b*x + a)^6*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^6*log(-sin(b*
x + a) + 1) + 2*(3*cos(b*x + a)^4 - 14*cos(b*x + a)^2 + 8)*sin(b*x + a))/(b
```

```
*cos(b*x + a)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**7*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1874, size = 99, normalized size = 1.27

$$-\frac{\frac{2\left(3\sin(bx+a)^5+8\sin(bx+a)^3-3\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^3}-3\log\left(|\sin\left(bx+a\right)+1|\right)+3\log\left(|\sin\left(bx+a\right)-1|\right)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/96*(2*(3*sin(b*x + a)^5 + 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x +
a)^2 - 1)^3 - 3*log(abs(sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1)))/
b
```

3.97 $\int \sec^5(a+bx)\tan^4(a+bx)\,dx$

Optimal. Leaf size=99

$$\frac{3\tanh^{-1}(\sin(a+bx))}{128b}+\frac{\tan^3(a+bx)\sec^5(a+bx)}{8b}-\frac{\tan(a+bx)\sec^5(a+bx)}{16b}+\frac{\tan(a+bx)\sec^3(a+bx)}{64b}+\frac{3\tan(a+b}{1}$$

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(128*b) + (3*Sec[a + b*x]*Tan[a + b*x])/(128*b) +
 (Sec[a + b*x]^3*Tan[a + b*x])/(64*b) - (Sec[a + b*x]^5*Tan[a + b*x])/(16*b
) + (Sec[a + b*x]^5*Tan[a + b*x]^3)/(8*b)
```

Rubi [A] time = 0.0893559, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2611, 3768, 3770}

$$\frac{3\tanh^{-1}(\sin(a+bx))}{128b}+\frac{\tan^3(a+bx)\sec^5(a+bx)}{8b}-\frac{\tan(a+bx)\sec^5(a+bx)}{16b}+\frac{\tan(a+bx)\sec^3(a+bx)}{64b}+\frac{3\tan(a+b}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5*Tan[a + b*x]^4,x]
```


```
[Out] (3*ArcTanh[Sin[a + b*x]])/(128*b) + (3*Sec[a + b*x]*Tan[a + b*x])/(128*b) +
 (Sec[a + b*x]^3*Tan[a + b*x])/(64*b) - (Sec[a + b*x]^5*Tan[a + b*x])/(16*b
) + (Sec[a + b*x]^5*Tan[a + b*x]^3)/(8*b)
```


Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = \frac{\sec^5(a+bx)\tan^3(a+bx)}{8b} - \frac{3}{8}\int \sec^5(a+bx)\tan^2(a+bx)\,dx$$
$$= -\frac{\sec^5(a+bx)\tan(a+bx)}{16b} + \frac{\sec^5(a+bx)\tan^3(a+bx)}{8b} + \frac{1}{16}\int \sec^5(a+bx)\,dx$$
$$= \frac{\sec^3(a+bx)\tan(a+bx)}{64b} - \frac{\sec^5(a+bx)\tan(a+bx)}{16b} + \frac{\sec^5(a+bx)\tan^3(a+bx)}{8b}$$
$$= \frac{3\sec(a+bx)\tan(a+bx)}{128b} + \frac{\sec^3(a+bx)\tan(a+bx)}{64b} - \frac{\sec^5(a+bx)\tan(a+bx)}{16b}$$
$$= \frac{3\tanh^{-1}(\sin(a+bx))}{128b} + \frac{3\sec(a+bx)\tan(a+bx)}{128b} + \frac{\sec^3(a+bx)\tan(a+bx)}{64b}$$

Mathematica [A] time = 0.312647, size = 64, normalized size = 0.65

$$\frac{96\tanh^{-1}(\sin(a+bx)) + (-307\cos(2(a+bx)) + 26\cos(4(a+bx)) + 3\cos(6(a+bx)) + 182)\tan(a+bx)\sec^7(a+bx)}{4096b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5*Tan[a + b*x]^4,x]
```

```
[Out] (96*ArcTanh[Sin[a + b*x]] + (182 - 307*Cos[2*(a + b*x)] + 26*Cos[4*(a + b*x
)] + 3*Cos[6*(a + b*x)])*Sec[a + b*x]^7*Tan[a + b*x])/(4096*b)
```

Maple [A] time = 0.022, size = 129, normalized size = 1.3

$$\frac{(\sin(bx+a))^5}{8b(\cos(bx+a))^8} + \frac{(\sin(bx+a))^5}{16b(\cos(bx+a))^6} + \frac{(\sin(bx+a))^5}{64b(\cos(bx+a))^4} - \frac{(\sin(bx+a))^5}{128b(\cos(bx+a))^2} - \frac{(\sin(bx+a))^3}{128b} - \frac{3\sin(bx+a)}{128b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^9*sin(b*x+a)^4,x)
```

```
[Out] 1/8/b*sin(b*x+a)^5/cos(b*x+a)^8+1/16/b*sin(b*x+a)^5/cos(b*x+a)^6+1/64/b*sin
(b*x+a)^5/cos(b*x+a)^4-1/128/b*sin(b*x+a)^5/cos(b*x+a)^2-1/128*sin(b*x+a)^3
/b-3/128*sin(b*x+a)/b+3/128/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 1.08345, size = 150, normalized size = 1.52

$$-\frac{\frac{2\left(3\sin(bx+a)^7 - 11\sin(bx+a)^5 - 11\sin(bx+a)^3 + 3\sin(bx+a)\right)}{\sin(bx+a)^8 - 4\sin(bx+a)^6 + 6\sin(bx+a)^4 - 4\sin(bx+a)^2 + 1} - 3\log(\sin(bx+a)+1) + 3\log(\sin(bx+a)-1)}{256b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/256*(2*(3*sin(b*x + a)^7 - 11*sin(b*x + a)^5 - 11*sin(b*x + a)^3 + 3*sin
(b*x + a))/(sin(b*x + a)^8 - 4*sin(b*x + a)^6 + 6*sin(b*x + a)^4 - 4*sin(b*
x + a)^2 + 1) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b
```

Fricas [A] time = 1.76918, size = 255, normalized size = 2.58

$$\frac{3\,\cos\left(bx+a\right)^{8}\log\left(\sin\left(bx+a\right)+1\right)-3\,\cos\left(bx+a\right)^{8}\log\left(-\sin\left(bx+a\right)+1\right)+2\left(3\,\cos\left(bx+a\right)^{6}+2\,\cos\left(bx+a\right)^{4}-24}{256\,b\cos\left(bx+a\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/256*(3*cos(b*x + a)^8*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^8*log(-sin(b
*x + a) + 1) + 2*(3*cos(b*x + a)^6 + 2*cos(b*x + a)^4 - 24*cos(b*x + a)^2 +
 16)*sin(b*x + a))/(b*cos(b*x + a)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**9*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21974, size = 144, normalized size = 1.45

$$-\frac{\frac{4\left(3\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)^{3}-\frac{20}{\sin(bx+a)}-20\,\sin(bx+a)\right)}{\left(\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)^{2}-4\right)^{2}}-3\,\log\left(\left|\frac{1}{\sin(bx+a)}+\sin\left(bx+a\right)+2\right|\right)+3\,\log\left(\left|\frac{1}{\sin(bx+a)}+\sin\left(bx+a\right)-2\right|\right)}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*(4*(3*(1/sin(b*x + a) + sin(b*x + a))^3 - 20/sin(b*x + a) - 20*sin(b
*x + a))/((1/sin(b*x + a) + sin(b*x + a))^2 - 4)^2 - 3*log(abs(1/sin(b*x +
a) + sin(b*x + a) + 2)) + 3*log(abs(1/sin(b*x + a) + sin(b*x + a) - 2)))/b
```

3.98 $\int \cos^7(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cos^{12}(a+bx)}{12b}+\frac{\cos^{10}(a+bx)}{5b}-\frac{\cos^8(a+bx)}{8b}$$

```
[Out] -Cos[a + b*x]^8/(8*b) + Cos[a + b*x]^10/(5*b) - Cos[a + b*x]^12/(12*b)
```

Rubi [A] time = 0.0410329, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2565, 266, 43}

$$-\frac{\cos^{12}(a+bx)}{12b}+\frac{\cos^{10}(a+bx)}{5b}-\frac{\cos^8(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^7*Sin[a + b*x]^5,x]
```

```
[Out] -Cos[a + b*x]^8/(8*b) + Cos[a + b*x]^10/(5*b) - Cos[a + b*x]^12/(12*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^7(a+bx)\sin^5(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^7\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int (1-x)^2x^3\,dx,x,\cos^2(a+bx)\right)}{2b}\\
&= -\frac{\text{Subst}\left(\int \left(x^3-2x^4+x^5\right)dx,x,\cos^2(a+bx)\right)}{2b}\\
&= -\frac{\cos^8(a+bx)}{8b}+\frac{\cos^{10}(a+bx)}{5b}-\frac{\cos^{12}(a+bx)}{12b}
\end{aligned}$$

Mathematica [A] time = 0.378623, size = 68, normalized size = 1.48

$$-\frac{600\cos(2(a+bx))+75\cos(4(a+bx))-100\cos(6(a+bx))-30\cos(8(a+bx))+12\cos(10(a+bx))+5\cos(12(a+bx$$
$$122880b$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^7*Sin[a + b*x]^5,x]
```

```
[Out] -(600*Cos[2*(a + b*x)] + 75*Cos[4*(a + b*x)] - 100*Cos[6*(a + b*x)] - 30*Co
s[8*(a + b*x)] + 12*Cos[10*(a + b*x)] + 5*Cos[12*(a + b*x)])/(122880*b)
```

Maple [A] time = 0.013, size = 52, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{(\sin(bx+a))^4(\cos(bx+a))^8}{12}-\frac{(\sin(bx+a))^2(\cos(bx+a))^8}{30}-\frac{(\cos(bx+a))^8}{120}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(-1/12*sin(b*x+a)^4*cos(b*x+a)^8-1/30*sin(b*x+a)^2*cos(b*x+a)^8-1/120*c
os(b*x+a)^8)
```

Maxima [A] time = 1.02676, size = 62, normalized size = 1.35

$$-\frac{10\sin(bx+a)^{12}-36\sin(bx+a)^{10}+45\sin(bx+a)^8-20\sin(bx+a)^6}{120\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/120*(10*sin(b*x + a)^12 - 36*sin(b*x + a)^10 + 45*sin(b*x + a)^8 - 20*si
n(b*x + a)^6)/b
```

Fricas [A] time = 1.70107, size = 97, normalized size = 2.11

$$-\frac{10\cos(bx+a)^{12}-24\cos(bx+a)^{10}+15\cos(bx+a)^8}{120\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/120*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

Sympy [A] time = 81.0591, size = 83, normalized size = 1.8

$$\begin{cases} \frac{\sin^{12}(a+bx)}{120b}+\frac{\sin^{10}(a+bx)\cos^2(a+bx)}{20b}+\frac{\sin^8(a+bx)\cos^4(a+bx)}{8b}+\frac{\sin^6(a+bx)\cos^6(a+bx)}{6b} & \text{for } b\neq 0\\ x\sin^5(a)\cos^7(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((sin(a + b*x)**12/(120*b) + sin(a + b*x)**10*cos(a + b*x)**2/(20*
b) + sin(a + b*x)**8*cos(a + b*x)**4/(8*b) + sin(a + b*x)**6*cos(a + b*x)**
6/(6*b), Ne(b, 0)), (x*sin(a)**5*cos(a)**7, True))
```

Giac [B] time = 1.17461, size = 115, normalized size = 2.5

$$-\frac{\cos(12\,bx+12\,a)}{24576\,b}-\frac{\cos(10\,bx+10\,a)}{10240\,b}+\frac{\cos(8\,bx+8\,a)}{4096\,b}+\frac{5\,\cos(6\,bx+6\,a)}{6144\,b}-\frac{5\,\cos(4\,bx+4\,a)}{8192\,b}-\frac{5\,\cos(2\,bx+2}{1024\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/24576*cos(12*b*x + 12*a)/b - 1/10240*cos(10*b*x + 10*a)/b + 1/4096*cos(8
*b*x + 8*a)/b + 5/6144*cos(6*b*x + 6*a)/b - 5/8192*cos(4*b*x + 4*a)/b - 5/1
024*cos(2*b*x + 2*a)/b
```

3.99 $\int \cos^6(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cos^{11}(a+bx)}{11b}+\frac{2\cos^9(a+bx)}{9b}-\frac{\cos^7(a+bx)}{7b}$$

```
[Out] -Cos[a + b*x]^7/(7*b) + (2*Cos[a + b*x]^9)/(9*b) - Cos[a + b*x]^11/(11*b)
```

Rubi [A] time = 0.0356549, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 270}

$$-\frac{\cos^{11}(a+bx)}{11b}+\frac{2\cos^9(a+bx)}{9b}-\frac{\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^6*Sin[a + b*x]^5,x]
```

```
[Out] -Cos[a + b*x]^7/(7*b) + (2*Cos[a + b*x]^9)/(9*b) - Cos[a + b*x]^11/(11*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^6(a+bx)\sin^5(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int x^6\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\operatorname{Subst}\left(\int \left(x^6-2x^8+x^{10}\right)dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\cos^7(a+bx)}{7b}+\frac{2\cos^9(a+bx)}{9b}-\frac{\cos^{11}(a+bx)}{11b}\end{aligned}$$

Mathematica [A] time = 0.272674, size = 37, normalized size = 0.8

$$\frac{\cos^7(a+bx)(364\cos(2(a+bx))-63\cos(4(a+bx))-365)}{5544b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^6*Sin[a + b*x]^5,x]
```

```
[Out] (Cos[a + b*x]^7*(-365 + 364*Cos[2*(a + b*x)] - 63*Cos[4*(a + b*x)]))/(5544*
b)
```

Maple [A] time = 0.013, size = 52, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^7(\sin(bx+a))^4}{11}-\frac{4(\cos(bx+a))^7(\sin(bx+a))^2}{99}-\frac{8(\cos(bx+a))^7}{693}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(-1/11*cos(b*x+a)^7*sin(b*x+a)^4-4/99*cos(b*x+a)^7*sin(b*x+a)^2-8/693*c
os(b*x+a)^7)
```

Maxima [A] time = 0.988885, size = 49, normalized size = 1.07

$$-\frac{63\cos(bx+a)^{11}-154\cos(bx+a)^9+99\cos(bx+a)^7}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

Fricas [A] time = 1.74097, size = 97, normalized size = 2.11

$$-\frac{63\cos(bx+a)^{11}-154\cos(bx+a)^9+99\cos(bx+a)^7}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

Sympy [A] time = 54.0469, size = 68, normalized size = 1.48

$$\begin{cases}-\frac{\sin^4(a+bx)\cos^7(a+bx)}{7b}-\frac{4\sin^2(a+bx)\cos^9(a+bx)}{63b}-\frac{8\cos^{11}(a+bx)}{693b} & \text{for } b\neq 0\\ x\sin^5(a)\cos^6(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)**7/(7*b) - 4*sin(a + b*x)**2*cos(a
 + b*x)**9/(63*b) - 8*cos(a + b*x)**11/(693*b), Ne(b, 0)), (x*sin(a)**5*cos
```

```
(a)**6, True))
```

<u>Giac</u> [B] time = 1.13953, size = 111, normalized size = 2.41

$$-\frac{\cos\left(11\,bx+11\,a\right)}{11264\,b}-\frac{\cos\left(9\,bx+9\,a\right)}{9216\,b}+\frac{5\,\cos\left(7\,bx+7\,a\right)}{7168\,b}+\frac{\cos\left(5\,bx+5\,a\right)}{1024\,b}-\frac{5\,\cos\left(3\,bx+3\,a\right)}{1536\,b}-\frac{5\,\cos\left(bx+a\right)}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/11264*cos(11*b*x + 11*a)/b - 1/9216*cos(9*b*x + 9*a)/b + 5/7168*cos(7*b*
x + 7*a)/b + 1/1024*cos(5*b*x + 5*a)/b - 5/1536*cos(3*b*x + 3*a)/b - 5/512*
cos(b*x + a)/b
```

3.100 $\int \cos^5(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sin^{10}(a+bx)}{10b}-\frac{\sin^8(a+bx)}{4b}+\frac{\sin^6(a+bx)}{6b}$$

```
[Out] Sin[a + b*x]^6/(6*b) - Sin[a + b*x]^8/(4*b) + Sin[a + b*x]^10/(10*b)
```

Rubi [A] time = 0.0402625, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2564, 266, 43}

$$\frac{\sin^{10}(a+bx)}{10b}-\frac{\sin^8(a+bx)}{4b}+\frac{\sin^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*Sin[a + b*x]^5,x]
```

```
[Out] Sin[a + b*x]^6/(6*b) - Sin[a + b*x]^8/(4*b) + Sin[a + b*x]^10/(10*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^5(a+bx)\sin^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^5\left(1-x^2\right)^2\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (1-x)^2x^2\,dx, x, \sin^2(a+bx)\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \left(x^2-2x^3+x^4\right)dx, x, \sin^2(a+bx)\right)}{2b}\\ &= \frac{\sin^6(a+bx)}{6b}-\frac{\sin^8(a+bx)}{4b}+\frac{\sin^{10}(a+bx)}{10b}\end{aligned}$$

Mathematica [A] time = 0.0267049, size = 50, normalized size = 1.09

$$\frac{1}{32}\left(-\frac{5\cos(2(a+bx))}{16b}+\frac{5\cos(6(a+bx))}{96b}-\frac{\cos(10(a+bx))}{160b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*Sin[a + b*x]^5,x]
```

```
[Out] ((-5*Cos[2*(a + b*x)])/(16*b) + (5*Cos[6*(a + b*x)])/(96*b) - Cos[10*(a + b
*x)]/(160*b))/32
```

Maple [A] time = 0.013, size = 52, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^{6}(\sin(bx+a))^{4}}{10}-\frac{(\cos(bx+a))^{6}(\sin(bx+a))^{2}}{20}-\frac{(\cos(bx+a))^{6}}{60}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(-1/10*cos(b*x+a)^6*sin(b*x+a)^4-1/20*cos(b*x+a)^6*sin(b*x+a)^2-1/60*co
s(b*x+a)^6)
```

Maxima [A] time = 0.996927, size = 49, normalized size = 1.07

$$\frac{6\sin(bx+a)^{10}-15\sin(bx+a)^{8}+10\sin(bx+a)^{6}}{60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/60*(6*sin(b*x + a)^10 - 15*sin(b*x + a)^8 + 10*sin(b*x + a)^6)/b
```

Fricas [A] time = 1.67699, size = 93, normalized size = 2.02

$$-\frac{6\cos(bx+a)^{10}-15\cos(bx+a)^{8}+10\cos(bx+a)^{6}}{60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(6*cos(b*x + a)^10 - 15*cos(b*x + a)^8 + 10*cos(b*x + a)^6)/b
```

Sympy [A] time = 31.496, size = 63, normalized size = 1.37

$$\begin{cases}\frac{\sin^{10}(a+bx)}{60b}+\frac{\sin^{8}(a+bx)\cos^{2}(a+bx)}{12b}+\frac{\sin^{6}(a+bx)\cos^{4}(a+bx)}{6b} & \text{for } b\neq 0\\ x\sin^{5}(a)\cos^{5}(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((sin(a + b*x)**10/(60*b) + sin(a + b*x)**8*cos(a + b*x)**2/(12*b)
 + sin(a + b*x)**6*cos(a + b*x)**4/(6*b), Ne(b, 0)), (x*sin(a)**5*cos(a)**5
, True))
```

Giac [A] time = 1.11881, size = 58, normalized size = 1.26

$$-\frac{\cos\left(10\,bx+10\,a\right)}{5120\,b}+\frac{5\,\cos\left(6\,bx+6\,a\right)}{3072\,b}-\frac{5\,\cos\left(2\,bx+2\,a\right)}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/5120*cos(10*b*x + 10*a)/b + 5/3072*cos(6*b*x + 6*a)/b - 5/512*cos(2*b*x
+ 2*a)/b
```

3.101 $\int \cos^4(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cos^9(a+bx)}{9b}+\frac{2\cos^7(a+bx)}{7b}-\frac{\cos^5(a+bx)}{5b}$$

```
[Out] -Cos[a + b*x]^5/(5*b) + (2*Cos[a + b*x]^7)/(7*b) - Cos[a + b*x]^9/(9*b)
```

Rubi [A] time = 0.0364733, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 270}

$$-\frac{\cos^9(a+bx)}{9b}+\frac{2\cos^7(a+bx)}{7b}-\frac{\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Sin[a + b*x]^5,x]

[Out] -Cos[a + b*x]^5/(5*b) + (2*Cos[a + b*x]^7)/(7*b) - Cos[a + b*x]^9/(9*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(a+bx)\sin^5(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^4\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\cos^5(a+bx)}{5b}+\frac{2\cos^7(a+bx)}{7b}-\frac{\cos^9(a+bx)}{9b}
\end{aligned}$$

Mathematica [A] time = 0.143163, size = 37, normalized size = 0.8

$$\frac{\cos^5(a+bx)(220\cos(2(a+bx))-35\cos(4(a+bx))-249)}{2520b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Sin[a + b*x]^5,x]
```

```
[Out] (Cos[a + b*x]^5*(-249 + 220*Cos[2*(a + b*x)] - 35*Cos[4*(a + b*x)]))/(2520*
b)
```

Maple [A] time = 0.015, size = 52, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^5(\sin(bx+a))^4}{9}-\frac{4(\cos(bx+a))^5(\sin(bx+a))^2}{63}-\frac{8(\cos(bx+a))^5}{315}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(-1/9*cos(b*x+a)^5*sin(b*x+a)^4-4/63*cos(b*x+a)^5*sin(b*x+a)^2-8/315*co
s(b*x+a)^5)
```

Maxima [A] time = 0.996456, size = 49, normalized size = 1.07

$$-\frac{35\cos(bx+a)^9-90\cos(bx+a)^7+63\cos(bx+a)^5}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

Fricas [A] time = 1.71003, size = 95, normalized size = 2.07

$$-\frac{35\cos(bx+a)^9-90\cos(bx+a)^7+63\cos(bx+a)^5}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

Sympy [A] time = 19.9037, size = 68, normalized size = 1.48

$$\begin{cases} -\frac{\sin^4(a+bx)\cos^5(a+bx)}{5b}-\frac{4\sin^2(a+bx)\cos^7(a+bx)}{35b}-\frac{8\cos^9(a+bx)}{315b} & \text{for } b \neq 0 \\ x\sin^5(a)\cos^4(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)**5/(5*b) - 4*sin(a + b*x)**2*cos(a
 + b*x)**7/(35*b) - 8*cos(a + b*x)**9/(315*b), Ne(b, 0)), (x*sin(a)**5*cos(
```

```
a)**4, True))
```

Giac [A] time = 1.13285, size = 92, normalized size = 2.

$$-\frac{\cos(9\,bx+9\,a)}{2304\,b}+\frac{\cos(7\,bx+7\,a)}{1792\,b}+\frac{\cos(5\,bx+5\,a)}{320\,b}-\frac{\cos(3\,bx+3\,a)}{192\,b}-\frac{3\,\cos(bx+a)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/2304*cos(9*b*x + 9*a)/b + 1/1792*cos(7*b*x + 7*a)/b + 1/320*cos(5*b*x +
5*a)/b - 1/192*cos(3*b*x + 3*a)/b - 3/128*cos(b*x + a)/b
```

3.102 $\int \cos^3(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sin^6(a+bx)}{6b}-\frac{\sin^8(a+bx)}{8b}$$

```
[Out] Sin[a + b*x]^6/(6*b) - Sin[a + b*x]^8/(8*b)
```

Rubi [A] time = 0.0323852, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 14}

$$\frac{\sin^6(a+bx)}{6b}-\frac{\sin^8(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[a + b*x]^5,x]
```

```
[Out] Sin[a + b*x]^6/(6*b) - Sin[a + b*x]^8/(8*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sin^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^5\left(1-x^2\right)dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^5-x^7\right)dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\sin^6(a+bx)}{6b}-\frac{\sin^8(a+bx)}{8b}\end{aligned}$$

Mathematica [A] time = 0.119553, size = 48, normalized size = 1.55

$$\frac{-72\cos(2(a+bx))+12\cos(4(a+bx))+8\cos(6(a+bx))-3\cos(8(a+bx))}{3072b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[a + b*x]^5,x]
```

```
[Out] (-72*Cos[2*(a + b*x)] + 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] - 3*Cos[8*
(a + b*x)])/(3072*b)
```

Maple [A] time = 0.011, size = 52, normalized size = 1.7

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^4(\sin(bx+a))^4}{8}-\frac{(\cos(bx+a))^4(\sin(bx+a))^2}{12}-\frac{(\cos(bx+a))^4}{24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(-1/8*cos(b*x+a)^4*sin(b*x+a)^4-1/12*cos(b*x+a)^4*sin(b*x+a)^2-1/24*cos
(b*x+a)^4)
```

Maxima [A] time = 1.00128, size = 35, normalized size = 1.13

$$-\frac{3\,\sin(bx+a)^8-4\,\sin(bx+a)^6}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(3*sin(b*x + a)^8 - 4*sin(b*x + a)^6)/b
```

Fricas [A] time = 1.69535, size = 89, normalized size = 2.87

$$-\frac{3\,\cos(bx+a)^8-8\,\cos(bx+a)^6+6\,\cos(bx+a)^4}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(3*cos(b*x + a)^8 - 8*cos(b*x + a)^6 + 6*cos(b*x + a)^4)/b
```

Sympy [A] time = 11.6092, size = 42, normalized size = 1.35

$$\begin{cases}\frac{\sin^8(a+bx)}{24b}+\frac{\sin^6(a+bx)\cos^2(a+bx)}{6b} & \text{for } b\neq 0\\ x\sin^5(a)\cos^3(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((sin(a + b*x)**8/(24*b) + sin(a + b*x)**6*cos(a + b*x)**2/(6*b),
Ne(b, 0)), (x*sin(a)**5*cos(a)**3, True))
```

Giac [A] time = 1.14817, size = 35, normalized size = 1.13

$$-\frac{3\sin(bx+a)^8-4\sin(bx+a)^6}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/24*(3*sin(b*x + a)^8 - 4*sin(b*x + a)^6)/b
```

3.103 $\int \cos^2(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cos^7(a+bx)}{7b}+\frac{2\cos^5(a+bx)}{5b}-\frac{\cos^3(a+bx)}{3b}$$

```
[Out] -Cos[a + b*x]^3/(3*b) + (2*Cos[a + b*x]^5)/(5*b) - Cos[a + b*x]^7/(7*b)
```

Rubi [A] time = 0.0354446, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 270}

$$-\frac{\cos^7(a+bx)}{7b}+\frac{2\cos^5(a+bx)}{5b}-\frac{\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x]^5,x]
```

```
[Out] -Cos[a + b*x]^3/(3*b) + (2*Cos[a + b*x]^5)/(5*b) - Cos[a + b*x]^7/(7*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin^5(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\cos^3(a+bx)}{3b}+\frac{2\cos^5(a+bx)}{5b}-\frac{\cos^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.0895744, size = 37, normalized size = 0.8

$$\frac{\cos^3(a+bx)(108\cos(2(a+bx))-15\cos(4(a+bx))-157)}{840b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x]^5,x]
```

```
[Out] (Cos[a + b*x]^3*(-157 + 108*Cos[2*(a + b*x)] - 15*Cos[4*(a + b*x)]))/(840*b
)
```

Maple [A] time = 0.012, size = 52, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^3(\sin(bx+a))^4}{7}-\frac{4(\cos(bx+a))^3(\sin(bx+a))^2}{35}-\frac{8(\cos(bx+a))^3}{105}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(-1/7*cos(b*x+a)^3*sin(b*x+a)^4-4/35*cos(b*x+a)^3*sin(b*x+a)^2-8/105*co
s(b*x+a)^3)
```

Maxima [A] time = 0.981742, size = 49, normalized size = 1.07

$$-\frac{15\cos(bx+a)^7-42\cos(bx+a)^5+35\cos(bx+a)^3}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/105*(15*cos(b*x + a)^7 - 42*cos(b*x + a)^5 + 35*cos(b*x + a)^3)/b
```

Fricas [A] time = 1.59817, size = 95, normalized size = 2.07

$$-\frac{15\cos(bx+a)^7-42\cos(bx+a)^5+35\cos(bx+a)^3}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/105*(15*cos(b*x + a)^7 - 42*cos(b*x + a)^5 + 35*cos(b*x + a)^3)/b
```

Sympy [A] time = 6.94841, size = 68, normalized size = 1.48

$$\begin{cases}-\frac{\sin^4(a+bx)\cos^3(a+bx)}{3b}-\frac{4\sin^2(a+bx)\cos^5(a+bx)}{15b}-\frac{8\cos^7(a+bx)}{105b} & \text{for } b\neq 0\\ x\sin^5(a)\cos^2(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)**3/(3*b) - 4*sin(a + b*x)**2*cos(a
 + b*x)**5/(15*b) - 8*cos(a + b*x)**7/(105*b), Ne(b, 0)), (x*sin(a)**5*cos(
```

```
a)**2, True))
```

Giac [A] time = 1.16678, size = 73, normalized size = 1.59

$$-\frac{\cos(7\,bx+7\,a)}{448\,b}+\frac{3\,\cos(5\,bx+5\,a)}{320\,b}-\frac{\cos(3\,bx+3\,a)}{192\,b}-\frac{5\,\cos(bx+a)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/448*cos(7*b*x + 7*a)/b + 3/320*cos(5*b*x + 5*a)/b - 1/192*cos(3*b*x + 3*
a)/b - 5/64*cos(b*x + a)/b
```

3.104 $\int \cos(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sin^6(a+bx)}{6b}$$

```
[Out] Sin[a + b*x]^6/(6*b)
```

Rubi [A] time = 0.0173331, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2564, 30}

$$\frac{\sin^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[a + b*x]^5,x]
```

```
[Out] Sin[a + b*x]^6/(6*b)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^5(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int x^5\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^6(a+bx)}{6b}\end{aligned}$$

Mathematica [A] time = 0.0037264, size = 15, normalized size = 1.

$$\frac{\sin^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[a + b*x]^5,x]
```

```
[Out] Sin[a + b*x]^6/(6*b)
```

Maple [A] time = 0.004, size = 14, normalized size = 0.9

$$\frac{(\sin (bx+a))^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^5,x)
```

```
[Out] 1/6*sin(b*x+a)^6/b
```

Maxima [A] time = 0.964409, size = 18, normalized size = 1.2

$$\frac{\sin (bx+a)^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/6*sin(b*x + a)^6/b
```

Fricas [B] time = 1.50817, size = 85, normalized size = 5.67

$$-\frac{\cos (bx+a)^6-3\cos (bx+a)^4+3\cos (bx+a)^2}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/6*(cos(b*x + a)^6 - 3*cos(b*x + a)^4 + 3*cos(b*x + a)^2)/b
```

Sympy [A] time = 3.41103, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sin^6(a+bx)}{6b} & \text{for } b\neq 0 \\ x\sin^5(a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((sin(a + b*x)**6/(6*b), Ne(b, 0)), (x*sin(a)**5*cos(a), True))
```

Giac [A] time = 1.14516, size = 18, normalized size = 1.2

$$\frac{\sin (bx+a)^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/6*sin(b*x + a)^6/b
```

3.105 $\int \sin^4(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=40

$$-\frac{\cos^4(a+bx)}{4b}+\frac{\cos^2(a+bx)}{b}-\frac{\log(\cos(a+bx))}{b}$$

```
[Out] Cos[a + b*x]^2/b - Cos[a + b*x]^4/(4*b) - Log[Cos[a + b*x]]/b
```

Rubi [A] time = 0.0256355, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2590, 266, 43}

$$-\frac{\cos^4(a+bx)}{4b}+\frac{\cos^2(a+bx)}{b}-\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4*Tan[a + b*x],x]
```

```
[Out] Cos[a + b*x]^2/b - Cos[a + b*x]^4/(4*b) - Log[Cos[a + b*x]]/b
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sin^4(a+bx)\tan(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x}\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx,x,\cos^2(a+bx)\right)}{2b}\\
&= -\frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)dx,x,\cos^2(a+bx)\right)}{2b}\\
&= \frac{\cos^2(a+bx)}{b}-\frac{\cos^4(a+bx)}{4b}-\frac{\log(\cos(a+bx))}{b}
\end{aligned}$$

Mathematica [A] time = 0.0330667, size = 35, normalized size = 0.88

$$-\frac{\frac{1}{4}\cos^4(a+bx)-\cos^2(a+bx)+\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4*Tan[a + b*x],x]
```

```
[Out] -((-Cos[a + b*x]^2 + Cos[a + b*x]^4/4 + Log[Cos[a + b*x]])/b)
```

Maple [A] time = 0.014, size = 40, normalized size = 1.

$$-\frac{(\sin(bx+a))^4}{4b}-\frac{(\sin(bx+a))^2}{2b}-\frac{\ln(\cos(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^5,x)
```

```
[Out] -1/4*sin(b*x+a)^4/b-1/2*sin(b*x+a)^2/b-ln(cos(b*x+a))/b
```

Maxima [A] time = 0.956812, size = 50, normalized size = 1.25

$$-\frac{\sin(bx+a)^4+2\sin(bx+a)^2+2\log\left(\sin(bx+a)^2-1\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*(sin(b*x + a)^4 + 2*sin(b*x + a)^2 + 2*log(sin(b*x + a)^2 - 1))/b
```

Fricas [A] time = 1.70497, size = 90, normalized size = 2.25

$$-\frac{\cos(bx+a)^4-4\cos(bx+a)^2+4\log(-\cos(bx+a))}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(cos(b*x + a)^4 - 4*cos(b*x + a)^2 + 4*log(-cos(b*x + a)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18879, size = 305, normalized size = 7.62

$$-\frac{\frac{3\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)^2-\frac{20(\cos(bx+a)+1)}{\cos(bx+a)-1}-\frac{20(\cos(bx+a)-1)}{\cos(bx+a)+1}+44}{\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-2\right)^2}-2\log\left(\left|-\frac{\cos(bx+a)+1}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+2\right|\right)+2\log\left(\left|-\frac{\cos(bx+a)+1}{\cos(bx+a)-1}-\cdots\right.\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4*((3*((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b
*x + a) + 1))^2 - 20*(cos(b*x + a) + 1)/(cos(b*x + a) - 1) - 20*(cos(b*x +
a) - 1)/(cos(b*x + a) + 1) + 44)/((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (
cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 2)^2 - 2*log(abs(-(cos(b*x + a) + 1)
/(cos(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 2)) + 2*log(a
bs(-(cos(b*x + a) + 1)/(cos(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a
) + 1) - 2)))/b
```

3.106 $\int \sin^3(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=37

$$-\frac{\cos^3(a+bx)}{3b}+\frac{2\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}$$

```
[Out] (2*Cos[a + b*x])/b - Cos[a + b*x]^3/(3*b) + Sec[a + b*x]/b
```

Rubi [A] time = 0.0342953, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2590, 270}

$$-\frac{\cos^3(a+bx)}{3b}+\frac{2\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Tan[a + b*x]^2,x]
```

```
[Out] (2*Cos[a + b*x])/b - Cos[a + b*x]^3/(3*b) + Sec[a + b*x]/b
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\tan^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2}\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)dx,x,\cos(a+bx)\right)}{b}\\ &= \frac{2\cos(a+bx)}{b}-\frac{\cos^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0328598, size = 39, normalized size = 1.05

$$\frac{7\cos(a+bx)}{4b}-\frac{\cos(3(a+bx))}{12b}+\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Tan[a + b*x]^2,x]
```

[Out] (7*Cos[a + b*x])/(4*b) - Cos[3*(a + b*x)]/(12*b) + Sec[a + b*x]/b

---

Maple [A] time = 0.017, size = 50, normalized size = 1.4

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^6}{\cos(bx+a)}+\left(\frac{8}{3}+(\sin(bx+a))^4+\frac{4(\sin(bx+a))^2}{3}\right)\cos(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)^2*sin(b*x+a)^5,x)

[Out] 1/b*(sin(b*x+a)^6/cos(b*x+a)+(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)
)

Maxima [A] time = 0.965872, size = 43, normalized size = 1.16

$$-\frac{\cos(bx+a)^3-\frac{3}{\cos(bx+a)}-6\cos(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*sin(b*x+a)^5,x, algorithm="maxima")

[Out] -1/3*(cos(b*x + a)^3 - 3/cos(b*x + a) - 6*cos(b*x + a))/b

---

Fricas [A] time = 1.57127, size = 85, normalized size = 2.3

$$-\frac{\cos(bx+a)^4-6\cos(bx+a)^2-3}{3b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*sin(b*x+a)^5,x, algorithm="fricas")

[Out] -1/3*(cos(b*x + a)^4 - 6*cos(b*x + a)^2 - 3)/(b*cos(b*x + a))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)**2*sin(b*x+a)**5,x)

[Out] Timed out

Giac [B] time = 1.14049, size = 134, normalized size = 3.62

$$\frac{2\left(\frac{3}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1}+\frac{\frac{12\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{3\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-5}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^3}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 2/3*(3/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + (12*(cos(b*x + a) - 1)
/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 - 5)/((co
s(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^3)/b
```

3.107 $\int \sin^2(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=43

$$-\frac{\cos^2(a+bx)}{2b}+\frac{\sec^2(a+bx)}{2b}+\frac{2\log(\cos(a+bx))}{b}$$

```
[Out] -Cos[a + b*x]^2/(2*b) + (2*Log[Cos[a + b*x]])/b + Sec[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0379609, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2590, 266, 43}

$$-\frac{\cos^2(a+bx)}{2b}+\frac{\sec^2(a+bx)}{2b}+\frac{2\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Tan[a + b*x]^3,x]
```

```
[Out] -Cos[a + b*x]^2/(2*b) + (2*Log[Cos[a + b*x]])/b + Sec[a + b*x]^2/(2*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sin^2(a+bx)\tan^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^3}\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx,x,\cos^2(a+bx)\right)}{2b}\\
&= -\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)dx,x,\cos^2(a+bx)\right)}{2b}\\
&= -\frac{\cos^2(a+bx)}{2b}+\frac{2\log(\cos(a+bx))}{b}+\frac{\sec^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.0403313, size = 33, normalized size = 0.77

$$\frac{\sin^2(a+bx)+\sec^2(a+bx)+4\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Tan[a + b*x]^3,x]
```

```
[Out] (4*Log[Cos[a + b*x]] + Sec[a + b*x]^2 + Sin[a + b*x]^2)/(2*b)
```

Maple [A] time = 0.021, size = 60, normalized size = 1.4

$$\frac{(\sin(bx+a))^6}{2b(\cos(bx+a))^2}+\frac{(\sin(bx+a))^4}{2b}+\frac{(\sin(bx+a))^2}{b}+2\frac{\ln(\cos(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*sin(b*x+a)^5,x)
```

```
[Out] 1/2/b*sin(b*x+a)^6/cos(b*x+a)^2+1/2*sin(b*x+a)^4/b+sin(b*x+a)^2/b+2*ln(cos(
b*x+a))/b
```

Maxima [A] time = 0.95793, size = 55, normalized size = 1.28

$$\frac{\sin(bx+a)^2-\frac{1}{\sin(bx+a)^2-1}+2\log\left(\sin(bx+a)^2-1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/2*(sin(b*x + a)^2 - 1/(sin(b*x + a)^2 - 1) + 2*log(sin(b*x + a)^2 - 1))/b
```

Fricas [A] time = 1.62585, size = 139, normalized size = 3.23

$$-\frac{2\cos(bx+a)^4-8\cos(bx+a)^2\log(-\cos(bx+a))-\cos(bx+a)^2-2}{4b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(b*x + a)^4 - 8*cos(b*x + a)^2*log(-cos(b*x + a)) - cos(b*x + a)
^2 - 2)/(b*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20022, size = 246, normalized size = 5.72

$$-\frac{\frac{4\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)^2-4}+\log\left(\left|-\frac{\cos(bx+a)+1}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+2\right|\right)-\log\left(\left|-\frac{\cos(bx+a)+1}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-2\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -(4*((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b*x +
a) + 1))/(((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(
b*x + a) + 1))^2 - 4) + log(abs(-(cos(b*x + a) + 1)/(cos(b*x + a) - 1) - (c
os(b*x + a) - 1)/(cos(b*x + a) + 1) + 2)) - log(abs(-(cos(b*x + a) + 1)/(co
s(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 2)))/b
```

3.108 $\int \sin(a+bx)\tan^4(a+bx)\,dx$

Optimal. Leaf size=38

$$-\frac{\cos(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}-\frac{2\sec(a+bx)}{b}$$

```
[Out] -(Cos[a + b*x]/b) - (2*Sec[a + b*x])/b + Sec[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0252488, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 270}

$$-\frac{\cos(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}-\frac{2\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[a + b*x]^4,x]
```

```
[Out] -(Cos[a + b*x]/b) - (2*Sec[a + b*x])/b + Sec[a + b*x]^3/(3*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin(a+bx)\tan^4(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\cos(a+bx)}{b}-\frac{2\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] time = 0.0245295, size = 38, normalized size = 1.

$$-\frac{\cos(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}-\frac{2\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[a + b*x]^4,x]
```

```
[Out] -(Cos[a + b*x]/b) - (2*Sec[a + b*x])/b + Sec[a + b*x]^3/(3*b)
```

Maple [A] time = 0.021, size = 70, normalized size = 1.8

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^6}{3(\cos(bx+a))^3}-\frac{(\sin(bx+a))^6}{\cos(bx+a)}-\left(\frac{8}{3}+(\sin(bx+a))^4+\frac{4(\sin(bx+a))^2}{3}\right)\cos(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(1/3*sin(b*x+a)^6/cos(b*x+a)^3-sin(b*x+a)^6/cos(b*x+a)-(8/3+sin(b*x+a)^
4+4/3*sin(b*x+a)^2)*cos(b*x+a))
```

Maxima [A] time = 0.97019, size = 47, normalized size = 1.24

$$-\frac{\frac{6\cos(bx+a)^2-1}{\cos(bx+a)^3}+3\cos(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/3*((6*cos(b*x + a)^2 - 1)/cos(b*x + a)^3 + 3*cos(b*x + a))/b
```

Fricas [A] time = 1.63776, size = 90, normalized size = 2.37

$$-\frac{3\cos(bx+a)^4+6\cos(bx+a)^2-1}{3b\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(b*x + a)^4 + 6*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17805, size = 135, normalized size = 3.55

$$\frac{2\left(\frac{3}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1}-\frac{\frac{12\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+5}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 2/3*(3/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - (12*(cos(b*x + a) - 1)
/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 5)/((co
s(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^3)/b
```

3.109 $\int \tan^5(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\tan^4(a+bx)}{4b}-\frac{\tan^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}$$

```
[Out] -(Log[Cos[a + b*x]]/b) - Tan[a + b*x]^2/(2*b) + Tan[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0210946, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$\frac{\tan^4(a+bx)}{4b}-\frac{\tan^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*x]^5,x]
```

```
[Out] -(Log[Cos[a + b*x]]/b) - Tan[a + b*x]^2/(2*b) + Tan[a + b*x]^4/(4*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^5(a+bx)\,dx&=\frac{\tan^4(a+bx)}{4b}-\int \tan^3(a+bx)\,dx\\&=-\frac{\tan^2(a+bx)}{2b}+\frac{\tan^4(a+bx)}{4b}+\int \tan(a+bx)\,dx\\&=-\frac{\log(\cos(a+bx))}{b}-\frac{\tan^2(a+bx)}{2b}+\frac{\tan^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.051624, size = 37, normalized size = 0.86

$$-\frac{-\tan^4(a+bx)+2\tan^2(a+bx)+4\log(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*x]^5,x]
```

```
[Out] -(4*Log[Cos[a + b*x]] + 2*Tan[a + b*x]^2 - Tan[a + b*x]^4)/(4*b)
```

Maple [A] time = 0.026, size = 40, normalized size = 0.9

$$-\frac{\ln\left(\cos\left(bx+a\right)\right)}{b}-\frac{\left(\tan\left(bx+a\right)\right)^{2}}{2b}+\frac{\left(\tan\left(bx+a\right)\right)^{4}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5*sin(b*x+a)^5,x)
```

```
[Out] -ln(cos(b*x+a))/b-1/2*tan(b*x+a)^2/b+1/4*tan(b*x+a)^4/b
```

Maxima [A] time = 0.965203, size = 73, normalized size = 1.7

$$\frac{\frac{4\sin(bx+a)^{2}-3}{\sin(bx+a)^{4}-2\sin(bx+a)^{2}+1}-2\log\left(\sin\left(bx+a\right)^{2}-1\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/4*((4*sin(b*x + a)^2 - 3)/(sin(b*x + a)^4 - 2*sin(b*x + a)^2 + 1) - 2*log
(sin(b*x + a)^2 - 1))/b
```

Fricas [A] time = 1.674, size = 116, normalized size = 2.7

$$-\frac{4\cos\left(bx+a\right)^{4}\log\left(-\cos\left(bx+a\right)\right)+4\cos\left(bx+a\right)^{2}-1}{4b\cos\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*cos(b*x + a)^4*log(-cos(b*x + a)) + 4*cos(b*x + a)^2 - 1)/(b*cos(b*
x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22642, size = 305, normalized size = 7.09

$$\frac{\frac{3\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)^2+\frac{20(\cos(bx+a)+1)}{\cos(bx+a)-1}+\frac{20(\cos(bx+a)-1)}{\cos(bx+a)+1}+44}{\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+2\right)^2}+2\log\left(\left|-\frac{\cos(bx+a)+1}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+2\right|\right)-2\log\left(\left|-\frac{\cos(bx+a)+1}{\cos(bx+a)-1}-\frac{\cos}{\cos}\right.\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/4*((3*((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b*
x + a) + 1))^2 + 20*(cos(b*x + a) + 1)/(cos(b*x + a) - 1) + 20*(cos(b*x + a
) - 1)/(cos(b*x + a) + 1) + 44)/((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (c
os(b*x + a) - 1)/(cos(b*x + a) + 1) + 2)^2 + 2*log(abs(-(cos(b*x + a) + 1)/
(cos(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 2)) - 2*log(ab
s(-(cos(b*x + a) + 1)/(cos(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a)
 + 1) - 2)))/b
```

3.110 $\int \sec(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{\sec^5(a+bx)}{5b}-\frac{2\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}$$

```
[Out] Sec[a + b*x]/b - (2*Sec[a + b*x]^3)/(3*b) + Sec[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.023039, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 194}

$$\frac{\sec^5(a+bx)}{5b}-\frac{2\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]/b - (2*Sec[a + b*x]^3)/(3*b) + Sec[a + b*x]^5/(5*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(a+bx)\tan^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \sec(a+bx)\right)}{b}\\ &= \frac{\sec(a+bx)}{b}-\frac{2\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.027876, size = 41, normalized size = 1.

$$\frac{\sec^5(a+bx)}{5b}-\frac{2\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]/b - (2*Sec[a + b*x]^3)/(3*b) + Sec[a + b*x]^5/(5*b)
```

Maple [B] time = 0.023, size = 88, normalized size = 2.2

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^6}{5\,(\cos(bx+a))^5}-\frac{(\sin(bx+a))^6}{15\,(\cos(bx+a))^3}+\frac{(\sin(bx+a))^6}{5\cos(bx+a)}+\frac{\cos(bx+a)}{5}\left(\frac{8}{3}+(\sin(bx+a))^4+\frac{4\,(\sin(bx+a))^2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(1/5*sin(b*x+a)^6/cos(b*x+a)^5-1/15*sin(b*x+a)^6/cos(b*x+a)^3+1/5*sin(b
*x+a)^6/cos(b*x+a)+1/5*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a))
```

Maxima [A] time = 1.00889, size = 47, normalized size = 1.15

$$\frac{15\cos(bx+a)^4-10\cos(bx+a)^2+3}{15\,b\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/15*(15*cos(b*x + a)^4 - 10*cos(b*x + a)^2 + 3)/(b*cos(b*x + a)^5)
```

Fricas [A] time = 1.58203, size = 93, normalized size = 2.27

$$\frac{15\cos(bx+a)^4-10\cos(bx+a)^2+3}{15\,b\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/15*(15*cos(b*x + a)^4 - 10*cos(b*x + a)^2 + 3)/(b*cos(b*x + a)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15996, size = 97, normalized size = 2.37

$$\frac{16\left(\frac{5\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{10\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)}{15\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 16/15*(5*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 10*(cos(b*x + a) - 1)^2/(c
os(b*x + a) + 1)^2 + 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^5)
```

3.111 $\int \sec^2(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\tan^6(a+bx)}{6b}$$

```
[Out] Tan[a + b*x]^6/(6*b)
```

Rubi [A] time = 0.026738, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$\frac{\tan^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2*Tan[a + b*x]^5,x]
```

```
[Out] Tan[a + b*x]^6/(6*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)\tan^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^5\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\tan^6(a+bx)}{6b}\end{aligned}$$

Mathematica [A] time = 0.0087566, size = 15, normalized size = 1.

$$\frac{\tan^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2*Tan[a + b*x]^5,x]
```

```
[Out] Tan[a + b*x]^6/(6*b)
```

Maple [A] time = 0.03, size = 22, normalized size = 1.5

$$\frac{\left(\sin\left(bx+a\right)\right)^{6}}{6\,b\left(\cos\left(bx+a\right)\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^7*sin(b*x+a)^5,x)
```

```
[Out] 1/6/b*sin(b*x+a)^6/cos(b*x+a)^6
```

Maxima [B] time = 0.965047, size = 80, normalized size = 5.33

$$-\frac{3\,\sin\left(bx+a\right)^{4}-3\,\sin\left(bx+a\right)^{2}+1}{6\left(\sin\left(bx+a\right)^{6}-3\,\sin\left(bx+a\right)^{4}+3\,\sin\left(bx+a\right)^{2}-1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/6*(3*sin(b*x + a)^4 - 3*sin(b*x + a)^2 + 1)/((sin(b*x + a)^6 - 3*sin(b*x
 + a)^4 + 3*sin(b*x + a)^2 - 1)*b)
```

Fricas [B] time = 1.52293, size = 89, normalized size = 5.93

$$\frac{3\,\cos\left(bx+a\right)^{4}-3\,\cos\left(bx+a\right)^{2}+1}{6\,b\cos\left(bx+a\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(3*cos(b*x + a)^4 - 3*cos(b*x + a)^2 + 1)/(b*cos(b*x + a)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**7*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18071, size = 65, normalized size = 4.33

$$-\frac{32\left(\cos\left(bx+a\right)-1\right)^{3}}{3\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^{6}\left(\cos\left(bx+a\right)+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -32/3*(cos(b*x + a) - 1)^3/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^6
*(cos(b*x + a) + 1)^3)
```

3.112 $\int \sec^3(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sec^7(a+bx)}{7b}-\frac{2\sec^5(a+bx)}{5b}+\frac{\sec^3(a+bx)}{3b}$$

```
[Out] Sec[a + b*x]^3/(3*b) - (2*Sec[a + b*x]^5)/(5*b) + Sec[a + b*x]^7/(7*b)
```

Rubi [A] time = 0.0358386, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 270}

$$\frac{\sec^7(a+bx)}{7b}-\frac{2\sec^5(a+bx)}{5b}+\frac{\sec^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^3/(3*b) - (2*Sec[a + b*x]^5)/(5*b) + Sec[a + b*x]^7/(7*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^3(a+bx)\tan^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx, x, \sec(a+bx)\right)}{b}\\ &= \frac{\sec^3(a+bx)}{3b}-\frac{2\sec^5(a+bx)}{5b}+\frac{\sec^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.0324024, size = 46, normalized size = 1.

$$\frac{\sec^7(a+bx)}{7b}-\frac{2\sec^5(a+bx)}{5b}+\frac{\sec^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*Tan[a + b*x]^5,x]
```

[Out] Sec[a + b*x]^3/(3*b) - (2*Sec[a + b*x]^5)/(5*b) + Sec[a + b*x]^7/(7*b)

Maple [B] time = 0.023, size = 106, normalized size = 2.3

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^6}{7(\cos(bx+a))^7}+\frac{(\sin(bx+a))^6}{35(\cos(bx+a))^5}-\frac{(\sin(bx+a))^6}{105(\cos(bx+a))^3}+\frac{(\sin(bx+a))^6}{35\cos(bx+a)}+\frac{\cos(bx+a)}{35}\left(\frac{8}{3}+(\sin(bx+a))^4+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)^8*sin(b*x+a)^5,x)

[Out] 1/b*(1/7*sin(b*x+a)^6/cos(b*x+a)^7+1/35*sin(b*x+a)^6/cos(b*x+a)^5-1/105*sin(b*x+a)^6/cos(b*x+a)^3+1/35*sin(b*x+a)^6/cos(b*x+a)+1/35*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a))

Maxima [A] time = 0.95826, size = 47, normalized size = 1.02

$$\frac{35\cos(bx+a)^4-42\cos(bx+a)^2+15}{105\,b\cos(bx+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^8*sin(b*x+a)^5,x, algorithm="maxima")

[Out] 1/105*(35*cos(b*x + a)^4 - 42*cos(b*x + a)^2 + 15)/(b*cos(b*x + a)^7)

Fricas [A] time = 1.51301, size = 96, normalized size = 2.09

$$\frac{35\cos(bx+a)^4-42\cos(bx+a)^2+15}{105\,b\cos(bx+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^8*sin(b*x+a)^5,x, algorithm="fricas")

[Out] 1/105*(35*cos(b*x + a)^4 - 42*cos(b*x + a)^2 + 15)/(b*cos(b*x + a)^7)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)**8*sin(b*x+a)**5,x)

[Out] Timed out

Giac [B] time = 1.16871, size = 157, normalized size = 3.41

$$\frac{16\left(\frac{7\left(\cos(bx+a)-1\right)}{\cos(bx+a)+1}+\frac{21\left(\cos(bx+a)-1\right)^{2}}{\left(\cos(bx+a)+1\right)^{2}}-\frac{35\left(\cos(bx+a)-1\right)^{3}}{\left(\cos(bx+a)+1\right)^{3}}+\frac{70\left(\cos(bx+a)-1\right)^{4}}{\left(\cos(bx+a)+1\right)^{4}}+1\right)}{105\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 16/105*(7*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 21*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 - 35*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 70*(co
s(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 1)/(b*((cos(b*x + a) - 1)/(cos(b*x
 + a) + 1) + 1)^7)
```

3.113 $\int \sec^4(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\tan^8(a+bx)}{8b}+\frac{\tan^6(a+bx)}{6b}$$

```
[Out] Tan[a + b*x]^6/(6*b) + Tan[a + b*x]^8/(8*b)
```

Rubi [A] time = 0.0311392, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 14}

$$\frac{\tan^8(a+bx)}{8b}+\frac{\tan^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4*Tan[a + b*x]^5,x]
```

```
[Out] Tan[a + b*x]^6/(6*b) + Tan[a + b*x]^8/(8*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sec^4(a+bx)\tan^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^5\left(1+x^2\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^5+x^7\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\tan^6(a+bx)}{6b}+\frac{\tan^8(a+bx)}{8b}\end{aligned}$$

Mathematica [A] time = 0.0552115, size = 38, normalized size = 1.23

$$\frac{3\sec^8(a+bx)-8\sec^6(a+bx)+6\sec^4(a+bx)}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4*Tan[a + b*x]^5,x]
```

```
[Out] (6*Sec[a + b*x]^4 - 8*Sec[a + b*x]^6 + 3*Sec[a + b*x]^8)/(24*b)
```

Maple [A] time = 0.023, size = 42, normalized size = 1.4

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^6}{8(\cos(bx+a))^8}+\frac{(\sin(bx+a))^6}{24(\cos(bx+a))^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^9*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(1/8*sin(b*x+a)^6/cos(b*x+a)^8+1/24*sin(b*x+a)^6/cos(b*x+a)^6)
```

Maxima [B] time = 0.988295, size = 93, normalized size = 3.

$$\frac{6\sin(bx+a)^4-4\sin(bx+a)^2+1}{24\left(\sin(bx+a)^8-4\sin(bx+a)^6+6\sin(bx+a)^4-4\sin(bx+a)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*(6*sin(b*x + a)^4 - 4*sin(b*x + a)^2 + 1)/((sin(b*x + a)^8 - 4*sin(b*x
 + a)^6 + 6*sin(b*x + a)^4 - 4*sin(b*x + a)^2 + 1)*b)
```

Fricas [A] time = 1.53711, size = 90, normalized size = 2.9

$$\frac{6\cos(bx+a)^4-8\cos(bx+a)^2+3}{24\,b\cos(bx+a)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/24*(6*cos(b*x + a)^4 - 8*cos(b*x + a)^2 + 3)/(b*cos(b*x + a)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**9*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19924, size = 126, normalized size = 4.06

$$-\frac{32\left(\frac{(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}\right)}{3\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -32/3*((cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - (cos(b*x + a) - 1)^4/(co
s(b*x + a) + 1)^4 + (cos(b*x + a) - 1)^5/(cos(b*x + a) + 1)^5)/(b*((cos(b*x
 + a) - 1)/(cos(b*x + a) + 1) + 1)^8)
```

3.114 $\int \sec^5(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sec^9(a+bx)}{9b}-\frac{2\sec^7(a+bx)}{7b}+\frac{\sec^5(a+bx)}{5b}$$

```
[Out] Sec[a + b*x]^5/(5*b) - (2*Sec[a + b*x]^7)/(7*b) + Sec[a + b*x]^9/(9*b)
```

Rubi [A] time = 0.0350931, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 270}

$$\frac{\sec^9(a+bx)}{9b}-\frac{2\sec^7(a+bx)}{7b}+\frac{\sec^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^5/(5*b) - (2*Sec[a + b*x]^7)/(7*b) + Sec[a + b*x]^9/(9*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^5(a+bx)\tan^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx,x,\sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= \frac{\sec^5(a+bx)}{5b}-\frac{2\sec^7(a+bx)}{7b}+\frac{\sec^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] time = 0.0331702, size = 46, normalized size = 1.

$$\frac{\sec^9(a+bx)}{9b}-\frac{2\sec^7(a+bx)}{7b}+\frac{\sec^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^5/(5*b) - (2*Sec[a + b*x]^7)/(7*b) + Sec[a + b*x]^9/(9*b)
```

Maple [B] time = 0.023, size = 124, normalized size = 2.7

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^6}{9(\cos(bx+a))^9}+\frac{(\sin(bx+a))^6}{21(\cos(bx+a))^7}+\frac{(\sin(bx+a))^6}{105(\cos(bx+a))^5}-\frac{(\sin(bx+a))^6}{315(\cos(bx+a))^3}+\frac{(\sin(bx+a))^6}{105\cos(bx+a)}+\frac{\cos(bx+a)}{105}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^10*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(1/9*sin(b*x+a)^6/cos(b*x+a)^9+1/21*sin(b*x+a)^6/cos(b*x+a)^7+1/105*sin
(b*x+a)^6/cos(b*x+a)^5-1/315*sin(b*x+a)^6/cos(b*x+a)^3+1/105*sin(b*x+a)^6/c
os(b*x+a)+1/105*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a))
```

Maxima [A] time = 0.986853, size = 47, normalized size = 1.02

$$\frac{63\cos(bx+a)^4-90\cos(bx+a)^2+35}{315\,b\cos(bx+a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/315*(63*cos(b*x + a)^4 - 90*cos(b*x + a)^2 + 35)/(b*cos(b*x + a)^9)
```

Fricas [A] time = 1.57627, size = 96, normalized size = 2.09

$$\frac{63\cos(bx+a)^4-90\cos(bx+a)^2+35}{315\,b\cos(bx+a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/315*(63*cos(b*x + a)^4 - 90*cos(b*x + a)^2 + 35)/(b*cos(b*x + a)^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**10*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17873, size = 216, normalized size = 4.7

$$\frac{16\left(\frac{9\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{36\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-\frac{126\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{441\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}-\frac{315\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}+\frac{210\,(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}+1\right)}{315\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 16/315*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 36*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 - 126*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 441*(
cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 315*(cos(b*x + a) - 1)^5/(cos(b*
x + a) + 1)^5 + 210*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 1)/(b*((cos
(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^9)
```

3.115 $\int \sec^6(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sec^{10}(a+bx)}{10b}-\frac{\sec^8(a+bx)}{4b}+\frac{\sec^6(a+bx)}{6b}$$

```
[Out] Sec[a + b*x]^6/(6*b) - Sec[a + b*x]^8/(4*b) + Sec[a + b*x]^10/(10*b)
```

Rubi [A] time = 0.0391812, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2606, 266, 43}

$$\frac{\sec^{10}(a+bx)}{10b}-\frac{\sec^8(a+bx)}{4b}+\frac{\sec^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^6/(6*b) - Sec[a + b*x]^8/(4*b) + Sec[a + b*x]^10/(10*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec^6(a+bx)\tan^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^5\left(-1+x^2\right)^2\,dx, x, \sec(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int(-1+x)^2x^2\,dx, x, \sec^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int\left(x^2-2x^3+x^4\right)\,dx, x, \sec^2(a+bx)\right)}{2b}\\
&= \frac{\sec^6(a+bx)}{6b}-\frac{\sec^8(a+bx)}{4b}+\frac{\sec^{10}(a+bx)}{10b}
\end{aligned}$$

Mathematica [A] time = 0.0586623, size = 38, normalized size = 0.83

$$\frac{6\sec^{10}(a+bx)-15\sec^8(a+bx)+10\sec^6(a+bx)}{60b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6*Tan[a + b*x]^5,x]
```

```
[Out] (10*Sec[a + b*x]^6 - 15*Sec[a + b*x]^8 + 6*Sec[a + b*x]^10)/(60*b)
```

Maple [A] time = 0.023, size = 60, normalized size = 1.3

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^6}{10\,(\cos(bx+a))^{10}}+\frac{(\sin(bx+a))^6}{20\,(\cos(bx+a))^8}+\frac{(\sin(bx+a))^6}{60\,(\cos(bx+a))^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^11*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(1/10*sin(b*x+a)^6/cos(b*x+a)^10+1/20*sin(b*x+a)^6/cos(b*x+a)^8+1/60*si
n(b*x+a)^6/cos(b*x+a)^6)
```

Maxima [A] time = 0.972727, size = 107, normalized size = 2.33

$$-\frac{10\sin(bx+a)^4-5\sin(bx+a)^2+1}{60\left(\sin(bx+a)^{10}-5\sin(bx+a)^8+10\sin(bx+a)^6-10\sin(bx+a)^4+5\sin(bx+a)^2-1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^11*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(10*sin(b*x + a)^4 - 5*sin(b*x + a)^2 + 1)/((sin(b*x + a)^10 - 5*sin(
b*x + a)^8 + 10*sin(b*x + a)^6 - 10*sin(b*x + a)^4 + 5*sin(b*x + a)^2 - 1)*
b)
```

Fricas [A] time = 1.59344, size = 95, normalized size = 2.07

$$\frac{10\cos(bx+a)^4-15\cos(bx+a)^2+6}{60\,b\cos(bx+a)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^11*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/60*(10*cos(b*x + a)^4 - 15*cos(b*x + a)^2 + 6)/(b*cos(b*x + a)^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**11*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19066, size = 188, normalized size = 4.09

$$-\frac{32\left(\frac{5\left(\cos(bx+a)-1\right)^3}{\left(\cos(bx+a)+1\right)^3}-\frac{10\left(\cos(bx+a)-1\right)^4}{\left(\cos(bx+a)+1\right)^4}+\frac{18\left(\cos(bx+a)-1\right)^5}{\left(\cos(bx+a)+1\right)^5}-\frac{10\left(\cos(bx+a)-1\right)^6}{\left(\cos(bx+a)+1\right)^6}+\frac{5\left(\cos(bx+a)-1\right)^7}{\left(\cos(bx+a)+1\right)^7}\right)}{15\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^11*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -32/15*(5*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 10*(cos(b*x + a) - 1)
^4/(cos(b*x + a) + 1)^4 + 18*(cos(b*x + a) - 1)^5/(cos(b*x + a) + 1)^5 - 10
*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 5*(cos(b*x + a) - 1)^7/(cos(b*
x + a) + 1)^7)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^10)
```

3.116 $\int \sec^7(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sec^{11}(a+bx)}{11b}-\frac{2\sec^9(a+bx)}{9b}+\frac{\sec^7(a+bx)}{7b}$$

```
[Out] Sec[a + b*x]^7/(7*b) - (2*Sec[a + b*x]^9)/(9*b) + Sec[a + b*x]^11/(11*b)
```

Rubi [A] time = 0.0345757, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 270}

$$\frac{\sec^{11}(a+bx)}{11b}-\frac{2\sec^9(a+bx)}{9b}+\frac{\sec^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^7*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^7/(7*b) - (2*Sec[a + b*x]^9)/(9*b) + Sec[a + b*x]^11/(11*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^7(a+bx)\tan^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^6\left(-1+x^2\right)^2\,dx,x,\sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^6-2x^8+x^{10}\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= \frac{\sec^7(a+bx)}{7b}-\frac{2\sec^9(a+bx)}{9b}+\frac{\sec^{11}(a+bx)}{11b}\end{aligned}$$

Mathematica [A] time = 0.0325798, size = 46, normalized size = 1.

$$\frac{\sec^{11}(a+bx)}{11b}-\frac{2\sec^9(a+bx)}{9b}+\frac{\sec^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^7*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^7/(7*b) - (2*Sec[a + b*x]^9)/(9*b) + Sec[a + b*x]^11/(11*b)
```

Maple [B] time = 0.024, size = 142, normalized size = 3.1

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^6}{11(\cos(bx+a))^{11}}+\frac{5(\sin(bx+a))^6}{99(\cos(bx+a))^9}+\frac{5(\sin(bx+a))^6}{231(\cos(bx+a))^7}+\frac{(\sin(bx+a))^6}{231(\cos(bx+a))^5}-\frac{(\sin(bx+a))^6}{693(\cos(bx+a))^3}+\frac{(\sin(b}{231\cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^12*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(1/11*sin(b*x+a)^6/cos(b*x+a)^11+5/99*sin(b*x+a)^6/cos(b*x+a)^9+5/231*s
in(b*x+a)^6/cos(b*x+a)^7+1/231*sin(b*x+a)^6/cos(b*x+a)^5-1/693*sin(b*x+a)^6
/cos(b*x+a)^3+1/231*sin(b*x+a)^6/cos(b*x+a)+1/231*(8/3+sin(b*x+a)^4+4/3*sin
(b*x+a)^2)*cos(b*x+a))
```

Maxima [A] time = 0.974391, size = 47, normalized size = 1.02

$$\frac{99\cos(bx+a)^4-154\cos(bx+a)^2+63}{693\,b\cos(bx+a)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^12*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/693*(99*cos(b*x + a)^4 - 154*cos(b*x + a)^2 + 63)/(b*cos(b*x + a)^11)
```

Fricas [A] time = 1.61024, size = 99, normalized size = 2.15

$$\frac{99\cos(bx+a)^4-154\cos(bx+a)^2+63}{693\,b\cos(bx+a)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^12*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/693*(99*cos(b*x + a)^4 - 154*cos(b*x + a)^2 + 63)/(b*cos(b*x + a)^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**12*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19285, size = 275, normalized size = 5.98

$$\frac{16\left(\frac{11\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{55\,(\cos(bx+a)-1)^{2}}{(\cos(bx+a)+1)^{2}}-\frac{297\,(\cos(bx+a)-1)^{3}}{(\cos(bx+a)+1)^{3}}+\frac{1485\,(\cos(bx+a)-1)^{4}}{(\cos(bx+a)+1)^{4}}-\frac{2079\,(\cos(bx+a)-1)^{5}}{(\cos(bx+a)+1)^{5}}+\frac{2541\,(\cos(bx+a)-1)^{6}}{(\cos(bx+a)+1)^{6}}-\frac{1155\,(}{(\cos}\right.}{693\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^12*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 16/693*(11*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 55*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 - 297*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 1485
*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 2079*(cos(b*x + a) - 1)^5/(cos
(b*x + a) + 1)^5 + 2541*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 - 1155*(c
os(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 + 462*(cos(b*x + a) - 1)^8/(cos(b*x
 + a) + 1)^8 + 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^11)
```

3.117 $\int \sec^8(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sec^{12}(a+bx)}{12b}-\frac{\sec^{10}(a+bx)}{5b}+\frac{\sec^8(a+bx)}{8b}$$

```
[Out] Sec[a + b*x]^8/(8*b) - Sec[a + b*x]^10/(5*b) + Sec[a + b*x]^12/(12*b)
```

Rubi [A] time = 0.0402263, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2606, 266, 43}

$$\frac{\sec^{12}(a+bx)}{12b}-\frac{\sec^{10}(a+bx)}{5b}+\frac{\sec^8(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^8*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^8/(8*b) - Sec[a + b*x]^10/(5*b) + Sec[a + b*x]^12/(12*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^8(a+bx)\tan^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^7\left(-1+x^2\right)^2\,dx,x,\sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (-1+x)^2x^3\,dx,x,\sec^2(a+bx)\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \left(x^3-2x^4+x^5\right)\,dx,x,\sec^2(a+bx)\right)}{2b}\\ &= \frac{\sec^8(a+bx)}{8b}-\frac{\sec^{10}(a+bx)}{5b}+\frac{\sec^{12}(a+bx)}{12b}\end{aligned}$$

Mathematica [A] time = 0.128708, size = 38, normalized size = 0.83

$$\frac{10\sec^{12}(a+bx)-24\sec^{10}(a+bx)+15\sec^{8}(a+bx)}{120b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^8*Tan[a + b*x]^5,x]
```

```
[Out] (15*Sec[a + b*x]^8 - 24*Sec[a + b*x]^10 + 10*Sec[a + b*x]^12)/(120*b)
```

Maple [A] time = 0.026, size = 78, normalized size = 1.7

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^{6}}{12\,(\cos(bx+a))^{12}}+\frac{(\sin(bx+a))^{6}}{20\,(\cos(bx+a))^{10}}+\frac{(\sin(bx+a))^{6}}{40\,(\cos(bx+a))^{8}}+\frac{(\sin(bx+a))^{6}}{120\,(\cos(bx+a))^{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^13*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(1/12*sin(b*x+a)^6/cos(b*x+a)^12+1/20*sin(b*x+a)^6/cos(b*x+a)^10+1/40*s
in(b*x+a)^6/cos(b*x+a)^8+1/120*sin(b*x+a)^6/cos(b*x+a)^6)
```

Maxima [B] time = 0.974354, size = 120, normalized size = 2.61

$$\frac{15\sin(bx+a)^{4}-6\sin(bx+a)^{2}+1}{120\left(\sin(bx+a)^{12}-6\sin(bx+a)^{10}+15\sin(bx+a)^{8}-20\sin(bx+a)^{6}+15\sin(bx+a)^{4}-6\sin(bx+a)^{2}+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^13*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/120*(15*sin(b*x + a)^4 - 6*sin(b*x + a)^2 + 1)/((sin(b*x + a)^12 - 6*sin(
b*x + a)^10 + 15*sin(b*x + a)^8 - 20*sin(b*x + a)^6 + 15*sin(b*x + a)^4 - 6
*sin(b*x + a)^2 + 1)*b)
```

Fricas [A] time = 1.64175, size = 97, normalized size = 2.11

$$\frac{15\cos(bx+a)^{4}-24\cos(bx+a)^{2}+10}{120\,b\cos(bx+a)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^13*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/120*(15*cos(b*x + a)^4 - 24*cos(b*x + a)^2 + 10)/(b*cos(b*x + a)^12)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**13*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20258, size = 247, normalized size = 5.37

$$-\frac{32\left(\frac{5(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{15(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{39(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}-\frac{42(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}+\frac{39(\cos(bx+a)-1)^7}{(\cos(bx+a)+1)^7}-\frac{15(\cos(bx+a)-1)^8}{(\cos(bx+a)+1)^8}+\frac{5(\cos(bx+a)-1)^9}{(\cos(bx+a)+1)^9}\right)}{15\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^13*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -32/15*(5*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 15*(cos(b*x + a) - 1)
^4/(cos(b*x + a) + 1)^4 + 39*(cos(b*x + a) - 1)^5/(cos(b*x + a) + 1)^5 - 42
*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 39*(cos(b*x + a) - 1)^7/(cos(b
*x + a) + 1)^7 - 15*(cos(b*x + a) - 1)^8/(cos(b*x + a) + 1)^8 + 5*(cos(b*x
+ a) - 1)^9/(cos(b*x + a) + 1)^9)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1)
 + 1)^12)
```

3.118 $\int \sin^3(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{5\sin^3(a+bx)}{6b}+\frac{5\sin(a+bx)}{2b}+\frac{\sin^3(a+bx)\tan^2(a+bx)}{2b}-\frac{5\tanh^{-1}(\sin(a+bx))}{2b}$$

```
[Out] (-5*ArcTanh[Sin[a + b*x]])/(2*b) + (5*Sin[a + b*x])/(2*b) + (5*Sin[a + b*x]
^3)/(6*b) + (Sin[a + b*x]^3*Tan[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0469644, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2592, 288, 302, 206}

$$\frac{5\sin^3(a+bx)}{6b}+\frac{5\sin(a+bx)}{2b}+\frac{\sin^3(a+bx)\tan^2(a+bx)}{2b}-\frac{5\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Tan[a + b*x]^3,x]
```

```
[Out] (-5*ArcTanh[Sin[a + b*x]])/(2*b) + (5*Sin[a + b*x])/(2*b) + (5*Sin[a + b*x]
^3)/(6*b) + (Sin[a + b*x]^3*Tan[a + b*x]^2)/(2*b)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{\sin^3(a+bx)\tan^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b}$$
$$= \frac{\sin^3(a+bx)\tan^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \sin(a+bx)\right)}{2b}$$
$$= \frac{5\sin(a+bx)}{2b} + \frac{5\sin^3(a+bx)}{6b} + \frac{\sin^3(a+bx)\tan^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \text{si}\right.}{2b}$$
$$= -\frac{5\tanh^{-1}(\sin(a+bx))}{2b} + \frac{5\sin(a+bx)}{2b} + \frac{5\sin^3(a+bx)}{6b} + \frac{\sin^3(a+bx)\tan^2(a+bx)}{2b}$$

Mathematica [A] time = 0.185798, size = 52, normalized size = 0.79

$$\frac{(24\cos(2(a+bx)) - \cos(4(a+bx)) + 37)\tan(a+bx)\sec(a+bx) - 60\tanh^{-1}(\sin(a+bx))}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Tan[a + b*x]^3,x]
```

```
[Out] (-60*ArcTanh[Sin[a + b*x]] + (37 + 24*Cos[2*(a + b*x)] - Cos[4*(a + b*x)])*
Sec[a + b*x]*Tan[a + b*x])/(24*b)
```

Maple [A] time = 0.022, size = 79, normalized size = 1.2

$$\frac{(\sin(bx+a))^7}{2b(\cos(bx+a))^2} + \frac{(\sin(bx+a))^5}{2b} + \frac{5(\sin(bx+a))^3}{6b} + \frac{5\sin(bx+a)}{2b} - \frac{5\ln(\sec(bx+a)+\tan(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*sin(b*x+a)^6,x)
```

```
[Out] 1/2/b*sin(b*x+a)^7/cos(b*x+a)^2+1/2*sin(b*x+a)^5/b+5/6*sin(b*x+a)^3/b+5/2*s
in(b*x+a)/b-5/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 0.959911, size = 89, normalized size = 1.35

$$\frac{4\sin(bx+a)^3 - \frac{6\sin(bx+a)}{\sin(bx+a)^2-1} - 15\log(\sin(bx+a)+1) + 15\log(\sin(bx+a)-1) + 24\sin(bx+a)}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] 1/12*(4*sin(b*x + a)^3 - 6*sin(b*x + a)/(sin(b*x + a)^2 - 1) - 15*log(sin(b
*x + a) + 1) + 15*log(sin(b*x + a) - 1) + 24*sin(b*x + a))/b
```

Fricas [A] time = 1.76429, size = 231, normalized size = 3.5

$$-\frac{15 \cos(bx+a)^2 \log(\sin(bx+a)+1) - 15 \cos(bx+a)^2 \log(-\sin(bx+a)+1) + 2\left(2 \cos(bx+a)^4 - 14 \cos(bx+\right.}{12\, b \cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/12*(15*cos(b*x + a)^2*log(sin(b*x + a) + 1) - 15*cos(b*x + a)^2*log(-sin
(b*x + a) + 1) + 2*(2*cos(b*x + a)^4 - 14*cos(b*x + a)^2 - 3)*sin(b*x + a))
/(b*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22717, size = 92, normalized size = 1.39

$$\frac{4 \sin(bx+a)^3 - \frac{6 \sin(bx+a)}{\sin(bx+a)^2-1} - 15 \log(|\sin(bx+a)+1|) + 15 \log(|\sin(bx+a)-1|) + 24 \sin(bx+a)}{12\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 1/12*(4*sin(b*x + a)^3 - 6*sin(b*x + a)/(sin(b*x + a)^2 - 1) - 15*log(abs(s
in(b*x + a) + 1)) + 15*log(abs(sin(b*x + a) - 1)) + 24*sin(b*x + a))/b
```

3.119 $\int \sin(a+bx)\tan^6(a+bx)\,dx$

Optimal. Leaf size=50

$$\frac{\cos(a+bx)}{b}+\frac{\sec^5(a+bx)}{5b}-\frac{\sec^3(a+bx)}{b}+\frac{3\sec(a+bx)}{b}$$

```
[Out] Cos[a + b*x]/b + (3*Sec[a + b*x])/b - Sec[a + b*x]^3/b + Sec[a + b*x]^5/(5*
b)
```

Rubi [A] time = 0.0279641, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 270}

$$\frac{\cos(a+bx)}{b}+\frac{\sec^5(a+bx)}{5b}-\frac{\sec^3(a+bx)}{b}+\frac{3\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[a + b*x]^6,x]
```

```
[Out] Cos[a + b*x]/b + (3*Sec[a + b*x])/b - Sec[a + b*x]^3/b + Sec[a + b*x]^5/(5*
b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\tan^6(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^6}\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^6}-\frac{3}{x^4}+\frac{3}{x^2}\right)dx,x,\cos(a+bx)\right)}{b}\\ &= \frac{\cos(a+bx)}{b}+\frac{3\sec(a+bx)}{b}-\frac{\sec^3(a+bx)}{b}+\frac{\sec^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0400032, size = 50, normalized size = 1.

$$\frac{\cos(a+bx)}{b}+\frac{\sec^5(a+bx)}{5b}-\frac{\sec^3(a+bx)}{b}+\frac{3\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[a + b*x]^6,x]
```

```
[Out] Cos[a + b*x]/b + (3*Sec[a + b*x])/b - Sec[a + b*x]^3/b + Sec[a + b*x]^5/(5*
b)
```

Maple [A] time = 0.026, size = 96, normalized size = 1.9

$$\frac{1}{b}\left(\frac{(\sin(bx+a))^8}{5(\cos(bx+a))^5}-\frac{(\sin(bx+a))^8}{5(\cos(bx+a))^3}+\frac{(\sin(bx+a))^8}{\cos(bx+a)}+\left(\frac{16}{5}+(\sin(bx+a))^6+\frac{6(\sin(bx+a))^4}{5}+\frac{8(\sin(bx+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6*sin(b*x+a)^7,x)
```

```
[Out] 1/b*(1/5*sin(b*x+a)^8/cos(b*x+a)^5-1/5*sin(b*x+a)^8/cos(b*x+a)^3+sin(b*x+a)
^8/cos(b*x+a)+(16/5+sin(b*x+a)^6+6/5*sin(b*x+a)^4+8/5*sin(b*x+a)^2)*cos(b*x
+a))
```

Maxima [A] time = 0.975018, size = 61, normalized size = 1.22

$$\frac{\frac{15\cos(bx+a)^4-5\cos(bx+a)^2+1}{\cos(bx+a)^5}+5\cos(bx+a)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/5*((15*cos(b*x + a)^4 - 5*cos(b*x + a)^2 + 1)/cos(b*x + a)^5 + 5*cos(b*x
+ a))/b
```

Fricas [A] time = 1.58501, size = 116, normalized size = 2.32

$$\frac{5\cos(bx+a)^6+15\cos(bx+a)^4-5\cos(bx+a)^2+1}{5b\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/5*(5*cos(b*x + a)^6 + 15*cos(b*x + a)^4 - 5*cos(b*x + a)^2 + 1)/(b*cos(b*
x + a)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6*sin(b*x+a)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21745, size = 194, normalized size = 3.88

$$-\frac{2\left(\frac{5}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1}-\frac{\frac{50\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{80\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{30\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{5\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+11}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^5}\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -2/5*(5/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - (50*(cos(b*x + a) - 1
)/(cos(b*x + a) + 1) + 80*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 30*(c
os(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 5*(cos(b*x + a) - 1)^4/(cos(b*x +
 a) + 1)^4 + 11)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^5)/b
```

3.120 $\int \cos^5(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{\cos^5(a+bx)}{5b}+\frac{\cos^3(a+bx)}{3b}+\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Cos[a + b*x]/b + Cos[a + b*x]^3/(3*b) + Cos[a
+ b*x]^5/(5*b)
```

Rubi [A] time = 0.029208, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2592, 302, 206}

$$\frac{\cos^5(a+bx)}{5b}+\frac{\cos^3(a+bx)}{3b}+\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*Cot[a + b*x],x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Cos[a + b*x]/b + Cos[a + b*x]^3/(3*b) + Cos[a
+ b*x]^5/(5*b)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^5(a+bx)\cot(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(-1-x^2-x^4+\frac{1}{1-x^2}\right)dx,x,\cos(a+bx)\right)}{b}\\
&= \frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}+\frac{\cos^5(a+bx)}{5b}-\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\tanh^{-1}(\cos(a+bx))}{b}+\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}+\frac{\cos^5(a+bx)}{5b}
\end{aligned}$$

Mathematica [A] time = 0.0316335, size = 75, normalized size = 1.42

$$\frac{11\cos(a+bx)}{8b}+\frac{7\cos(3(a+bx))}{48b}+\frac{\cos(5(a+bx))}{80b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*Cot[a + b*x],x]
```

```
[Out] (11*Cos[a + b*x])/(8*b) + (7*Cos[3*(a + b*x)])/(48*b) + Cos[5*(a + b*x)]/(8
0*b) - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b
```

Maple [A] time = 0.016, size = 58, normalized size = 1.1

$$\frac{(\cos(bx+a))^5}{5b}+\frac{(\cos(bx+a))^3}{3b}+\frac{\cos(bx+a)}{b}+\frac{\ln(\csc(bx+a)-\cot(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6/sin(b*x+a),x)
```

```
[Out] 1/5*cos(b*x+a)^5/b+1/3*cos(b*x+a)^3/b+cos(b*x+a)/b+1/b*ln(csc(b*x+a)-cot(b*
x+a))
```

Maxima [A] time = 1.03192, size = 76, normalized size = 1.43

$$\frac{6\cos(bx+a)^5+10\cos(bx+a)^3+30\cos(bx+a)-15\log(\cos(bx+a)+1)+15\log(\cos(bx+a)-1)}{30b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/30*(6*cos(b*x + a)^5 + 10*cos(b*x + a)^3 + 30*cos(b*x + a) - 15*log(cos(b
*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b
```

Fricas [A] time = 1.73491, size = 178, normalized size = 3.36

$$\frac{6\cos(bx+a)^5+10\cos(bx+a)^3+30\cos(bx+a)-15\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+15\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)}{30b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/30*(6*cos(b*x + a)^5 + 10*cos(b*x + a)^3 + 30*cos(b*x + a) - 15*log(1/2*c
os(b*x + a) + 1/2) + 15*log(-1/2*cos(b*x + a) + 1/2))/b
```

Sympy [A] time = 11.0172, size = 1085, normalized size = 20.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6/sin(b*x+a),x)
```

```
[Out] Piecewise((15*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**10/(15*b*tan(a/2 + b*
x/2)**10 + 75*b*tan(a/2 + b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan
(a/2 + b*x/2)**4 + 75*b*tan(a/2 + b*x/2)**2 + 15*b) + 75*log(tan(a/2 + b*x/
2))*tan(a/2 + b*x/2)**8/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)*
*8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 +
 b*x/2)**2 + 15*b) + 150*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(15*b*ta
n(a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 +
 150*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 + b*x/2)**2 + 15*b) + 150*log(tan
(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/
2 + b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*
b*tan(a/2 + b*x/2)**2 + 15*b) + 75*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**
2/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)**8 + 150*b*tan(a/2 + b
*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 + b*x/2)**2 + 15*b) + 1
5*log(tan(a/2 + b*x/2))/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)*
*8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 +
 b*x/2)**2 + 15*b) + 90*tan(a/2 + b*x/2)**8/(15*b*tan(a/2 + b*x/2)**10 + 75
*b*tan(a/2 + b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)
**4 + 75*b*tan(a/2 + b*x/2)**2 + 15*b) + 180*tan(a/2 + b*x/2)**6/(15*b*tan(
a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 1
50*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 + b*x/2)**2 + 15*b) + 280*tan(a/2 +
 b*x/2)**4/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)**8 + 150*b*ta
n(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 + b*x/2)**2 +
15*b) + 140*tan(a/2 + b*x/2)**2/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 +
 b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*t
an(a/2 + b*x/2)**2 + 15*b) + 46/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 +
 b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*t
an(a/2 + b*x/2)**2 + 15*b), Ne(b, 0)), (x*cos(a)**6/sin(a), True))
```

Giac [B] time = 1.17656, size = 196, normalized size = 3.7

$$\frac{\frac{4\left(\frac{70\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{140\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{90\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{45\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}-23\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^5}+15\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/30*(4*(70*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 140*(cos(b*x + a) - 1)^
2/(cos(b*x + a) + 1)^2 + 90*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 45*
(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 23)/((cos(b*x + a) - 1)/(cos(b*
x + a) + 1) - 1)^5 + 15*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/
b
```

3.121 $\int \cos^4(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{\sin^4(a+bx)}{4b}-\frac{\sin^2(a+bx)}{b}+\frac{\log(\sin(a+bx))}{b}$$

```
[Out] Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/b + Sin[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0273101, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2590, 266, 43}

$$\frac{\sin^4(a+bx)}{4b}-\frac{\sin^2(a+bx)}{b}+\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Cot[a + b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/b + Sin[a + b*x]^4/(4*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^4(a+bx)\cot(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x}\,dx,x,-\sin(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx,x,\sin^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)\,dx,x,\sin^2(a+bx)\right)}{2b}\\
&= \frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{b}+\frac{\sin^4(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.0148495, size = 40, normalized size = 1.

$$\frac{\sin^4(a+bx)}{4b} - \frac{\sin^2(a+bx)}{b} + \frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Cot[a + b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/b + Sin[a + b*x]^4/(4*b)
```

Maple [A] time = 0.016, size = 39, normalized size = 1.

$$\frac{(\cos(bx+a))^4}{4b} + \frac{(\cos(bx+a))^2}{2b} + \frac{\ln(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5/sin(b*x+a),x)
```

```
[Out] 1/4*cos(b*x+a)^4/b+1/2*cos(b*x+a)^2/b+ln(sin(b*x+a))/b
```

Maxima [A] time = 0.973472, size = 47, normalized size = 1.18

$$\frac{\sin(bx+a)^4 - 4\sin(bx+a)^2 + 2\log\left(\sin(bx+a)^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(sin(b*x + a)^4 - 4*sin(b*x + a)^2 + 2*log(sin(b*x + a)^2))/b
```

Fricas [A] time = 1.72162, size = 93, normalized size = 2.32

$$\frac{\cos(bx+a)^4 + 2\cos(bx+a)^2 + 4\log\left(\frac{1}{2}\sin(bx+a)\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(cos(b*x + a)^4 + 2*cos(b*x + a)^2 + 4*log(1/2*sin(b*x + a)))/b
```

Sympy [A] time = 6.07945, size = 1086, normalized size = 27.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5/sin(b*x+a),x)
```

```
[Out] Piecewise((-log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b
*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2
+ b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(b*ta
n(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b
*tan(a/2 + b*x/2)**2 + b) - 6*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)
**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)
**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2
 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2
 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2)**2 + 1)/
(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4
+ 4*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/
(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4
+ 4*b*tan(a/2 + b*x/2)**2 + b) + 4*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**
6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**
4 + 4*b*tan(a/2 + b*x/2)**2 + b) + 6*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)
**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)
**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + 4*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/
2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/
2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2))/(b*tan(a/2 + b
*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2
+ b*x/2)**2 + b) - 4*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a
/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4
*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b
*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*tan(a/2 + b*x/2)**2
/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4
 + 4*b*tan(a/2 + b*x/2)**2 + b), Ne(b, 0)), (x*cos(a)**5/sin(a), True))
```

Giac [B] time = 1.16468, size = 230, normalized size = 5.75

$$-\frac{\frac{\frac{52\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{102\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{52\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{25\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}-25}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^4}-6\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)+12\,\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right|\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/12*((52*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 102*(cos(b*x + a) - 1)^2
/(cos(b*x + a) + 1)^2 + 52*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 25*(
cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 25)/((cos(b*x + a) - 1)/(cos(b*x
 + a) + 1) - 1)^4 - 6*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 1
2*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

3.122 $\int \cos^3(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\cos^3(a+bx)}{3b}+\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Cos[a + b*x]/b + Cos[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.025659, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2592, 302, 206}

$$\frac{\cos^3(a+bx)}{3b}+\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Cot[a + b*x],x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Cos[a + b*x]/b + Cos[a + b*x]^3/(3*b)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^3(a+bx)\cot(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx,x,\cos(a+bx)\right)}{b}\\
&= \frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}-\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\tanh^{-1}(\cos(a+bx))}{b}+\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] time = 0.024518, size = 60, normalized size = 1.58

$$\frac{5\cos(a+bx)}{4b}+\frac{\cos(3(a+bx))}{12b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Cot[a + b*x],x]
```

```
[Out] (5*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b) - Log[Cos[(a + b*x)/2]]/b
+ Log[Sin[(a + b*x)/2]]/b
```

Maple [A] time = 0.011, size = 45, normalized size = 1.2

$$\frac{(\cos(bx+a))^3}{3b}+\frac{\cos(bx+a)}{b}+\frac{\ln(\csc(bx+a)-\cot(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/sin(b*x+a),x)
```

```
[Out] 1/3*cos(b*x+a)^3/b+cos(b*x+a)/b+1/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [A] time = 0.95757, size = 62, normalized size = 1.63

$$\frac{2\cos(bx+a)^3+6\cos(bx+a)-3\log(\cos(bx+a)+1)+3\log(\cos(bx+a)-1)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*cos(b*x + a)^3 + 6*cos(b*x + a) - 3*log(cos(b*x + a) + 1) + 3*log(co
s(b*x + a) - 1))/b
```

Fricas [A] time = 1.67045, size = 146, normalized size = 3.84

$$\frac{2\cos(bx+a)^3+6\cos(bx+a)-3\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+3\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(b*x + a)^3 + 6*cos(b*x + a) - 3*log(1/2*cos(b*x + a) + 1/2) + 3*
log(-1/2*cos(b*x + a) + 1/2))/b
```

Sympy [A] time = 3.41468, size = 410, normalized size = 10.79

$$\begin{cases} \frac{3\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)}{3b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+9b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+9b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+3b} + \frac{9\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{3b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+9b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+9b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+3b} + \frac{9\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan}{3b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+9b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)} \\ \frac{x\cos^4(a)}{\sin(a)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/sin(b*x+a),x)
```

```
[Out] Piecewise((3*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(3*b*tan(a/2 + b*x/2
)**6 + 9*b*tan(a/2 + b*x/2)**4 + 9*b*tan(a/2 + b*x/2)**2 + 3*b) + 9*log(tan
(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(3*b*tan(a/2 + b*x/2)**6 + 9*b*tan(a/2 +
 b*x/2)**4 + 9*b*tan(a/2 + b*x/2)**2 + 3*b) + 9*log(tan(a/2 + b*x/2))*tan(a
/2 + b*x/2)**2/(3*b*tan(a/2 + b*x/2)**6 + 9*b*tan(a/2 + b*x/2)**4 + 9*b*tan
(a/2 + b*x/2)**2 + 3*b) + 3*log(tan(a/2 + b*x/2))/(3*b*tan(a/2 + b*x/2)**6
+ 9*b*tan(a/2 + b*x/2)**4 + 9*b*tan(a/2 + b*x/2)**2 + 3*b) - 4*tan(a/2 + b*
x/2)**6/(3*b*tan(a/2 + b*x/2)**6 + 9*b*tan(a/2 + b*x/2)**4 + 9*b*tan(a/2 +
b*x/2)**2 + 3*b) + 4/(3*b*tan(a/2 + b*x/2)**6 + 9*b*tan(a/2 + b*x/2)**4 + 9
*b*tan(a/2 + b*x/2)**2 + 3*b), Ne(b, 0)), (x*cos(a)**4/sin(a), True))
```

Giac [B] time = 1.16256, size = 136, normalized size = 3.58

$$\frac{\frac{8\left(\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-2\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^3}+3\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(8*(3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(c
os(b*x + a) + 1)^2 - 2)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^3 + 3*l
og(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.123 $\int \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}$$

```
[Out] Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0211221, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cot[a + b*x],x]

[Out] Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/(2*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\cot(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, -\sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)dx, x, -\sin(a+bx)\right)}{b}\\ &= \frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0142919, size = 27, normalized size = 1.

$$\frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/(2*b)
```

Maple [A] time = 0.016, size = 26, normalized size = 1.

$$\frac{(\cos(bx+a))^2}{2b}+\frac{\ln(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(b*x+a),x)
```

```
[Out] 1/2*cos(b*x+a)^2/b+ln(sin(b*x+a))/b
```

Maxima [A] time = 0.960211, size = 34, normalized size = 1.26

$$-\frac{\sin(bx+a)^2-\log\left(\sin(bx+a)^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(sin(b*x + a)^2 - log(sin(b*x + a)^2))/b
```

Fricas [A] time = 1.67852, size = 68, normalized size = 2.52

$$\frac{\cos(bx+a)^2+2\log\left(\frac{1}{2}\sin(bx+a)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(b*x + a)^2 + 2*log(1/2*sin(b*x + a)))/b
```

Sympy [A] time = 1.93497, size = 369, normalized size = 13.67

$$\begin{cases} -\frac{\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}-\frac{2\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}-\frac{\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}+\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^4\left(\frac{a}{2}+\right.}{b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^2\left(\frac{a}{2}+\right.} \\ \frac{x\cos^3(a)}{\sin(a)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(b*x+a),x)
```

```
[Out] Piecewise((-log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b
*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - 2*log(tan(a/2 + b*x/2)**2 + 1)*ta
n(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - l
```

```
og(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**
2 + b) + log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 +
 2*b*tan(a/2 + b*x/2)**2 + b) + 2*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2
/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/
2))/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - 2*tan(a/2 + b*x
/2)**2/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b), Ne(b, 0)), (x
*cos(a)**3/sin(a), True))
```

Giac [A] time = 1.17104, size = 34, normalized size = 1.26

$$-\frac{\sin\left(bx+a\right)^{2}-\log\left(\sin\left(bx+a\right)^{2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(sin(b*x + a)^2 - log(sin(b*x + a)^2))/b
```

3.124 $\int \cos(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Cos[a + b*x]/b
```

Rubi [A] time = 0.0147569, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2592, 321, 206}

$$\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Cos[a + b*x]/b
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos(a+bx)\cot(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b}\\
&= \frac{\cos(a+bx)}{b}-\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{\tanh^{-1}(\cos(a+bx))}{b}+\frac{\cos(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0154529, size = 42, normalized size = 1.83

$$\frac{\cos(a+bx)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] Cos[a + b*x]/b - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b
```

Maple [A] time = 0.013, size = 32, normalized size = 1.4

$$\frac{\cos(bx+a)}{b}+\frac{\ln(\csc(bx+a)-\cot(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(b*x+a),x)
```

```
[Out] cos(b*x+a)/b+1/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [A] time = 0.977233, size = 46, normalized size = 2.

$$\frac{2\cos(bx+a)-\log(\cos(bx+a)+1)+\log(\cos(bx+a)-1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
```

Fricas [A] time = 1.60282, size = 115, normalized size = 5.

$$\frac{2\cos(bx+a)-\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cos(b*x + a) - log(1/2*cos(b*x + a) + 1/2) + log(-1/2*cos(b*x + a) +
 1/2))/b
```

Sympy [A] time = 1.07189, size = 92, normalized size = 4.

$$\begin{cases}\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}+\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}+\frac{2}{b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} & \text{for } b\neq 0\\ \frac{x\cos^2(a)}{\sin(a)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(b*x+a),x)
```

```
[Out] Piecewise((log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**2
 + b) + log(tan(a/2 + b*x/2))/(b*tan(a/2 + b*x/2)**2 + b) + 2/(b*tan(a/2 +
b*x/2)**2 + b), Ne(b, 0)), (x*cos(a)**2/sin(a), True))
```

Giac [B] time = 1.21611, size = 77, normalized size = 3.35

$$-\frac{\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1}-\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - log(abs(-cos(b*x + a)
 + 1)/abs(cos(b*x + a) + 1)))/b
```

3.125 $\int \cot(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\sin(a+bx))}{b}$$

```
[Out] Log[Sin[a + b*x]]/b
```

Rubi [A] time = 0.0040922, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(a+bx)\,dx = \frac{\log(\sin(a+bx))}{b}$$

Mathematica [A] time = 0.0106428, size = 19, normalized size = 1.73

$$\frac{\log(\tan(a+bx))+\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x],x]
```

```
[Out] (Log[Cos[a + b*x]] + Log[Tan[a + b*x]])/b
```

Maple [A] time = 0.004, size = 12, normalized size = 1.1

$$\frac{\ln(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+a),x)
```

```
[Out] ln(sin(b*x+a))/b
```

Maxima [A] time = 0.970729, size = 15, normalized size = 1.36

$$\frac{\log\left(\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] log(sin(b*x + a))/b
```

Fricas [A] time = 1.65546, size = 34, normalized size = 3.09

$$\frac{\log\left(\frac{1}{2}\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] log(1/2*sin(b*x + a))/b
```

Sympy [A] time = 0.411503, size = 17, normalized size = 1.55

$$\begin{cases} \frac{\log\left(\sin\left(a+bx\right)\right)}{b} & \text{for}\ b \neq 0 \\ \frac{x\cos\left(a\right)}{\sin\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a),x)
```

```
[Out] Piecewise((log(sin(a + b*x))/b, Ne(b, 0)), (x*cos(a)/sin(a), True))
```

Giac [A] time = 1.18395, size = 16, normalized size = 1.45

$$\frac{\log\left(|\sin\left(bx+a\right)|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a),x, algorithm="giac")
```

```
[Out] log(abs(sin(b*x + a)))/b
```

3.126 $\int \csc(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\tan(a+bx))}{b}$$

```
[Out] Log[Tan[a + b*x]]/b
```

Rubi [A] time = 0.0099491, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2620, 29}

$$\frac{\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] Log[Tan[a + b*x]]/b
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sec(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\log(\tan(a+bx))}{b}\end{aligned}$$

Mathematica [B] time = 0.0214335, size = 31, normalized size = 2.82

$$2\left(\frac{\log(\sin(a+bx))}{2b}-\frac{\log(\cos(a+bx))}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] 2*(-Log[Cos[a + b*x]]/(2*b) + Log[Sin[a + b*x]]/(2*b))
```

Maple [A] time = 0.017, size = 12, normalized size = 1.1

$$\frac{\ln\left(\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/sin(b*x+a),x)
```

```
[Out] ln(tan(b*x+a))/b
```

Maxima [B] time = 0.977127, size = 38, normalized size = 3.45

$$-\frac{\log\left(\sin\left(bx+a\right)^{2}-1\right)-\log\left(\sin\left(bx+a\right)^{2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b
```

Fricas [B] time = 1.89355, size = 85, normalized size = 7.73

$$-\frac{\log\left(\cos\left(bx+a\right)^{2}\right)-\log\left(-\frac{1}{4}\,\cos\left(bx+a\right)^{2}+\frac{1}{4}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(log(cos(b*x + a)^2) - log(-1/4*cos(b*x + a)^2 + 1/4))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(a+bx\right)}{\sin\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a),x)
```

```
[Out] Integral(sec(a + b*x)/sin(a + b*x), x)
```

Giac [B] time = 1.1987, size = 76, normalized size = 6.91

$$\frac{\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)-2\,\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b*
x + a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

3.127 $\int \csc(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\sec(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Sec[a + b*x]/b
```

Rubi [A] time = 0.0216238, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2622, 321, 207}

$$\frac{\sec(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Sec[a + b*x]/b
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sec^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\sec(a+bx)\right)}{b}\\ &= \frac{\sec(a+bx)}{b}+\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sec(a+bx)\right)}{b}\\ &= -\frac{\tanh^{-1}(\cos(a+bx))}{b}+\frac{\sec(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0259297, size = 42, normalized size = 1.83

$$\frac{\sec(a+bx)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] -(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b
```

Maple [A] time = 0.017, size = 34, normalized size = 1.5

$$\frac{1}{b\cos(bx+a)}+\frac{\ln(\csc(bx+a)-\cot(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/sin(b*x+a),x)
```

```
[Out] 1/b/cos(b*x+a)+1/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [A] time = 0.986469, size = 49, normalized size = 2.13

$$\frac{\frac{2}{\cos(bx+a)}-\log(\cos(bx+a)+1)+\log(\cos(bx+a)-1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
```

Fricas [B] time = 1.60872, size = 154, normalized size = 6.7

$$-\frac{\cos(bx+a)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-\cos(bx+a)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-2}{2b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(b*x + a)*log(1/2*cos(b*x + a) + 1/2) - cos(b*x + a)*log(-1/2*cos(
b*x + a) + 1/2) - 2)/(b*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^2(a+bx)}{\sin(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/sin(b*x+a),x)
```

```
[Out] Integral(sec(a + b*x)**2/sin(a + b*x), x)
```

Giac [B] time = 1.19753, size = 74, normalized size = 3.22

$$\frac{\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1}+\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + log(abs(-cos(b*x + a)
+ 1)/abs(cos(b*x + a) + 1)))/b
```

3.128 $\int \csc(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\tan^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}$$

```
[Out] Log[Tan[a + b*x]]/b + Tan[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0202023, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2620, 14}

$$\frac{\tan^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^3,x]

[Out] Log[Tan[a + b*x]]/b + Tan[a + b*x]^2/(2*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sec^3(a+bx)\,dx&=\frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx,x,\tan(a+bx)\right)}{b}\\&=\frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)dx,x,\tan(a+bx)\right)}{b}\\&=\frac{\log(\tan(a+bx))}{b}+\frac{\tan^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0309752, size = 36, normalized size = 1.33

$$-\frac{-\sec^2(a+bx)-2\log(\sin(a+bx))+2\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] -(2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]] - Sec[a + b*x]^2)/(2*b)
```

Maple [A] time = 0.022, size = 26, normalized size = 1.

$$\frac{1}{2\,b\,(\cos(bx+a))^2}+\frac{\ln(\tan(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3/sin(b*x+a),x)
```

```
[Out] 1/2/b/cos(b*x+a)^2+ln(tan(b*x+a))/b
```

Maxima [A] time = 1.04927, size = 54, normalized size = 2.

$$-\frac{\frac{1}{\sin(bx+a)^2-1}+\log\left(\sin(bx+a)^2-1\right)-\log\left(\sin(bx+a)^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2
))/b
```

Fricas [B] time = 1.68809, size = 154, normalized size = 5.7

$$-\frac{\cos(bx+a)^2\log\left(\cos(bx+a)^2\right)-\cos(bx+a)^2\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right)-1}{2\,b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(b*x + a)^2*log(cos(b*x + a)^2) - cos(b*x + a)^2*log(-1/4*cos(b*x
+ a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^3(a+bx)}{\sin(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/sin(b*x+a),x)
```

```
[Out] Integral(sec(a + b*x)**3/sin(a + b*x), x)
```

Giac [B] time = 1.22026, size = 167, normalized size = 6.19

$$\frac{\frac{\frac{2\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+3}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^2}+\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)-2\,\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*((2*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos
(b*x + a) + 1)^2 + 3)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^2 + log(a
bs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b*x + a) - 1
)/(cos(b*x + a) + 1) - 1)))/b
```

3.129 $\int \csc(a+bx)\sec^4(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Sec[a + b*x]/b + Sec[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0256347, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2622, 302, 207}

$$\frac{\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^4,x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Sec[a + b*x]/b + Sec[a + b*x]^3/(3*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc(a+bx)\sec^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx,x,\sec(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx,x,\sec(a+bx)\right)}{b}\\
&= \frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}+\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sec(a+bx)\right)}{b}\\
&= -\frac{\tanh^{-1}(\cos(a+bx))}{b}+\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] time = 0.022725, size = 57, normalized size = 1.5

$$\frac{\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^4,x]
```

```
[Out] -(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b + Sec
[a + b*x]^3/(3*b)
```

Maple [A] time = 0.022, size = 47, normalized size = 1.2

$$\frac{1}{3\,b\,(\cos(bx+a))^3}+\frac{1}{b\cos(bx+a)}+\frac{\ln(\csc(bx+a)-\cot(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4/sin(b*x+a),x)
```

```
[Out] 1/3/b/cos(b*x+a)^3+1/b/cos(b*x+a)+1/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [A] time = 1.01395, size = 68, normalized size = 1.79

$$\frac{\frac{2\left(3\cos(bx+a)^2+1\right)}{\cos(bx+a)^3}-3\log(\cos(bx+a)+1)+3\log(\cos(bx+a)-1)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(3*cos(b*x + a)^2 + 1)/cos(b*x + a)^3 - 3*log(cos(b*x + a) + 1) + 3*
log(cos(b*x + a) - 1))/b
```

Fricas [A] time = 1.62311, size = 193, normalized size = 5.08

$$-\frac{3\cos(bx+a)^3\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-3\cos(bx+a)^3\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-6\cos(bx+a)^2-2}{6\,b\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*cos(b*x + a)^3*log(1/2*cos(b*x + a) + 1/2) - 3*cos(b*x + a)^3*log(-
1/2*cos(b*x + a) + 1/2) - 6*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(a+bx)}{\sin(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4/sin(b*x+a),x)
```

```
[Out] Integral(sec(a + b*x)**4/sin(a + b*x), x)
```

Giac [B] time = 1.23461, size = 136, normalized size = 3.58

$$\frac{\frac{8\left(\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+2\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}+3\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(8*(3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(c
os(b*x + a) + 1)^2 + 2)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^3 + 3*l
og(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.130 $\int \csc(a+bx)\sec^5(a+bx)\,dx$

Optimal. Leaf size=39

$$\frac{\tan^4(a+bx)}{4b}+\frac{\tan^2(a+bx)}{b}+\frac{\log(\tan(a+bx))}{b}$$

```
[Out] Log[Tan[a + b*x]]/b + Tan[a + b*x]^2/b + Tan[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0262298, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2620, 266, 43}

$$\frac{\tan^4(a+bx)}{4b}+\frac{\tan^2(a+bx)}{b}+\frac{\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^5,x]

[Out] Log[Tan[a + b*x]]/b + Tan[a + b*x]^2/b + Tan[a + b*x]^4/(4*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc(a+bx)\sec^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x}\,dx, x, \tan(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x}\,dx, x, \tan^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x}+x\right)\,dx, x, \tan^2(a+bx)\right)}{2b}\\
&= \frac{\log(\tan(a+bx))}{b}+\frac{\tan^2(a+bx)}{b}+\frac{\tan^4(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.0907424, size = 46, normalized size = 1.18

$$-\frac{-\sec^4(a+bx)-2\sec^2(a+bx)-4\log(\sin(a+bx))+4\log(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^5,x]
```

```
[Out] -(4*Log[Cos[a + b*x]] - 4*Log[Sin[a + b*x]] - 2*Sec[a + b*x]^2 - Sec[a + b*
x]^4)/(4*b)
```

Maple [A] time = 0.023, size = 39, normalized size = 1.

$$\frac{1}{4\,b\left(\cos\left(bx+a\right)\right)^4}+\frac{1}{2\,b\left(\cos\left(bx+a\right)\right)^2}+\frac{\ln\left(\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5/sin(b*x+a),x)
```

```
[Out] 1/4/b/cos(b*x+a)^4+1/2/b/cos(b*x+a)^2+ln(tan(b*x+a))/b
```

Maxima [A] time = 0.974171, size = 88, normalized size = 2.26

$$-\frac{\frac{2\,\sin(bx+a)^2-3}{\sin(bx+a)^4-2\,\sin(bx+a)^2+1}+2\,\log\left(\sin\left(bx+a\right)^2-1\right)-2\,\log\left(\sin\left(bx+a\right)^2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*((2*sin(b*x + a)^2 - 3)/(sin(b*x + a)^4 - 2*sin(b*x + a)^2 + 1) + 2*lo
g(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b
```

Fricas [A] time = 1.74276, size = 185, normalized size = 4.74

$$-\frac{2\,\cos\left(bx+a\right)^4\log\left(\cos\left(bx+a\right)^2\right)-2\,\cos\left(bx+a\right)^4\log\left(-\frac{1}{4}\,\cos\left(bx+a\right)^2+\frac{1}{4}\right)-2\,\cos\left(bx+a\right)^2-1}{4\,b\cos\left(bx+a\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(b*x + a)^4*log(cos(b*x + a)^2) - 2*cos(b*x + a)^4*log(-1/4*cos(
b*x + a)^2 + 1/4) - 2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5(a+bx)}{\sin(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5/sin(b*x+a),x)
```

```
[Out] Integral(sec(a + b*x)**5/sin(a + b*x), x)
```

Giac [B] time = 1.19017, size = 230, normalized size = 5.9

$$\frac{\frac{\frac{52(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{102(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{52(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{25(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+25}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^4}+6\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)-12\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right|\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*((52*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 102*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 52*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 25*(c
os(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 25)/((cos(b*x + a) - 1)/(cos(b*x
+ a) + 1) + 1)^4 + 6*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 12
*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

3.131 $\int \csc(a+bx)\sec^6(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{\sec^5(a+bx)}{5b}+\frac{\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Sec[a + b*x]/b + Sec[a + b*x]^3/(3*b) + Sec[a
+ b*x]^5/(5*b)
```

Rubi [A] time = 0.0279177, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2622, 302, 207}

$$\frac{\sec^5(a+bx)}{5b}+\frac{\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^6,x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Sec[a + b*x]/b + Sec[a + b*x]^3/(3*b) + Sec[a
+ b*x]^5/(5*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc(a+bx)\sec^6(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx,x,\sec(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right)dx,x,\sec(a+bx)\right)}{b}\\
&= \frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}+\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sec(a+bx)\right)}{b}\\
&= -\frac{\tanh^{-1}(\cos(a+bx))}{b}+\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}
\end{aligned}$$

Mathematica [A] time = 0.022692, size = 72, normalized size = 1.36

$$\frac{\sec^5(a+bx)}{5b}+\frac{\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^6,x]
```

```
[Out] -(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b + Sec
[a + b*x]^3/(3*b) + Sec[a + b*x]^5/(5*b)
```

Maple [A] time = 0.026, size = 60, normalized size = 1.1

$$\frac{1}{5\,b\left(\cos\left(bx+a\right)\right)^{5}}+\frac{1}{3\,b\left(\cos\left(bx+a\right)\right)^{3}}+\frac{1}{b\cos\left(bx+a\right)}+\frac{\ln\left(\csc\left(bx+a\right)-\cot\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6/sin(b*x+a),x)
```

```
[Out] 1/5/b/cos(b*x+a)^5+1/3/b/cos(b*x+a)^3+1/b/cos(b*x+a)+1/b*ln(csc(b*x+a)-cot(
b*x+a))
```

Maxima [A] time = 0.96447, size = 81, normalized size = 1.53

$$\frac{\frac{2\left(15\,\cos\left(bx+a\right)^{4}+5\,\cos\left(bx+a\right)^{2}+3\right)}{\cos\left(bx+a\right)^{5}}-15\,\log\left(\cos\left(bx+a\right)+1\right)+15\,\log\left(\cos\left(bx+a\right)-1\right)}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/30*(2*(15*cos(b*x + a)^4 + 5*cos(b*x + a)^2 + 3)/cos(b*x + a)^5 - 15*log(
cos(b*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b
```

Fricas [A] time = 1.69654, size = 225, normalized size = 4.25

$$-\frac{15\,\cos\left(bx+a\right)^{5}\log\left(\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)-15\,\cos\left(bx+a\right)^{5}\log\left(-\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)-30\,\cos\left(bx+a\right)^{4}-10\,\cos\left(bx-\right.}{30\,b\cos\left(bx+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/30*(15*cos(b*x + a)^5*log(1/2*cos(b*x + a) + 1/2) - 15*cos(b*x + a)^5*lo
g(-1/2*cos(b*x + a) + 1/2) - 30*cos(b*x + a)^4 - 10*cos(b*x + a)^2 - 6)/(b*
cos(b*x + a)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6/sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19563, size = 196, normalized size = 3.7

$$\frac{\frac{4\left(\frac{70(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{140(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{90(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{45(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+23\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^5}+15\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/30*(4*(70*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 140*(cos(b*x + a) - 1)^
2/(cos(b*x + a) + 1)^2 + 90*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 45*
(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 23)/((cos(b*x + a) - 1)/(cos(b*
x + a) + 1) + 1)^5 + 15*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/
b
```

3.132 $\int \csc(a+bx)\sec^7(a+bx)\,dx$

Optimal. Leaf size=57

$$\frac{\tan^6(a+bx)}{6b}+\frac{3\tan^4(a+bx)}{4b}+\frac{3\tan^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}$$

```
[Out] Log[Tan[a + b*x]]/b + (3*Tan[a + b*x]^2)/(2*b) + (3*Tan[a + b*x]^4)/(4*b) +
 Tan[a + b*x]^6/(6*b)
```

Rubi [A] time = 0.0312009, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2620, 266, 43}

$$\frac{\tan^6(a+bx)}{6b}+\frac{3\tan^4(a+bx)}{4b}+\frac{3\tan^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^7,x]
```

```
[Out] Log[Tan[a + b*x]]/b + (3*Tan[a + b*x]^2)/(2*b) + (3*Tan[a + b*x]^4)/(4*b) +
 Tan[a + b*x]^6/(6*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc(a+bx)\sec^7(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x}\,dx, x, \tan(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x}\,dx, x, \tan^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x}+3x+x^2\right)dx, x, \tan^2(a+bx)\right)}{2b}\\
&= \frac{\log(\tan(a+bx))}{b}+\frac{3\tan^2(a+bx)}{2b}+\frac{3\tan^4(a+bx)}{4b}+\frac{\tan^6(a+bx)}{6b}
\end{aligned}$$

Mathematica [A] time = 0.145671, size = 56, normalized size = 0.98

$$-\frac{-2\sec^6(a+bx)-3\sec^4(a+bx)-6\sec^2(a+bx)-12\log(\sin(a+bx))+12\log(\cos(a+bx))}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^7,x]
```

```
[Out] -(12*Log[Cos[a + b*x]] - 12*Log[Sin[a + b*x]] - 6*Sec[a + b*x]^2 - 3*Sec[a
+ b*x]^4 - 2*Sec[a + b*x]^6)/(12*b)
```

Maple [A] time = 0.025, size = 52, normalized size = 0.9

$$\frac{1}{6\,b\left(\cos\left(bx+a\right)\right)^{6}}+\frac{1}{4\,b\left(\cos\left(bx+a\right)\right)^{4}}+\frac{1}{2\,b\left(\cos\left(bx+a\right)\right)^{2}}+\frac{\ln\left(\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^7/sin(b*x+a),x)
```

```
[Out] 1/6/b/cos(b*x+a)^6+1/4/b/cos(b*x+a)^4+1/2/b/cos(b*x+a)^2+ln(tan(b*x+a))/b
```

Maxima [A] time = 1.00839, size = 115, normalized size = 2.02

$$-\frac{\frac{6\,\sin(bx+a)^4-15\,\sin(bx+a)^2+11}{\sin(bx+a)^6-3\,\sin(bx+a)^4+3\,\sin(bx+a)^2-1}+6\,\log\left(\sin\left(bx+a\right)^2-1\right)-6\,\log\left(\sin\left(bx+a\right)^2\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/12*((6*sin(b*x + a)^4 - 15*sin(b*x + a)^2 + 11)/(sin(b*x + a)^6 - 3*sin(
b*x + a)^4 + 3*sin(b*x + a)^2 - 1) + 6*log(sin(b*x + a)^2 - 1) - 6*log(sin(
b*x + a)^2))/b
```

Fricas [A] time = 1.71733, size = 212, normalized size = 3.72

$$-\frac{6\,\cos\left(bx+a\right)^6\log\left(\cos\left(bx+a\right)^2\right)-6\,\cos\left(bx+a\right)^6\log\left(-\frac{1}{4}\,\cos\left(bx+a\right)^2+\frac{1}{4}\right)-6\,\cos\left(bx+a\right)^4-3\,\cos\left(bx+a\right)^2-}{12\,b\cos\left(bx+a\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(6*cos(b*x + a)^6*log(cos(b*x + a)^2) - 6*cos(b*x + a)^6*log(-1/4*cos
(b*x + a)^2 + 1/4) - 6*cos(b*x + a)^4 - 3*cos(b*x + a)^2 - 2)/(b*cos(b*x +
a)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**7/sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22457, size = 289, normalized size = 5.07

$$\frac{\frac{\frac{522\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{1485\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{1580\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{1485\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{522\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}+\frac{147\,(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}+147}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^6}+30\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)-60}{60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/60*((522*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1485*(cos(b*x + a) - 1)^
2/(cos(b*x + a) + 1)^2 + 1580*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 1
485*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 522*(cos(b*x + a) - 1)^5/(c
os(b*x + a) + 1)^5 + 147*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 147)/(
(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^6 + 30*log(abs(-cos(b*x + a) + 1
)/abs(cos(b*x + a) + 1)) - 60*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1
) - 1)))/b
```

3.133 $\int \cos^5(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=50

$$-\frac{\sin^5(a+bx)}{5b}+\frac{\sin^3(a+bx)}{b}-\frac{3\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

```
[Out] -(Csc[a + b*x]/b) - (3*Sin[a + b*x])/b + Sin[a + b*x]^3/b - Sin[a + b*x]^5/
(5*b)
```

Rubi [A] time = 0.038405, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2590, 270}

$$-\frac{\sin^5(a+bx)}{5b}+\frac{\sin^3(a+bx)}{b}-\frac{3\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*Cot[a + b*x]^2,x]
```

```
[Out] -(Csc[a + b*x]/b) - (3*Sin[a + b*x])/b + Sin[a + b*x]^3/b - Sin[a + b*x]^5/
(5*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(a+bx)\cot^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^2}\,dx,x,-\sin(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-3+\frac{1}{x^2}+3x^2-x^4\right)dx,x,-\sin(a+bx)\right)}{b}\\ &= -\frac{\csc(a+bx)}{b}-\frac{3\sin(a+bx)}{b}+\frac{\sin^3(a+bx)}{b}-\frac{\sin^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.025517, size = 50, normalized size = 1.

$$-\frac{\sin^5(a+bx)}{5b}+\frac{\sin^3(a+bx)}{b}-\frac{3\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*Cot[a + b*x]^2,x]
```

```
[Out] -(Csc[a + b*x]/b) - (3*Sin[a + b*x])/b + Sin[a + b*x]^3/b - Sin[a + b*x]^5/
(5*b)
```

Maple [A] time = 0.013, size = 62, normalized size = 1.2

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^{8}}{\sin(bx+a)}-\left(\frac{16}{5}+(\cos(bx+a))^{6}+\frac{6\,(\cos(bx+a))^{4}}{5}+\frac{8\,(\cos(bx+a))^{2}}{5}\right)\sin(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7/sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/sin(b*x+a)*cos(b*x+a)^8-(16/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*cos
(b*x+a)^2)*sin(b*x+a))
```

Maxima [A] time = 0.985622, size = 57, normalized size = 1.14

$$-\frac{\sin(bx+a)^{5}-5\,\sin(bx+a)^{3}+\frac{5}{\sin(bx+a)}+15\,\sin(bx+a)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/5*(sin(b*x + a)^5 - 5*sin(b*x + a)^3 + 5/sin(b*x + a) + 15*sin(b*x + a))
/b
```

Fricas [A] time = 1.9589, size = 111, normalized size = 2.22

$$\frac{\cos(bx+a)^{6}+2\,\cos(bx+a)^{4}+8\,\cos(bx+a)^{2}-16}{5\,b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*(cos(b*x + a)^6 + 2*cos(b*x + a)^4 + 8*cos(b*x + a)^2 - 16)/(b*sin(b*x
+ a))
```

Sympy [A] time = 8.10902, size = 82, normalized size = 1.64

$$\begin{cases} -\frac{16\sin^{5}(a+bx)}{5b}-\frac{8\sin^{3}(a+bx)\cos^{2}(a+bx)}{b}-\frac{6\sin(a+bx)\cos^{4}(a+bx)}{b}-\frac{\cos^{6}(a+bx)}{b\sin(a+bx)} & \text{for } b\neq 0 \\ \frac{x\cos^{7}(a)}{\sin^{2}(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-16*sin(a + b*x)**5/(5*b) - 8*sin(a + b*x)**3*cos(a + b*x)**2/b
- 6*sin(a + b*x)*cos(a + b*x)**4/b - cos(a + b*x)**6/(b*sin(a + b*x)), Ne(b
, 0)), (x*cos(a)**7/sin(a)**2, True))
```

Giac [A] time = 1.19002, size = 57, normalized size = 1.14

$$-\frac{\sin\left(bx+a\right)^{5}-5\,\sin\left(bx+a\right)^{3}+\frac{5}{\sin(bx+a)}+15\,\sin\left(bx+a\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/5*(sin(b*x + a)^5 - 5*sin(b*x + a)^3 + 5/sin(b*x + a) + 15*sin(b*x + a))
/b
```

3.134 $\int \cos^4(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=61

$$-\frac{15\cot(a+bx)}{8b}+\frac{\cos^4(a+bx)\cot(a+bx)}{4b}+\frac{5\cos^2(a+bx)\cot(a+bx)}{8b}-\frac{15x}{8}$$

```
[Out] (-15*x)/8 - (15*Cot[a + b*x])/(8*b) + (5*Cos[a + b*x]^2*Cot[a + b*x])/(8*b)
 + (Cos[a + b*x]^4*Cot[a + b*x])/(4*b)
```

Rubi [A] time = 0.0455062, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2591, 288, 321, 203}

$$-\frac{15\cot(a+bx)}{8b}+\frac{\cos^4(a+bx)\cot(a+bx)}{4b}+\frac{5\cos^2(a+bx)\cot(a+bx)}{8b}-\frac{15x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Cot[a + b*x]^2,x]
```

```
[Out] (-15*x)/8 - (15*Cot[a + b*x])/(8*b) + (5*Cos[a + b*x]^2*Cot[a + b*x])/(8*b)
 + (Cos[a + b*x]^4*Cot[a + b*x])/(4*b)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^3}\,dx,x,\cot(a+bx)\right)}{b}$$
$$= \frac{\cos^4(a+bx)\cot(a+bx)}{4b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx,x,\cot(a+bx)\right)}{4b}$$
$$= \frac{5\cos^2(a+bx)\cot(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot(a+bx)}{4b} - \frac{15\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,\text{co}\right.}{8b}$$
$$= -\frac{15\cot(a+bx)}{8b} + \frac{5\cos^2(a+bx)\cot(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot(a+bx)}{4b} + \frac{15\,\text{Sub}}{}$$
$$= -\frac{15x}{8} - \frac{15\cot(a+bx)}{8b} + \frac{5\cos^2(a+bx)\cot(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot(a+bx)}{4b}$$

Mathematica [A] time = 0.138363, size = 41, normalized size = 0.67

$$-\frac{16\sin(2(a+bx))+\sin(4(a+bx))+32\cot(a+bx)+60a+60bx}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Cot[a + b*x]^2,x]
```

```
[Out] -(60*a + 60*b*x + 32*Cot[a + b*x] + 16*Sin[2*(a + b*x)] + Sin[4*(a + b*x)])
/(32*b)
```

Maple [A] time = 0.011, size = 66, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^7}{\sin(bx+a)} - \left((\cos(bx+a))^5 + \frac{5(\cos(bx+a))^3}{4} + \frac{15\cos(bx+a)}{8}\right)\sin(bx+a) - \frac{15bx}{8} - \frac{15a}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6/sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/sin(b*x+a)*cos(b*x+a)^7-(cos(b*x+a)^5+5/4*cos(b*x+a)^3+15/8*cos(b*x
+a))*sin(b*x+a)-15/8*b*x-15/8*a)
```

Maxima [A] time = 1.48495, size = 85, normalized size = 1.39

$$-\frac{15\,bx + 15\,a + \frac{15\tan(bx+a)^4 + 25\tan(bx+a)^2 + 8}{\tan(bx+a)^5 + 2\tan(bx+a)^3 + \tan(bx+a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(15*b*x + 15*a + (15*tan(b*x + a)^4 + 25*tan(b*x + a)^2 + 8)/(tan(b*x
+ a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a)))/b
```

Fricas [A] time = 1.84227, size = 135, normalized size = 2.21

$$\frac{2\cos(bx+a)^5+5\cos(bx+a)^3-15\,bx\sin(bx+a)-15\cos(bx+a)}{8\,b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(b*x + a)^5 + 5*cos(b*x + a)^3 - 15*b*x*sin(b*x + a) - 15*cos(b*x
 + a))/(b*sin(b*x + a))
```

Sympy [A] time = 4.80812, size = 119, normalized size = 1.95

$$\begin{cases} -\frac{15x\sin^4(a+bx)}{8}-\frac{15x\sin^2(a+bx)\cos^2(a+bx)}{4}-\frac{15x\cos^4(a+bx)}{8}-\frac{15\sin^3(a+bx)\cos(a+bx)}{8b}-\frac{25\sin(a+bx)\cos^3(a+bx)}{8b}-\frac{\cos^5(a+bx)}{b\sin(a+bx)} & \text{for } b \neq \\ \frac{x\cos^6(a)}{\sin^2(a)} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-15*x*sin(a + b*x)**4/8 - 15*x*sin(a + b*x)**2*cos(a + b*x)**2/4
 - 15*x*cos(a + b*x)**4/8 - 15*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 25*sin(
a + b*x)*cos(a + b*x)**3/(8*b) - cos(a + b*x)**5/(b*sin(a + b*x)), Ne(b, 0)
), (x*cos(a)**6/sin(a)**2, True))
```

Giac [A] time = 1.24547, size = 74, normalized size = 1.21

$$-\frac{15\,bx+15\,a+\frac{7\tan(bx+a)^3+9\tan(bx+a)}{\left(\tan(bx+a)^2+1\right)^2}+\frac{8}{\tan(bx+a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(15*b*x + 15*a + (7*tan(b*x + a)^3 + 9*tan(b*x + a))/(tan(b*x + a)^2 +
 1)^2 + 8/tan(b*x + a))/b
```

3.135 $\int \cos^3(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\sin^3(a+bx)}{3b}-\frac{2\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

```
[Out] -(Csc[a + b*x]/b) - (2*Sin[a + b*x])/b + Sin[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0344497, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2590, 270}

$$\frac{\sin^3(a+bx)}{3b}-\frac{2\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Cot[a + b*x]^2,x]
```

```
[Out] -(Csc[a + b*x]/b) - (2*Sin[a + b*x])/b + Sin[a + b*x]^3/(3*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\cot^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx,x,-\sin(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)dx,x,-\sin(a+bx)\right)}{b}\\ &= -\frac{\csc(a+bx)}{b}-\frac{2\sin(a+bx)}{b}+\frac{\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0165735, size = 38, normalized size = 1.

$$\frac{\sin^3(a+bx)}{3b}-\frac{2\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Cot[a + b*x]^2,x]
```

```
[Out] -(Csc[a + b*x]/b) - (2*Sin[a + b*x])/b + Sin[a + b*x]^3/(3*b)
```

Maple [A] time = 0.012, size = 52, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^6}{\sin(bx+a)}-\left(\frac{8}{3}+(\cos(bx+a))^4+\frac{4\,(\cos(bx+a))^2}{3}\right)\sin(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5/sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-cos(b*x+a)^6/sin(b*x+a)-(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a
))
```

Maxima [A] time = 0.990273, size = 43, normalized size = 1.13

$$\frac{\sin(bx+a)^3-\frac{3}{\sin(bx+a)}-6\,\sin(bx+a)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(sin(b*x + a)^3 - 3/sin(b*x + a) - 6*sin(b*x + a))/b
```

Fricas [A] time = 1.78223, size = 84, normalized size = 2.21

$$\frac{\cos(bx+a)^4+4\,\cos(bx+a)^2-8}{3\,b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(b*x + a)^4 + 4*cos(b*x + a)^2 - 8)/(b*sin(b*x + a))
```

Sympy [A] time = 2.675, size = 61, normalized size = 1.61

$$\begin{cases}-\frac{8\sin^3(a+bx)}{3b}-\frac{4\sin(a+bx)\cos^2(a+bx)}{b}-\frac{\cos^4(a+bx)}{b\sin(a+bx)} & \text{for } b\neq 0\\ \frac{x\cos^5(a)}{\sin^2(a)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-8*sin(a + b*x)**3/(3*b) - 4*sin(a + b*x)*cos(a + b*x)**2/b - co
s(a + b*x)**4/(b*sin(a + b*x)), Ne(b, 0)), (x*cos(a)**5/sin(a)**2, True))
```

Giac [A] time = 1.15322, size = 43, normalized size = 1.13

$$\frac{\sin(bx+a)^3 - \frac{3}{\sin(bx+a)} - 6\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(sin(b*x + a)^3 - 3/sin(b*x + a) - 6*sin(b*x + a))/b
```

3.136 $\int \cos^2(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=40

$$-\frac{3\cot(a+bx)}{2b}+\frac{\cos^2(a+bx)\cot(a+bx)}{2b}-\frac{3x}{2}$$

```
[Out] (-3*x)/2 - (3*Cot[a + b*x])/(2*b) + (Cos[a + b*x]^2*Cot[a + b*x])/(2*b)
```

Rubi [A] time = 0.0367128, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2591, 288, 321, 203}

$$-\frac{3\cot(a+bx)}{2b}+\frac{\cos^2(a+bx)\cot(a+bx)}{2b}-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cot[a + b*x]^2,x]
```

```
[Out] (-3*x)/2 - (3*Cot[a + b*x])/(2*b) + (Cos[a + b*x]^2*Cot[a + b*x])/(2*b)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \cot(a+bx)\right)}{b}$$
$$= \frac{\cos^2(a+bx)\cot(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \cot(a+bx)\right)}{2b}$$
$$= -\frac{3\cot(a+bx)}{2b} + \frac{\cos^2(a+bx)\cot(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cot(a+bx)\right)}{2b}$$
$$= -\frac{3x}{2} - \frac{3\cot(a+bx)}{2b} + \frac{\cos^2(a+bx)\cot(a+bx)}{2b}$$

Mathematica [A] time = 0.134405, size = 31, normalized size = 0.78

$$-\frac{6(a+bx)+\sin(2(a+bx))+4\cot(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cot[a + b*x]^2,x]
```

```
[Out] -(6*(a + b*x) + 4*Cot[a + b*x] + Sin[2*(a + b*x)])/(4*b)
```

Maple [A] time = 0.012, size = 56, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^5}{\sin(bx+a)} - \left((\cos(bx+a))^3 + \frac{3\cos(bx+a)}{2}\right)\sin(bx+a) - \frac{3bx}{2} - \frac{3a}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-cos(b*x+a)^5/sin(b*x+a)-(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)-3/2*
b*x-3/2*a)
```

Maxima [A] time = 1.48251, size = 58, normalized size = 1.45

$$-\frac{3bx+3a+\frac{3\tan(bx+a)^2+2}{\tan(bx+a)^3+\tan(bx+a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x + 3*a + (3*tan(b*x + a)^2 + 2)/(tan(b*x + a)^3 + tan(b*x + a)))
/b
```

Fricas [A] time = 1.82469, size = 104, normalized size = 2.6

$$\frac{\cos(bx+a)^3 - 3bx\sin(bx+a) - 3\cos(bx+a)}{2b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(b*x + a)^3 - 3*b*x*sin(b*x + a) - 3*cos(b*x + a))/(b*sin(b*x + a))
```

Sympy [A] time = 1.63634, size = 75, normalized size = 1.88

$$\begin{cases} -\frac{3x\sin^2(a+bx)}{2} - \frac{3x\cos^2(a+bx)}{2} - \frac{3\sin(a+bx)\cos(a+bx)}{2b} - \frac{\cos^3(a+bx)}{b\sin(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^4(a)}{\sin^2(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-3*x*sin(a + b*x)**2/2 - 3*x*cos(a + b*x)**2/2 - 3*sin(a + b*x)*
cos(a + b*x)/(2*b) - cos(a + b*x)**3/(b*sin(a + b*x)), Ne(b, 0)), (x*cos(a)
**4/sin(a)**2, True))
```

Giac [A] time = 1.15988, size = 58, normalized size = 1.45

$$-\frac{3\,bx + 3\,a + \frac{3\,\tan(bx+a)^2+2}{\tan(bx+a)^3+\tan(bx+a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b*x + 3*a + (3*tan(b*x + a)^2 + 2)/(tan(b*x + a)^3 + tan(b*x + a)))
/b
```

3.137 $\int \cos(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=23

$$-\frac{\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

```
[Out] -(Csc[a + b*x]/b) - Sin[a + b*x]/b
```

Rubi [A] time = 0.0203114, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$-\frac{\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -(Csc[a + b*x]/b) - Sin[a + b*x]/b
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cot^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,-\sin(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx,x,-\sin(a+bx)\right)}{b}\\ &= -\frac{\csc(a+bx)}{b}-\frac{\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0118145, size = 23, normalized size = 1.

$$-\frac{\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -(Csc[a + b*x]/b) - Sin[a + b*x]/b
```

Maple [A] time = 0.011, size = 42, normalized size = 1.8

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^4}{\sin(bx+a)} - \left(2+(\cos(bx+a))^2\right)\sin(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-cos(b*x+a)^4/sin(b*x+a)-(2+cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [A] time = 0.992325, size = 27, normalized size = 1.17

$$-\frac{\frac{1}{\sin(bx+a)} + \sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(1/sin(b*x + a) + sin(b*x + a))/b
```

Fricas [A] time = 1.83447, size = 53, normalized size = 2.3

$$\frac{\cos(bx+a)^2 - 2}{b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (cos(b*x + a)^2 - 2)/(b*sin(b*x + a))
```

Sympy [A] time = 0.983448, size = 39, normalized size = 1.7

$$\begin{cases} -\frac{2\sin(a+bx)}{b} - \frac{\cos^2(a+bx)}{b\sin(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^3(a)}{\sin^2(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*sin(a + b*x)/b - cos(a + b*x)**2/(b*sin(a + b*x)), Ne(b, 0)),
 (x*cos(a)**3/sin(a)**2, True))
```

Giac [A] time = 1.13176, size = 27, normalized size = 1.17

$$-\frac{\frac{1}{\sin(bx+a)} + \sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(1/sin(b*x + a) + sin(b*x + a))/b
```

3.138 $\int \cot^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\cot(a+bx)}{b}-x$$

```
[Out] -x - Cot[a + b*x]/b
```

Rubi [A] time = 0.0078847, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$-\frac{\cot(a+bx)}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^2,x]
```

```
[Out] -x - Cot[a + b*x]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(a+bx)\,dx &= -\frac{\cot(a+bx)}{b}-\int 1\,dx\\ &= -x-\frac{\cot(a+bx)}{b}\end{aligned}$$

Mathematica [C] time = 0.0132666, size = 29, normalized size = 1.93

$$-\frac{\cot(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^2,x]
```

```
[Out] -((Cot[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a + b*x]^2])/b)
```

Maple [A] time = 0.01, size = 21, normalized size = 1.4

$$\frac{-\cot(bx+a)-bx-a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-cot(b*x+a)-b*x-a)
```

Maxima [A] time = 1.47918, size = 24, normalized size = 1.6

$$-\frac{bx+a+\frac{1}{\tan(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b*x + a + 1/tan(b*x + a))/b
```

Fricas [A] time = 1.93702, size = 72, normalized size = 4.8

$$-\frac{bx\sin(bx+a)+\cos(bx+a)}{b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b*x*sin(b*x + a) + cos(b*x + a))/(b*sin(b*x + a))
```

Sympy [A] time = 0.693617, size = 29, normalized size = 1.93

$$\begin{cases} -x-\frac{\cos(a+bx)}{b\sin(a+bx)} & \text{for } b\neq 0 \\ \frac{x\cos^2(a)}{\sin^2(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-x - cos(a + b*x)/(b*sin(a + b*x)), Ne(b, 0)), (x*cos(a)**2/sin(
a)**2, True))
```

Giac [B] time = 1.16921, size = 47, normalized size = 3.13

$$-\frac{2\,bx+2\,a+\frac{1}{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + 2*a + 1/tan(1/2*b*x + 1/2*a) - tan(1/2*b*x + 1/2*a))/b
```

3.139 $\int \cot(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\csc(a+bx)}{b}$$

```
[Out] -(Csc[a + b*x]/b)
```

Rubi [A] time = 0.0093198, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2606, 8}

$$-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]*Csc[a + b*x],x]

[Out] -(Csc[a + b*x]/b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot(a+bx)\csc(a+bx)\,dx &= -\frac{\text{Subst}\left(\int 1\,dx, x, \csc(a+bx)\right)}{b}\\ &= -\frac{\csc(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0083335, size = 11, normalized size = 1.

$$-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]*Csc[a + b*x],x]

[Out] -(Csc[a + b*x]/b)
```

Maple [A] time = 0.002, size = 14, normalized size = 1.3

$$-\frac{1}{b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+a)^2,x)
```

```
[Out] -1/b/sin(b*x+a)
```

Maxima [A] time = 0.994688, size = 18, normalized size = 1.64

$$-\frac{1}{b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/(b*sin(b*x + a))
```

Fricas [A] time = 1.81136, size = 28, normalized size = 2.55

$$-\frac{1}{b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/(b*sin(b*x + a))
```

Sympy [A] time = 0.684761, size = 20, normalized size = 1.82

$$\begin{cases} -\frac{1}{b\sin\left(a+bx\right)} & \text{for}\ b \neq 0 \\ \frac{x\cos\left(a\right)}{\sin^{2}\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-1/(b*sin(a + b*x)), Ne(b, 0)), (x*cos(a)/sin(a)**2, True))
```

Giac [A] time = 1.15179, size = 18, normalized size = 1.64

$$-\frac{1}{b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/(b*sin(b*x + a))
```

3.140 $\int \csc^2(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}-\frac{\csc(a+bx)}{b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/b - Csc[a + b*x]/b
```

Rubi [A] time = 0.0217056, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2621, 321, 207}

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b - Csc[a + b*x]/b
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc^2(a+bx)\sec(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(a+bx)\right)}{b}\\
&= -\frac{\csc(a+bx)}{b}-\frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(a+bx)\right)}{b}\\
&= \frac{\tanh^{-1}(\sin(a+bx))}{b}-\frac{\csc(a+bx)}{b}
\end{aligned}$$

Mathematica [C] time = 0.0146672, size = 27, normalized size = 1.17

$$-\frac{\csc(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] -((Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b)
```

Maple [A] time = 0.017, size = 33, normalized size = 1.4

$$-\frac{1}{b\sin(bx+a)}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/sin(b*x+a)^2,x)
```

```
[Out] -1/b/sin(b*x+a)+1/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 0.989706, size = 49, normalized size = 2.13

$$-\frac{\frac{2}{\sin(bx+a)}-\log(\sin(bx+a)+1)+\log(\sin(bx+a)-1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b
```

Fricas [B] time = 1.9893, size = 136, normalized size = 5.91

$$\frac{\log(\sin(bx+a)+1)\sin(bx+a)-\log(-\sin(bx+a)+1)\sin(bx+a)-2}{2b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(b*x + a) + 1)*sin(b*x + a) - log(-sin(b*x + a) + 1)*sin(b*x +
a) - 2)/(b*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec(a+bx)}{\sin^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)**2,x)
```

```
[Out] Integral(sec(a + b*x)/sin(a + b*x)**2, x)
```

Giac [A] time = 1.19789, size = 51, normalized size = 2.22

$$-\frac{\frac{2}{\sin(bx+a)} - \log\left(\left|\sin\left(bx+a\right)+1\right|\right) + \log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2/sin(b*x + a) - log(abs(sin(b*x + a) + 1)) + log(abs(sin(b*x + a) -
1)))/b
```

3.141 $\int \csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=22

$$\frac{\tan(a+bx)}{b}-\frac{\cot(a+bx)}{b}$$

```
[Out] -(Cot[a + b*x]/b) + Tan[a + b*x]/b
```

Rubi [A] time = 0.0303198, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 14}

$$\frac{\tan(a+bx)}{b}-\frac{\cot(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] -(Cot[a + b*x]/b) + Tan[a + b*x]/b
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sec^2(a+bx)\,dx&=\frac{\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx,x,\tan(a+bx)\right)}{b}\\&=\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx,x,\tan(a+bx)\right)}{b}\\&=-\frac{\cot(a+bx)}{b}+\frac{\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0166702, size = 13, normalized size = 0.59

$$-\frac{2\cot(2(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] (-2*Cot[2*(a + b*x)])/b
```

Maple [A] time = 0.044, size = 31, normalized size = 1.4

$$\frac{1}{b}\left(\frac{1}{\cos\left(bx+a\right)\sin\left(bx+a\right)}-2\,\cot\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/sin(b*x+a)/cos(b*x+a)-2*cot(b*x+a))
```

Maxima [A] time = 0.986127, size = 30, normalized size = 1.36

$$-\frac{\frac{1}{\tan(bx+a)}-\tan\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(1/tan(b*x + a) - tan(b*x + a))/b
```

Fricas [A] time = 1.79507, size = 74, normalized size = 3.36

$$-\frac{2\,\cos\left(bx+a\right)^2-1}{b\cos\left(bx+a\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^2\left(a+bx\right)}{\sin^2\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/sin(b*x+a)**2,x)
```

```
[Out] Integral(sec(a + b*x)**2/sin(a + b*x)**2, x)
```

<u>Giac</u> [A] time = 1.14322, size = 22, normalized size = 1.

$$-\frac{2}{b\tan\left(2\,bx+2\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/(b*tan(2*b*x + 2*a))
```

3.142 $\int \csc^2(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{3\csc(a+bx)}{2b}+\frac{3\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\csc(a+bx)\sec^2(a+bx)}{2b}$$

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(2*b) - (3*Csc[a + b*x])/(2*b) + (Csc[a + b*x]*Se
c[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0431818, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 321, 207}

$$-\frac{3\csc(a+bx)}{2b}+\frac{3\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\csc(a+bx)\sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(2*b) - (3*Csc[a + b*x])/(2*b) + (Csc[a + b*x]*Se
c[a + b*x]^2)/(2*b)
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \csc(a+bx)\right)}{b}$$
$$= \frac{\csc(a+bx)\sec^2(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}$$
$$= -\frac{3\csc(a+bx)}{2b} + \frac{\csc(a+bx)\sec^2(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}$$
$$= \frac{3\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3\csc(a+bx)}{2b} + \frac{\csc(a+bx)\sec^2(a+bx)}{2b}$$

Mathematica [C] time = 0.0131511, size = 27, normalized size = 0.55

$$-\frac{\csc(a+bx)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\sin^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] -((Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/b)
```

Maple [A] time = 0.023, size = 55, normalized size = 1.1

$$\frac{1}{2\,b\sin(bx+a)(\cos(bx+a))^2} - \frac{3}{2\,b\sin(bx+a)} + \frac{3\,\ln(\sec(bx+a)+\tan(bx+a))}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3/sin(b*x+a)^2,x)
```

```
[Out] 1/2/b/sin(b*x+a)/cos(b*x+a)^2-3/2/b/sin(b*x+a)+3/2/b*ln(sec(b*x+a)+tan(b*x+
a))
```

Maxima [A] time = 1.01712, size = 82, normalized size = 1.67

$$-\frac{\frac{2\left(3\sin(bx+a)^2-2\right)}{\sin(bx+a)^3-\sin(bx+a)} - 3\,\log(\sin(bx+a)+1) + 3\,\log(\sin(bx+a)-1)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin(
b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b
```

Fricas [A] time = 1.98169, size = 228, normalized size = 4.65

$$\frac{3 \cos(bx+a)^2 \log(\sin(bx+a)+1)\sin(bx+a) - 3 \cos(bx+a)^2 \log(-\sin(bx+a)+1)\sin(bx+a) - 6 \cos(bx+a)^2 +}{4\,b\cos(bx+a)^2 \sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*cos(b*x + a)^2*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*cos(b*x + a)^2
*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 2)/(b*cos(b*x + a
)^2*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(a+bx)}{\sin^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/sin(b*x+a)**2,x)
```

```
[Out] Integral(sec(a + b*x)**3/sin(a + b*x)**2, x)
```

Giac [A] time = 1.21955, size = 85, normalized size = 1.73

$$-\frac{\frac{2\left(3\sin(bx+a)^2-2\right)}{\sin(bx+a)^3-\sin(bx+a)} - 3\log(|\sin(bx+a)+1|) + 3\log(|\sin(bx+a)-1|)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(abs(
sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1)))/b
```

3.143 $\int \csc^2(a+bx)\sec^4(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\tan^3(a+bx)}{3b}+\frac{2\tan(a+bx)}{b}-\frac{\cot(a+bx)}{b}$$

```
[Out] -(Cot[a + b*x]/b) + (2*Tan[a + b*x])/b + Tan[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0367211, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 270}

$$\frac{\tan^3(a+bx)}{3b}+\frac{2\tan(a+bx)}{b}-\frac{\cot(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sec[a + b*x]^4,x]
```

```
[Out] -(Cot[a + b*x]/b) + (2*Tan[a + b*x])/b + Tan[a + b*x]^3/(3*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sec^4(a+bx)\,dx&=\frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2}\,dx,x,\tan(a+bx)\right)}{b}\\&=\frac{\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)dx,x,\tan(a+bx)\right)}{b}\\&=-\frac{\cot(a+bx)}{b}+\frac{2\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0330181, size = 46, normalized size = 1.21

$$\frac{5\tan(a+bx)}{3b}-\frac{\cot(a+bx)}{b}+\frac{\tan(a+bx)\sec^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sec[a + b*x]^4,x]
```

```
[Out] -(Cot[a + b*x]/b) + (5*Tan[a + b*x])/(3*b) + (Sec[a + b*x]^2*Tan[a + b*x])/
(3*b)
```

Maple [A] time = 0.022, size = 50, normalized size = 1.3

$$\frac{1}{b}\left(\frac{1}{3\left(\cos\left(bx+a\right)\right)^{3}\sin\left(bx+a\right)}+\frac{4}{3\cos\left(bx+a\right)\sin\left(bx+a\right)}-\frac{8\cot\left(bx+a\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4/sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/3/sin(b*x+a)/cos(b*x+a)^3+4/3/sin(b*x+a)/cos(b*x+a)-8/3*cot(b*x+a))
```

Maxima [A] time = 0.985613, size = 43, normalized size = 1.13

$$\frac{\tan\left(bx+a\right)^{3}-\frac{3}{\tan\left(bx+a\right)}+6\tan\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(b*x + a)^3 - 3/tan(b*x + a) + 6*tan(b*x + a))/b
```

Fricas [A] time = 1.78534, size = 108, normalized size = 2.84

$$-\frac{8\cos\left(bx+a\right)^{4}-4\cos\left(bx+a\right)^{2}-1}{3\,b\cos\left(bx+a\right)^{3}\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(8*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3*sin(b*x +
a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{4}\left(a+bx\right)}{\sin^{2}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4/sin(b*x+a)**2,x)
```

```
[Out] Integral(sec(a + b*x)**4/sin(a + b*x)**2, x)
```

Giac [A] time = 1.19946, size = 43, normalized size = 1.13

$$\frac{\tan\left(bx+a\right)^3-\frac{3}{\tan(bx+a)}+6\,\tan\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(tan(b*x + a)^3 - 3/tan(b*x + a) + 6*tan(b*x + a))/b
```

3.144 $\int \csc^2(a+bx)\sec^5(a+bx)\,dx$

Optimal. Leaf size=70

$$-\frac{15\csc(a+bx)}{8b}+\frac{15\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\csc(a+bx)\sec^4(a+bx)}{4b}+\frac{5\csc(a+bx)\sec^2(a+bx)}{8b}$$

```
[Out] (15*ArcTanh[Sin[a + b*x]])/(8*b) - (15*Csc[a + b*x])/(8*b) + (5*Csc[a + b*x
]*Sec[a + b*x]^2)/(8*b) + (Csc[a + b*x]*Sec[a + b*x]^4)/(4*b)
```

Rubi [A] time = 0.0467545, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 321, 207}

$$-\frac{15\csc(a+bx)}{8b}+\frac{15\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\csc(a+bx)\sec^4(a+bx)}{4b}+\frac{5\csc(a+bx)\sec^2(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sec[a + b*x]^5,x]
```

```
[Out] (15*ArcTanh[Sin[a + b*x]])/(8*b) - (15*Csc[a + b*x])/(8*b) + (5*Csc[a + b*x
]*Sec[a + b*x]^2)/(8*b) + (Csc[a + b*x]*Sec[a + b*x]^4)/(4*b)
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, \csc(a+bx)\right)}{b}$$
$$= \frac{\csc(a+bx)\sec^4(a+bx)}{4b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{4b}$$
$$= \frac{5\csc(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc(a+bx)\sec^4(a+bx)}{4b} - \frac{15\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \text{cs}\right.}{8b}$$
$$= -\frac{15\csc(a+bx)}{8b} + \frac{5\csc(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc(a+bx)\sec^4(a+bx)}{4b} - \frac{15\,\text{Subs}}{}$$
$$= \frac{15\tanh^{-1}(\sin(a+bx))}{8b} - \frac{15\csc(a+bx)}{8b} + \frac{5\csc(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc(a+bx}{}$$

Mathematica [C] time = 0.0135408, size = 27, normalized size = 0.39

$$-\frac{\csc(a+bx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\sin^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sec[a + b*x]^5,x]
```

```
[Out] -((Csc[a + b*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[a + b*x]^2])/b)
```

Maple [A] time = 0.029, size = 76, normalized size = 1.1

$$\frac{1}{4\,b\sin(bx+a)(\cos(bx+a))^4} + \frac{5}{8\,b\sin(bx+a)(\cos(bx+a))^2} - \frac{15}{8\,b\sin(bx+a)} + \frac{15\ln(\sec(bx+a)+\tan(bx+a}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5/sin(b*x+a)^2,x)
```

```
[Out] 1/4/b/sin(b*x+a)/cos(b*x+a)^4+5/8/b/sin(b*x+a)/cos(b*x+a)^2-15/8/b/sin(b*x+
a)+15/8/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 1.00028, size = 107, normalized size = 1.53

$$-\frac{\frac{2\left(15\sin(bx+a)^4-25\sin(bx+a)^2+8\right)}{\sin(bx+a)^5-2\sin(bx+a)^3+\sin(bx+a)} - 15\log(\sin(bx+a)+1) + 15\log(\sin(bx+a)-1)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(15*sin(b*x + a)^4 - 25*sin(b*x + a)^2 + 8)/(sin(b*x + a)^5 - 2*si
n(b*x + a)^3 + sin(b*x + a)) - 15*log(sin(b*x + a) + 1) + 15*log(sin(b*x +
```

```
a) - 1))/b
```

Fricas [A] time = 2.09936, size = 261, normalized size = 3.73

$$\frac{15\cos(bx+a)^4\log(\sin(bx+a)+1)\sin(bx+a)-15\cos(bx+a)^4\log(-\sin(bx+a)+1)\sin(bx+a)-30\cos(bx+a}{16\,b\cos(bx+a)^4\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(15*cos(b*x + a)^4*log(sin(b*x + a) + 1)*sin(b*x + a) - 15*cos(b*x + a
)^4*log(-sin(b*x + a) + 1)*sin(b*x + a) - 30*cos(b*x + a)^4 + 10*cos(b*x +
a)^2 + 4)/(b*cos(b*x + a)^4*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5/sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22019, size = 99, normalized size = 1.41

$$-\frac{\frac{2\left(7\sin(bx+a)^3-9\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^2}+\frac{16}{\sin(bx+a)}-15\log(|\sin(bx+a)+1|)+15\log(|\sin(bx+a)-1|)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*(7*sin(b*x + a)^3 - 9*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 + 16/si
n(b*x + a) - 15*log(abs(sin(b*x + a) + 1)) + 15*log(abs(sin(b*x + a) - 1)))
/b
```

3.145 $\int \cos^4(a+bx)\cot^3(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{\sin^4(a+bx)}{4b}+\frac{3\sin^2(a+bx)}{2b}-\frac{\csc^2(a+bx)}{2b}-\frac{3\log(\sin(a+bx))}{b}$$

```
[Out] -Csc[a + b*x]^2/(2*b) - (3*Log[Sin[a + b*x]])/b + (3*Sin[a + b*x]^2)/(2*b)
- Sin[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.04388, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2590, 266, 43}

$$-\frac{\sin^4(a+bx)}{4b}+\frac{3\sin^2(a+bx)}{2b}-\frac{\csc^2(a+bx)}{2b}-\frac{3\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Cot[a + b*x]^3,x]
```

```
[Out] -Csc[a + b*x]^2/(2*b) - (3*Log[Sin[a + b*x]])/b + (3*Sin[a + b*x]^2)/(2*b)
- Sin[a + b*x]^4/(4*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^4(a+bx)\cot^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^3}\,dx, x, -\sin(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^3}{x^2}\,dx, x, \sin^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}-\frac{3}{x}-x\right)dx, x, \sin^2(a+bx)\right)}{2b}\\
&= -\frac{\csc^2(a+bx)}{2b}-\frac{3\log(\sin(a+bx))}{b}+\frac{3\sin^2(a+bx)}{2b}-\frac{\sin^4(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.102288, size = 45, normalized size = 0.78

$$-\frac{\sin^4(a+bx)-6\sin^2(a+bx)+2\csc^2(a+bx)+12\log(\sin(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Cot[a + b*x]^3,x]
```

```
[Out] -(2*Csc[a + b*x]^2 + 12*Log[Sin[a + b*x]] - 6*Sin[a + b*x]^2 + Sin[a + b*x]
^4)/(4*b)
```

Maple [A] time = 0.014, size = 74, normalized size = 1.3

$$-\frac{(\cos(bx+a))^8}{2b(\sin(bx+a))^2}-\frac{(\cos(bx+a))^6}{2b}-\frac{3(\cos(bx+a))^4}{4b}-\frac{3(\cos(bx+a))^2}{2b}-3\frac{\ln(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7/sin(b*x+a)^3,x)
```

```
[Out] -1/2/b/sin(b*x+a)^2*cos(b*x+a)^8-1/2*cos(b*x+a)^6/b-3/4*cos(b*x+a)^4/b-3/2*
cos(b*x+a)^2/b-3*ln(sin(b*x+a))/b
```

Maxima [A] time = 0.971924, size = 61, normalized size = 1.05

$$-\frac{\sin(bx+a)^4-6\sin(bx+a)^2+\frac{2}{\sin(bx+a)^2}+6\log\left(\sin(bx+a)^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(sin(b*x + a)^4 - 6*sin(b*x + a)^2 + 2/sin(b*x + a)^2 + 6*log(sin(b*x
+ a)^2))/b
```

Fricas [A] time = 2.02392, size = 190, normalized size = 3.28

$$-\frac{8\cos(bx+a)^6+24\cos(bx+a)^4-51\cos(bx+a)^2+96\left(\cos(bx+a)^2-1\right)\log\left(\frac{1}{2}\sin(bx+a)\right)+3}{32\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*cos(b*x + a)^6 + 24*cos(b*x + a)^4 - 51*cos(b*x + a)^2 + 96*(cos(b
*x + a)^2 - 1)*log(1/2*sin(b*x + a)) + 3)/(b*cos(b*x + a)^2 - b)
```

Sympy [A] time = 22.4741, size = 1484, normalized size = 25.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((24*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**10/(8*b*tan(a/
2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b
*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 96*log(tan(a/2 + b*x/2)**
2 + 1)*tan(a/2 + b*x/2)**8/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2
)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 +
b*x/2)**2) + 144*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(8*b*tan(
a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32
*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 96*log(tan(a/2 + b*x/2)
**2 + 1)*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x
/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2
+ b*x/2)**2) + 24*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(8*b*tan
(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 3
2*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 24*log(tan(a/2 + b*x/2
))*tan(a/2 + b*x/2)**10/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**
8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x
/2)**2) - 96*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/(8*b*tan(a/2 + b*x/2
)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2
+ b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 144*log(tan(a/2 + b*x/2))*tan(a/2
+ b*x/2)**6/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan
(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 96
*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**10 + 32*b
*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4
+ 8*b*tan(a/2 + b*x/2)**2) - 24*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(
8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)
**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - tan(a/2 + b*x/2
)**12/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 +
 b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 57*tan(a
/2 + b*x/2)**8/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*
tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) +
 80*tan(a/2 + b*x/2)**6/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**
8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x
/2)**2) + 57*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 +
 b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(
a/2 + b*x/2)**2) - 1/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 +
 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)
**2), Ne(b, 0)), (x*cos(a)**7/sin(a)**3, True))
```

Giac [B] time = 1.24494, size = 312, normalized size = 5.38

$$\frac{\frac{\left(\frac{12\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{2\left(\frac{76\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{118\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{76\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{25\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}-25\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^4}-12\log\left(\frac{|-\cos\cdots}{|\cos(\cdots}\right.}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*((12*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)*(cos(b*x + a) + 1)/(cos
(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 2*(76*(cos(b*x + a
) - 1)/(cos(b*x + a) + 1) - 118*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 +
 76*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 25*(cos(b*x + a) - 1)^4/(co
s(b*x + a) + 1)^4 - 25)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^4 - 12*
log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 24*log(abs(-(cos(b*x +
a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

3.146 $\int \cos^3(a+bx)\cot^3(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{5\cos^3(a+bx)}{6b}-\frac{5\cos(a+bx)}{2b}-\frac{\cos^3(a+bx)\cot^2(a+bx)}{2b}+\frac{5\tanh^{-1}(\cos(a+bx))}{2b}$$

```
[Out] (5*ArcTanh[Cos[a + b*x]])/(2*b) - (5*Cos[a + b*x])/(2*b) - (5*Cos[a + b*x]^
3)/(6*b) - (Cos[a + b*x]^3*Cot[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0449874, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2592, 288, 302, 206}

$$-\frac{5\cos^3(a+bx)}{6b}-\frac{5\cos(a+bx)}{2b}-\frac{\cos^3(a+bx)\cot^2(a+bx)}{2b}+\frac{5\tanh^{-1}(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Cot[a + b*x]^3,x]
```

```
[Out] (5*ArcTanh[Cos[a + b*x]])/(2*b) - (5*Cos[a + b*x])/(2*b) - (5*Cos[a + b*x]^
3)/(6*b) - (Cos[a + b*x]^3*Cot[a + b*x]^2)/(2*b)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{\cos^3(a+bx)\cot^2(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}$$
$$= -\frac{\cos^3(a+bx)\cot^2(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \cos(a+bx)\right)}{2b}$$
$$= -\frac{5\cos(a+bx)}{2b} - \frac{5\cos^3(a+bx)}{6b} - \frac{\cos^3(a+bx)\cot^2(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x,\right.}{2b}$$
$$= \frac{5\tanh^{-1}(\cos(a+bx))}{2b} - \frac{5\cos(a+bx)}{2b} - \frac{5\cos^3(a+bx)}{6b} - \frac{\cos^3(a+bx)\cot^2(a+bx)}{2b}$$

Mathematica [A] time = 0.038896, size = 103, normalized size = 1.56

$$-\frac{9\cos(a+bx)}{4b} - \frac{\cos(3(a+bx))}{12b} - \frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{5\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{5\log\left(\cos\left(\frac{1}{2}(a+bx\right.\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Cot[a + b*x]^3,x]
```

```
[Out] (-9*Cos[a + b*x])/(4*b) - Cos[3*(a + b*x)]/(12*b) - Csc[(a + b*x)/2]^2/(8*b
) + (5*Log[Cos[(a + b*x)/2]])/(2*b) - (5*Log[Sin[(a + b*x)/2]])/(2*b) + Sec
[(a + b*x)/2]^2/(8*b)
```

Maple [A] time = 0.013, size = 81, normalized size = 1.2

$$-\frac{(\cos(bx+a))^7}{2b(\sin(bx+a))^2} - \frac{(\cos(bx+a))^5}{2b} - \frac{5(\cos(bx+a))^3}{6b} - \frac{5\cos(bx+a)}{2b} - \frac{5\ln(\csc(bx+a)-\cot(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6/sin(b*x+a)^3,x)
```

```
[Out] -1/2/b*cos(b*x+a)^7/sin(b*x+a)^2-1/2*cos(b*x+a)^5/b-5/6*cos(b*x+a)^3/b-5/2*
cos(b*x+a)/b-5/2/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [A] time = 0.977276, size = 89, normalized size = 1.35

$$-\frac{4\cos(bx+a)^3 - \frac{6\cos(bx+a)}{\cos(bx+a)^2-1} + 24\cos(bx+a) - 15\log(\cos(bx+a)+1) + 15\log(\cos(bx+a)-1)}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(4*cos(b*x + a)^3 - 6*cos(b*x + a)/(cos(b*x + a)^2 - 1) + 24*cos(b*x
+ a) - 15*log(cos(b*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b
```

Fricas [A] time = 2.04597, size = 265, normalized size = 4.02

$$-\frac{4\cos\left(bx+a\right)^{5}+20\cos\left(bx+a\right)^{3}-15\left(\cos\left(bx+a\right)^{2}-1\right)\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)+15\left(\cos\left(bx+a\right)^{2}-1\right)\log\left(-\frac{1}{2}\right.}{12\left(b\cos\left(bx+a\right)^{2}-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(4*cos(b*x + a)^5 + 20*cos(b*x + a)^3 - 15*(cos(b*x + a)^2 - 1)*log(1
/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^2 - 1)*log(-1/2*cos(b*x + a) + 1/
2) - 30*cos(b*x + a))/(b*cos(b*x + a)^2 - b)
```

Sympy [A] time = 11.7231, size = 719, normalized size = 10.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-60*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/(24*b*tan(a/2 + b*
x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*tan(a/
2 + b*x/2)**2) - 180*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(24*b*tan(a/
2 + b*x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*
tan(a/2 + b*x/2)**2) - 180*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(24*b*
tan(a/2 + b*x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 +
 24*b*tan(a/2 + b*x/2)**2) - 60*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(
24*b*tan(a/2 + b*x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)
**4 + 24*b*tan(a/2 + b*x/2)**2) + 3*tan(a/2 + b*x/2)**10/(24*b*tan(a/2 + b*
x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*tan(a/
2 + b*x/2)**2) - 165*tan(a/2 + b*x/2)**6/(24*b*tan(a/2 + b*x/2)**8 + 72*b*t
an(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*tan(a/2 + b*x/2)**2) -
 225*tan(a/2 + b*x/2)**4/(24*b*tan(a/2 + b*x/2)**8 + 72*b*tan(a/2 + b*x/2)*
*6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*tan(a/2 + b*x/2)**2) - 130*tan(a/2 + b
*x/2)**2/(24*b*tan(a/2 + b*x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/
2 + b*x/2)**4 + 24*b*tan(a/2 + b*x/2)**2) - 3/(24*b*tan(a/2 + b*x/2)**8 + 7
2*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*tan(a/2 + b*x/2)*
*2), Ne(b, 0)), (x*cos(a)**6/sin(a)**3, True))
```

Giac [B] time = 1.16818, size = 220, normalized size = 3.33

$$\frac{\frac{3\left(\frac{10(\cos(bx+a)-1)}{\cos(bx+a)+1}+1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}-\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{16\left(\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{9(\cos(bx+a)-1)^{2}}{(\cos(bx+a)+1)^{2}}-7\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^{3}}-30\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*(10*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)*(cos(b*x + a) + 1)/(
cos(b*x + a) - 1) - 3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 16*(12*(cos(b
*x + a) - 1)/(cos(b*x + a) + 1) - 9*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)
^2 - 7)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^3 - 30*log(abs(-cos(b*x
 + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.147 $\int \cos^2(a+bx)\cot^3(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\sin^2(a+bx)}{2b}-\frac{\csc^2(a+bx)}{2b}-\frac{2\log(\sin(a+bx))}{b}$$

```
[Out] -Csc[a + b*x]^2/(2*b) - (2*Log[Sin[a + b*x]])/b + Sin[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0388649, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2590, 266, 43}

$$\frac{\sin^2(a+bx)}{2b}-\frac{\csc^2(a+bx)}{2b}-\frac{2\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cot[a + b*x]^3,x]
```

```
[Out] -Csc[a + b*x]^2/(2*b) - (2*Log[Sin[a + b*x]])/b + Sin[a + b*x]^2/(2*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^2(a+bx)\cot^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^3}\,dx,x,-\sin(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx,x,\sin^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)dx,x,\sin^2(a+bx)\right)}{2b}\\
&= -\frac{\csc^2(a+bx)}{2b}-\frac{2\log(\sin(a+bx))}{b}+\frac{\sin^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.0617029, size = 35, normalized size = 0.81

$$-\frac{-\sin^2(a+bx)+\csc^2(a+bx)+4\log(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cot[a + b*x]^3,x]
```

```
[Out] -(Csc[a + b*x]^2 + 4*Log[Sin[a + b*x]] - Sin[a + b*x]^2)/(2*b)
```

Maple [A] time = 0.013, size = 61, normalized size = 1.4

$$-\frac{(\cos(bx+a))^6}{2b(\sin(bx+a))^2}-\frac{(\cos(bx+a))^4}{2b}-\frac{(\cos(bx+a))^2}{b}-2\frac{\ln(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5/sin(b*x+a)^3,x)
```

```
[Out] -1/2/b*cos(b*x+a)^6/sin(b*x+a)^2-1/2*cos(b*x+a)^4/b-cos(b*x+a)^2/b-2*ln(sin
(b*x+a))/b
```

Maxima [A] time = 0.97445, size = 47, normalized size = 1.09

$$\frac{\sin(bx+a)^2-\frac{1}{\sin(bx+a)^2}-2\log\left(\sin(bx+a)^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(sin(b*x + a)^2 - 1/sin(b*x + a)^2 - 2*log(sin(b*x + a)^2))/b
```

Fricas [A] time = 1.94162, size = 159, normalized size = 3.7

$$-\frac{2\cos(bx+a)^4-3\cos(bx+a)^2+8\left(\cos(bx+a)^2-1\right)\log\left(\frac{1}{2}\sin(bx+a)\right)-1}{4\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(b*x + a)^4 - 3*cos(b*x + a)^2 + 8*(cos(b*x + a)^2 - 1)*log(1/2*
sin(b*x + a)) - 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [A] time = 6.34138, size = 614, normalized size = 14.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((16*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(8*b*tan(a/2
 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 32*log
(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**6 + 16
*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 16*log(tan(a/2 + b*x/2)
**2 + 1)*tan(a/2 + b*x/2)**2/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/
2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 16*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2
)**6/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*
x/2)**2) - 32*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/
2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 16*log(tan(a/
2 + b*x/2))*tan(a/2 + b*x/2)**2/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b
*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - tan(a/2 + b*x/2)**8/(8*b*tan(a/2 + b*
x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 18*tan(a/2
+ b*x/2)**4/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a
/2 + b*x/2)**2) - 1/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8
*b*tan(a/2 + b*x/2)**2), Ne(b, 0)), (x*cos(a)**5/sin(a)**3, True))
```

Giac [B] time = 1.21101, size = 252, normalized size = 5.86

$$\frac{\frac{\left(\frac{8(\cos(bx+a)-1)}{\cos(bx+a)+1}+1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{8\left(\frac{4(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-3\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^2}-8\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)+16\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right.\right.}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*((8*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)*(cos(b*x + a) + 1)/(cos(
b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 8*(4*(cos(b*x + a)
- 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 - 3)/
((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^2 - 8*log(abs(-cos(b*x + a) + 1
)/abs(cos(b*x + a) + 1)) + 16*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1
) + 1)))/b
```

3.148 $\int \cos(a + bx) \cot^3(a + bx) \, dx$

Optimal. Leaf size=49

$$-\frac{3\cos(a+bx)}{2b} - \frac{\cos(a+bx)\cot^2(a+bx)}{2b} + \frac{3\tanh^{-1}(\cos(a+bx))}{2b}$$

```
[Out] (3*ArcTanh[Cos[a + b*x]])/(2*b) - (3*Cos[a + b*x])/(2*b) - (Cos[a + b*x]*Co
t[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0292992, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2592, 288, 321, 206}

$$-\frac{3\cos(a+bx)}{2b} - \frac{\cos(a+bx)\cot^2(a+bx)}{2b} + \frac{3\tanh^{-1}(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[a + b*x]^3,x]
```

```
[Out] (3*ArcTanh[Cos[a + b*x]])/(2*b) - (3*Cos[a + b*x])/(2*b) - (Cos[a + b*x]*Co
t[a + b*x]^2)/(2*b)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos(a+bx)\cot^3(a+bx)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{\cos(a+bx)\cot^2(a+bx)}{2b} + \frac{3\operatorname{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}$$
$$= -\frac{3\cos(a+bx)}{2b} - \frac{\cos(a+bx)\cot^2(a+bx)}{2b} + \frac{3\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}$$
$$= \frac{3\tanh^{-1}(\cos(a+bx))}{2b} - \frac{3\cos(a+bx)}{2b} - \frac{\cos(a+bx)\cot^2(a+bx)}{2b}$$

Mathematica [A] time = 0.0268677, size = 86, normalized size = 1.76

$$-\frac{\cos(a+bx)}{b} - \frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{3\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{3\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[a + b*x]^3,x]
```

```
[Out] -(Cos[a + b*x]/b) - Csc[(a + b*x)/2]^2/(8*b) + (3*Log[Cos[(a + b*x)/2]])/(2
*b) - (3*Log[Sin[(a + b*x)/2]])/(2*b) + Sec[(a + b*x)/2]^2/(8*b)
```

Maple [A] time = 0.013, size = 68, normalized size = 1.4

$$-\frac{(\cos(bx+a))^5}{2b(\sin(bx+a))^2} - \frac{(\cos(bx+a))^3}{2b} - \frac{3\cos(bx+a)}{2b} - \frac{3\ln(\csc(bx+a) - \cot(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/sin(b*x+a)^3,x)
```

```
[Out] -1/2/b*cos(b*x+a)^5/sin(b*x+a)^2-1/2*cos(b*x+a)^3/b-3/2*cos(b*x+a)/b-3/2/b*
ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [A] time = 1.01758, size = 76, normalized size = 1.55

$$\frac{\frac{2\cos(bx+a)}{\cos(bx+a)^2-1} - 4\cos(bx+a) + 3\log(\cos(bx+a)+1) - 3\log(\cos(bx+a)-1)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - 4*cos(b*x + a) + 3*log(cos(b*x +
 a) + 1) - 3*log(cos(b*x + a) - 1))/b
```

Fricas [A] time = 2.035, size = 232, normalized size = 4.73

$$-\frac{4\cos(bx+a)^3-3\left(\cos(bx+a)^2-1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+3\left(\cos(bx+a)^2-1\right)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-6\cos(bx+a)}{4\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*cos(b*x + a)^3 - 3*(cos(b*x + a)^2 - 1)*log(1/2*cos(b*x + a) + 1/2)
 + 3*(cos(b*x + a)^2 - 1)*log(-1/2*cos(b*x + a) + 1/2) - 6*cos(b*x + a))/(b
*cos(b*x + a)^2 - b)
```

Sympy [A] time = 3.35844, size = 241, normalized size = 4.92

$$\begin{cases}-\frac{12\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}-\frac{12\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}+\frac{\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}-\frac{18\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}-\frac{1}{8b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)} & \text{for } b\neq 0\\ \frac{x\cos^4(a)}{\sin^3(a)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-12*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x
/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 12*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/
2)**2/(8*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + tan(a/2 + b*x/2
)**6/(8*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 18*tan(a/2 + b*x
/2)**2/(8*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 1/(8*b*tan(a/2
 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2), Ne(b, 0)), (x*cos(a)**4/sin(a)**3,
 True))
```

Giac [B] time = 1.18454, size = 189, normalized size = 3.86

$$-\frac{\frac{\frac{14(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+6\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*((14*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(c
os(b*x + a) + 1)^2 - 1)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x +
 a) - 1)^2/(cos(b*x + a) + 1)^2) + (cos(b*x + a) - 1)/(cos(b*x + a) + 1) +
6*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.149 $\int \cot^3(a+bx)\,dx$

Optimal. Leaf size=28

$$-\frac{\cot^2(a+bx)}{2b}-\frac{\log(\sin(a+bx))}{b}$$

```
[Out] -Cot[a + b*x]^2/(2*b) - Log[Sin[a + b*x]]/b
```

Rubi [A] time = 0.0129672, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$-\frac{\cot^2(a+bx)}{2b}-\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^3,x]

[Out] -Cot[a + b*x]^2/(2*b) - Log[Sin[a + b*x]]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^3(a+bx)\,dx &= -\frac{\cot^2(a+bx)}{2b}-\int \cot(a+bx)\,dx\\ &= -\frac{\cot^2(a+bx)}{2b}-\frac{\log(\sin(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.0948977, size = 34, normalized size = 1.21

$$-\frac{\cot^2(a+bx)+2\log(\tan(a+bx))+2\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^3,x]

[Out] -(Cot[a + b*x]^2 + 2*Log[Cos[a + b*x]] + 2*Log[Tan[a + b*x]])/(2*b)
```

Maple [A] time = 0.011, size = 27, normalized size = 1.

$$-\frac{(\cot (bx+a))^2}{2b}-\frac{\ln (\sin (bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(b*x+a)^3,x)
```

```
[Out] -1/2*cot(b*x+a)^2/b-ln(sin(b*x+a))/b
```

Maxima [A] time = 0.977599, size = 31, normalized size = 1.11

$$-\frac{\frac{1}{\sin(bx+a)^2}+\log\left(\sin (bx+a)^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b
```

Fricas [A] time = 2.39515, size = 108, normalized size = 3.86

$$-\frac{2\left(\cos (bx+a)^2-1\right)\log\left(\frac{1}{2}\sin (bx+a)\right)-1}{2\left(b\cos (bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cos(b*x + a)^2 - 1)*log(1/2*sin(b*x + a)) - 1)/(b*cos(b*x + a)^2 -
 b)
```

Sympy [A] time = 1.0051, size = 42, normalized size = 1.5

$$\begin{cases} -\frac{\log (\sin (a+bx))}{b}-\frac{\cos^2 (a+bx)}{2b\sin^2 (a+bx)} & \text{for}\ b\neq 0 \\ \frac{x\cos^3 (a)}{\sin^3 (a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-log(sin(a + b*x))/b - cos(a + b*x)**2/(2*b*sin(a + b*x)**2), Ne
(b, 0)), (x*cos(a)**3/sin(a)**3, True))
```

Giac [A] time = 1.13655, size = 49, normalized size = 1.75

$$\frac{\frac{\sin(bx+a)^2-1}{\sin(bx+a)^2} - \log\left(\sin\left(bx+a\right)^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*((sin(b*x + a)^2 - 1)/sin(b*x + a)^2 - log(sin(b*x + a)^2))/b
```

3.150 $\int \cot^2(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tanh^{-1}(\cos(a+bx))}{2b}-\frac{\cot(a+bx)\csc(a+bx)}{2b}$$

```
[Out] ArcTanh[Cos[a + b*x]]/(2*b) - (Cot[a + b*x]*Csc[a + b*x])/(2*b)
```

Rubi [A] time = 0.0226236, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$\frac{\tanh^{-1}(\cos(a+bx))}{2b}-\frac{\cot(a+bx)\csc(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] ArcTanh[Cos[a + b*x]]/(2*b) - (Cot[a + b*x]*Csc[a + b*x])/(2*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(a+bx)\csc(a+bx)\,dx &= -\frac{\cot(a+bx)\csc(a+bx)}{2b}-\frac{1}{2}\int \csc(a+bx)\,dx\\ &= \frac{\tanh^{-1}(\cos(a+bx))}{2b}-\frac{\cot(a+bx)\csc(a+bx)}{2b}\end{aligned}$$

Mathematica [B] time = 0.0268574, size = 75, normalized size = 2.21

$$-\frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -Csc[(a + b*x)/2]^2/(8*b) + Log[Cos[(a + b*x)/2]]/(2*b) - Log[Sin[(a + b*x)
/2]]/(2*b) + Sec[(a + b*x)/2]^2/(8*b)
```

Maple [A] time = 0.01, size = 55, normalized size = 1.6

$$-\frac{(\cos(bx+a))^3}{2b(\sin(bx+a))^2}-\frac{\cos(bx+a)}{2b}-\frac{\ln(\csc(bx+a)-\cot(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(b*x+a)^3,x)
```

```
[Out] -1/2/b*cos(b*x+a)^3/sin(b*x+a)^2-1/2*cos(b*x+a)/b-1/2/b*ln(csc(b*x+a)-cot(b
*x+a))
```

Maxima [A] time = 0.981926, size = 62, normalized size = 1.82

$$\frac{\frac{2\cos(bx+a)}{\cos(bx+a)^2-1}+\log(\cos(bx+a)+1)-\log(\cos(bx+a)-1)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(
b*x + a) - 1))/b
```

Fricas [B] time = 2.57544, size = 200, normalized size = 5.88

$$\frac{\left(\cos(bx+a)^2-1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-\left(\cos(bx+a)^2-1\right)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+2\cos(bx+a)}{4\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((cos(b*x + a)^2 - 1)*log(1/2*cos(b*x + a) + 1/2) - (cos(b*x + a)^2 - 1
)*log(-1/2*cos(b*x + a) + 1/2) + 2*cos(b*x + a))/(b*cos(b*x + a)^2 - b)
```

Sympy [A] time = 1.6412, size = 58, normalized size = 1.71

$$\begin{cases} -\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{2b}+\frac{\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}-\frac{1}{8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)} & \text{for } b\neq 0 \\ \frac{x\cos^2(a)}{\sin^3(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-log(tan(a/2 + b*x/2))/(2*b) + tan(a/2 + b*x/2)**2/(8*b) - 1/(8*
b*tan(a/2 + b*x/2)**2), Ne(b, 0)), (x*cos(a)**2/sin(a)**3, True))
```

Giac [B] time = 1.17846, size = 126, normalized size = 3.71

$$\frac{\frac{\left(\frac{2\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-2\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*((2*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)*(cos(b*x + a) + 1)/(cos(
b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 2*log(abs(-cos(b*x
+ a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.151 $\int \cot(a+bx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\csc^2(a+bx)}{2b}$$

```
[Out] -Csc[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0178605, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$-\frac{\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -Csc[a + b*x]^2/(2*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot(a+bx)\csc^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x\,dx, x, \csc(a+bx)\right)}{b}\\ &= -\frac{\csc^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0121944, size = 15, normalized size = 1.

$$-\frac{\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -Csc[a + b*x]^2/(2*b)
```

Maple [A] time = 0.004, size = 14, normalized size = 0.9

$$-\frac{1}{2\left(\sin\left(bx+a\right)\right)^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+a)^3,x)
```

```
[Out] -1/2/sin(b*x+a)^2/b
```

Maxima [A] time = 0.961035, size = 18, normalized size = 1.2

$$-\frac{1}{2\,b\sin\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(b*sin(b*x + a)^2)
```

Fricas [A] time = 2.48954, size = 38, normalized size = 2.53

$$\frac{1}{2\left(b\cos\left(bx+a\right)^{2}-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2/(b*cos(b*x + a)^2 - b)
```

Sympy [A] time = 0.976703, size = 24, normalized size = 1.6

$$\begin{cases} -\frac{1}{2b\sin^{2}\left(a+bx\right)} & \text{for}\ b\neq 0 \\ \frac{x\cos\left(a\right)}{\sin^{3}\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-1/(2*b*sin(a + b*x)**2), Ne(b, 0)), (x*cos(a)/sin(a)**3, True))
```

Giac [A] time = 1.13086, size = 18, normalized size = 1.2

$$-\frac{1}{2\,b\sin\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2/(b*sin(b*x + a)^2)
```

3.152 $\int \csc^3(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\log(\tan(a+bx))}{b}-\frac{\cot^2(a+bx)}{2b}$$

```
[Out] -Cot[a + b*x]^2/(2*b) + Log[Tan[a + b*x]]/b
```

Rubi [A] time = 0.0218528, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2620, 14}

$$\frac{\log(\tan(a+bx))}{b}-\frac{\cot^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -Cot[a + b*x]^2/(2*b) + Log[Tan[a + b*x]]/b
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sec(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{x^3}\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)dx,x,\tan(a+bx)\right)}{b}\\ &= -\frac{\cot^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.041896, size = 34, normalized size = 1.26

$$-\frac{\csc^2(a+bx)-2\log(\sin(a+bx))+2\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -(Csc[a + b*x]^2 + 2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]])/(2*b)
```

Maple [A] time = 0.018, size = 26, normalized size = 1.

$$-\frac{1}{2\left(\sin\left(bx+a\right)\right)^{2}b}+\frac{\ln\left(\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/sin(b*x+a)^3,x)
```

```
[Out] -1/2/sin(b*x+a)^2/b+ln(tan(b*x+a))/b
```

Maxima [A] time = 0.99941, size = 49, normalized size = 1.81

$$-\frac{\frac{1}{\sin(bx+a)^{2}}+\log\left(\sin\left(bx+a\right)^{2}-1\right)-\log\left(\sin\left(bx+a\right)^{2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b
```

Fricas [B] time = 2.38227, size = 176, normalized size = 6.52

$$-\frac{\left(\cos\left(bx+a\right)^{2}-1\right)\log\left(\cos\left(bx+a\right)^{2}\right)-\left(\cos\left(bx+a\right)^{2}-1\right)\log\left(-\frac{1}{4}\cos\left(bx+a\right)^{2}+\frac{1}{4}\right)-1}{2\left(b\cos\left(bx+a\right)^{2}-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((cos(b*x + a)^2 - 1)*log(cos(b*x + a)^2) - (cos(b*x + a)^2 - 1)*log(-
1/4*cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(a+bx\right)}{\sin^{3}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)**3,x)
```

```
[Out] Integral(sec(a + b*x)/sin(a + b*x)**3, x)
```

Giac [B] time = 1.16666, size = 161, normalized size = 5.96

$$-\frac{\frac{\left(\frac{4\left(\cos(bx+a)-1\right)}{\cos(bx+a)+1}-1\right)\left(\cos(bx+a)+1\right)}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-4\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)+8\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right|\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*((4*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/(cos
(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 4*log(abs(-cos(b*x
 + a) + 1)/abs(cos(b*x + a) + 1)) + 8*log(abs(-(cos(b*x + a) - 1)/(cos(b*x
+ a) + 1) - 1)))/b
```

3.153 $\int \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=49

$$\frac{3\sec(a+bx)}{2b}-\frac{3\tanh^{-1}(\cos(a+bx))}{2b}-\frac{\csc^2(a+bx)\sec(a+bx)}{2b}$$

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(2*b) + (3*Sec[a + b*x])/(2*b) - (Csc[a + b*x]^2
*Sec[a + b*x])/(2*b)
```

Rubi [A] time = 0.0429918, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 321, 207}

$$\frac{3\sec(a+bx)}{2b}-\frac{3\tanh^{-1}(\cos(a+bx))}{2b}-\frac{\csc^2(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(2*b) + (3*Sec[a + b*x])/(2*b) - (Csc[a + b*x]^2
*Sec[a + b*x])/(2*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{b}$$
$$= -\frac{\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{2b}$$
$$= \frac{3\sec(a+bx)}{2b} - \frac{\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{2b}$$
$$= -\frac{3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3\sec(a+bx)}{2b} - \frac{\csc^2(a+bx)\sec(a+bx)}{2b}$$

Mathematica [B] time = 0.248013, size = 143, normalized size = 2.92

$$\frac{\csc^4(a+bx)\left(-6\cos(2(a+bx)) + 2\cos(3(a+bx)) + 3\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right) - 3\cos(3(a+bx))\log\left(\sin\left(\frac{1}{2}\right.\right.\right.}{2b\left(\csc^2\left(\frac{1}{2}(a+bx)\right) - \sec^2\left(\frac{1}{2}(a+bx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (Csc[a + b*x]^4*(2 - 6*Cos[2*(a + b*x)] + 2*Cos[3*(a + b*x)] + 3*Cos[3*(a +
 b*x)]*Log[Cos[(a + b*x)/2]] - 3*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + C
os[a + b*x]*(-2 - 3*Log[Cos[(a + b*x)/2]] + 3*Log[Sin[(a + b*x)/2]])))/(2*b
*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2))
```

Maple [A] time = 0.017, size = 57, normalized size = 1.2

$$-\frac{1}{2\,b\,(\sin(bx+a))^2\cos(bx+a)} + \frac{3}{2\,b\cos(bx+a)} + \frac{3\,\ln(\csc(bx+a) - \cot(bx+a))}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/sin(b*x+a)^3,x)
```

```
[Out] -1/2/b/sin(b*x+a)^2/cos(b*x+a)+3/2/b/cos(b*x+a)+3/2/b*ln(csc(b*x+a)-cot(b*x
+a))
```

Maxima [A] time = 0.970174, size = 82, normalized size = 1.67

$$\frac{\frac{2\left(3\cos(bx+a)^2-2\right)}{\cos(bx+a)^3-\cos(bx+a)} - 3\,\log(\cos(bx+a)+1) + 3\,\log(\cos(bx+a)-1)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(b
*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b
```

Fricas [B] time = 2.25569, size = 261, normalized size = 5.33

$$\frac{6\cos(bx+a)^2 - 3\left(\cos(bx+a)^3 - \cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + 3\left(\cos(bx+a)^3 - \cos(bx+a)\right)\log\left(-\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) - 4}{4\left(b\cos(bx+a)^3 - b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*cos(b*x + a)^2 - 3*(cos(b*x + a)^3 - cos(b*x + a))*log(1/2*cos(b*x +
 a) + 1/2) + 3*(cos(b*x + a)^3 - cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2)
 - 4)/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(a+bx)}{\sin^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/sin(b*x+a)**3,x)
```

```
[Out] Integral(sec(a + b*x)**2/sin(a + b*x)**3, x)
```

Giac [B] time = 1.20574, size = 189, normalized size = 3.86

$$\frac{\frac{\frac{14(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + 1}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1} + \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}} - \frac{\cos(bx+a)-1}{\cos(bx+a)+1} + 6\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*((14*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(co
s(b*x + a) + 1)^2 + 1)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x +
a) - 1)^2/(cos(b*x + a) + 1)^2) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 6
*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.154 $\int \csc^3(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\tan^2(a+bx)}{2b}-\frac{\cot^2(a+bx)}{2b}+\frac{2\log(\tan(a+bx))}{b}$$

```
[Out] -Cot[a + b*x]^2/(2*b) + (2*Log[Tan[a + b*x]])/b + Tan[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0381614, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$\frac{\tan^2(a+bx)}{2b}-\frac{\cot^2(a+bx)}{2b}+\frac{2\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] -Cot[a + b*x]^2/(2*b) + (2*Log[Tan[a + b*x]])/b + Tan[a + b*x]^2/(2*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc^3(a+bx)\sec^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^3}\,dx,x,\tan(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^2}\,dx,x,\tan^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}+\frac{2}{x}\right)dx,x,\tan^2(a+bx)\right)}{2b}\\
&= -\frac{\cot^2(a+bx)}{2b}+\frac{2\log(\tan(a+bx))}{b}+\frac{\tan^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.0176275, size = 61, normalized size = 1.42

$$8\left(-\frac{\csc^2(a+bx)}{16b}+\frac{\sec^2(a+bx)}{16b}+\frac{\log(\sin(a+bx))}{4b}-\frac{\log(\cos(a+bx))}{4b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] 8*(-Csc[a + b*x]^2/(16*b) - Log[Cos[a + b*x]]/(4*b) + Log[Sin[a + b*x]]/(4*
b) + Sec[a + b*x]^2/(16*b))
```

Maple [A] time = 0.02, size = 48, normalized size = 1.1

$$\frac{1}{2\,b\left(\sin\left(bx+a\right)\right)^{2}\left(\cos\left(bx+a\right)\right)^{2}}-\frac{1}{b\left(\sin\left(bx+a\right)\right)^{2}}+2\,\frac{\ln\left(\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3/sin(b*x+a)^3,x)
```

```
[Out] 1/2/b/sin(b*x+a)^2/cos(b*x+a)^2-1/sin(b*x+a)^2/b+2*ln(tan(b*x+a))/b
```

Maxima [A] time = 1.00213, size = 86, normalized size = 2.

$$-\frac{\frac{2\,\sin(bx+a)^{2}-1}{\sin(bx+a)^{4}-\sin(bx+a)^{2}}+2\,\log\left(\sin\left(bx+a\right)^{2}-1\right)-2\,\log\left(\sin\left(bx+a\right)^{2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(
b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b
```

Fricas [B] time = 2.21115, size = 261, normalized size = 6.07

$$\frac{2\,\cos\left(bx+a\right)^{2}-2\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}\right)\log\left(\cos\left(bx+a\right)^{2}\right)+2\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}\right)\log\left(-\frac{1}{4}\,\cos\left(\right.\right.}{2\left(b\cos\left(bx+a\right)^{4}-b\cos\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cos(b*x + a)^2 - 2*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(cos(b*x + a
)^2) + 2*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) -
 1)/(b*cos(b*x + a)^4 - b*cos(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(a+bx)}{\sin^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/sin(b*x+a)**3,x)
```

```
[Out] Integral(sec(a + b*x)**3/sin(a + b*x)**3, x)
```

Giac [B] time = 1.24046, size = 254, normalized size = 5.91

$$-\frac{\frac{\left(\frac{8(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-\frac{8\left(\frac{4(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+3\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^2}-8\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)+16\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-\right.\right.}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*((8*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/(cos
(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 8*(4*(cos(b*x + a)
 - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 3)
/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^2 - 8*log(abs(-cos(b*x + a) +
1)/abs(cos(b*x + a) + 1)) + 16*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) +
1) - 1)))/b
```

3.155 $\int \csc^3(a+bx)\sec^4(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{5\sec^3(a+bx)}{6b}+\frac{5\sec(a+bx)}{2b}-\frac{5\tanh^{-1}(\cos(a+bx))}{2b}-\frac{\csc^2(a+bx)\sec^3(a+bx)}{2b}$$

```
[Out] (-5*ArcTanh[Cos[a + b*x]])/(2*b) + (5*Sec[a + b*x])/(2*b) + (5*Sec[a + b*x]
^3)/(6*b) - (Csc[a + b*x]^2*Sec[a + b*x]^3)/(2*b)
```

Rubi [A] time = 0.0427421, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 302, 207}

$$\frac{5\sec^3(a+bx)}{6b}+\frac{5\sec(a+bx)}{2b}-\frac{5\tanh^{-1}(\cos(a+bx))}{2b}-\frac{\csc^2(a+bx)\sec^3(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sec[a + b*x]^4,x]
```

```
[Out] (-5*ArcTanh[Cos[a + b*x]])/(2*b) + (5*Sec[a + b*x])/(2*b) + (5*Sec[a + b*x]
^3)/(6*b) - (Csc[a + b*x]^2*Sec[a + b*x]^3)/(2*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{b}$$
$$= -\frac{\csc^2(a+bx)\sec^3(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{2b}$$
$$= -\frac{\csc^2(a+bx)\sec^3(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \sec(a+bx)\right)}{2b}$$
$$= \frac{5\sec(a+bx)}{2b} + \frac{5\sec^3(a+bx)}{6b} - \frac{\csc^2(a+bx)\sec^3(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{s}\right.}{2b}$$
$$= -\frac{5\tanh^{-1}(\cos(a+bx))}{2b} + \frac{5\sec(a+bx)}{2b} + \frac{5\sec^3(a+bx)}{6b} - \frac{\csc^2(a+bx)\sec^3(a+bx)}{2b}$$

Mathematica [B] time = 0.421722, size = 205, normalized size = 3.11

$$\frac{2\csc^8(a+bx)\left(-40\cos(2(a+bx))+13\cos(3(a+bx))-30\cos(4(a+bx))+13\cos(5(a+bx))+15\cos(3(a+bx))\log\left(\text{c}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sec[a + b*x]^4,x]
```

```
[Out] (2*Csc[a + b*x]^8*(22 - 40*Cos[2*(a + b*x)] + 13*Cos[3*(a + b*x)] - 30*Cos[
4*(a + b*x)] + 13*Cos[5*(a + b*x)] + 15*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/
2]] + 15*Cos[5*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 15*Cos[3*(a + b*x)]*Log[S
in[(a + b*x)/2]] - 15*Cos[5*(a + b*x)]*Log[Sin[(a + b*x)/2]] + Cos[a + b*x]
*(-26 - 30*Log[Cos[(a + b*x)/2]] + 30*Log[Sin[(a + b*x)/2]])))/(3*b*(Csc[(a
 + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Maple [A] time = 0.023, size = 78, normalized size = 1.2

$$\frac{1}{3\,b\,(\sin(bx+a))^2\,(\cos(bx+a))^3} - \frac{5}{6\,b\,(\sin(bx+a))^2\cos(bx+a)} + \frac{5}{2\,b\cos(bx+a)} + \frac{5\,\ln(\csc(bx+a)-\cot(bx+a))}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4/sin(b*x+a)^3,x)
```

```
[Out] 1/3/b/sin(b*x+a)^2/cos(b*x+a)^3-5/6/b/sin(b*x+a)^2/cos(b*x+a)+5/2/b/cos(b*x
+a)+5/2/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [A] time = 0.997612, size = 99, normalized size = 1.5

$$\frac{\frac{2\left(15\cos(bx+a)^4-10\cos(bx+a)^2-2\right)}{\cos(bx+a)^5-\cos(bx+a)^3} - 15\log(\cos(bx+a)+1) + 15\log(\cos(bx+a)-1)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(2*(15*cos(b*x + a)^4 - 10*cos(b*x + a)^2 - 2)/(cos(b*x + a)^5 - cos(b
*x + a)^3) - 15*log(cos(b*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b
```

Fricas [A] time = 2.32969, size = 301, normalized size = 4.56

$$\frac{30\cos(bx+a)^4-20\cos(bx+a)^2-15\left(\cos(bx+a)^5-\cos(bx+a)^3\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+15\left(\cos(bx+a)^5-\right.}{12\left(b\cos(bx+a)^5-b\cos(bx+a)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(30*cos(b*x + a)^4 - 20*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - cos(b*x
+ a)^3)*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5 - cos(b*x + a)^3)*
log(-1/2*cos(b*x + a) + 1/2) - 4)/(b*cos(b*x + a)^5 - b*cos(b*x + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^4(a+bx)}{\sin^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4/sin(b*x+a)**3,x)
```

```
[Out] Integral(sec(a + b*x)**4/sin(a + b*x)**3, x)
```

Giac [B] time = 1.17862, size = 220, normalized size = 3.33

$$-\frac{\frac{3\left(\frac{10(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}+\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{16\left(\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{9(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+7\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}-30\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(10*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/
(cos(b*x + a) - 1) + 3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 16*(12*(cos(
b*x + a) - 1)/(cos(b*x + a) + 1) + 9*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1
)^2 + 7)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^3 - 30*log(abs(-cos(b*
x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.156 $\int \csc^3(a+bx)\sec^5(a+bx)\,dx$

Optimal. Leaf size=58

$$\frac{\tan^4(a+bx)}{4b}+\frac{3\tan^2(a+bx)}{2b}-\frac{\cot^2(a+bx)}{2b}+\frac{3\log(\tan(a+bx))}{b}$$

```
[Out] -Cot[a + b*x]^2/(2*b) + (3*Log[Tan[a + b*x]])/b + (3*Tan[a + b*x]^2)/(2*b)
+ Tan[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0431127, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$\frac{\tan^4(a+bx)}{4b}+\frac{3\tan^2(a+bx)}{2b}-\frac{\cot^2(a+bx)}{2b}+\frac{3\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sec[a + b*x]^5,x]
```

```
[Out] -Cot[a + b*x]^2/(2*b) + (3*Log[Tan[a + b*x]])/b + (3*Tan[a + b*x]^2)/(2*b)
+ Tan[a + b*x]^4/(4*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sec^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^3}\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx,x,\tan^2(a+bx)\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)dx,x,\tan^2(a+bx)\right)}{2b}\\ &= -\frac{\cot^2(a+bx)}{2b}+\frac{3\log(\tan(a+bx))}{b}+\frac{3\tan^2(a+bx)}{2b}+\frac{\tan^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.225134, size = 56, normalized size = 0.97

$$-\frac{2\csc^2(a+bx)-\sec^4(a+bx)-4\sec^2(a+bx)-12\log(\sin(a+bx))+12\log(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sec[a + b*x]^5,x]
```

```
[Out] -(2*Csc[a + b*x]^2 + 12*Log[Cos[a + b*x]] - 12*Log[Sin[a + b*x]] - 4*Sec[a
+ b*x]^2 - Sec[a + b*x]^4)/(4*b)
```

Maple [A] time = 0.024, size = 69, normalized size = 1.2

$$\frac{1}{4\,b\left(\sin\left(bx+a\right)\right)^{2}\left(\cos\left(bx+a\right)\right)^{4}}+\frac{3}{4\,b\left(\sin\left(bx+a\right)\right)^{2}\left(\cos\left(bx+a\right)\right)^{2}}-\frac{3}{2\,b\left(\sin\left(bx+a\right)\right)^{2}}+3\,\frac{\ln\left(\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5/sin(b*x+a)^3,x)
```

```
[Out] 1/4/b/sin(b*x+a)^2/cos(b*x+a)^4+3/4/b/sin(b*x+a)^2/cos(b*x+a)^2-3/2/sin(b*x
+a)^2/b+3*ln(tan(b*x+a))/b
```

Maxima [A] time = 1.00627, size = 111, normalized size = 1.91

$$-\frac{\frac{6\,\sin(bx+a)^{4}-9\,\sin(bx+a)^{2}+2}{\sin(bx+a)^{6}-2\,\sin(bx+a)^{4}+\sin(bx+a)^{2}}+6\,\log\left(\sin\left(bx+a\right)^{2}-1\right)-6\,\log\left(\sin\left(bx+a\right)^{2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((6*sin(b*x + a)^4 - 9*sin(b*x + a)^2 + 2)/(sin(b*x + a)^6 - 2*sin(b*x
 + a)^4 + sin(b*x + a)^2) + 6*log(sin(b*x + a)^2 - 1) - 6*log(sin(b*x + a)^
2))/b
```

Fricas [B] time = 2.37828, size = 286, normalized size = 4.93

$$\frac{6\,\cos\left(bx+a\right)^{4}-3\,\cos\left(bx+a\right)^{2}-6\left(\cos\left(bx+a\right)^{6}-\cos\left(bx+a\right)^{4}\right)\log\left(\cos\left(bx+a\right)^{2}\right)+6\left(\cos\left(bx+a\right)^{6}-\cos\left(bx+\right.\right.}{4\left(b\cos\left(bx+a\right)^{6}-b\cos\left(bx+a\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*cos(b*x + a)^4 - 3*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - cos(b*x + a)
^4)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - cos(b*x + a)^4)*log(-1/4*cos(
b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^6 - b*cos(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5/sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23017, size = 313, normalized size = 5.4

$$-\frac{\frac{\left(\frac{12\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-\frac{2\left(\frac{76\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{118\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{76\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{25\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+25\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^4}-12\log\left(\frac{|-\cos(bx}{|\cos(bx}\right.}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*((12*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/(co
s(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 2*(76*(cos(b*x +
a) - 1)/(cos(b*x + a) + 1) + 118*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2
+ 76*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 25*(cos(b*x + a) - 1)^4/(c
os(b*x + a) + 1)^4 + 25)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^4 - 12
*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 24*log(abs(-(cos(b*x +
 a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

3.157 $\int \cos^5(a+bx)\cot^4(a+bx)\,dx$

Optimal. Leaf size=68

$$\frac{\sin^5(a+bx)}{5b}-\frac{4\sin^3(a+bx)}{3b}+\frac{6\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{4\csc(a+bx)}{b}$$

```
[Out] (4*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (6*Sin[a + b*x])/b - (4*Sin[a +
 b*x]^3)/(3*b) + Sin[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0440119, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2590, 270}

$$\frac{\sin^5(a+bx)}{5b}-\frac{4\sin^3(a+bx)}{3b}+\frac{6\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{4\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*Cot[a + b*x]^4,x]
```

```
[Out] (4*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (6*Sin[a + b*x])/b - (4*Sin[a +
 b*x]^3)/(3*b) + Sin[a + b*x]^5/(5*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(a+bx)\cot^4(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{x^4}\,dx, x, -\sin(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(6+\frac{1}{x^4}-\frac{4}{x^2}-4x^2+x^4\right)dx, x, -\sin(a+bx)\right)}{b}\\ &= \frac{4\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{6\sin(a+bx)}{b}-\frac{4\sin^3(a+bx)}{3b}+\frac{\sin^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0376901, size = 68, normalized size = 1.

$$\frac{\sin^5(a+bx)}{5b}-\frac{4\sin^3(a+bx)}{3b}+\frac{6\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{4\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*Cot[a + b*x]^4,x]
```

```
[Out] (4*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (6*Sin[a + b*x])/b - (4*Sin[a +
 b*x]^3)/(3*b) + Sin[a + b*x]^5/(5*b)
```

Maple [A] time = 0.043, size = 90, normalized size = 1.3

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^{10}}{3(\sin(bx+a))^3}+\frac{7(\cos(bx+a))^{10}}{3\sin(bx+a)}+\frac{7\sin(bx+a)}{3}\left(\frac{128}{35}+(\cos(bx+a))^8+\frac{8(\cos(bx+a))^6}{7}+\frac{48(\cos(bx+a)}{35}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^9/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3/sin(b*x+a)^3*cos(b*x+a)^10+7/3/sin(b*x+a)*cos(b*x+a)^10+7/3*(128/
35+cos(b*x+a)^8+8/7*cos(b*x+a)^6+48/35*cos(b*x+a)^4+64/35*cos(b*x+a)^2)*sin
(b*x+a))
```

Maxima [A] time = 0.990353, size = 76, normalized size = 1.12

$$\frac{3\sin(bx+a)^5-20\sin(bx+a)^3+\frac{5\left(12\sin(bx+a)^2-1\right)}{\sin(bx+a)^3}+90\sin(bx+a)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/15*(3*sin(b*x + a)^5 - 20*sin(b*x + a)^3 + 5*(12*sin(b*x + a)^2 - 1)/sin(
b*x + a)^3 + 90*sin(b*x + a))/b
```

Fricas [A] time = 2.27207, size = 176, normalized size = 2.59

$$-\frac{3\cos(bx+a)^8+8\cos(bx+a)^6+48\cos(bx+a)^4-192\cos(bx+a)^2+128}{15\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(3*cos(b*x + a)^8 + 8*cos(b*x + a)^6 + 48*cos(b*x + a)^4 - 192*cos(b*
x + a)^2 + 128)/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [A] time = 24.745, size = 105, normalized size = 1.54

$$\begin{cases}\frac{128\sin^5(a+bx)}{15b}+\frac{64\sin^3(a+bx)\cos^2(a+bx)}{3b}+\frac{16\sin(a+bx)\cos^4(a+bx)}{b}+\frac{8\cos^6(a+bx)}{3b\sin(a+bx)}-\frac{\cos^8(a+bx)}{3b\sin^3(a+bx)} & \text{for } b\neq 0\\ \frac{x\cos^9(a)}{\sin^4(a)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**9/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((128*sin(a + b*x)**5/(15*b) + 64*sin(a + b*x)**3*cos(a + b*x)**2/
(3*b) + 16*sin(a + b*x)*cos(a + b*x)**4/b + 8*cos(a + b*x)**6/(3*b*sin(a +
b*x)) - cos(a + b*x)**8/(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)**9/sin(
a)**4, True))
```

Giac [A] time = 1.19047, size = 76, normalized size = 1.12

$$\frac{3 \sin\left(bx+a\right)^{5} - 20 \sin\left(bx+a\right)^{3} + \frac{5\left(12 \sin(bx+a)^{2}-1\right)}{\sin(bx+a)^{3}} + 90 \sin\left(bx+a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/15*(3*sin(b*x + a)^5 - 20*sin(b*x + a)^3 + 5*(12*sin(b*x + a)^2 - 1)/sin(
b*x + a)^3 + 90*sin(b*x + a))/b
```

3.158 $\int \cos^4(a+bx)\cot^4(a+bx)\,dx$

Optimal. Leaf size=80

$$-\frac{35\cot^3(a+bx)}{24b}+\frac{35\cot(a+bx)}{8b}+\frac{\cos^4(a+bx)\cot^3(a+bx)}{4b}+\frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b}+\frac{35x}{8}$$

```
[Out] (35*x)/8 + (35*Cot[a + b*x])/(8*b) - (35*Cot[a + b*x]^3)/(24*b) + (7*Cos[a
+ b*x]^2*Cot[a + b*x]^3)/(8*b) + (Cos[a + b*x]^4*Cot[a + b*x]^3)/(4*b)
```

Rubi [A] time = 0.0484905, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2591, 288, 302, 203}

$$-\frac{35\cot^3(a+bx)}{24b}+\frac{35\cot(a+bx)}{8b}+\frac{\cos^4(a+bx)\cot^3(a+bx)}{4b}+\frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b}+\frac{35x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Cot[a + b*x]^4,x]
```

```
[Out] (35*x)/8 + (35*Cot[a + b*x])/(8*b) - (35*Cot[a + b*x]^3)/(24*b) + (7*Cos[a
+ b*x]^2*Cot[a + b*x]^3)/(8*b) + (Cos[a + b*x]^4*Cot[a + b*x]^3)/(4*b)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^8}{(1+x^2)^3}\,dx, x, \cot(a+bx)\right)}{b}$$

$$= \frac{\cos^4(a+bx)\cot^3(a+bx)}{4b} - \frac{7\,\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \cot(a+bx)\right)}{4b}$$

$$= \frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot^3(a+bx)}{4b} - \frac{35\,\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \cdots\right.}{8b}$$

$$= \frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot^3(a+bx)}{4b} - \frac{35\,\text{Subst}\left(\int \left(-1+x^2+\cdots\right.\right.}{8\cdots}$$

$$= \frac{35\cot(a+bx)}{8b} - \frac{35\cot^3(a+bx)}{24b} + \frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot\cdots}{4b}$$

$$= \frac{35x}{8} + \frac{35\cot(a+bx)}{8b} - \frac{35\cot^3(a+bx)}{24b} + \frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b} + \frac{\cos^4(a+\cdots}{}$$

Mathematica [A] time = 0.280761, size = 53, normalized size = 0.66

$$\frac{420(a+bx)+72\sin(2(a+bx))+3\sin(4(a+bx))-32\cot(a+bx)\left(\csc^2(a+bx)-10\right)}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Cot[a + b*x]^4,x]
```

```
[Out] (420*(a + b*x) - 32*Cot[a + b*x]*(-10 + Csc[a + b*x]^2) + 72*Sin[2*(a + b*x
)] + 3*Sin[4*(a + b*x)])/(96*b)
```

Maple [A] time = 0.042, size = 94, normalized size = 1.2

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^9}{3(\sin(bx+a))^3}+2\frac{(\cos(bx+a))^9}{\sin(bx+a)}+2\left((\cos(bx+a))^7+7/6\,(\cos(bx+a))^5+\frac{35\,(\cos(bx+a))^3}{24}+\frac{35\cos(bx\cdots}{16}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^8/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3/sin(b*x+a)^3*cos(b*x+a)^9+2/sin(b*x+a)*cos(b*x+a)^9+2*(cos(b*x+a)
^7+7/6*cos(b*x+a)^5+35/24*cos(b*x+a)^3+35/16*cos(b*x+a))*sin(b*x+a)+35/8*b*
x+35/8*a)
```

Maxima [A] time = 1.49196, size = 101, normalized size = 1.26

$$\frac{105\,bx+105\,a+\frac{105\tan(bx+a)^6+175\tan(bx+a)^4+56\tan(bx+a)^2-8}{\tan(bx+a)^7+2\tan(bx+a)^5+\tan(bx+a)^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/24*(105*b*x + 105*a + (105*tan(b*x + a)^6 + 175*tan(b*x + a)^4 + 56*tan(b
*x + a)^2 - 8)/(tan(b*x + a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3))/b
```

Fricas [A] time = 2.32568, size = 230, normalized size = 2.88

$$-\frac{6\cos(bx+a)^7+21\cos(bx+a)^5-140\cos(bx+a)^3-105\left(bx\cos(bx+a)^2-bx\right)\sin(bx+a)+105\cos(bx+a)}{24\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/24*(6*cos(b*x + a)^7 + 21*cos(b*x + a)^5 - 140*cos(b*x + a)^3 - 105*(b*x
*cos(b*x + a)^2 - b*x)*sin(b*x + a) + 105*cos(b*x + a))/((b*cos(b*x + a)^2
- b)*sin(b*x + a))
```

Sympy [A] time = 15.6449, size = 141, normalized size = 1.76

$$\begin{cases}\frac{35x\sin^4(a+bx)}{8}+\frac{35x\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{35x\cos^4(a+bx)}{8}+\frac{35\sin^3(a+bx)\cos(a+bx)}{8b}+\frac{175\sin(a+bx)\cos^3(a+bx)}{24b}+\frac{7\cos^5(a+bx)}{3b\sin(a+bx)}-\frac{\cos^7(a}{3b\sin^3} \\ \frac{x\cos^8(a)}{\sin^4(a)}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**8/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((35*x*sin(a + b*x)**4/8 + 35*x*sin(a + b*x)**2*cos(a + b*x)**2/4
+ 35*x*cos(a + b*x)**4/8 + 35*sin(a + b*x)**3*cos(a + b*x)/(8*b) + 175*sin(
a + b*x)*cos(a + b*x)**3/(24*b) + 7*cos(a + b*x)**5/(3*b*sin(a + b*x)) - co
s(a + b*x)**7/(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)**8/sin(a)**4, Tru
e))
```

Giac [A] time = 1.18604, size = 92, normalized size = 1.15

$$\frac{105\,bx+105\,a+\frac{3\left(11\,\tan(bx+a)^3+13\,\tan(bx+a)\right)}{\left(\tan(bx+a)^2+1\right)^2}+\frac{8\left(9\,\tan(bx+a)^2-1\right)}{\tan(bx+a)^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/24*(105*b*x + 105*a + 3*(11*tan(b*x + a)^3 + 13*tan(b*x + a))/(tan(b*x +
a)^2 + 1)^2 + 8*(9*tan(b*x + a)^2 - 1)/tan(b*x + a)^3)/b
```

3.159 $\int \cos^3(a+bx)\cot^4(a+bx)\,dx$

Optimal. Leaf size=53

$$-\frac{\sin^3(a+bx)}{3b}+\frac{3\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{3\csc(a+bx)}{b}$$

```
[Out] (3*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (3*Sin[a + b*x])/b - Sin[a + b*
x]^3/(3*b)
```

Rubi [A] time = 0.0402648, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2590, 270}

$$-\frac{\sin^3(a+bx)}{3b}+\frac{3\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{3\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Cot[a + b*x]^4,x]
```

```
[Out] (3*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (3*Sin[a + b*x])/b - Sin[a + b*
x]^3/(3*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(a+bx)\cot^4(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^4}\,dx, x, -\sin(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(3+\frac{1}{x^4}-\frac{3}{x^2}-x^2\right)dx, x, -\sin(a+bx)\right)}{b}\\
&= \frac{3\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{3\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] time = 0.0255077, size = 53, normalized size = 1.

$$-\frac{\sin^3(a+bx)}{3b}+\frac{3\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{3\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Cot[a + b*x]^4,x]
```

```
[Out] (3*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (3*Sin[a + b*x])/b - Sin[a + b*
x]^3/(3*b)
```

Maple [A] time = 0.012, size = 80, normalized size = 1.5

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^8}{3(\sin(bx+a))^3}+\frac{5(\cos(bx+a))^8}{3\sin(bx+a)}+\frac{5\sin(bx+a)}{3}\left(\frac{16}{5}+(\cos(bx+a))^6+\frac{6(\cos(bx+a))^4}{5}+\frac{8(\cos(bx+a))^2}{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3/sin(b*x+a)^3*cos(b*x+a)^8+5/3/sin(b*x+a)*cos(b*x+a)^8+5/3*(16/5+c
os(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [A] time = 0.98447, size = 59, normalized size = 1.11

$$-\frac{\sin(bx+a)^3-\frac{9\sin(bx+a)^2-1}{\sin(bx+a)^3}-9\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(sin(b*x + a)^3 - (9*sin(b*x + a)^2 - 1)/sin(b*x + a)^3 - 9*sin(b*x +
a))/b
```

Fricas [A] time = 2.34696, size = 142, normalized size = 2.68

$$-\frac{\cos(bx+a)^6+6\cos(bx+a)^4-24\cos(bx+a)^2+16}{3\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(b*x + a)^6 + 6*cos(b*x + a)^4 - 24*cos(b*x + a)^2 + 16)/((b*cos(b
*x + a)^2 - b)*sin(b*x + a))
```

Sympy [A] time = 8.07703, size = 82, normalized size = 1.55

$$\begin{cases}\frac{16\sin^3(a+bx)}{3b}+\frac{8\sin(a+bx)\cos^2(a+bx)}{b}+\frac{2\cos^4(a+bx)}{b\sin(a+bx)}-\frac{\cos^6(a+bx)}{3b\sin^3(a+bx)} & \text{for } b\neq 0\\ \frac{x\cos^7(a)}{\sin^4(a)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((16*sin(a + b*x)**3/(3*b) + 8*sin(a + b*x)*cos(a + b*x)**2/b + 2*
cos(a + b*x)**4/(b*sin(a + b*x)) - cos(a + b*x)**6/(3*b*sin(a + b*x)**3), N
e(b, 0)), (x*cos(a)**7/sin(a)**4, True))
```

Giac [A] time = 1.16567, size = 55, normalized size = 1.04

$$-\frac{\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)^{3}-\frac{12}{\sin(bx+a)}-12\sin(bx+a)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*((1/sin(b*x + a) + sin(b*x + a))^3 - 12/sin(b*x + a) - 12*sin(b*x + a)
)/b
```

3.160 $\int \cos^2(a+bx)\cot^4(a+bx)\,dx$

Optimal. Leaf size=57

$$-\frac{5\cot^3(a+bx)}{6b}+\frac{5\cot(a+bx)}{2b}+\frac{\cos^2(a+bx)\cot^3(a+bx)}{2b}+\frac{5x}{2}$$

```
[Out] (5*x)/2 + (5*Cot[a + b*x])/(2*b) - (5*Cot[a + b*x]^3)/(6*b) + (Cos[a + b*x]
^2*Cot[a + b*x]^3)/(2*b)
```

Rubi [A] time = 0.0410902, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2591, 288, 302, 203}

$$-\frac{5\cot^3(a+bx)}{6b}+\frac{5\cot(a+bx)}{2b}+\frac{\cos^2(a+bx)\cot^3(a+bx)}{2b}+\frac{5x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cot[a + b*x]^4,x]
```

```
[Out] (5*x)/2 + (5*Cot[a + b*x])/(2*b) - (5*Cot[a + b*x]^3)/(6*b) + (Cos[a + b*x]
^2*Cot[a + b*x]^3)/(2*b)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \cot(a+bx)\right)}{b}$$
$$= \frac{\cos^2(a+bx)\cot^3(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \cot(a+bx)\right)}{2b}$$
$$= \frac{\cos^2(a+bx)\cot^3(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)dx, x, \cot(a+bx)\right)}{2b}$$
$$= \frac{5\cot(a+bx)}{2b} - \frac{5\cot^3(a+bx)}{6b} + \frac{\cos^2(a+bx)\cot^3(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x\right.}{2b}$$
$$= \frac{5x}{2} + \frac{5\cot(a+bx)}{2b} - \frac{5\cot^3(a+bx)}{6b} + \frac{\cos^2(a+bx)\cot^3(a+bx)}{2b}$$

Mathematica [A] time = 0.177181, size = 43, normalized size = 0.75

$$\frac{30(a+bx)+3\sin(2(a+bx))-4\cot(a+bx)\left(\csc^2(a+bx)-7\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cot[a + b*x]^4,x]
```

```
[Out] (30*(a + b*x) - 4*Cot[a + b*x]*(-7 + Csc[a + b*x]^2) + 3*Sin[2*(a + b*x)])/
(12*b)
```

Maple [A] time = 0.012, size = 84, normalized size = 1.5

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^7}{3(\sin(bx+a))^3} + \frac{4(\cos(bx+a))^7}{3\sin(bx+a)} + \frac{4\sin(bx+a)}{3}\left((\cos(bx+a))^5 + \frac{5(\cos(bx+a))^3}{4} + \frac{15\cos(bx+a)}{8}\right) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3*cos(b*x+a)^7/sin(b*x+a)^3+4/3/sin(b*x+a)*cos(b*x+a)^7+4/3*(cos(b*
x+a)^5+5/4*cos(b*x+a)^3+15/8*cos(b*x+a))*sin(b*x+a)+5/2*b*x+5/2*a)
```

Maxima [A] time = 1.48475, size = 74, normalized size = 1.3

$$\frac{15\,bx + 15\,a + \frac{15\,\tan(bx+a)^4 + 10\,\tan(bx+a)^2 - 2}{\tan(bx+a)^5 + \tan(bx+a)^3}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(15*b*x + 15*a + (15*tan(b*x + a)^4 + 10*tan(b*x + a)^2 - 2)/(tan(b*x +
 a)^5 + tan(b*x + a)^3))/b
```

Fricas [A] time = 2.23404, size = 197, normalized size = 3.46

$$-\frac{3\,\cos\left(bx+a\right)^{5}-20\,\cos\left(bx+a\right)^{3}-15\left(bx\cos\left(bx+a\right)^{2}-bx\right)\sin\left(bx+a\right)+15\,\cos\left(bx+a\right)}{6\left(b\cos\left(bx+a\right)^{2}-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*cos(b*x + a)^5 - 20*cos(b*x + a)^3 - 15*(b*x*cos(b*x + a)^2 - b*x)*
sin(b*x + a) + 15*cos(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [A] time = 4.73463, size = 97, normalized size = 1.7

$$\begin{cases} \frac{5x\sin^{2}(a+bx)}{2}+\frac{5x\cos^{2}(a+bx)}{2}+\frac{5\sin(a+bx)\cos(a+bx)}{2b}+\frac{5\cos^{3}(a+bx)}{3b\sin(a+bx)}-\frac{\cos^{5}(a+bx)}{3b\sin^{3}(a+bx)} & \text{for } b\neq 0 \\ \frac{x\cos^{6}(a)}{\sin^{4}(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((5*x*sin(a + b*x)**2/2 + 5*x*cos(a + b*x)**2/2 + 5*sin(a + b*x)*c
os(a + b*x)/(2*b) + 5*cos(a + b*x)**3/(3*b*sin(a + b*x)) - cos(a + b*x)**5/
(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)**6/sin(a)**4, True))
```

Giac [A] time = 1.15451, size = 74, normalized size = 1.3

$$\frac{15\,bx+15\,a+\frac{3\,\tan(bx+a)}{\tan(bx+a)^{2}+1}+\frac{2\left(6\,\tan(bx+a)^{2}-1\right)}{\tan(bx+a)^{3}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/6*(15*b*x + 15*a + 3*tan(b*x + a)/(tan(b*x + a)^2 + 1) + 2*(6*tan(b*x + a
)^2 - 1)/tan(b*x + a)^3)/b
```

3.161 $\int \cos(a+bx)\cot^4(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{2\csc(a+bx)}{b}$$

```
[Out] (2*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + Sin[a + b*x]/b
```

Rubi [A] time = 0.0254015, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 270}

$$\frac{\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[a + b*x]^4,x]
```

```
[Out] (2*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + Sin[a + b*x]/b
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cot^4(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx,x,-\sin(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx,x,-\sin(a+bx)\right)}{b}\\ &= \frac{2\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0187425, size = 37, normalized size = 1.

$$\frac{\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[a + b*x]^4,x]
```

```
[Out] (2*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + Sin[a + b*x]/b
```

Maple [A] time = 0.012, size = 68, normalized size = 1.8

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^6}{3(\sin(bx+a))^3}+\frac{(\cos(bx+a))^6}{\sin(bx+a)}+\left(\frac{8}{3}+(\cos(bx+a))^4+\frac{4(\cos(bx+a))^2}{3}\right)\sin(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3*cos(b*x+a)^6/sin(b*x+a)^3+cos(b*x+a)^6/sin(b*x+a)+(8/3+cos(b*x+a)
^4+4/3*cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [A] time = 0.989545, size = 47, normalized size = 1.27

$$\frac{\frac{6\sin(bx+a)^2-1}{\sin(bx+a)^3}+3\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((6*sin(b*x + a)^2 - 1)/sin(b*x + a)^3 + 3*sin(b*x + a))/b
```

Fricas [A] time = 2.17163, size = 117, normalized size = 3.16

$$-\frac{3\cos(bx+a)^4-12\cos(bx+a)^2+8}{3\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 8)/((b*cos(b*x + a)^2 - b)*sin
(b*x + a))
```

Sympy [A] time = 2.75586, size = 63, normalized size = 1.7

$$\begin{cases} \frac{8\sin(a+bx)}{3b}+\frac{4\cos^2(a+bx)}{3b\sin(a+bx)}-\frac{\cos^4(a+bx)}{3b\sin^3(a+bx)} & \text{for } b\neq 0 \\ \frac{x\cos^5(a)}{\sin^4(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((8*sin(a + b*x)/(3*b) + 4*cos(a + b*x)**2/(3*b*sin(a + b*x)) - co
s(a + b*x)**4/(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)**5/sin(a)**4, Tru
e))
```

Giac [A] time = 1.16345, size = 47, normalized size = 1.27

$$\frac{\frac{6 \sin(bx+a)^2-1}{\sin(bx+a)^3} + 3 \sin(bx + a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/3*((6*sin(b*x + a)^2 - 1)/sin(b*x + a)^3 + 3*sin(b*x + a))/b
```

3.162 $\int \cot^4(a+bx)\,dx$

Optimal. Leaf size=27

$$-\frac{\cot^3(a+bx)}{3b}+\frac{\cot(a+bx)}{b}+x$$

```
[Out] x + Cot[a + b*x]/b - Cot[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0165922, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$-\frac{\cot^3(a+bx)}{3b}+\frac{\cot(a+bx)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^4,x]
```

```
[Out] x + Cot[a + b*x]/b - Cot[a + b*x]^3/(3*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot^4(a+bx)\,dx &= -\frac{\cot^3(a+bx)}{3b}-\int \cot^2(a+bx)\,dx\\ &= \frac{\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}+\int 1\,dx\\ &= x+\frac{\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}\end{aligned}$$

Mathematica [C] time = 0.0091097, size = 33, normalized size = 1.22

$$-\frac{\cot^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^4,x]
```

```
[Out] -(Cot[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[a + b*x]^2])/(3*b)
```

Maple [A] time = 0.014, size = 26, normalized size = 1.

$$\frac{1}{b}\left(-\frac{(\cot(bx+a))^3}{3}+\cot(bx+a)+bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3*cot(b*x+a)^3+cot(b*x+a)+b*x+a)
```

Maxima [A] time = 1.48012, size = 46, normalized size = 1.7

$$\frac{3\,bx+3\,a+\frac{3\,\tan(bx+a)^2-1}{\tan(bx+a)^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*b*x + 3*a + (3*tan(b*x + a)^2 - 1)/tan(b*x + a)^3)/b
```

Fricas [B] time = 2.08996, size = 166, normalized size = 6.15

$$\frac{4\cos(bx+a)^3+3\left(bx\cos(bx+a)^2-bx\right)\sin(bx+a)-3\cos(bx+a)}{3\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(b*x + a)^3 + 3*(b*x*cos(b*x + a)^2 - b*x)*sin(b*x + a) - 3*cos(b
*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [A] time = 1.67347, size = 48, normalized size = 1.78

$$\begin{cases} x+\frac{\cos(a+bx)}{b\sin(a+bx)}-\frac{\cos^3(a+bx)}{3b\sin^3(a+bx)} & \text{for } b\neq 0\\ \frac{x\cos^4(a)}{\sin^4(a)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((x + cos(a + b*x)/(b*sin(a + b*x)) - cos(a + b*x)**3/(3*b*sin(a +
 b*x)**3), Ne(b, 0)), (x*cos(a)**4/sin(a)**4, True))
```

Giac [B] time = 1.16704, size = 84, normalized size = 3.11

$$\frac{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{3}+24\,bx+24\,a+\frac{15\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}-1}{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{3}}-15\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/24*(tan(1/2*b*x + 1/2*a)^3 + 24*b*x + 24*a + (15*tan(1/2*b*x + 1/2*a)^2 -
 1)/tan(1/2*b*x + 1/2*a)^3 - 15*tan(1/2*b*x + 1/2*a))/b
```

3.163 $\int \cot^3(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}$$

```
[Out] Csc[a + b*x]/b - Csc[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0209366, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2606}

$$\frac{\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^3*Csc[a + b*x],x]
```

```
[Out] Csc[a + b*x]/b - Csc[a + b*x]^3/(3*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot^3(a+bx)\csc(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\csc(a+bx)\right)}{b}\\ &= \frac{\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0136318, size = 26, normalized size = 1.

$$\frac{\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^3*Csc[a + b*x],x]
```

```
[Out] Csc[a + b*x]/b - Csc[a + b*x]^3/(3*b)
```

Maple [B] time = 0.013, size = 60, normalized size = 2.3

$$\frac{1}{b}\left(-\frac{(\cos(bx+a))^4}{3\,(\sin(bx+a))^3}+\frac{(\cos(bx+a))^4}{3\sin(bx+a)}+\frac{\left(2+(\cos(bx+a))^2\right)\sin(bx+a)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3*cos(b*x+a)^4/sin(b*x+a)^3+1/3*cos(b*x+a)^4/sin(b*x+a)+1/3*(2+cos(
b*x+a)^2)*sin(b*x+a))
```

Maxima [A] time = 0.993042, size = 34, normalized size = 1.31

$$\frac{3\sin\left(bx+a\right)^{2}-1}{3\,b\sin\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^3)
```

Fricas [A] time = 2.09488, size = 89, normalized size = 3.42

$$\frac{3\cos\left(bx+a\right)^{2}-2}{3\left(b\cos\left(bx+a\right)^{2}-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*cos(b*x + a)^2 - 2)/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [A] time = 1.62873, size = 42, normalized size = 1.62

$$\begin{cases} \frac{2}{3b\sin\left(a+bx\right)} - \frac{\cos^{2}\left(a+bx\right)}{3b\sin^{3}\left(a+bx\right)} & \text{for}\: b \neq 0 \\ \frac{x\cos^{3}\left(a\right)}{\sin^{4}\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((2/(3*b*sin(a + b*x)) - cos(a + b*x)**2/(3*b*sin(a + b*x)**3), Ne
(b, 0)), (x*cos(a)**3/sin(a)**4, True))
```

Giac [A] time = 1.12884, size = 34, normalized size = 1.31

$$\frac{3\sin\left(bx+a\right)^{2}-1}{3\,b\sin\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^4,x, algorithm="giac")

[Out] 1/3*(3*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^3)
```

3.164 $\int \cot^2(a+bx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\cot^3(a+bx)}{3b}$$

```
[Out] -Cot[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.029278, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$-\frac{\cot^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^2*Csc[a + b*x]^2,x]
```

```
[Out] -Cot[a + b*x]^3/(3*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^2(a+bx)\csc^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, -\cot(a+bx)\right)}{b}\\ &= -\frac{\cot^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0095005, size = 15, normalized size = 1.

$$-\frac{\cot^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^2*Csc[a + b*x]^2,x]
```

```
[Out] -Cot[a + b*x]^3/(3*b)
```

Maple [A] time = 0.012, size = 22, normalized size = 1.5

$$-\frac{(\cos(bx+a))^3}{3(\sin(bx+a))^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(b*x+a)^4,x)
```

```
[Out] -1/3*cos(b*x+a)^3/sin(b*x+a)^3/b
```

Maxima [A] time = 0.992593, size = 18, normalized size = 1.2

$$-\frac{1}{3b\tan(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3/(b*tan(b*x + a)^3)
```

Fricas [B] time = 2.1128, size = 78, normalized size = 5.2

$$\frac{\cos(bx+a)^3}{3\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*cos(b*x + a)^3/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [A] time = 2.84079, size = 71, normalized size = 4.73

$$\begin{cases} \frac{\tan^3\left(\frac{a}{2}+\frac{bx}{2}\right)}{24b} - \frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} + \frac{1}{8b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)} - \frac{1}{24b\tan^3\left(\frac{a}{2}+\frac{bx}{2}\right)} & \text{for } b \neq 0 \\ \frac{x\cos^2(a)}{\sin^4(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((tan(a/2 + b*x/2)**3/(24*b) - tan(a/2 + b*x/2)/(8*b) + 1/(8*b*tan
(a/2 + b*x/2)) - 1/(24*b*tan(a/2 + b*x/2)**3), Ne(b, 0)), (x*cos(a)**2/sin(
a)**4, True))
```

Giac [A] time = 1.17765, size = 18, normalized size = 1.2

$$-\frac{1}{3\,b\tan\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3/(b*tan(b*x + a)^3)
```

3.165 $\int \cot(a+bx)\csc^3(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\csc^3(a+bx)}{3b}$$

```
[Out] -Csc[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0184351, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$-\frac{\csc^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]*Csc[a + b*x]^3,x]
```

```
[Out] -Csc[a + b*x]^3/(3*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot(a+bx)\csc^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^2\,dx, x, \csc(a+bx)\right)}{b}\\ &= -\frac{\csc^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0096359, size = 15, normalized size = 1.

$$-\frac{\csc^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]*Csc[a + b*x]^3,x]
```

```
[Out] -Csc[a + b*x]^3/(3*b)
```

Maple [A] time = 0.003, size = 14, normalized size = 0.9

$$-\frac{1}{3\left(\sin\left(bx+a\right)\right)^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+a)^4,x)
```

```
[Out] -1/3/sin(b*x+a)^3/b
```

Maxima [A] time = 0.973422, size = 18, normalized size = 1.2

$$-\frac{1}{3\,b\sin\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3/(b*sin(b*x + a)^3)
```

Fricas [A] time = 2.07392, size = 58, normalized size = 3.87

$$\frac{1}{3\left(b\cos\left(bx+a\right)^{2}-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [A] time = 1.47669, size = 24, normalized size = 1.6

$$\begin{cases} -\frac{1}{3b\sin^{3}\left(a+bx\right)} & \text{for}\ b\neq 0 \\ \frac{x\cos\left(a\right)}{\sin^{4}\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((-1/(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)/sin(a)**4, True))
```

Giac [A] time = 1.11615, size = 18, normalized size = 1.2

$$-\frac{1}{3\,b\sin\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3/(b*sin(b*x + a)^3)
```

3.166 $\int \csc^4(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=38

$$-\frac{\csc^3(a+bx)}{3b}-\frac{\csc(a+bx)}{b}+\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/b - Csc[a + b*x]/b - Csc[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0258361, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2621, 302, 207}

$$-\frac{\csc^3(a+bx)}{3b}-\frac{\csc(a+bx)}{b}+\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*Sec[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b - Csc[a + b*x]/b - Csc[a + b*x]^3/(3*b)
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc^4(a+bx)\sec(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx,x,\csc(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx,x,\csc(a+bx)\right)}{b}\\
&= -\frac{\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}-\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(a+bx)\right)}{b}\\
&= \frac{\tanh^{-1}(\sin(a+bx))}{b}-\frac{\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [C] time = 0.012873, size = 31, normalized size = 0.82

$$-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*Sec[a + b*x],x]
```

```
[Out] -(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[a + b*x]^2])/(3*b)
```

Maple [A] time = 0.021, size = 46, normalized size = 1.2

$$-\frac{1}{3\left(\sin\left(bx+a\right)\right)^{3}b}-\frac{1}{b\sin\left(bx+a\right)}+\frac{\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/sin(b*x+a)^4,x)
```

```
[Out] -1/3/sin(b*x+a)^3/b-1/b/sin(b*x+a)+1/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 0.980149, size = 68, normalized size = 1.79

$$-\frac{\frac{2\left(3\,\sin\left(bx+a\right)^{2}+1\right)}{\sin\left(bx+a\right)^{3}}-3\,\log\left(\sin\left(bx+a\right)+1\right)+3\,\log\left(\sin\left(bx+a\right)-1\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(3*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 3*log(sin(b*x + a) + 1) + 3
*log(sin(b*x + a) - 1))/b
```

Fricas [B] time = 2.24198, size = 252, normalized size = 6.63

$$\frac{3\left(\cos\left(bx+a\right)^{2}-1\right)\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-3\left(\cos\left(bx+a\right)^{2}-1\right)\log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-6\,c}{6\left(b\cos\left(bx+a\right)^{2}-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(cos(b*x + a)^2 - 1)*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*(cos(b*x
 + a)^2 - 1)*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 8)/((
b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx)}{\sin^4(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)**4,x)
```

```
[Out] Integral(sec(a + b*x)/sin(a + b*x)**4, x)
```

Giac [A] time = 1.17546, size = 70, normalized size = 1.84

$$-\frac{\frac{2\left(3\,\sin(bx+a)^2+1\right)}{\sin(bx+a)^3}-3\,\log\left(|\sin(bx+a)+1|\right)+3\,\log\left(|\sin(bx+a)-1|\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*(3*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 3*log(abs(sin(b*x + a) + 1)
) + 3*log(abs(sin(b*x + a) - 1)))/b
```

3.167 $\int \csc^4(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{\tan(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}-\frac{2\cot(a+bx)}{b}$$

```
[Out] (-2*Cot[a + b*x])/b - Cot[a + b*x]^3/(3*b) + Tan[a + b*x]/b
```

Rubi [A] time = 0.0351279, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 270}

$$\frac{\tan(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}-\frac{2\cot(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*Sec[a + b*x]^2,x]
```

```
[Out] (-2*Cot[a + b*x])/b - Cot[a + b*x]^3/(3*b) + Tan[a + b*x]/b
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^4(a+bx)\sec^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4}\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)dx,x,\tan(a+bx)\right)}{b}\\ &= -\frac{2\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}+\frac{\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0349905, size = 45, normalized size = 1.22

$$\frac{\tan(a+bx)}{b}-\frac{5\cot(a+bx)}{3b}-\frac{\cot(a+bx)\csc^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*Sec[a + b*x]^2,x]
```

```
[Out] (-5*Cot[a + b*x])/(3*b) - (Cot[a + b*x]*Csc[a + b*x]^2)/(3*b) + Tan[a + b*x
]/b
```

Maple [A] time = 0.021, size = 50, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{1}{3\cos\left(bx+a\right)\left(\sin\left(bx+a\right)\right)^{3}}+\frac{4}{3\cos\left(bx+a\right)\sin\left(bx+a\right)}-\frac{8\cot\left(bx+a\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3/sin(b*x+a)^3/cos(b*x+a)+4/3/sin(b*x+a)/cos(b*x+a)-8/3*cot(b*x+a))
```

Maxima [A] time = 0.968858, size = 47, normalized size = 1.27

$$-\frac{\frac{6\tan\left(bx+a\right)^{2}+1}{\tan\left(bx+a\right)^{3}}-3\tan\left(bx+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b
```

Fricas [A] time = 2.09653, size = 135, normalized size = 3.65

$$-\frac{8\cos\left(bx+a\right)^{4}-12\cos\left(bx+a\right)^{2}+3}{3\left(b\cos\left(bx+a\right)^{3}-b\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(8*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 3)/((b*cos(b*x + a)^3 - b*cos(
b*x + a))*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{2}\left(a+bx\right)}{\sin^{4}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/sin(b*x+a)**4,x)
```

```
[Out] Integral(sec(a + b*x)**2/sin(a + b*x)**4, x)
```

Giac [A] time = 1.15963, size = 47, normalized size = 1.27

$$-\frac{\frac{6 \tan(bx+a)^2+1}{\tan(bx+a)^3} - 3 \tan(bx+a)}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b
```

3.168 $\int \csc^4(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{5\csc^3(a+bx)}{6b}-\frac{5\csc(a+bx)}{2b}+\frac{5\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{2b}$$

```
[Out] (5*ArcTanh[Sin[a + b*x]])/(2*b) - (5*Csc[a + b*x])/(2*b) - (5*Csc[a + b*x]^
3)/(6*b) + (Csc[a + b*x]^3*Sec[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.042476, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 302, 207}

$$-\frac{5\csc^3(a+bx)}{6b}-\frac{5\csc(a+bx)}{2b}+\frac{5\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*Sec[a + b*x]^3,x]
```

```
[Out] (5*ArcTanh[Sin[a + b*x]])/(2*b) - (5*Csc[a + b*x])/(2*b) - (5*Csc[a + b*x]^
3)/(6*b) + (Csc[a + b*x]^3*Sec[a + b*x]^2)/(2*b)
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \csc^4(a+bx)\sec^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \csc(a+bx)\right)}{b} \\
&= \frac{\csc^3(a+bx)\sec^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b} \\
&= \frac{\csc^3(a+bx)\sec^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(a+bx)\right)}{2b} \\
&= -\frac{5\csc(a+bx)}{2b} - \frac{5\csc^3(a+bx)}{6b} + \frac{\csc^3(a+bx)\sec^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx \right.}{2b} \\
&= \frac{5\tanh^{-1}(\sin(a+bx))}{2b} - \frac{5\csc(a+bx)}{2b} - \frac{5\csc^3(a+bx)}{6b} + \frac{\csc^3(a+bx)\sec^2(a+bx}{2b}
\end{aligned}
$$

Mathematica [C] time = 0.013114, size = 31, normalized size = 0.47

$$
-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\sin^2(a+bx)\right)}{3b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*Sec[a + b*x]^3,x]
```

```
[Out] -(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, Sin[a + b*x]^2])/(3*b)
```

Maple [A] time = 0.026, size = 76, normalized size = 1.2

$$
-\frac{1}{3\,b\left(\sin(bx+a)\right)^3\left(\cos(bx+a)\right)^2} + \frac{5}{6\,b\sin(bx+a)\left(\cos(bx+a)\right)^2} - \frac{5}{2\,b\sin(bx+a)} + \frac{5\,\ln\left(\sec(bx+a)+\tan(bx\right.}{2\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3/sin(b*x+a)^4,x)
```

```
[Out] -1/3/b/sin(b*x+a)^3/cos(b*x+a)^2+5/6/b/sin(b*x+a)/cos(b*x+a)^2-5/2/b/sin(b*
x+a)+5/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 1.00858, size = 99, normalized size = 1.5

$$
-\frac{\frac{2\left(15\,\sin(bx+a)^4-10\,\sin(bx+a)^2-2\right)}{\sin(bx+a)^5-\sin(bx+a)^3} - 15\,\log\left(\sin(bx+a)+1\right) + 15\,\log\left(\sin(bx+a)-1\right)}{12\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/12*(2*(15*sin(b*x + a)^4 - 10*sin(b*x + a)^2 - 2)/(sin(b*x + a)^5 - sin(
b*x + a)^3) - 15*log(sin(b*x + a) + 1) + 15*log(sin(b*x + a) - 1))/b
```

Fricas [B] time = 2.2853, size = 342, normalized size = 5.18

$$-\frac{30\,\cos\left(bx+a\right)^{4}-15\,\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}\right)\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)+15\,\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right.}{12\,\left(b\cos\left(bx+a\right)^{4}-b\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/12*(30*cos(b*x + a)^4 - 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(sin(b*x
 + a) + 1)*sin(b*x + a) + 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-sin(b*x
 + a) + 1)*sin(b*x + a) - 40*cos(b*x + a)^2 + 6)/((b*cos(b*x + a)^4 - b*cos
(b*x + a)^2)*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{3}\left(a+bx\right)}{\sin^{4}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/sin(b*x+a)**4,x)
```

```
[Out] Integral(sec(a + b*x)**3/sin(a + b*x)**4, x)
```

Giac [A] time = 1.20445, size = 97, normalized size = 1.47

$$-\frac{\frac{6\,\sin\left(bx+a\right)}{\sin\left(bx+a\right)^{2}-1}+\frac{4\,\left(6\,\sin\left(bx+a\right)^{2}+1\right)}{\sin\left(bx+a\right)^{3}}-15\,\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+15\,\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/12*(6*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 4*(6*sin(b*x + a)^2 + 1)/sin(b
*x + a)^3 - 15*log(abs(sin(b*x + a) + 1)) + 15*log(abs(sin(b*x + a) - 1)))/
b
```

3.169 $\int \csc^4(a+bx)\sec^4(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{\tan^3(a+bx)}{3b}+\frac{3\tan(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}-\frac{3\cot(a+bx)}{b}$$

```
[Out] (-3*Cot[a + b*x])/b - Cot[a + b*x]^3/(3*b) + (3*Tan[a + b*x])/b + Tan[a + b
*x]^3/(3*b)
```

Rubi [A] time = 0.0383726, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 270}

$$\frac{\tan^3(a+bx)}{3b}+\frac{3\tan(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}-\frac{3\cot(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*Sec[a + b*x]^4,x]
```

```
[Out] (-3*Cot[a + b*x])/b - Cot[a + b*x]^3/(3*b) + (3*Tan[a + b*x])/b + Tan[a + b
*x]^3/(3*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^4(a+bx)\sec^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^4}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^4}+\frac{3}{x^2}+x^2\right)dx, x, \tan(a+bx)\right)}{b}\\ &= -\frac{3\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}+\frac{3\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.018937, size = 43, normalized size = 0.81

$$16\left(-\frac{\cot(2(a+bx))}{3b}-\frac{\cot(2(a+bx))\csc^2(2(a+bx))}{6b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*Sec[a + b*x]^4,x]
```

```
[Out] 16*(-Cot[2*(a + b*x)]/(3*b) - (Cot[2*(a + b*x)]*Csc[2*(a + b*x)]^2)/(6*b))
```

Maple [A] time = 0.026, size = 68, normalized size = 1.3

$$\frac{1}{b}\left(\frac{1}{3\left(\cos\left(bx+a\right)\right)^{3}\left(\sin\left(bx+a\right)\right)^{3}}-\frac{2}{3\cos\left(bx+a\right)\left(\sin\left(bx+a\right)\right)^{3}}+\frac{8}{3\cos\left(bx+a\right)\sin\left(bx+a\right)}-\frac{16\cot\left(bx+a\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(1/3/sin(b*x+a)^3/cos(b*x+a)^3-2/3/sin(b*x+a)^3/cos(b*x+a)+8/3/sin(b*x+
a)/cos(b*x+a)-16/3*cot(b*x+a))
```

Maxima [A] time = 1.01605, size = 59, normalized size = 1.11

$$\frac{\tan\left(bx+a\right)^{3}-\frac{9\tan\left(bx+a\right)^{2}+1}{\tan\left(bx+a\right)^{3}}+9\tan\left(bx+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(b*x + a)^3 - (9*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 + 9*tan(b*x + a
))/b
```

Fricas [A] time = 2.14864, size = 165, normalized size = 3.11

$$-\frac{16\cos\left(bx+a\right)^{6}-24\cos\left(bx+a\right)^{4}+6\cos\left(bx+a\right)^{2}+1}{3\left(b\cos\left(bx+a\right)^{5}-b\cos\left(bx+a\right)^{3}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(16*cos(b*x + a)^6 - 24*cos(b*x + a)^4 + 6*cos(b*x + a)^2 + 1)/((b*cos
(b*x + a)^5 - b*cos(b*x + a)^3)*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{4}\left(a+bx\right)}{\sin^{4}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4/sin(b*x+a)**4,x)
```

```
[Out] Integral(sec(a + b*x)**4/sin(a + b*x)**4, x)
```

Giac [A] time = 1.14615, size = 42, normalized size = 0.79

$$-\frac{8\left(3\tan\left(2bx+2a\right)^2+1\right)}{3b\tan\left(2bx+2a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -8/3*(3*tan(2*b*x + 2*a)^2 + 1)/(b*tan(2*b*x + 2*a)^3)
```

3.170 $\int \csc^4(a+bx)\sec^5(a+bx)\,dx$

Optimal. Leaf size=89

$$-\frac{35\csc^3(a+bx)}{24b}-\frac{35\csc(a+bx)}{8b}+\frac{35\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\csc^3(a+bx)\sec^4(a+bx)}{4b}+\frac{7\csc^3(a+bx)\sec^2(a+bx)}{8b}$$

```
[Out] (35*ArcTanh[Sin[a + b*x]])/(8*b) - (35*Csc[a + b*x])/(8*b) - (35*Csc[a + b*
x]^3)/(24*b) + (7*Csc[a + b*x]^3*Sec[a + b*x]^2)/(8*b) + (Csc[a + b*x]^3*Se
c[a + b*x]^4)/(4*b)
```

Rubi [A] time = 0.0493638, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 302, 207}

$$-\frac{35\csc^3(a+bx)}{24b}-\frac{35\csc(a+bx)}{8b}+\frac{35\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\csc^3(a+bx)\sec^4(a+bx)}{4b}+\frac{7\csc^3(a+bx)\sec^2(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*Sec[a + b*x]^5,x]
```

```
[Out] (35*ArcTanh[Sin[a + b*x]])/(8*b) - (35*Csc[a + b*x])/(8*b) - (35*Csc[a + b*
x]^3)/(24*b) + (7*Csc[a + b*x]^3*Sec[a + b*x]^2)/(8*b) + (Csc[a + b*x]^3*Se
c[a + b*x]^4)/(4*b)
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^3}\,dx, x, \csc(a+bx)\right)}{b}$$

$$= \frac{\csc^3(a+bx)\sec^4(a+bx)}{4b} - \frac{7\,\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{4b}$$

$$= \frac{7\csc^3(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc^3(a+bx)\sec^4(a+bx)}{4b} - \frac{35\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \right.}{8b}$$

$$= \frac{7\csc^3(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc^3(a+bx)\sec^4(a+bx)}{4b} - \frac{35\,\text{Subst}\left(\int \left(1+x^2+ \right.\right.}{8b}$$

$$= -\frac{35\csc(a+bx)}{8b} - \frac{35\csc^3(a+bx)}{24b} + \frac{7\csc^3(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc^3(a+bx)\sec}{4b}$$

$$= \frac{35\tanh^{-1}(\sin(a+bx))}{8b} - \frac{35\csc(a+bx)}{8b} - \frac{35\csc^3(a+bx)}{24b} + \frac{7\csc^3(a+bx)\sec^2(a}{8b}$$

Mathematica [C] time = 0.0139726, size = 31, normalized size = 0.35

$$-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*Sec[a + b*x]^5,x]
```

```
[Out] -(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, Sin[a + b*x]^2])/(3*b)
```

Maple [A] time = 0.026, size = 97, normalized size = 1.1

$$\frac{1}{4\,b\,(\sin(bx+a))^3\,(\cos(bx+a))^4} - \frac{7}{12\,b\,(\sin(bx+a))^3\,(\cos(bx+a))^2} + \frac{35}{24\,b\sin(bx+a)\,(\cos(bx+a))^2} - \frac{35}{8\,b\sin(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5/sin(b*x+a)^4,x)
```

```
[Out] 1/4/b/sin(b*x+a)^3/cos(b*x+a)^4-7/12/b/sin(b*x+a)^3/cos(b*x+a)^2+35/24/b/si
n(b*x+a)/cos(b*x+a)^2-35/8/b/sin(b*x+a)+35/8/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 0.996, size = 123, normalized size = 1.38

$$-\frac{\frac{2\left(105\sin(bx+a)^6-175\sin(bx+a)^4+56\sin(bx+a)^2+8\right)}{\sin(bx+a)^7-2\sin(bx+a)^5+\sin(bx+a)^3} - 105\log(\sin(bx+a)+1) + 105\log(\sin(bx+a)-1)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(105*sin(b*x + a)^6 - 175*sin(b*x + a)^4 + 56*sin(b*x + a)^2 + 8)/
(sin(b*x + a)^7 - 2*sin(b*x + a)^5 + sin(b*x + a)^3) - 105*log(sin(b*x + a)
 + 1) + 105*log(sin(b*x + a) - 1))/b
```

Fricas [A] time = 2.33682, size = 375, normalized size = 4.21

$$-\frac{210\,\cos\left(bx+a\right)^{6}-280\,\cos\left(bx+a\right)^{4}-105\left(\cos\left(bx+a\right)^{6}-\cos\left(bx+a\right)^{4}\right)\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)+105\left(\cos\right.}{48\left(b\cos\left(bx+a\right)^{6}-b\cos\left(bx+a\right)^{4}\right)\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/48*(210*cos(b*x + a)^6 - 280*cos(b*x + a)^4 - 105*(cos(b*x + a)^6 - cos(
b*x + a)^4)*log(sin(b*x + a) + 1)*sin(b*x + a) + 105*(cos(b*x + a)^6 - cos(
b*x + a)^4)*log(-sin(b*x + a) + 1)*sin(b*x + a) + 42*cos(b*x + a)^2 + 12)/(
(b*cos(b*x + a)^6 - b*cos(b*x + a)^4)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5/sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21251, size = 115, normalized size = 1.29

$$-\frac{\frac{6\left(11\,\sin(bx+a)^{3}-13\,\sin(bx+a)\right)}{\left(\sin(bx+a)^{2}-1\right)^{2}}+\frac{16\left(9\,\sin(bx+a)^{2}+1\right)}{\sin(bx+a)^{3}}-105\,\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+105\,\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/48*(6*(11*sin(b*x + a)^3 - 13*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 + 16*
(9*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 105*log(abs(sin(b*x + a) + 1)) + 10
5*log(abs(sin(b*x + a) - 1)))/b
```

3.171 $\int \cos^4(a+bx)\cot^5(a+bx)\,dx$

Optimal. Leaf size=69

$$\frac{\sin^4(a+bx)}{4b}-\frac{2\sin^2(a+bx)}{b}-\frac{\csc^4(a+bx)}{4b}+\frac{2\csc^2(a+bx)}{b}+\frac{6\log(\sin(a+bx))}{b}$$

```
[Out] (2*Csc[a + b*x]^2)/b - Csc[a + b*x]^4/(4*b) + (6*Log[Sin[a + b*x]])/b - (2*
Sin[a + b*x]^2)/b + Sin[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0475192, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2590, 266, 43}

$$\frac{\sin^4(a+bx)}{4b}-\frac{2\sin^2(a+bx)}{b}-\frac{\csc^4(a+bx)}{4b}+\frac{2\csc^2(a+bx)}{b}+\frac{6\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Cot[a + b*x]^5,x]
```

```
[Out] (2*Csc[a + b*x]^2)/b - Csc[a + b*x]^4/(4*b) + (6*Log[Sin[a + b*x]])/b - (2*
Sin[a + b*x]^2)/b + Sin[a + b*x]^4/(4*b)
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^4(a+bx)\cot^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{x^5}\,dx, x, -\sin(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^4}{x^3}\,dx, x, \sin^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(-4+\frac{1}{x^3}-\frac{4}{x^2}+\frac{6}{x}+x\right)dx, x, \sin^2(a+bx)\right)}{2b}\\
&= \frac{2\csc^2(a+bx)}{b}-\frac{\csc^4(a+bx)}{4b}+\frac{6\log(\sin(a+bx))}{b}-\frac{2\sin^2(a+bx)}{b}+\frac{\sin^4(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.10923, size = 55, normalized size = 0.8

$$\frac{\sin^4(a+bx)-8\sin^2(a+bx)-\csc^4(a+bx)+8\csc^2(a+bx)+24\log(\sin(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Cot[a + b*x]^5,x]
```

```
[Out] (8*Csc[a + b*x]^2 - Csc[a + b*x]^4 + 24*Log[Sin[a + b*x]] - 8*Sin[a + b*x]^
2 + Sin[a + b*x]^4)/(4*b)
```

Maple [A] time = 0.012, size = 107, normalized size = 1.6

$$-\frac{(\cos(bx+a))^{10}}{4b(\sin(bx+a))^4}+\frac{3(\cos(bx+a))^{10}}{4b(\sin(bx+a))^2}+\frac{3(\cos(bx+a))^8}{4b}+\frac{(\cos(bx+a))^6}{b}+\frac{3(\cos(bx+a))^4}{2b}+3\frac{(\cos(bx+a))^2}{b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^9/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b/sin(b*x+a)^4*cos(b*x+a)^10+3/4/b/sin(b*x+a)^2*cos(b*x+a)^10+3/4*cos(
b*x+a)^8/b+cos(b*x+a)^6/b+3/2*cos(b*x+a)^4/b+3*cos(b*x+a)^2/b+6*ln(sin(b*x+
a))/b
```

Maxima [A] time = 0.96805, size = 76, normalized size = 1.1

$$\frac{\sin(bx+a)^4-8\sin(bx+a)^2+\frac{8\sin(bx+a)^2-1}{\sin(bx+a)^4}+12\log\left(\sin(bx+a)^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(sin(b*x + a)^4 - 8*sin(b*x + a)^2 + (8*sin(b*x + a)^2 - 1)/sin(b*x + a
)^4 + 12*log(sin(b*x + a)^2))/b
```

Fricas [A] time = 2.42775, size = 274, normalized size = 3.97

$$\frac{8\cos(bx+a)^8+32\cos(bx+a)^6-115\cos(bx+a)^4+38\cos(bx+a)^2+192\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log}{32\left(b\cos(bx+a)^4-2b\cos(bx+a)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/32*(8*cos(b*x + a)^8 + 32*cos(b*x + a)^6 - 115*cos(b*x + a)^4 + 38*cos(b*
x + a)^2 + 192*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*sin(b*x + a)
) + 29)/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [A] time = 59.7554, size = 1664, normalized size = 24.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**9/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-384*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**12/(64*b*tan
(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8
+ 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 1536*log(tan(a/2
+ b*x/2)**2 + 1)*tan(a/2 + b*x/2)**10/(64*b*tan(a/2 + b*x/2)**12 + 256*b*ta
n(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6
+ 64*b*tan(a/2 + b*x/2)**4) - 2304*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b
*x/2)**8/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*ta
n(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) -
 1536*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(64*b*tan(a/2 + b*x/
2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan
(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 384*log(tan(a/2 + b*x/2)**2
+ 1)*tan(a/2 + b*x/2)**4/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2
)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/
2 + b*x/2)**4) + 384*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**12/(64*b*tan(a
/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 +
256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 1536*log(tan(a/2 +
b*x/2))*tan(a/2 + b*x/2)**10/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b
*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*ta
n(a/2 + b*x/2)**4) + 2304*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/(64*b*t
an(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**
8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 1536*log(tan(a/
2 + b*x/2))*tan(a/2 + b*x/2)**6/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2
+ b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b
*tan(a/2 + b*x/2)**4) + 384*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(64*b
*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)
**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - tan(a/2 + b*x
/2)**16/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan
(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) +
24*tan(a/2 + b*x/2)**14/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)
**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2
 + b*x/2)**4) - 744*tan(a/2 + b*x/2)**10/(64*b*tan(a/2 + b*x/2)**12 + 256*b
*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)*
*6 + 64*b*tan(a/2 + b*x/2)**4) - 1182*tan(a/2 + b*x/2)**8/(64*b*tan(a/2 + b
*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*
tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 744*tan(a/2 + b*x/2)**6/(
64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*
x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 24*tan(a/
2 + b*x/2)**2/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384
*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)*
*4) - 1/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan
(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4), N
e(b, 0)), (x*cos(a)**9/sin(a)**5, True))
```

Giac [B] time = 1.21353, size = 374, normalized size = 5.42

$$-\frac{\frac{\left(\frac{28\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{288\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}+\frac{28\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{32\left(\frac{84\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{126\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{84\,(\cos(bx+a)\cdots}{(\cos(bx+a)+\cdots}\cdots\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^4}\cdots}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/64*((28*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 288*(cos(b*x + a) - 1)^2
/(cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 + 28*(
cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) +
 1)^2 + 32*(84*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 126*(cos(b*x + a) -
1)^2/(cos(b*x + a) + 1)^2 + 84*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 -
25*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 25)/((cos(b*x + a) - 1)/(cos
(b*x + a) + 1) - 1)^4 - 192*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1
)) + 384*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

3.172 $\int \cos^3(a+bx)\cot^5(a+bx)\,dx$

Optimal. Leaf size=89

$$\frac{35\cos^3(a+bx)}{24b}+\frac{35\cos(a+bx)}{8b}-\frac{\cos^3(a+bx)\cot^4(a+bx)}{4b}+\frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b}-\frac{35\tanh^{-1}(\cos(a+bx}{8b}$$

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(8*b) + (35*Cos[a + b*x])/(8*b) + (35*Cos[a + b
*x]^3)/(24*b) + (7*Cos[a + b*x]^3*Cot[a + b*x]^2)/(8*b) - (Cos[a + b*x]^3*C
ot[a + b*x]^4)/(4*b)
```

Rubi [A] time = 0.0501559, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2592, 288, 302, 206}

$$\frac{35\cos^3(a+bx)}{24b}+\frac{35\cos(a+bx)}{8b}-\frac{\cos^3(a+bx)\cot^4(a+bx)}{4b}+\frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b}-\frac{35\tanh^{-1}(\cos(a+bx}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Cot[a + b*x]^5,x]
```

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(8*b) + (35*Cos[a + b*x])/(8*b) + (35*Cos[a + b
*x]^3)/(24*b) + (7*Cos[a + b*x]^3*Cot[a + b*x]^2)/(8*b) - (Cos[a + b*x]^3*C
ot[a + b*x]^4)/(4*b)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^8}{(1-x^2)^3}\,dx, x, \cos(a+bx)\right)}{b}$$

$$= -\frac{\cos^3(a+bx)\cot^4(a+bx)}{4b} + \frac{7\,\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cos(a+bx)\right)}{4b}$$

$$= \frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos^3(a+bx)\cot^4(a+bx)}{4b} - \frac{35\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos\right.}{8b}$$

$$= \frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos^3(a+bx)\cot^4(a+bx)}{4b} - \frac{35\,\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x}\right.\right.}{8b}$$

$$= \frac{35\cos(a+bx)}{8b} + \frac{35\cos^3(a+bx)}{24b} + \frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos^3(a+bx)\cot^4(a}{4b}$$

$$= -\frac{35\tanh^{-1}(\cos(a+bx))}{8b} + \frac{35\cos(a+bx)}{8b} + \frac{35\cos^3(a+bx)}{24b} + \frac{7\cos^3(a+bx)\cot^2(a}{8b}$$

Mathematica [A] time = 0.0406061, size = 141, normalized size = 1.58

$$\frac{13\cos(a+bx)}{4b} + \frac{\cos(3(a+bx))}{12b} - \frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b} + \frac{13\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b} - \frac{13\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b} + -\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Cot[a + b*x]^5,x]
```

```
[Out] (13*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b) + (13*Csc[(a + b*x)/2]^2)
/(32*b) - Csc[(a + b*x)/2]^4/(64*b) - (35*Log[Cos[(a + b*x)/2]])/(8*b) + (3
5*Log[Sin[(a + b*x)/2]])/(8*b) - (13*Sec[(a + b*x)/2]^2)/(32*b) + Sec[(a +
b*x)/2]^4/(64*b)
```

Maple [A] time = 0.011, size = 115, normalized size = 1.3

$$-\frac{(\cos(bx+a))^9}{4b(\sin(bx+a))^4} + \frac{5(\cos(bx+a))^9}{8b(\sin(bx+a))^2} + \frac{5(\cos(bx+a))^7}{8b} + \frac{7(\cos(bx+a))^5}{8b} + \frac{35(\cos(bx+a))^3}{24b} + \frac{35\cos(bx+a)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^8/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b*cos(b*x+a)^9/sin(b*x+a)^4+5/8/b/sin(b*x+a)^2*cos(b*x+a)^9+5/8*cos(b*
x+a)^7/b+7/8*cos(b*x+a)^5/b+35/24*cos(b*x+a)^3/b+35/8*cos(b*x+a)/b+35/8/b*l
n(csc(b*x+a)-cot(b*x+a))
```

Maxima [A] time = 0.967444, size = 120, normalized size = 1.35

$$\frac{16\cos(bx+a)^3 - \frac{6\left(13\cos(bx+a)^3-11\cos(bx+a)\right)}{\cos(bx+a)^4-2\cos(bx+a)^2+1} + 144\cos(bx+a) - 105\log(\cos(bx+a)+1) + 105\log(\cos(bx+a)-1)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(16*cos(b*x + a)^3 - 6*(13*cos(b*x + a)^3 - 11*cos(b*x + a))/(cos(b*x
+ a)^4 - 2*cos(b*x + a)^2 + 1) + 144*cos(b*x + a) - 105*log(cos(b*x + a) +
1) + 105*log(cos(b*x + a) - 1))/b
```

Fricas [A] time = 2.44378, size = 378, normalized size = 4.25

$$\frac{16\,\cos(bx+a)^7+112\,\cos(bx+a)^5-350\,\cos(bx+a)^3-105\left(\cos(bx+a)^4-2\,\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\,\cos(bx+a\ldots\right.}{48\left(b\cos(bx+a)^4-2\,b\cos(bx\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(16*cos(b*x + a)^7 + 112*cos(b*x + a)^5 - 350*cos(b*x + a)^3 - 105*(co
s(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*cos(b*x + a) + 1/2) + 105*(cos
(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/2*cos(b*x + a) + 1/2) + 210*cos(
b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [A] time = 33.4691, size = 869, normalized size = 9.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**8/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((840*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**10/(192*b*tan(a/2 +
b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)**6 + 192*b*
tan(a/2 + b*x/2)**4) + 2520*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/(192*
b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)
**6 + 192*b*tan(a/2 + b*x/2)**4) + 2520*log(tan(a/2 + b*x/2))*tan(a/2 + b*x
/2)**6/(192*b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(
a/2 + b*x/2)**6 + 192*b*tan(a/2 + b*x/2)**4) + 840*log(tan(a/2 + b*x/2))*ta
n(a/2 + b*x/2)**4/(192*b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 +
 576*b*tan(a/2 + b*x/2)**6 + 192*b*tan(a/2 + b*x/2)**4) + 3*tan(a/2 + b*x/2
)**14/(192*b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a
/2 + b*x/2)**6 + 192*b*tan(a/2 + b*x/2)**4) - 63*tan(a/2 + b*x/2)**12/(192*
b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)
**6 + 192*b*tan(a/2 + b*x/2)**4) + 2016*tan(a/2 + b*x/2)**8/(192*b*tan(a/2
+ b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)**6 + 192*
b*tan(a/2 + b*x/2)**4) + 3066*tan(a/2 + b*x/2)**6/(192*b*tan(a/2 + b*x/2)**
10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)**6 + 192*b*tan(a/2
+ b*x/2)**4) + 1694*tan(a/2 + b*x/2)**4/(192*b*tan(a/2 + b*x/2)**10 + 576*b
*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)**6 + 192*b*tan(a/2 + b*x/2)**
4) + 63*tan(a/2 + b*x/2)**2/(192*b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b
*x/2)**8 + 576*b*tan(a/2 + b*x/2)**6 + 192*b*tan(a/2 + b*x/2)**4) - 3/(192*
b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)
**6 + 192*b*tan(a/2 + b*x/2)**4), Ne(b, 0)), (x*cos(a)**8/sin(a)**5, True))
```

Giac [B] time = 1.20934, size = 282, normalized size = 3.17

$$-\frac{\frac{3\left(\frac{24\left(\cos(bx+a)-1\right)}{\cos(bx+a)+1}+\frac{210\left(\cos(bx+a)-1\right)^2}{\left(\cos(bx+a)+1\right)^2}+1\right)\left(\cos(bx+a)+1\right)^2}{\left(\cos(bx+a)-1\right)^2}-\frac{72\left(\cos(bx+a)-1\right)}{\cos(bx+a)+1}-\frac{3\left(\cos(bx+a)-1\right)^2}{\left(\cos(bx+a)+1\right)^2}-\frac{256\left(\frac{9\left(\cos(bx+a)-1\right)}{\cos(bx+a)+1}-\frac{6\left(\cos(bx+a)-1\right)^2}{\left(\cos(bx+a)+1\right)^2}-5\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^3}-420\log}{192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/192*(3*(24*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 210*(cos(b*x + a) - 1
)^2/(cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 7
2*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(cos(b*x +
 a) + 1)^2 - 256*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 6*(cos(b*x + a)
 - 1)^2/(cos(b*x + a) + 1)^2 - 5)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) -
1)^3 - 420*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.173 $\int \cos^2(a+bx)\cot^5(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{\sin^2(a+bx)}{2b}-\frac{\csc^4(a+bx)}{4b}+\frac{3\csc^2(a+bx)}{2b}+\frac{3\log(\sin(a+bx))}{b}$$

```
[Out] (3*Csc[a + b*x]^2)/(2*b) - Csc[a + b*x]^4/(4*b) + (3*Log[Sin[a + b*x]])/b -
 Sin[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0426495, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2590, 266, 43}

$$-\frac{\sin^2(a+bx)}{2b}-\frac{\csc^4(a+bx)}{4b}+\frac{3\csc^2(a+bx)}{2b}+\frac{3\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cot[a + b*x]^5,x]
```


```
[Out] (3*Csc[a + b*x]^2)/(2*b) - Csc[a + b*x]^4/(4*b) + (3*Log[Sin[a + b*x]])/b -
 Sin[a + b*x]^2/(2*b)
```


Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^2(a+bx)\cot^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^5}\,dx,x,-\sin(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^3}{x^3}\,dx,x,\sin^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^3}-\frac{3}{x^2}+\frac{3}{x}\right)dx,x,\sin^2(a+bx)\right)}{2b}\\
&= \frac{3\csc^2(a+bx)}{2b}-\frac{\csc^4(a+bx)}{4b}+\frac{3\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.154481, size = 47, normalized size = 0.81

$$\frac{-2\sin^2(a+bx)-\csc^4(a+bx)+6\csc^2(a+bx)+12\log(\sin(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cot[a + b*x]^5,x]
```

```
[Out] (6*Csc[a + b*x]^2 - Csc[a + b*x]^4 + 12*Log[Sin[a + b*x]] - 2*Sin[a + b*x]^
2)/(4*b)
```

Maple [A] time = 0.013, size = 95, normalized size = 1.6

$$-\frac{(\cos(bx+a))^8}{4\,b\,(\sin(bx+a))^4}+\frac{(\cos(bx+a))^8}{2\,b\,(\sin(bx+a))^2}+\frac{(\cos(bx+a))^6}{2\,b}+\frac{3\,(\cos(bx+a))^4}{4\,b}+\frac{3\,(\cos(bx+a))^2}{2\,b}+3\,\frac{\ln(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b*cos(b*x+a)^8/sin(b*x+a)^4+1/2/b/sin(b*x+a)^2*cos(b*x+a)^8+1/2*cos(b*
x+a)^6/b+3/4*cos(b*x+a)^4/b+3/2*cos(b*x+a)^2/b+3*ln(sin(b*x+a))/b
```

Maxima [A] time = 0.961055, size = 66, normalized size = 1.14

$$-\frac{2\,\sin(bx+a)^2-\frac{6\,\sin(bx+a)^2-1}{\sin(bx+a)^4}-6\,\log\left(\sin(bx+a)^2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sin(b*x + a)^2 - (6*sin(b*x + a)^2 - 1)/sin(b*x + a)^4 - 6*log(sin(
b*x + a)^2))/b
```

Fricas [A] time = 2.24891, size = 239, normalized size = 4.12

$$\frac{2\,\cos(bx+a)^6-5\,\cos(bx+a)^4-2\,\cos(bx+a)^2+12\left(\cos(bx+a)^4-2\,\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\,\sin(bx+a)\right)+4}{4\left(b\cos(bx+a)^4-2\,b\cos(bx+a)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*cos(b*x + a)^6 - 5*cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 12*(cos(b*x +
 a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*sin(b*x + a)) + 4)/(b*cos(b*x + a)^4
- 2*b*cos(b*x + a)^2 + b)
```

Sympy [A] time = 18.9652, size = 733, normalized size = 12.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-192*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**8/(64*b*tan(
a/2 + b*x/2)**8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 3
84*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(64*b*tan(a/2 + b*x/2)*
*8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 192*log(tan(a/
2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(64*b*tan(a/2 + b*x/2)**8 + 128*b*ta
n(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 192*log(tan(a/2 + b*x/2))*t
an(a/2 + b*x/2)**8/(64*b*tan(a/2 + b*x/2)**8 + 128*b*tan(a/2 + b*x/2)**6 +
64*b*tan(a/2 + b*x/2)**4) + 384*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(
64*b*tan(a/2 + b*x/2)**8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2
)**4) + 192*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(64*b*tan(a/2 + b*x/2
)**8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - tan(a/2 + b*
x/2)**12/(64*b*tan(a/2 + b*x/2)**8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a
/2 + b*x/2)**4) + 18*tan(a/2 + b*x/2)**10/(64*b*tan(a/2 + b*x/2)**8 + 128*b
*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 166*tan(a/2 + b*x/2)**6/
(64*b*tan(a/2 + b*x/2)**8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/
2)**4) + 18*tan(a/2 + b*x/2)**2/(64*b*tan(a/2 + b*x/2)**8 + 128*b*tan(a/2 +
 b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 1/(64*b*tan(a/2 + b*x/2)**8 + 128*
b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4), Ne(b, 0)), (x*cos(a)**7/
sin(a)**5, True))
```

Giac [B] time = 1.20473, size = 313, normalized size = 5.4

$$-\frac{\frac{20(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{\frac{18(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{111(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{36(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} - \frac{72(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} + 1}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1} - \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}\right)^2} - 96\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) + 19}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/64*(20*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos
(b*x + a) + 1)^2 + (18*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 111*(cos(b*x
 + a) - 1)^2/(cos(b*x + a) + 1)^2 + 36*(cos(b*x + a) - 1)^3/(cos(b*x + a) +
 1)^3 - 72*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 1)/((cos(b*x + a) -
1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2)^2 - 96*l
og(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 192*log(abs(-(cos(b*x +
a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

3.174 $\int \cos(a+bx)\cot^5(a+bx)\,dx$

Optimal. Leaf size=70

$$\frac{15\cos(a+bx)}{8b}-\frac{\cos(a+bx)\cot^4(a+bx)}{4b}+\frac{5\cos(a+bx)\cot^2(a+bx)}{8b}-\frac{15\tanh^{-1}(\cos(a+bx))}{8b}$$

```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(8*b) + (15*Cos[a + b*x])/(8*b) + (5*Cos[a + b*
x]*Cot[a + b*x]^2)/(8*b) - (Cos[a + b*x]*Cot[a + b*x]^4)/(4*b)
```

Rubi [A] time = 0.0361286, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2592, 288, 321, 206}

$$\frac{15\cos(a+bx)}{8b}-\frac{\cos(a+bx)\cot^4(a+bx)}{4b}+\frac{5\cos(a+bx)\cot^2(a+bx)}{8b}-\frac{15\tanh^{-1}(\cos(a+bx))}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[a + b*x]^5,x]
```


```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(8*b) + (15*Cos[a + b*x])/(8*b) + (5*Cos[a + b*
x]*Cot[a + b*x]^2)/(8*b) - (Cos[a + b*x]*Cot[a + b*x]^4)/(4*b)
```


Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos(a+bx)\cot^5(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{\cos(a+bx)\cot^4(a+bx)}{4b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cos(a+bx)\right)}{4b}$$
$$= \frac{5\cos(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos(a+bx)\cot^4(a+bx)}{4b} - \frac{15\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos\right.}{8b}$$
$$= \frac{15\cos(a+bx)}{8b} + \frac{5\cos(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos(a+bx)\cot^4(a+bx)}{4b} - \frac{15\,\text{Subst}}{}$$
$$= -\frac{15\tanh^{-1}(\cos(a+bx))}{8b} + \frac{15\cos(a+bx)}{8b} + \frac{5\cos(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos(a+b}{}$$

Mathematica [A] time = 0.0306889, size = 123, normalized size = 1.76

$$\frac{\cos(a+bx)}{b} - \frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b} + \frac{9\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b} - \frac{9\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{15\log\left(\sin\left(\frac{1}{2}(a+bx\right.\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[a + b*x]^5,x]
```

```
[Out] Cos[a + b*x]/b + (9*Csc[(a + b*x)/2]^2)/(32*b) - Csc[(a + b*x)/2]^4/(64*b)
- (15*Log[Cos[(a + b*x)/2]])/(8*b) + (15*Log[Sin[(a + b*x)/2]])/(8*b) - (9*
Sec[(a + b*x)/2]^2)/(32*b) + Sec[(a + b*x)/2]^4/(64*b)
```

Maple [A] time = 0.012, size = 102, normalized size = 1.5

$$-\frac{(\cos(bx+a))^7}{4\,b\,(\sin(bx+a))^4} + \frac{3\,(\cos(bx+a))^7}{8\,b\,(\sin(bx+a))^2} + \frac{3\,(\cos(bx+a))^5}{8\,b} + \frac{5\,(\cos(bx+a))^3}{8\,b} + \frac{15\,\cos(bx+a)}{8\,b} + \frac{15\,\ln(\csc(bx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b*cos(b*x+a)^7/sin(b*x+a)^4+3/8/b*cos(b*x+a)^7/sin(b*x+a)^2+3/8*cos(b*
x+a)^5/b+5/8*cos(b*x+a)^3/b+15/8*cos(b*x+a)/b+15/8/b*ln(csc(b*x+a)-cot(b*x+
a))
```

Maxima [A] time = 0.977546, size = 107, normalized size = 1.53

$$-\frac{\frac{2\left(9\cos(bx+a)^3-7\cos(bx+a)\right)}{\cos(bx+a)^4-2\cos(bx+a)^2+1} - 16\cos(bx+a) + 15\log(\cos(bx+a)+1) - 15\log(\cos(bx+a)-1)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(9*cos(b*x + a)^3 - 7*cos(b*x + a))/(cos(b*x + a)^4 - 2*cos(b*x +
a)^2 + 1) - 16*cos(b*x + a) + 15*log(cos(b*x + a) + 1) - 15*log(cos(b*x + a
) - 1))/b
```

Fricas [A] time = 2.30116, size = 344, normalized size = 4.91

$$\frac{16\cos(bx+a)^5 - 50\cos(bx+a)^3 - 15\left(\cos(bx+a)^4 - 2\cos(bx+a)^2 + 1\right)\log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + 15\left(\cos(bx+a)^4\right.}{16\left(b\cos(bx+a)^4 - 2\,b\cos(bx+a)^2 + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(16*cos(b*x + a)^5 - 50*cos(b*x + a)^3 - 15*(cos(b*x + a)^4 - 2*cos(b*
x + a)^2 + 1)*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^4 - 2*cos(b*x
+ a)^2 + 1)*log(-1/2*cos(b*x + a) + 1/2) + 30*cos(b*x + a))/(b*cos(b*x + a)
^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [A] time = 9.93402, size = 330, normalized size = 4.71

$$\begin{cases} \frac{120\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^{6}\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b\tan^{6}\left(\frac{a}{2}+\frac{bx}{2}\right)+64b\tan^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)} + \frac{120\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b\tan^{6}\left(\frac{a}{2}+\frac{bx}{2}\right)+64b\tan^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)} + \frac{\tan^{10}\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b\tan^{6}\left(\frac{a}{2}+\frac{bx}{2}\right)+64b\tan^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)} - \frac{15\tan^{8}\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b\tan^{6}\left(\frac{a}{2}+\frac{bx}{2}\right)+64b\tan^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)} \\ \frac{x\cos^{6}(a)}{\sin^{5}(a)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((120*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(64*b*tan(a/2 + b*
x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 120*log(tan(a/2 + b*x/2))*tan(a/2 + b
*x/2)**4/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + tan(a/2 +
b*x/2)**10/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 15*tan(a
/2 + b*x/2)**8/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 160*
tan(a/2 + b*x/2)**4/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) +
 15*tan(a/2 + b*x/2)**2/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**
4) - 1/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4), Ne(b, 0)), (x
*cos(a)**6/sin(a)**5, True))
```

Giac [B] time = 1.19111, size = 221, normalized size = 3.16

$$-\frac{\frac{\left(\frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{90(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + 1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2} - \frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{128}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1} - 60\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/64*((16*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 90*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 16*(c
```

```
os(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(b*x + a) +
1)^2 + 128/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - 60*log(abs(-cos(b*
x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.175 $\int \cot^5(a+bx)\,dx$

Optimal. Leaf size=42

$$-\frac{\cot^4(a+bx)}{4b}+\frac{\cot^2(a+bx)}{2b}+\frac{\log(\sin(a+bx))}{b}$$

```
[Out] Cot[a + b*x]^2/(2*b) - Cot[a + b*x]^4/(4*b) + Log[Sin[a + b*x]]/b
```

Rubi [A] time = 0.0210047, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$-\frac{\cot^4(a+bx)}{4b}+\frac{\cot^2(a+bx)}{2b}+\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^5,x]
```

```
[Out] Cot[a + b*x]^2/(2*b) - Cot[a + b*x]^4/(4*b) + Log[Sin[a + b*x]]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^5(a+bx)\,dx &= -\frac{\cot^4(a+bx)}{4b}-\int \cot^3(a+bx)\,dx\\ &= \frac{\cot^2(a+bx)}{2b}-\frac{\cot^4(a+bx)}{4b}+\int \cot(a+bx)\,dx\\ &= \frac{\cot^2(a+bx)}{2b}-\frac{\cot^4(a+bx)}{4b}+\frac{\log(\sin(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.105084, size = 46, normalized size = 1.1

$$\frac{-\cot^4(a+bx)+2\cot^2(a+bx)+4\log(\tan(a+bx))+4\log(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^5,x]
```

```
[Out] (2*Cot[a + b*x]^2 - Cot[a + b*x]^4 + 4*Log[Cos[a + b*x]] + 4*Log[Tan[a + b*
x]])/(4*b)
```

Maple [A] time = 0.013, size = 39, normalized size = 0.9

$$\frac{(\cot(bx+a))^2}{2b}-\frac{(\cot(bx+a))^4}{4b}+\frac{\ln(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5/sin(b*x+a)^5,x)
```

```
[Out] 1/2*cot(b*x+a)^2/b-1/4*cot(b*x+a)^4/b+ln(sin(b*x+a))/b
```

Maxima [A] time = 0.9705, size = 51, normalized size = 1.21

$$\frac{\frac{4\sin(bx+a)^2-1}{\sin(bx+a)^4}+2\log\left(\sin(bx+a)^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/4*((4*sin(b*x + a)^2 - 1)/sin(b*x + a)^4 + 2*log(sin(b*x + a)^2))/b
```

Fricas [A] time = 2.34446, size = 188, normalized size = 4.48

$$-\frac{4\cos(bx+a)^2-4\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\sin(bx+a)\right)-3}{4\left(b\cos(bx+a)^4-2b\cos(bx+a)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*cos(b*x + a)^2 - 4*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*
sin(b*x + a)) - 3)/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [A] time = 2.7029, size = 61, normalized size = 1.45

$$\begin{cases}\frac{\log(\sin(a+bx))}{b}+\frac{\cos^2(a+bx)}{2b\sin^2(a+bx)}-\frac{\cos^4(a+bx)}{4b\sin^4(a+bx)} & \text{for } b\neq 0\\ \frac{x\cos^5(a)}{\sin^5(a)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((log(sin(a + b*x))/b + cos(a + b*x)**2/(2*b*sin(a + b*x)**2) - co
s(a + b*x)**4/(4*b*sin(a + b*x)**4), Ne(b, 0)), (x*cos(a)**5/sin(a)**5, Tru
e))
```

Giac [B] time = 1.21436, size = 221, normalized size = 5.26

$$-\frac{\frac{\left(\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{48(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}+\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-32\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)+64\log\left(\left|-\frac{\cos(bx+a)-}{\cos(bx+a)+}\right.\right.}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/64*((12*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 48*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 + 12*(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) +
1)^2 - 32*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 64*log(abs(-(
cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

3.176 $\int \cot^4(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=55

$$-\frac{3\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\cot^3(a+bx)\csc(a+bx)}{4b}+\frac{3\cot(a+bx)\csc(a+bx)}{8b}$$

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(8*b) + (3*Cot[a + b*x]*Csc[a + b*x])/(8*b) - (C
ot[a + b*x]^3*Csc[a + b*x])/(4*b)
```

Rubi [A] time = 0.0421048, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$-\frac{3\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\cot^3(a+bx)\csc(a+bx)}{4b}+\frac{3\cot(a+bx)\csc(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^4*Csc[a + b*x],x]
```

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(8*b) + (3*Cot[a + b*x]*Csc[a + b*x])/(8*b) - (C
ot[a + b*x]^3*Csc[a + b*x])/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^4(a+bx)\csc(a+bx)\,dx &= -\frac{\cot^3(a+bx)\csc(a+bx)}{4b}-\frac{3}{4}\int \cot^2(a+bx)\csc(a+bx)\,dx\\ &= \frac{3\cot(a+bx)\csc(a+bx)}{8b}-\frac{\cot^3(a+bx)\csc(a+bx)}{4b}+\frac{3}{8}\int \csc(a+bx)\,dx\\ &= -\frac{3\tanh^{-1}(\cos(a+bx))}{8b}+\frac{3\cot(a+bx)\csc(a+bx)}{8b}-\frac{\cot^3(a+bx)\csc(a+bx)}{4b}\end{aligned}$$

Mathematica [B] time = 0.031234, size = 113, normalized size = 2.05

$$-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{5\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{5\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{3\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{8b}-\frac{3\log\left(\mathrm{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^4*Csc[a + b*x],x]
```

```
[Out] (5*Csc[(a + b*x)/2]^2)/(32*b) - Csc[(a + b*x)/2]^4/(64*b) - (3*Log[Cos[(a +
 b*x)/2]])/(8*b) + (3*Log[Sin[(a + b*x)/2]])/(8*b) - (5*Sec[(a + b*x)/2]^2)
/(32*b) + Sec[(a + b*x)/2]^4/(64*b)
```

Maple [A] time = 0.012, size = 89, normalized size = 1.6

$$-\frac{(\cos(bx+a))^5}{4b(\sin(bx+a))^4}+\frac{(\cos(bx+a))^5}{8b(\sin(bx+a))^2}+\frac{(\cos(bx+a))^3}{8b}+\frac{3\cos(bx+a)}{8b}+\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b*cos(b*x+a)^5/sin(b*x+a)^4+1/8/b*cos(b*x+a)^5/sin(b*x+a)^2+1/8*cos(b*
x+a)^3/b+3/8*cos(b*x+a)/b+3/8/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [A] time = 0.981819, size = 96, normalized size = 1.75

$$-\frac{\frac{2\left(5\cos(bx+a)^3-3\cos(bx+a)\right)}{\cos(bx+a)^4-2\cos(bx+a)^2+1}+3\log(\cos(bx+a)+1)-3\log(\cos(bx+a)-1)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(5*cos(b*x + a)^3 - 3*cos(b*x + a))/(cos(b*x + a)^4 - 2*cos(b*x +
a)^2 + 1) + 3*log(cos(b*x + a) + 1) - 3*log(cos(b*x + a) - 1))/b
```

Fricas [B] time = 2.14132, size = 315, normalized size = 5.73

$$-\frac{10\cos(bx+a)^3+3\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-3\left(\cos(bx+a)^4-2\cos(bx+a)^2+\cdots\right.}{16\left(b\cos(bx+a)^4-2b\cos(bx+a)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/16*(10*cos(b*x + a)^3 + 3*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/
2*cos(b*x + a) + 1/2) - 3*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/2*
cos(b*x + a) + 1/2) - 6*cos(b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^
2 + b)
```

Sympy [A] time = 4.94042, size = 92, normalized size = 1.67

$$\begin{cases}\frac{3\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{8b}+\frac{\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b}-\frac{\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}+\frac{1}{8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}-\frac{1}{64b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)} & \text{for } b\neq 0\\ \frac{x\cos^4(a)}{\sin^5(a)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((3*log(tan(a/2 + b*x/2))/(8*b) + tan(a/2 + b*x/2)**4/(64*b) - tan
(a/2 + b*x/2)**2/(8*b) + 1/(8*b*tan(a/2 + b*x/2)**2) - 1/(64*b*tan(a/2 + b*
x/2)**4), Ne(b, 0)), (x*cos(a)**4/sin(a)**5, True))
```

Giac [B] time = 1.18685, size = 188, normalized size = 3.42

$$-\frac{\frac{\left(\frac{8\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{18\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}-\frac{8\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-12\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/64*((8*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 18*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 8*(cos
(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)
^2 - 12*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.177 $\int \cot^3(a+bx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\cot^4(a+bx)}{4b}$$

```
[Out] -Cot[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0276831, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$-\frac{\cot^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^3*Csc[a + b*x]^2,x]
```

```
[Out] -Cot[a + b*x]^4/(4*b)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^3(a+bx)\csc^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^3\,dx, x, -\cot(a+bx)\right)}{b}\\ &= -\frac{\cot^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0068362, size = 15, normalized size = 1.

$$-\frac{\cot^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^3*Csc[a + b*x]^2,x]
```

```
[Out] -Cot[a + b*x]^4/(4*b)
```

Maple [A] time = 0.01, size = 22, normalized size = 1.5

$$-\frac{(\cos(bx+a))^4}{4(\sin(bx+a))^4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(b*x+a)^5,x)
```

```
[Out] -1/4*cos(b*x+a)^4/sin(b*x+a)^4/b
```

Maxima [A] time = 0.989208, size = 34, normalized size = 2.27

$$\frac{2\sin(bx+a)^2-1}{4b\sin(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^4)
```

Fricas [B] time = 1.72945, size = 99, normalized size = 6.6

$$-\frac{2\cos(bx+a)^2-1}{4\left(b\cos(bx+a)^4-2b\cos(bx+a)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [A] time = 2.66975, size = 44, normalized size = 2.93

$$\begin{cases} \frac{1}{4b\sin^2(a+bx)} - \frac{\cos^2(a+bx)}{4b\sin^4(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^3(a)}{\sin^5(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((1/(4*b*sin(a + b*x)**2) - cos(a + b*x)**2/(4*b*sin(a + b*x)**4),
 Ne(b, 0)), (x*cos(a)**3/sin(a)**5, True))
```

Giac [A] time = 1.1294, size = 34, normalized size = 2.27

$$\frac{2\sin(bx+a)^2-1}{4b\sin(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/4*(2*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^4)
```

3.178 $\int \cot^2(a+bx)\csc^3(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}+\frac{\cot(a+bx)\csc(a+bx)}{8b}$$

```
[Out] ArcTanh[Cos[a + b*x]]/(8*b) + (Cot[a + b*x]*Csc[a + b*x])/(8*b) - (Cot[a +
b*x]*Csc[a + b*x]^3)/(4*b)
```

Rubi [A] time = 0.0451343, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2611, 3768, 3770}

$$\frac{\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}+\frac{\cot(a+bx)\csc(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^2*Csc[a + b*x]^3,x]
```

```
[Out] ArcTanh[Cos[a + b*x]]/(8*b) + (Cot[a + b*x]*Csc[a + b*x])/(8*b) - (Cot[a +
b*x]*Csc[a + b*x]^3)/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(a+bx)\csc^3(a+bx)\,dx&=-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}-\frac{1}{4}\int \csc^3(a+bx)\,dx\\&=\frac{\cot(a+bx)\csc(a+bx)}{8b}-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}-\frac{1}{8}\int \csc(a+bx)\,dx\\&=\frac{\tanh^{-1}(\cos(a+bx))}{8b}+\frac{\cot(a+bx)\csc(a+bx)}{8b}-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}\end{aligned}$$

Mathematica [B] time = 0.0342107, size = 113, normalized size = 2.05

$$-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{8b}+\frac{\log\left(\cos\left(\frac{1}{2}(a+\right.\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^2*Csc[a + b*x]^3,x]
```

```
[Out] Csc[(a + b*x)/2]^2/(32*b) - Csc[(a + b*x)/2]^4/(64*b) + Log[Cos[(a + b*x)/2
]]/(8*b) - Log[Sin[(a + b*x)/2]]/(8*b) - Sec[(a + b*x)/2]^2/(32*b) + Sec[(a
 + b*x)/2]^4/(64*b)
```

Maple [A] time = 0.011, size = 76, normalized size = 1.4

$$-\frac{(\cos(bx+a))^3}{4b(\sin(bx+a))^4}-\frac{(\cos(bx+a))^3}{8b(\sin(bx+a))^2}-\frac{\cos(bx+a)}{8b}-\frac{\ln(\csc(bx+a)-\cot(bx+a))}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b*cos(b*x+a)^3/sin(b*x+a)^4-1/8/b*cos(b*x+a)^3/sin(b*x+a)^2-1/8*cos(b*
x+a)/b-1/8/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [A] time = 0.970983, size = 88, normalized size = 1.6

$$-\frac{\frac{2\left(\cos(bx+a)^3+\cos(bx+a)\right)}{\cos(bx+a)^4-2\cos(bx+a)^2+1}-\log(\cos(bx+a)+1)+\log(\cos(bx+a)-1)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(cos(b*x + a)^3 + cos(b*x + a))/(cos(b*x + a)^4 - 2*cos(b*x + a)^2
 + 1) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
```

Fricas [B] time = 1.95067, size = 308, normalized size = 5.6

$$-\frac{2\cos(bx+a)^3-\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\mathrm{l}}{16\left(b\cos(bx+a)^4-2b\cos(bx+a)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/16*(2*cos(b*x + a)^3 - (cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*c
os(b*x + a) + 1/2) + (cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/2*cos(b
*x + a) + 1/2) + 2*cos(b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b
```

)

Sympy [A] time = 4.73168, size = 58, normalized size = 1.05

$$\begin{cases} -\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{8b}+\frac{\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b}-\frac{1}{64b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)} & \text{for } b\neq 0 \\ \frac{x\cos^2(a)}{\sin^5(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-log(tan(a/2 + b*x/2))/(8*b) + tan(a/2 + b*x/2)**4/(64*b) - 1/(6
4*b*tan(a/2 + b*x/2)**4), Ne(b, 0)), (x*cos(a)**2/sin(a)**5, True))
```

Giac [A] time = 1.16238, size = 132, normalized size = 2.4

$$\frac{\frac{\left(\frac{2(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-4\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/64*((2*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^
2/(cos(b*x + a) - 1)^2 + (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 - 4*log(
abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.179 $\int \cot(a+bx)\csc^4(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\csc^4(a+bx)}{4b}$$

[Out] -Csc[a + b*x]^4/(4*b)

Rubi [A] time = 0.0183253, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$-\frac{\csc^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

[In] Int[Cot[a + b*x]*Csc[a + b*x]^4,x]

[Out] -Csc[a + b*x]^4/(4*b)

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot(a+bx)\csc^4(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^3\,dx, x, \csc(a+bx)\right)}{b}\\ &= -\frac{\csc^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0090331, size = 15, normalized size = 1.

$$-\frac{\csc^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + b*x]*Csc[a + b*x]^4,x]

[Out] -Csc[a + b*x]^4/(4*b)

Maple [A] time = 0.003, size = 14, normalized size = 0.9

$$-\frac{1}{4\left(\sin\left(bx+a\right)\right)^{4}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+a)^5,x)
```

```
[Out] -1/4/sin(b*x+a)^4/b
```

Maxima [A] time = 0.98342, size = 18, normalized size = 1.2

$$-\frac{1}{4\,b\sin\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4/(b*sin(b*x + a)^4)
```

Fricas [A] time = 1.69782, size = 68, normalized size = 4.53

$$-\frac{1}{4\left(b\cos\left(bx+a\right)^{4}-2\,b\cos\left(bx+a\right)^{2}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [A] time = 2.46108, size = 24, normalized size = 1.6

$$\begin{cases} -\frac{1}{4b\sin^{4}(a+bx)} & \text{for}\ b\neq 0\\ \frac{x\cos(a)}{\sin^{5}(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-1/(4*b*sin(a + b*x)**4), Ne(b, 0)), (x*cos(a)/sin(a)**5, True))
```

Giac [A] time = 1.14118, size = 18, normalized size = 1.2

$$-\frac{1}{4\,b\sin\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4/(b*sin(b*x + a)^4)
```

3.180 $\int \csc^5(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=40

$$-\frac{\cot^4(a+bx)}{4b}-\frac{\cot^2(a+bx)}{b}+\frac{\log(\tan(a+bx))}{b}$$

```
[Out] -(Cot[a + b*x]^2/b) - Cot[a + b*x]^4/(4*b) + Log[Tan[a + b*x]]/b
```

Rubi [A] time = 0.0271886, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2620, 266, 43}

$$-\frac{\cot^4(a+bx)}{4b}-\frac{\cot^2(a+bx)}{b}+\frac{\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^5*Sec[a + b*x],x]
```

```
[Out] -(Cot[a + b*x]^2/b) - Cot[a + b*x]^4/(4*b) + Log[Tan[a + b*x]]/b
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc^5(a+bx)\sec(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^5}\,dx,x,\tan(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3}\,dx,x,\tan^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{2}{x^2}+\frac{1}{x}\right)dx,x,\tan^2(a+bx)\right)}{2b}\\
&= -\frac{\cot^2(a+bx)}{b}-\frac{\cot^4(a+bx)}{4b}+\frac{\log(\tan(a+bx))}{b}
\end{aligned}$$

Mathematica [A] time = 0.111257, size = 44, normalized size = 1.1

$$-\frac{\csc^4(a+bx)+2\csc^2(a+bx)-4\log(\sin(a+bx))+4\log(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^5*Sec[a + b*x],x]
```

```
[Out] -(2*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 4*Log[Cos[a + b*x]] - 4*Log[Sin[a + b
*x]])/(4*b)
```

Maple [A] time = 0.02, size = 39, normalized size = 1.

$$-\frac{1}{4\left(\sin\left(bx+a\right)\right)^{4}b}-\frac{1}{2\left(\sin\left(bx+a\right)\right)^{2}b}+\frac{\ln\left(\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/sin(b*x+a)^5,x)
```

```
[Out] -1/4/sin(b*x+a)^4/b-1/2/sin(b*x+a)^2/b+ln(tan(b*x+a))/b
```

Maxima [A] time = 0.971075, size = 69, normalized size = 1.72

$$-\frac{\frac{2\sin\left(bx+a\right)^{2}+1}{\sin\left(bx+a\right)^{4}}+2\log\left(\sin\left(bx+a\right)^{2}-1\right)-2\log\left(\sin\left(bx+a\right)^{2}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*((2*sin(b*x + a)^2 + 1)/sin(b*x + a)^4 + 2*log(sin(b*x + a)^2 - 1) - 2
*log(sin(b*x + a)^2))/b
```

Fricas [B] time = 1.86571, size = 285, normalized size = 7.12

$$\frac{2\cos\left(bx+a\right)^{2}-2\left(\cos\left(bx+a\right)^{4}-2\cos\left(bx+a\right)^{2}+1\right)\log\left(\cos\left(bx+a\right)^{2}\right)+2\left(\cos\left(bx+a\right)^{4}-2\cos\left(bx+a\right)^{2}+1\right)\log\Big(}{4\left(b\cos\left(bx+a\right)^{4}-2b\cos\left(bx+a\right)^{2}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*cos(b*x + a)^2 - 2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(cos(b
*x + a)^2) + 2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/4*cos(b*x + a
)^2 + 1/4) - 3)/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33617, size = 223, normalized size = 5.58

$$\frac{\frac{\left(\frac{12\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{48\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}+\frac{12\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+32\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)-64\,\log\left(\left|-\frac{\cos(bx+a}{\cos(bx+a}\right.\right.}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/64*((12*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 48*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 + 12*(co
s(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1
)^2 + 32*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 64*log(abs(-(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

3.181 $\int \csc^5(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=70

$$\frac{15\sec(a+bx)}{8b}-\frac{15\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\csc^4(a+bx)\sec(a+bx)}{4b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{8b}$$

```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(8*b) + (15*Sec[a + b*x])/(8*b) - (5*Csc[a + b*
x]^2*Sec[a + b*x])/(8*b) - (Csc[a + b*x]^4*Sec[a + b*x])/(4*b)
```

Rubi [A] time = 0.0428248, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 321, 207}

$$\frac{15\sec(a+bx)}{8b}-\frac{15\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\csc^4(a+bx)\sec(a+bx)}{4b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^5*Sec[a + b*x]^2,x]
```

```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(8*b) + (15*Sec[a + b*x])/(8*b) - (5*Csc[a + b*
x]^2*Sec[a + b*x])/(8*b) - (Csc[a + b*x]^4*Sec[a + b*x])/(4*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, \sec(a+bx)\right)}{b}$$

$$= -\frac{\csc^4(a+bx)\sec(a+bx)}{4b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{4b}$$

$$= -\frac{5\csc^2(a+bx)\sec(a+bx)}{8b} - \frac{\csc^4(a+bx)\sec(a+bx)}{4b} + \frac{15\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, s\ldots\right.}{8b}$$

$$= \frac{15\sec(a+bx)}{8b} - \frac{5\csc^2(a+bx)\sec(a+bx)}{8b} - \frac{\csc^4(a+bx)\sec(a+bx)}{4b} + \frac{15\,\text{Subst}\ldots}{\ldots}$$

$$= -\frac{15\tanh^{-1}(\cos(a+bx))}{8b} + \frac{15\sec(a+bx)}{8b} - \frac{5\csc^2(a+bx)\sec(a+bx)}{8b} - \frac{\csc^4(a+\ldots}{\ldots}$$

Mathematica [A] time = 4.55911, size = 129, normalized size = 1.84

$$-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right) + 14\csc^2\left(\frac{1}{2}(a+bx)\right) + \frac{\sec^2\left(\frac{1}{2}(a+bx)\right)\left(-14\tan^2\left(\frac{1}{2}(a+bx)\right)+\cos(a+bx)\left(\sec^4\left(\frac{1}{2}(a+bx)\right)-8\left(-15\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)+15\log\ldots\right.\right.\right.}{\tan^2\left(\frac{1}{2}(a+bx)\right)-1}}{64b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^5*Sec[a + b*x]^2,x]
```

```
[Out] -(14*Csc[(a + b*x)/2]^2 + Csc[(a + b*x)/2]^4 + (Sec[(a + b*x)/2]^2*(78 + Co
s[a + b*x]*(-8*(8 + 15*Log[Cos[(a + b*x)/2]] - 15*Log[Sin[(a + b*x)/2]]) +
Sec[(a + b*x)/2]^4) - 14*Tan[(a + b*x)/2]^2))/(-1 + Tan[(a + b*x)/2]^2))/(6
4*b)
```

Maple [A] time = 0.022, size = 78, normalized size = 1.1

$$-\frac{1}{4\,b\,(\sin(bx+a))^4\cos(bx+a)} - \frac{5}{8\,b\,(\sin(bx+a))^2\cos(bx+a)} + \frac{15}{8\,b\cos(bx+a)} + \frac{15\ln(\csc(bx+a)-\cot(bx+\ldots}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b/sin(b*x+a)^4/cos(b*x+a)-5/8/b/sin(b*x+a)^2/cos(b*x+a)+15/8/b/cos(b*x
+a)+15/8/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [A] time = 0.968699, size = 107, normalized size = 1.53

$$\frac{\frac{2\left(15\cos(bx+a)^4-25\cos(bx+a)^2+8\right)}{\cos(bx+a)^5-2\cos(bx+a)^3+\cos(bx+a)} - 15\log(\cos(bx+a)+1) + 15\log(\cos(bx+a)-1)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(15*cos(b*x + a)^4 - 25*cos(b*x + a)^2 + 8)/(cos(b*x + a)^5 - 2*cos
(b*x + a)^3 + cos(b*x + a)) - 15*log(cos(b*x + a) + 1) + 15*log(cos(b*x + a
) - 1))/b
```

Fricas [B] time = 1.98859, size = 374, normalized size = 5.34

$$\frac{30\cos(bx+a)^4-50\cos(bx+a)^2-15\left(\cos(bx+a)^5-2\cos(bx+a)^3+\cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+15\left(c\right.}{16\left(b\cos(bx+a)^5-2\,b\cos(bx+a)^3+b\cos(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(30*cos(b*x + a)^4 - 50*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - 2*cos(b*
x + a)^3 + cos(b*x + a))*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5 -
 2*cos(b*x + a)^3 + cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2) + 16)/(b*cos
(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18195, size = 220, normalized size = 3.14

$$\frac{\frac{\left(\frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{90(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}-\frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{128}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1}+60\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/64*((16*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 90*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 16*(co
s(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1
)^2 + 128/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + 60*log(abs(-cos(b*x
 + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.182 $\int \csc^5(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=58

$$\frac{\tan^2(a+bx)}{2b}-\frac{\cot^4(a+bx)}{4b}-\frac{3\cot^2(a+bx)}{2b}+\frac{3\log(\tan(a+bx))}{b}$$

```
[Out] (-3*Cot[a + b*x]^2)/(2*b) - Cot[a + b*x]^4/(4*b) + (3*Log[Tan[a + b*x]])/b
+ Tan[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0398449, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$\frac{\tan^2(a+bx)}{2b}-\frac{\cot^4(a+bx)}{4b}-\frac{3\cot^2(a+bx)}{2b}+\frac{3\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^5*Sec[a + b*x]^3,x]
```

```
[Out] (-3*Cot[a + b*x]^2)/(2*b) - Cot[a + b*x]^4/(4*b) + (3*Log[Tan[a + b*x]])/b
+ Tan[a + b*x]^2/(2*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc^5(a+bx)\sec^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^5}\,dx,x,\tan(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^3}\,dx,x,\tan^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^3}+\frac{3}{x^2}+\frac{3}{x}\right)dx,x,\tan^2(a+bx)\right)}{2b}\\
&= -\frac{3\cot^2(a+bx)}{2b}-\frac{\cot^4(a+bx)}{4b}+\frac{3\log(\tan(a+bx))}{b}+\frac{\tan^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.340545, size = 54, normalized size = 0.93

$$-\frac{\csc^4(a+bx)+4\csc^2(a+bx)-2\sec^2(a+bx)-12\log(\sin(a+bx))+12\log(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^5*Sec[a + b*x]^3,x]
```

```
[Out] -(4*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 12*Log[Cos[a + b*x]] - 12*Log[Sin[a +
 b*x]] - 2*Sec[a + b*x]^2)/(4*b)
```

Maple [A] time = 0.026, size = 69, normalized size = 1.2

$$-\frac{1}{4\,b\,(\sin(bx+a))^4\,(\cos(bx+a))^2}+\frac{3}{4\,b\,(\sin(bx+a))^2\,(\cos(bx+a))^2}-\frac{3}{2\,b\,(\sin(bx+a))^2}+3\,\frac{\ln(\tan(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b/sin(b*x+a)^4/cos(b*x+a)^2+3/4/b/sin(b*x+a)^2/cos(b*x+a)^2-3/2/sin(b*
x+a)^2/b+3*ln(tan(b*x+a))/b
```

Maxima [A] time = 0.985309, size = 100, normalized size = 1.72

$$-\frac{\frac{6\,\sin(bx+a)^4-3\,\sin(bx+a)^2-1}{\sin(bx+a)^6-\sin(bx+a)^4}+6\,\log\left(\sin(bx+a)^2-1\right)-6\,\log\left(\sin(bx+a)^2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*((6*sin(b*x + a)^4 - 3*sin(b*x + a)^2 - 1)/(sin(b*x + a)^6 - sin(b*x +
 a)^4) + 6*log(sin(b*x + a)^2 - 1) - 6*log(sin(b*x + a)^2))/b
```

Fricas [B] time = 1.92825, size = 366, normalized size = 6.31

$$\frac{6\,\cos(bx+a)^4-9\,\cos(bx+a)^2-6\left(\cos(bx+a)^6-2\,\cos(bx+a)^4+\cos(bx+a)^2\right)\log\left(\cos(bx+a)^2\right)+6\left(\cos(bx+a\right.}{4\left(b\cos(bx+a)^6-2\,b\cos(bx+a)^4+b\cos(bx+a)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - 2*cos(b*x +
a)^4 + cos(b*x + a)^2)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - 2*cos(b*x
+ a)^4 + cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) + 2)/(b*cos(b*x + a
)^6 - 2*b*cos(b*x + a)^4 + b*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24591, size = 313, normalized size = 5.4

$$\frac{\frac{20(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{\frac{18(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{111(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{36(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} + \frac{72(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} - 1}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1} + \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}\right)^2} + 96\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) - 192}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/64*(20*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(
b*x + a) + 1)^2 + (18*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 111*(cos(b*x
+ a) - 1)^2/(cos(b*x + a) + 1)^2 + 36*(cos(b*x + a) - 1)^3/(cos(b*x + a) +
1)^3 + 72*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 1)/((cos(b*x + a) - 1
)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2)^2 + 96*lo
g(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 192*log(abs(-(cos(b*x + a
) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

3.183 $\int \csc^5(a+bx)\sec^4(a+bx)\,dx$

Optimal. Leaf size=89

$$\frac{35\sec^3(a+bx)}{24b}+\frac{35\sec(a+bx)}{8b}-\frac{35\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{4b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b}$$

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(8*b) + (35*Sec[a + b*x])/(8*b) + (35*Sec[a + b
*x]^3)/(24*b) - (7*Csc[a + b*x]^2*Sec[a + b*x]^3)/(8*b) - (Csc[a + b*x]^4*S
ec[a + b*x]^3)/(4*b)
```

Rubi [A] time = 0.0462182, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 302, 207}

$$\frac{35\sec^3(a+bx)}{24b}+\frac{35\sec(a+bx)}{8b}-\frac{35\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{4b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^5*Sec[a + b*x]^4,x]
```

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(8*b) + (35*Sec[a + b*x])/(8*b) + (35*Sec[a + b
*x]^3)/(24*b) - (7*Csc[a + b*x]^2*Sec[a + b*x]^3)/(8*b) - (Csc[a + b*x]^4*S
ec[a + b*x]^3)/(4*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^3}\,dx, x, \sec(a+bx)\right)}{b}$$
$$= -\frac{\csc^4(a+bx)\sec^3(a+bx)}{4b} + \frac{7\,\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{4b}$$
$$= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{4b} + \frac{35\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x\right.}{8b}$$
$$= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{4b} + \frac{35\,\text{Subst}\left(\int \left(1+x^2+\right.\right.}{}$$
$$= \frac{35\sec(a+bx)}{8b} + \frac{35\sec^3(a+bx)}{24b} - \frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b} - \frac{\csc^4(a+bx)\sec^3}{4b}$$
$$= -\frac{35\tanh^{-1}(\cos(a+bx))}{8b} + \frac{35\sec(a+bx)}{8b} + \frac{35\sec^3(a+bx)}{24b} - \frac{7\csc^2(a+bx)\sec^3}{8b}$$

Mathematica [B] time = 0.504024, size = 268, normalized size = 3.01

$$-\frac{\csc^{10}(a+bx)\left(658\cos(2(a+bx)) - 228\cos(3(a+bx)) + 140\cos(4(a+bx)) - 76\cos(5(a+bx)) - 210\cos(6(a+bx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^5*Sec[a + b*x]^4,x]
```

```
[Out] -(Csc[a + b*x]^10*(-204 + 658*Cos[2*(a + b*x)] - 228*Cos[3*(a + b*x)] + 140
*Cos[4*(a + b*x)] - 76*Cos[5*(a + b*x)] - 210*Cos[6*(a + b*x)] + 76*Cos[7*(
a + b*x)] - 315*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 105*Cos[5*(a + b*x
)]*Log[Cos[(a + b*x)/2]] + 105*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)/2]] + 3*C
os[a + b*x]*(76 + 105*Log[Cos[(a + b*x)/2]] - 105*Log[Sin[(a + b*x)/2]]) +
315*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 105*Cos[5*(a + b*x)]*Log[Sin[(
a + b*x)/2]] - 105*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]]))/(24*b*(Csc[(a +
 b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Maple [A] time = 0.025, size = 99, normalized size = 1.1

$$-\frac{1}{4\,b\,(\sin(bx+a))^4\,(\cos(bx+a))^3} + \frac{7}{12\,b\,(\sin(bx+a))^2\,(\cos(bx+a))^3} - \frac{35}{24\,b\,(\sin(bx+a))^2\cos(bx+a)} + \frac{35}{8\,b\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b/sin(b*x+a)^4/cos(b*x+a)^3+7/12/b/sin(b*x+a)^2/cos(b*x+a)^3-35/24/b/s
in(b*x+a)^2/cos(b*x+a)+35/8/b/cos(b*x+a)+35/8/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [A] time = 0.973315, size = 123, normalized size = 1.38

$$\frac{\frac{2\left(105\cos(bx+a)^6 - 175\cos(bx+a)^4 + 56\cos(bx+a)^2 + 8\right)}{\cos(bx+a)^7 - 2\cos(bx+a)^5 + \cos(bx+a)^3} - 105\log(\cos(bx+a)+1) + 105\log(\cos(bx+a)-1)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(2*(105*cos(b*x + a)^6 - 175*cos(b*x + a)^4 + 56*cos(b*x + a)^2 + 8)/(
cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3) - 105*log(cos(b*x + a)
+ 1) + 105*log(cos(b*x + a) - 1))/b
```

Fricas [A] time = 1.89678, size = 416, normalized size = 4.67

$$\frac{210 \cos(bx+a)^6 - 350 \cos(bx+a)^4 + 112 \cos(bx+a)^2 - 105\left(\cos(bx+a)^7 - 2\cos(bx+a)^5 + \cos(bx+a)^3\right)\log\left(\frac{1}{2} \cdots\right.}{48\left(b\cos(bx+a)^7 - 2b\cos(bx+a)^{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(210*cos(b*x + a)^6 - 350*cos(b*x + a)^4 + 112*cos(b*x + a)^2 - 105*(c
os(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(1/2*cos(b*x + a) + 1
/2) + 105*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(-1/2*cos
(b*x + a) + 1/2) + 16)/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b*x +
 a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4/sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22033, size = 282, normalized size = 3.17

$$\frac{\frac{3\left(\frac{24(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{210(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - 1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2} - \frac{72(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{256\left(\frac{9(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{6(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + 5\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1} + 1\right)^3} + 420\log\left(\cdots\right.}{192b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/192*(3*(24*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 210*(cos(b*x + a) - 1)
^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 72
*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos(b*x +
a) + 1)^2 + 256*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 6*(cos(b*x + a)
- 1)^2/(cos(b*x + a) + 1)^2 + 5)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1
)^3 + 420*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.184 $\int \csc^5(a+bx)\sec^5(a+bx)\,dx$

Optimal. Leaf size=69

$$\frac{\tan^4(a+bx)}{4b}+\frac{2\tan^2(a+bx)}{b}-\frac{\cot^4(a+bx)}{4b}-\frac{2\cot^2(a+bx)}{b}+\frac{6\log(\tan(a+bx))}{b}$$

```
[Out] (-2*Cot[a + b*x]^2)/b - Cot[a + b*x]^4/(4*b) + (6*Log[Tan[a + b*x]])/b + (2
*Tan[a + b*x]^2)/b + Tan[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0459103, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$\frac{\tan^4(a+bx)}{4b}+\frac{2\tan^2(a+bx)}{b}-\frac{\cot^4(a+bx)}{4b}-\frac{2\cot^2(a+bx)}{b}+\frac{6\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^5*Sec[a + b*x]^5,x]
```

```
[Out] (-2*Cot[a + b*x]^2)/b - Cot[a + b*x]^4/(4*b) + (6*Log[Tan[a + b*x]])/b + (2
*Tan[a + b*x]^2)/b + Tan[a + b*x]^4/(4*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc^5(a+bx)\sec^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{x^5}\,dx,x,\tan(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^4}{x^3}\,dx,x,\tan^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(4+\frac{1}{x^3}+\frac{4}{x^2}+\frac{6}{x}+x\right)dx,x,\tan^2(a+bx)\right)}{2b}\\
&= -\frac{2\cot^2(a+bx)}{b}-\frac{\cot^4(a+bx)}{4b}+\frac{6\log(\tan(a+bx))}{b}+\frac{2\tan^2(a+bx)}{b}+\frac{\tan^4(a+}{4b}
\end{aligned}$$

Mathematica [A] time = 0.0283338, size = 91, normalized size = 1.32

$$32\left(-\frac{\csc^4(a+bx)}{128b}-\frac{3\csc^2(a+bx)}{64b}+\frac{\sec^4(a+bx)}{128b}+\frac{3\sec^2(a+bx)}{64b}+\frac{3\log(\sin(a+bx))}{16b}-\frac{3\log(\cos(a+bx))}{16b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^5*Sec[a + b*x]^5,x]
```

```
[Out] 32*((-3*Csc[a + b*x]^2)/(64*b) - Csc[a + b*x]^4/(128*b) - (3*Log[Cos[a + b*
x]])/(16*b) + (3*Log[Sin[a + b*x]])/(16*b) + (3*Sec[a + b*x]^2)/(64*b) + Se
c[a + b*x]^4/(128*b))
```

Maple [A] time = 0.026, size = 90, normalized size = 1.3

$$\frac{1}{4\,b\,(\sin(bx+a))^4\,(\cos(bx+a))^4}-\frac{1}{2\,b\,(\sin(bx+a))^4\,(\cos(bx+a))^2}+\frac{3}{2\,b\,(\sin(bx+a))^2\,(\cos(bx+a))^2}-3\,\frac{1}{b\,(\sin(bx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5/sin(b*x+a)^5,x)
```

```
[Out] 1/4/b/sin(b*x+a)^4/cos(b*x+a)^4-1/2/b/sin(b*x+a)^4/cos(b*x+a)^2+3/2/b/sin(b
*x+a)^2/cos(b*x+a)^2-3/sin(b*x+a)^2/b+6*ln(tan(b*x+a))/b
```

Maxima [A] time = 0.972823, size = 124, normalized size = 1.8

$$-\frac{\frac{12\,\sin(bx+a)^6-18\,\sin(bx+a)^4+4\,\sin(bx+a)^2+1}{\sin(bx+a)^8-2\,\sin(bx+a)^6+\sin(bx+a)^4}+12\,\log\left(\sin(bx+a)^2-1\right)-12\,\log\left(\sin(bx+a)^2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*((12*sin(b*x + a)^6 - 18*sin(b*x + a)^4 + 4*sin(b*x + a)^2 + 1)/(sin(b
*x + a)^8 - 2*sin(b*x + a)^6 + sin(b*x + a)^4) + 12*log(sin(b*x + a)^2 - 1)
 - 12*log(sin(b*x + a)^2))/b
```

Fricas [B] time = 1.94674, size = 397, normalized size = 5.75

$$\frac{12\,\cos(bx+a)^6-18\,\cos(bx+a)^4+4\,\cos(bx+a)^2-12\left(\cos(bx+a)^8-2\,\cos(bx+a)^6+\cos(bx+a)^4\right)\log\left(\cos(bx+\right.}{4\left(b\cos(bx+a)^8-2\,b\cos(bx+a)^6+b\,\mathrm{cc}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(12*cos(b*x + a)^6 - 18*cos(b*x + a)^4 + 4*cos(b*x + a)^2 - 12*(cos(b*x
 + a)^8 - 2*cos(b*x + a)^6 + cos(b*x + a)^4)*log(cos(b*x + a)^2) + 12*(cos(
b*x + a)^8 - 2*cos(b*x + a)^6 + cos(b*x + a)^4)*log(-1/4*cos(b*x + a)^2 + 1
```

```
/4) + 1)/(b*cos(b*x + a)^8 - 2*b*cos(b*x + a)^6 + b*cos(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5/sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24231, size = 375, normalized size = 5.43

$$\frac{\frac{\left(\frac{28(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{288(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}+\frac{28(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{32\left(\frac{84(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{126(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{84(\cos(bx+a)-1)}{(\cos(bx+a)+1)^3}\right.}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^4}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/64*((28*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 288*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 + 28*(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(b*x + a) +
1)^2 + 32*(84*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 126*(cos(b*x + a) - 1
)^2/(cos(b*x + a) + 1)^2 + 84*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 2
5*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 25)/((cos(b*x + a) - 1)/(cos(
b*x + a) + 1) + 1)^4 + 192*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)
) - 384*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

3.185 $\int \cot^2(x)\csc^4(x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{5}\cot^5(x)-\frac{\cot^3(x)}{3}$$

```
[Out] -Cot[x]^3/3 - Cot[x]^5/5
```

Rubi [A] time = 0.025849, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 14}

$$-\frac{1}{5}\cot^5(x)-\frac{\cot^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*Csc[x]^4,x]

[Out] -Cot[x]^3/3 - Cot[x]^5/5
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cot^2(x)\csc^4(x)\,dx &= \text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,-\cot(x)\right)\\ &= \text{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,-\cot(x)\right)\\ &= -\frac{1}{3}\cot^3(x)-\frac{\cot^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.0216094, size = 27, normalized size = 1.59

$$\frac{2\cot(x)}{15}-\frac{1}{5}\cot(x)\csc^4(x)+\frac{1}{15}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2*Csc[x]^4,x]

[Out] (2*Cot[x])/15 + (Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5
```

Maple [A] time = 0.007, size = 22, normalized size = 1.3

$$-\frac{(\cos(x))^3}{5(\sin(x))^5}-\frac{2(\cos(x))^3}{15(\sin(x))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/sin(x)^6,x)
```

```
[Out] -1/5*cos(x)^3/sin(x)^5-2/15*cos(x)^3/sin(x)^3
```

Maxima [A] time = 0.959092, size = 19, normalized size = 1.12

$$-\frac{5\tan(x)^2+3}{15\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/sin(x)^6,x, algorithm="maxima")
```

```
[Out] -1/15*(5*tan(x)^2 + 3)/tan(x)^5
```

Fricas [B] time = 1.77145, size = 93, normalized size = 5.47

$$\frac{2\cos(x)^5-5\cos(x)^3}{15\left(\cos(x)^4-2\cos(x)^2+1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/sin(x)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(2*cos(x)^5 - 5*cos(x)^3)/((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x))
```

Sympy [B] time = 0.060839, size = 29, normalized size = 1.71

$$\frac{2\cos(x)}{15\sin(x)}+\frac{\cos(x)}{15\sin^3(x)}-\frac{\cos(x)}{5\sin^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/sin(x)**6,x)
```

```
[Out] 2*cos(x)/(15*sin(x)) + cos(x)/(15*sin(x)**3) - cos(x)/(5*sin(x)**5)
```

Giac [A] time = 1.12669, size = 19, normalized size = 1.12

$$-\frac{5\tan(x)^2+3}{15\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/sin(x)^6,x, algorithm="giac")
```

```
[Out] -1/15*(5*tan(x)^2 + 3)/tan(x)^5
```

3.186 $\int \cot^3(x)\csc^4(x)\,dx$

Optimal. Leaf size=17

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rubi [A] time = 0.0260525, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cot^3(x)\csc^4(x)\,dx &= -\text{Subst}\left(\int x^3\left(-1+x^2\right)\,dx,x,\csc(x)\right)\\ &= -\text{Subst}\left(\int \left(-x^3+x^5\right)\,dx,x,\csc(x)\right)\\ &= \frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}\end{aligned}$$

Mathematica [A] time = 0.0075872, size = 17, normalized size = 1.

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Maple [A] time = 0.007, size = 22, normalized size = 1.3

$$-\frac{(\cos(x))^4}{6(\sin(x))^6}-\frac{(\cos(x))^4}{12(\sin(x))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/sin(x)^7,x)
```

```
[Out] -1/6/sin(x)^6*cos(x)^4-1/12*cos(x)^4/sin(x)^4
```

Maxima [A] time = 0.960337, size = 19, normalized size = 1.12

$$\frac{3\sin(x)^2-2}{12\sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="maxima")
```

```
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

Fricas [B] time = 1.74161, size = 86, normalized size = 5.06

$$\frac{3\cos(x)^2-1}{12\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

Sympy [A] time = 0.101693, size = 14, normalized size = 0.82

$$\frac{3\sin^2(x)-2}{12\sin^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/sin(x)**7,x)
```

```
[Out] (3*sin(x)**2 - 2)/(12*sin(x)**6)
```

Giac [A] time = 1.1373, size = 24, normalized size = 1.41

$$\frac{3\cos(x)^2-1}{12\left(\cos(x)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="giac")
```

```
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^2 - 1)^3
```

3.187 $\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx$

Optimal. Leaf size=22

$$-\frac{2(d\cos(a+bx))^{5/2}}{5bd}$$

```
[Out] (-2*(d*Cos[a + b*x])^(5/2))/(5*b*d)
```

Rubi [A] time = 0.0252584, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$-\frac{2(d\cos(a+bx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x],x]
```

```
[Out] (-2*(d*Cos[a + b*x])^(5/2))/(5*b*d)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^{3/2}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{2(d\cos(a+bx))^{5/2}}{5bd}\end{aligned}$$

Mathematica [A] time = 0.0325464, size = 22, normalized size = 1.

$$-\frac{2(d\cos(a+bx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x],x]
```

```
[Out] (-2*(d*Cos[a + b*x])^(5/2))/(5*b*d)
```

Maple [A] time = 0.007, size = 19, normalized size = 0.9

$$-\frac{2}{5\,bd}\,(d\cos(bx+a))^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*sin(b*x+a),x)
```

```
[Out] -2/5*(d*cos(b*x+a))^(5/2)/b/d
```

Maxima [A] time = 0.974904, size = 24, normalized size = 1.09

$$-\frac{2\,(d\cos(bx+a))^{\frac{5}{2}}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/5*(d*cos(b*x + a))^(5/2)/(b*d)
```

Fricas [A] time = 1.95647, size = 62, normalized size = 2.82

$$-\frac{2\,\sqrt{d\cos(bx+a)}d\cos(bx+a)^{2}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(d*cos(b*x + a))*d*cos(b*x + a)^2/b
```

Sympy [A] time = 63.4996, size = 34, normalized size = 1.55

$$\begin{cases} -\frac{2d^{\frac{3}{2}}\cos^{\frac{5}{2}}(a+bx)}{5b} & \text{for } b \neq 0 \\ x\,(d\cos(a))^{\frac{3}{2}}\sin(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*sin(b*x+a),x)
```

```
[Out] Piecewise((-2*d**(3/2)*cos(a + b*x)**(5/2)/(5*b), Ne(b, 0)), (x*(d*cos(a))*
*(3/2)*sin(a), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a), x)
```

3.188 $\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx$

Optimal. Leaf size=22

$$-\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

[Out] (-2*(d*Cos[a + b*x])^(3/2))/(3*b*d)

Rubi [A] time = 0.0217415, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$-\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x],x]

[Out] (-2*(d*Cos[a + b*x])^(3/2))/(3*b*d)

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \sqrt{x}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{2(d\cos(a+bx))^{3/2}}{3bd}\end{aligned}$$

Mathematica [A] time = 0.0165863, size = 22, normalized size = 1.

$$-\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x],x]

[Out] (-2*(d*Cos[a + b*x])^(3/2))/(3*b*d)

Maple [A] time = 0.006, size = 19, normalized size = 0.9

$$-\frac{2}{3bd}(d\cos(bx+a))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*sin(b*x+a),x)
```

```
[Out] -2/3*(d*cos(b*x+a))^(3/2)/b/d
```

Maxima [A] time = 0.958472, size = 24, normalized size = 1.09

$$-\frac{2(d\cos(bx+a))^{\frac{3}{2}}}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/3*(d*cos(b*x + a))^(3/2)/(b*d)
```

Fricas [A] time = 1.91013, size = 57, normalized size = 2.59

$$-\frac{2\sqrt{d\cos(bx+a)}\cos(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(d*cos(b*x + a))*cos(b*x + a)/b
```

Sympy [A] time = 1.58721, size = 34, normalized size = 1.55

$$\begin{cases} -\frac{2\sqrt{d}\cos^{\frac{3}{2}}(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sqrt{d\cos(a)}\sin(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a),x)
```

```
[Out] Piecewise((-2*sqrt(d)*cos(a + b*x)**(3/2)/(3*b), Ne(b, 0)), (x*sqrt(d*cos(a
))*sin(a), True))
```

Giac [A] time = 1.12548, size = 28, normalized size = 1.27

$$-\frac{2\sqrt{d\cos(bx+a)}\cos(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(d*cos(b*x + a))*cos(b*x + a)/b
```

3.189 $\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=20

$$-\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

```
[Out] (-2*Sqrt[d*Cos[a + b*x]])/(b*d)
```

Rubi [A] time = 0.0232669, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$-\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sqrt[d*Cos[a + b*x]],x]

[Out] (-2*Sqrt[d*Cos[a + b*x]])/(b*d)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{2\sqrt{d\cos(a+bx)}}{bd}\end{aligned}$$

Mathematica [A] time = 0.0145341, size = 20, normalized size = 1.

$$-\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sqrt[d*Cos[a + b*x]],x]

[Out] (-2*Sqrt[d*Cos[a + b*x]])/(b*d)
```

Maple [A] time = 0.008, size = 19, normalized size = 1.

$$-2\frac{\sqrt{d\cos(bx+a)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] -2*(d*cos(b*x+a))^(1/2)/b/d
```

Maxima [A] time = 0.983249, size = 24, normalized size = 1.2

$$-\frac{2\sqrt{d\cos(bx+a)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(d*cos(b*x + a))/(b*d)
```

Fricas [A] time = 1.90865, size = 42, normalized size = 2.1

$$-\frac{2\sqrt{d\cos(bx+a)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(d*cos(b*x + a))/(b*d)
```

Sympy [A] time = 1.23599, size = 32, normalized size = 1.6

$$\begin{cases} -\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{d}} & \text{for } b \neq 0 \\ \frac{x\sin(a)}{\sqrt{d\cos(a)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(cos(a + b*x))/(b*sqrt(d)), Ne(b, 0)), (x*sin(a)/sqrt(d*c
os(a)), True))
```

Giac [A] time = 1.20944, size = 24, normalized size = 1.2

$$-\frac{2\sqrt{d\cos(bx+a)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(d*cos(b*x + a))/(b*d)
```

3.190 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=20

$$\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] 2/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.0258174, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] 2/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{bd\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0235167, size = 20, normalized size = 1.

$$\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] 2/(b*d*Sqrt[d*Cos[a + b*x]])
```

Maple [A] time = 0.004, size = 19, normalized size = 1.

$$2\frac{1}{bd\sqrt{d\cos(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] 2/b/d/(d*cos(b*x+a))^(1/2)
```

Maxima [A] time = 0.978707, size = 24, normalized size = 1.2

$$\frac{2}{\sqrt{d\cos(bx+a)}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/(sqrt(d*cos(b*x + a))*b*d)
```

Fricas [A] time = 1.8419, size = 61, normalized size = 3.05

$$\frac{2\sqrt{d\cos(bx+a)}}{bd^2\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(d*cos(b*x + a))/(b*d^2*cos(b*x + a))
```

Sympy [A] time = 6.07928, size = 31, normalized size = 1.55

$$\begin{cases} \frac{2}{bd^{\frac{3}{2}}\sqrt{\cos(a+bx)}} & \text{for } b \neq 0 \\ \frac{x\sin(a)}{(d\cos(a))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((2/(b*d**(3/2)*sqrt(cos(a + b*x))), Ne(b, 0)), (x*sin(a)/(d*cos(a
))**(3/2), True))
```

Giac [A] time = 1.17022, size = 24, normalized size = 1.2

$$\frac{2}{\sqrt{d\cos(bx+a)}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/(sqrt(d*cos(b*x + a))*b*d)
```

3.191 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.0268243, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$\frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] 2/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{3bd(d\cos(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0276373, size = 22, normalized size = 1.

$$\frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] 2/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Maple [A] time = 0.006, size = 19, normalized size = 0.9

$$\frac{2}{3\,bd}\left(d\cos\left(bx+a\right)\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] 2/3/b/d/(d*cos(b*x+a))^(3/2)
```

Maxima [A] time = 0.976698, size = 24, normalized size = 1.09

$$\frac{2}{3\,\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3/((d*cos(b*x + a))^(3/2)*b*d)
```

Fricas [A] time = 1.82662, size = 66, normalized size = 3.

$$\frac{2\,\sqrt{d\cos\left(bx+a\right)}}{3\,bd^{3}\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(d*cos(b*x + a))/(b*d^3*cos(b*x + a)^2)
```

Sympy [A] time = 66.8572, size = 32, normalized size = 1.45

$$\begin{cases} \frac{2}{3bd^{\frac{5}{2}}\cos^{\frac{3}{2}}{(a+bx)}} & \text{for}\: b \neq 0 \\ \frac{x\sin{(a)}}{(d\cos{(a)})^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((2/(3*b*d**(5/2)*cos(a + b*x)**(3/2)), Ne(b, 0)), (x*sin(a)/(d*co
s(a))**(5/2), True))
```

<u>Giac</u> [A] time = 1.16396, size = 35, normalized size = 1.59

$$\frac{2}{3\sqrt{d\cos\left(bx+a\right)}bd^{2}\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3/(sqrt(d*cos(b*x + a))*b*d^2*cos(b*x + a))
```

3.192 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

```
[Out] 2/(5*b*d*(d*Cos[a + b*x])^(5/2))
```

Rubi [A] time = 0.0263088, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$\frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] 2/(5*b*d*(d*Cos[a + b*x])^(5/2))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{7/2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{5bd(d\cos(a+bx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0390279, size = 22, normalized size = 1.

$$\frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] 2/(5*b*d*(d*Cos[a + b*x])^(5/2))
```

Maple [A] time = 0.004, size = 19, normalized size = 0.9

$$\frac{2}{5bd}(d\cos(bx+a))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] 2/5/b/d/(d*cos(b*x+a))^(5/2)
```

Maxima [A] time = 0.961368, size = 24, normalized size = 1.09

$$\frac{2}{5(d\cos(bx+a))^{\frac{5}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5/((d*cos(b*x + a))^(5/2)*b*d)
```

Fricas [A] time = 1.76498, size = 66, normalized size = 3.

$$\frac{2\sqrt{d\cos(bx+a)}}{5bd^4\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(d*cos(b*x + a))/(b*d^4*cos(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15505, size = 35, normalized size = 1.59

$$\frac{2}{5\sqrt{d\cos(bx+a)}bd^3\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] 2/5/(sqrt(d*cos(b*x + a))*b*d^3*cos(b*x + a)^2)
```

3.193 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

```
[Out] 2/(7*b*d*(d*Cos[a + b*x])^(7/2))
```

Rubi [A] time = 0.0270828, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$\frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] 2/(7*b*d*(d*Cos[a + b*x])^(7/2))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{9/2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{7bd(d\cos(a+bx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.0625983, size = 22, normalized size = 1.

$$\frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] 2/(7*b*d*(d*Cos[a + b*x])^(7/2))
```

Maple [A] time = 0.004, size = 19, normalized size = 0.9

$$\frac{2}{7\,bd}\,(d\cos(bx+a))^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x)
```

```
[Out] 2/7/b/d/(d*cos(b*x+a))^(7/2)
```

Maxima [A] time = 0.975963, size = 24, normalized size = 1.09

$$\frac{2}{7\,(d\cos(bx+a))^{\frac{7}{2}}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] 2/7/((d*cos(b*x + a))^(7/2)*b*d)
```

Fricas [A] time = 1.72055, size = 66, normalized size = 3.

$$\frac{2\,\sqrt{d\cos(bx+a)}}{7\,bd^5\cos(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] 2/7*sqrt(d*cos(b*x + a))/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18283, size = 35, normalized size = 1.59

$$\frac{2}{7\,\sqrt{d\cos(bx+a)}bd^4\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

```
[Out] 2/7/(sqrt(d*cos(b*x + a))*b*d^4*cos(b*x + a)^3)
```

3.194 $\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx$

Optimal. Leaf size=126

$$\frac{28d^3\sin(a+bx)(d\cos(a+bx))^{3/2}}{585b}+\frac{28d^4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{195b\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}+\frac{4d\sin}{}$$

```
[Out] (28*d^4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(195*b*Sqrt[Cos[a +
 b*x]]) + (28*d^3*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(585*b) + (4*d*(d*Co
s[a + b*x])^(7/2)*Sin[a + b*x])/(117*b) - (2*(d*Cos[a + b*x])^(11/2)*Sin[a
+ b*x])/(13*b*d)
```

Rubi [A] time = 0.100993, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2568, 2635, 2640, 2639}

$$\frac{28d^3\sin(a+bx)(d\cos(a+bx))^{3/2}}{585b}+\frac{28d^4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{195b\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}+\frac{4d\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^2,x]
```

```
[Out] (28*d^4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(195*b*Sqrt[Cos[a +
 b*x]]) + (28*d^3*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(585*b) + (4*d*(d*Co
s[a + b*x])^(7/2)*Sin[a + b*x])/(117*b) - (2*(d*Cos[a + b*x])^(11/2)*Sin[a
+ b*x])/(13*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{11/2}\sin(a+bx)}{13bd} + \frac{2}{13}\int (d\cos(a+bx))^{9/2}\,dx$$
$$= \frac{4d(d\cos(a+bx))^{7/2}\sin(a+bx)}{117b} - \frac{2(d\cos(a+bx))^{11/2}\sin(a+bx)}{13bd} + \frac{1}{117}\left(14d^2\right)\int$$
$$= \frac{28d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{585b} + \frac{4d(d\cos(a+bx))^{7/2}\sin(a+bx)}{117b} - \frac{2(d\cos(a-}{}$$
$$= \frac{28d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{585b} + \frac{4d(d\cos(a+bx))^{7/2}\sin(a+bx)}{117b} - \frac{2(d\cos(a-}{}$$
$$= \frac{28d^4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{195b\sqrt{\cos(a+bx)}} + \frac{28d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{585b} + \frac{4d(d\,c}{}$$

Mathematica [C] time = 0.135806, size = 60, normalized size = 0.48

$$\frac{d^2\sqrt[4]{\cos^2(a+bx)}\tan^3(a+bx)(d\cos(a+bx))^{5/2}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{2};\frac{5}{2};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^2,x]
```

```
[Out] (d^2*(d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4,
3/2, 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)
```

Maple [A] time = 0.095, size = 249, normalized size = 2.

$$\frac{4\,d^5}{585\,b}\sqrt{d\left(2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(2880\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{15}-11520\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{13}+1928\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x)
```

```
[Out] 4/585*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^5*(2880*c
os(1/2*b*x+1/2*a)^15-11520*cos(1/2*b*x+1/2*a)^13+19280*cos(1/2*b*x+1/2*a)^1
1-17520*cos(1/2*b*x+1/2*a)^9+9284*cos(1/2*b*x+1/2*a)^7-2808*cos(1/2*b*x+1/2
*a)^5+425*cos(1/2*b*x+1/2*a)^3+21*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*
b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-21*cos(1/2*b*x+
1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*
x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{9}{2}}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(d^4\cos\left(bx+a\right)^6-d^4\cos\left(bx+a\right)^4\right)\sqrt{d\cos\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(d^4*cos(b*x + a)^6 - d^4*cos(b*x + a)^4)*sqrt(d*cos(b*x + a)), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(9/2)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}\sin\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^2, x)
```

3.195 $\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx$

Optimal. Leaf size=126

$$\frac{20d^3\sin(a+bx)\sqrt{d\cos(a+bx)}}{231b}+\frac{20d^4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{231b\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}+\frac{4d\sin(a+bx)(}{7}$$

```
[Out] (20*d^4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(231*b*Sqrt[d*Cos[a +
 b*x]]) + (20*d^3*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(231*b) + (4*d*(d*Cos[
a + b*x])^(5/2)*Sin[a + b*x])/(77*b) - (2*(d*Cos[a + b*x])^(9/2)*Sin[a + b*
x])/(11*b*d)
```

Rubi [A] time = 0.0977436, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2568, 2635, 2642, 2641}

$$\frac{20d^3\sin(a+bx)\sqrt{d\cos(a+bx)}}{231b}+\frac{20d^4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{231b\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}+\frac{4d\sin(a+bx)(}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^2,x]
```

```
[Out] (20*d^4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(231*b*Sqrt[d*Cos[a +
 b*x]]) + (20*d^3*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(231*b) + (4*d*(d*Cos[
a + b*x])^(5/2)*Sin[a + b*x])/(77*b) - (2*(d*Cos[a + b*x])^(9/2)*Sin[a + b*
x])/(11*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{9/2}\sin(a+bx)}{11bd} + \frac{2}{11}\int (d\cos(a+bx))^{7/2}\,dx$$
$$= \frac{4d(d\cos(a+bx))^{5/2}\sin(a+bx)}{77b} - \frac{2(d\cos(a+bx))^{9/2}\sin(a+bx)}{11bd} + \frac{1}{77}\left(10d^2\right)$$
$$= \frac{20d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b} + \frac{4d(d\cos(a+bx))^{5/2}\sin(a+bx)}{77b} - \frac{2(d\cos(a}{}$$
$$= \frac{20d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b} + \frac{4d(d\cos(a+bx))^{5/2}\sin(a+bx)}{77b} - \frac{2(d\cos(a}{}$$
$$= \frac{20d^4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{231b\sqrt{d\cos(a+bx)}} + \frac{20d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b} + \frac{4d(d\,c}{}$$

Mathematica [C] time = 0.133755, size = 60, normalized size = 0.48

$$\frac{d^2\cos^2(a+bx)^{3/4}\tan^3(a+bx)(d\cos(a+bx))^{3/2}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};\frac{5}{2};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^2,x]
```

```
[Out] (d^2*(d*Cos[a + b*x])^(3/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4,
3/2, 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)
```

Maple [A] time = 0.059, size = 236, normalized size = 1.9

$$\frac{4\,d^4}{231\,b}\sqrt{d\left(2\,(\cos(1/2\,bx+a/2))^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(672\,(\cos(1/2\,bx+a/2))^{13}-2352\,(\cos(1/2\,bx+a/2))^{11}+331\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x)
```

```
[Out] 4/231*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^4*(672*co
s(1/2*b*x+1/2*a)^13-2352*cos(1/2*b*x+1/2*a)^11+3312*cos(1/2*b*x+1/2*a)^9-24
00*cos(1/2*b*x+1/2*a)^7+922*cos(1/2*b*x+1/2*a)^5-159*cos(1/2*b*x+1/2*a)^3-5
*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticF(c
os(1/2*b*x+1/2*a),2^(1/2))+5*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^
4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^
2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{7}{2}}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(d^3\cos\left(bx+a\right)^5 - d^3\cos\left(bx+a\right)^3\right)\sqrt{d\cos\left(bx+a\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(d^3*cos(b*x + a)^5 - d^3*cos(b*x + a)^3)*sqrt(d*cos(b*x + a)), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(7/2)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}\sin\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^2, x)
```

3.196 $\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx$

Optimal. Leaf size=98

$$\frac{4d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{15b\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd}+\frac{4d\sin(a+bx)(d\cos(a+bx))^{3/2}}{45b}$$

```
[Out] (4*d^2*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(15*b*Sqrt[Cos[a + b
*x]]) + (4*d*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(45*b) - (2*(d*Cos[a + b*
x])^(7/2)*Sin[a + b*x])/(9*b*d)
```

Rubi [A] time = 0.0782281, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2568, 2635, 2640, 2639}

$$\frac{4d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{15b\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd}+\frac{4d\sin(a+bx)(d\cos(a+bx))^{3/2}}{45b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^2,x]
```

```
[Out] (4*d^2*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(15*b*Sqrt[Cos[a + b
*x]]) + (4*d*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(45*b) - (2*(d*Cos[a + b*
x])^(7/2)*Sin[a + b*x])/(9*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{7/2}\sin(a+bx)}{9bd} + \frac{2}{9}\int (d\cos(a+bx))^{5/2}\,dx$$
$$= \frac{4d(d\cos(a+bx))^{3/2}\sin(a+bx)}{45b} - \frac{2(d\cos(a+bx))^{7/2}\sin(a+bx)}{9bd} + \frac{1}{15}\left(2d^2\right)\int \sqrt{}$$
$$= \frac{4d(d\cos(a+bx))^{3/2}\sin(a+bx)}{45b} - \frac{2(d\cos(a+bx))^{7/2}\sin(a+bx)}{9bd} + \frac{\left(2d^2\sqrt{d\cos(a}\right.}{15}$$
$$= \frac{4d^2\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b\sqrt{\cos(a+bx)}} + \frac{4d(d\cos(a+bx))^{3/2}\sin(a+bx)}{45b} - 2(d\cos(a$$

Mathematica [C] time = 0.0499047, size = 57, normalized size = 0.58

$$\frac{\sqrt[4]{\cos^2(a+bx)}\tan^3(a+bx)(d\cos(a+bx))^{5/2}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{5}{2};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^2,x]
```

```
[Out] ((d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, 3/2,
 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)
```

Maple [B] time = 0.063, size = 223, normalized size = 2.3

$$\frac{4d^3}{45b}\sqrt{d\left(2\left(\cos\left(1/2\,bx+a/2\right)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(80\left(\cos\left(1/2\,bx+a/2\right)\right)^{11}-240\left(\cos\left(1/2\,bx+a/2\right)\right)^9+272\left(\cos(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x)
```

```
[Out] 4/45*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^3*(80*cos(
1/2*b*x+1/2*a)^11-240*cos(1/2*b*x+1/2*a)^9+272*cos(1/2*b*x+1/2*a)^7-144*cos
(1/2*b*x+1/2*a)^5+35*cos(1/2*b*x+1/2*a)^3+3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-
2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-3*cos
(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/s
in(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{5}{2}}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(d^2\cos\left(bx+a\right)^4-d^2\cos\left(bx+a\right)^2\right)\sqrt{d\cos\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*sqrt(d*cos(b*x + a)), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(5/2)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}\sin\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^2, x)
```

3.197 $\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx$

Optimal. Leaf size=98

$$\frac{4d^2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}+\frac{4d\sin(a+bx)\sqrt{d\cos(a+bx)}}{21b}$$

```
[Out] (4*d^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(21*b*Sqrt[d*Cos[a + b
*x]]) + (4*d*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(21*b) - (2*(d*Cos[a + b*x]
)^(5/2)*Sin[a + b*x])/(7*b*d)
```

Rubi [A] time = 0.0788385, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2568, 2635, 2642, 2641}

$$\frac{4d^2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}+\frac{4d\sin(a+bx)\sqrt{d\cos(a+bx)}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^2,x]
```

```
[Out] (4*d^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(21*b*Sqrt[d*Cos[a + b
*x]]) + (4*d*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(21*b) - (2*(d*Cos[a + b*x]
)^(5/2)*Sin[a + b*x])/(7*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{5/2}\sin(a+bx)}{7bd} + \frac{2}{7}\int (d\cos(a+bx))^{3/2}\,dx$$
$$= \frac{4d\sqrt{d\cos(a+bx)}\sin(a+bx)}{21b} - \frac{2(d\cos(a+bx))^{5/2}\sin(a+bx)}{7bd} + \frac{1}{21}\left(2d^2\right)\int$$
$$= \frac{4d\sqrt{d\cos(a+bx)}\sin(a+bx)}{21b} - \frac{2(d\cos(a+bx))^{5/2}\sin(a+bx)}{7bd} + \frac{\left(2d^2\sqrt{\cos(a}\right.}{21\sqrt{}}$$
$$= \frac{4d^2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b\sqrt{d\cos(a+bx)}} + \frac{4d\sqrt{d\cos(a+bx)}\sin(a+bx)}{21b} - 2(d\cos(a$$

Mathematica [C] time = 0.0707112, size = 57, normalized size = 0.58

$$\frac{\cos^2(a+bx)^{3/4}\tan^3(a+bx)(d\cos(a+bx))^{3/2}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{5}{2};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^2,x]
```

```
[Out] ((d*Cos[a + b*x])^(3/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 3/2,
 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)
```

Maple [A] time = 0.052, size = 208, normalized size = 2.1

$$\frac{4\,d^2}{21\,b}\sqrt{d\left(2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(24\,\left(\cos\left(1/2\,bx+a/2\right)\right)^9-60\,\left(\cos\left(1/2\,bx+a/2\right)\right)^7+50\,\left(\cos\left(1/\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x)
```

```
[Out] 4/21*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^2*(24*cos(
1/2*b*x+1/2*a)^9-60*cos(1/2*b*x+1/2*a)^7+50*cos(1/2*b*x+1/2*a)^5-15*cos(1/2
*b*x+1/2*a)^3-(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2
)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*
b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*
b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(d\cos(bx+a)^{3}-d\cos(bx+a)\right)\sqrt{d\cos(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*cos(b*x + a)^3 - d*cos(b*x + a))*sqrt(d*cos(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.198 $\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx$

Optimal. Leaf size=69

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}$$

```
[Out] (4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]])
 - (2*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(5*b*d)
```

Rubi [A] time = 0.0562589, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2568, 2640, 2639}

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^2,x]
```

```
[Out] (4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]])
 - (2*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(5*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx &= -\frac{2(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd}+\frac{2}{5}\int \sqrt{d\cos(a+bx)}\,dx\\ &= -\frac{2(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd}+\frac{\left(2\sqrt{d\cos(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{5\sqrt{\cos(a+bx)}}\\ &= \frac{4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}}-\frac{2(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd}\end{aligned}$$

Mathematica [C] time = 0.0861385, size = 58, normalized size = 0.84

$$\frac{d\sin^3(a+bx)\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{3}{2};\frac{5}{2};\sin^2(a+bx)\right)}{3b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^2,x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 3/2, 5/2, Sin[a + b*x]^2]*
Sin[a + b*x]^3)/(3*b*Sqrt[d*Cos[a + b*x]])
```

Maple [B] time = 0.052, size = 194, normalized size = 2.8

$$\frac{4d}{5b}\sqrt{d\left(2\left(\cos(1/2\,bx+a/2)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(4\left(\cos(1/2\,bx+a/2)\right)^7-8\left(\cos(1/2\,bx+a/2)\right)^5+5\left(\cos(1/2\,bx+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x)
```

```
[Out] 4/5*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d*(4*cos(1/2*
b*x+1/2*a)^7-8*cos(1/2*b*x+1/2*a)^5+5*cos(1/2*b*x+1/2*a)^3+(sin(1/2*b*x+1/2
*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a)
,2^(1/2))-cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a
)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\cos(bx+a)}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\sqrt{d\cos(bx+a)}\left(\cos(bx+a)^2-1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cos(bx+a)}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^2, x)
```

3.199 $\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=69

$$\frac{4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}$$

```
[Out] (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]])
 - (2*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(3*b*d)
```

Rubi [A] time = 0.0578891, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2568, 2642, 2641}

$$\frac{4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]])
 - (2*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(3*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx &= -\frac{2\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd}+\frac{2}{3}\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx\\
&= -\frac{2\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd}+\frac{\left(2\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{3\sqrt{d\cos(a+bx)}}\\
&= \frac{4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{d\cos(a+bx)}}-\frac{2\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd}
\end{aligned}$$

Mathematica [C] time = 0.105022, size = 58, normalized size = 0.84

$$\frac{d\sin^3(a+bx)\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{5}{2};\sin^2(a+bx)\right)}{3b(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/2, 5/2, Sin[a + b*x]^2]*
Sin[a + b*x]^3)/(3*b*(d*Cos[a + b*x])^(3/2))
```

Maple [B] time = 0.053, size = 188, normalized size = 2.7

$$\frac{4}{3b}\sqrt{d\left(2\left(\cos\left(1/2\,bx+a/2\right)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(2\cos\left(1/2\,bx+a/2\right)\left(\sin\left(1/2\,bx+a/2\right)\right)^4-\sqrt{\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] 4/3*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*cos(1/2*b*
x+1/2*a)*sin(1/2*b*x+1/2*a)^4-(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1
/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))-sin(1/2*b*x+1/2*a)^2
*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/
2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(bx+a\right)^2}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/sqrt(d*cos(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{d\cos\left(bx+a\right)}\left(\cos\left(bx+a\right)^2-1\right)}{d\cos\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 1)/(d*cos(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^2}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/sqrt(d*cos(b*x + a)), x)
```

3.200 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\cos(a+bx)}}$$

```
[Out] (-4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*d^2*Sqrt[Cos[a + b*x
]]) + (2*Sin[a + b*x])/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.0637746, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2566, 2640, 2639}

$$\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (-4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*d^2*Sqrt[Cos[a + b*x
]]) + (2*Sin[a + b*x])/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx &= \frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{2\int \sqrt{d\cos(a+bx)}\,dx}{d^2}\\ &= \frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\left(2\sqrt{d\cos(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{d^2\sqrt{\cos(a+bx)}}\\ &= -\frac{4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{bd^2\sqrt{\cos(a+bx)}}+\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [C] time = 0.0817393, size = 60, normalized size = 0.88

$$\frac{\sin^3(a+bx)\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{5}{4},\frac{3}{2};\frac{5}{2};\sin^2(a+bx)\right)}{3bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 3/2, 5/2, Sin[a + b*x]^2]*Si
n[a + b*x]^3)/(3*b*d*Sqrt[d*Cos[a + b*x]])
```

Maple [A] time = 0.065, size = 168, normalized size = 2.5

$$-4\frac{\sqrt{-2\left(\sin(1/2\,bx+a/2)\right)^4d+\left(\sin(1/2\,bx+a/2)\right)^2d}\left(\sqrt{2\left(\sin(1/2\,bx+a/2)\right)^2-1}\sqrt{\left(\sin(1/2\,bx+a/2)\right)^2}\mathrm{EllipticE}\left(\cos\right.\right.}{d\sqrt{-d\left(2\left(\sin(1/2\,bx+a/2)\right)^4-\left(\sin(1/2\,bx+a/2)\right)^2\right)}\sin(1/2\,bx+a/2)\sqrt{d\left(2\,(\mathrm{c}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] -4/d*(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(1/2)*((2*sin(1/2*b
*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2
*a),2^(1/2))-sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/
2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/
2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{d\cos(bx+a)}\left(\cos(bx+a)^2-1\right)}{d^2\cos(bx+a)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 1)/(d^2*cos(b*x + a)^2), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

3.201 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}$$

```
[Out] (-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a + b
*x]]) + (2*Sin[a + b*x])/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.0644769, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2566, 2642, 2641}

$$\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a + b
*x]]) + (2*Sin[a + b*x])/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx &= \frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx}{3d^2}\\ &= \frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{\left(2\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{3d^2\sqrt{d\cos(a+bx)}}\\ &= -\frac{4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.0809506, size = 60, normalized size = 0.83

$$\frac{\sin^3(a+bx)\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{3}{2},\frac{7}{4};\frac{5}{2};\sin^2(a+bx)\right)}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/2, 7/4, 5/2, Sin[a + b*x]^2]*Si
n[a + b*x]^3)/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Maple [B] time = 0.063, size = 242, normalized size = 3.4

$$-\frac{4}{3\,d^2b}\left(2\sqrt{(\sin(1/2\,bx+a/2))^2}\sqrt{2\,(\sin(1/2\,bx+a/2))^2-1}\text{EllipticF}\left(\cos(1/2\,bx+a/2),\sqrt{2}\right)(\sin(1/2\,bx+a/2))^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] -4/3*(2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Ellip
ticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2-(sin(1/2*b*x+1/2*a)^2
)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/
2))-sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/d^2*(d*(2*cos(1/2*b*x+1/2*a)^2
-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)/(-d*(2*sin(1/2*b
*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b
*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{d\cos(bx+a)}\left(\cos(bx+a)^2-1\right)}{d^3\cos(bx+a)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 1)/(d^3*cos(b*x + a)^3), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

3.202 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\cos(a+bx)}}+\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}$$

```
[Out] (4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*d^4*Sqrt[Cos[a + b*
x]]) + (2*Sin[a + b*x])/(5*b*d*(d*Cos[a + b*x])^(5/2)) - (4*Sin[a + b*x])/(
5*b*d^3*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.0817502, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2566, 2636, 2640, 2639}

$$-\frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\cos(a+bx)}}+\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*d^4*Sqrt[Cos[a + b*
x]]) + (2*Sin[a + b*x])/(5*b*d*(d*Cos[a + b*x])^(5/2)) - (4*Sin[a + b*x])/(
5*b*d^3*Sqrt[d*Cos[a + b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{2\int \frac{1}{(d\cos(a+bx))^{3/2}}\,dx}{5d^2}$$
$$= \frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{2\int \sqrt{d\cos(a+bx)}\,dx}{5d^4}$$
$$= \frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{\left(2\sqrt{d\cos(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{5d^4\sqrt{\cos(a+bx)}}$$
$$= \frac{4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^4\sqrt{\cos(a+bx)}} + \frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}$$

Mathematica [C] time = 0.0776264, size = 59, normalized size = 0.59

$$\frac{\sin^3(2(a+bx))\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{3}{2},\frac{9}{4};\frac{5}{2};\sin^2(a+bx)\right)}{24b(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[3/2, 9/4, 5/2, Sin[a + b*x]^2]*Si
n[2*(a + b*x)]^3)/(24*b*(d*Cos[a + b*x])^(7/2))
```

Maple [B] time = 0.091, size = 365, normalized size = 3.7

$$-\frac{4}{5\,d^4b}\sqrt{d\left(2\,(\cos(1/2\,bx+a/2))^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(4\sqrt{2\,(\sin(1/2\,bx+a/2))^2-1}\sqrt{(\sin(1/2\,bx+a/2))^2}\mathrm{EllipticE}\right($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] -4/5*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^4/sin(1/2*
b*x+1/2*a)^3/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+
1/2*a)^2-1)*(4*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2
)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^4-8*sin(1/2*b*x+
1/2*a)^6*cos(1/2*b*x+1/2*a)-4*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x
+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2
+8*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)
*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-sin(1/2
*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))*(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/
2*a)^2*d)^(1/2)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{d\cos\left(bx+a\right)}\left(\cos\left(bx+a\right)^2-1\right)}{d^4\cos\left(bx+a\right)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 1)/(d^4*cos(b*x + a)^4), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(bx+a\right)^2}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

3.203 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}}-\frac{4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21bd^4\sqrt{d\cos(a+bx)}}+\frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}}$$

```
[Out] (-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(21*b*d^4*Sqrt[d*Cos[a +
b*x]]) + (2*Sin[a + b*x])/(7*b*d*(d*Cos[a + b*x])^(7/2)) - (4*Sin[a + b*x])
/(21*b*d^3*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.0819411, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2566, 2636, 2642, 2641}

$$-\frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}}-\frac{4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21bd^4\sqrt{d\cos(a+bx)}}+\frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(9/2),x]
```


```
[Out] (-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(21*b*d^4*Sqrt[d*Cos[a +
b*x]]) + (2*Sin[a + b*x])/(7*b*d*(d*Cos[a + b*x])^(7/2)) - (4*Sin[a + b*x])
/(21*b*d^3*(d*Cos[a + b*x])^(3/2))
```


Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{2\int \frac{1}{(d\cos(a+bx))^{5/2}}\,dx}{7d^2}$$
$$= \frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}} - \frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx}{21d^4}$$
$$= \frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}} - \frac{\left(2\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{21d^4\sqrt{d\cos(a+bx)}}$$
$$= -\frac{4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21bd^4\sqrt{d\cos(a+bx)}} + \frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}}$$

Mathematica [C] time = 0.0641523, size = 59, normalized size = 0.59

$$\frac{\sin^3(2(a+bx))\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{3}{2},\frac{11}{4};\frac{5}{2};\sin^2(a+bx)\right)}{24b(d\cos(a+bx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/2, 11/4, 5/2, Sin[a + b*x]^2]*S
in[2*(a + b*x)]^3)/(24*b*(d*Cos[a + b*x])^(9/2))
```

Maple [B] time = 0.068, size = 396, normalized size = 4.

$$\frac{4}{21\,d^4b}\left(-8\sqrt{(\sin(1/2\,bx+a/2))^2}\sqrt{2\,(\sin(1/2\,bx+a/2))^2-1}\mathrm{EllipticF}\left(\cos(1/2\,bx+a/2),\sqrt{2}\right)(\sin(1/2\,bx+a/2))^6+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x)
```

```
[Out] 4/21*(-8*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Elli
pticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^6+12*(sin(1/2*b*x+1/2*
a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2
^(1/2))*sin(1/2*b*x+1/2*a)^4-8*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)-6*(s
in(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1
/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2+8*cos(1/2*b*x+1/2*a)*sin(1/2*b*
x+1/2*a)^4+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*El
lipticF(cos(1/2*b*x+1/2*a),2^(1/2))+sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)
)/d^4*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(-d*(2*sin(
1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)^3/
sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{d\cos\left(bx+a\right)}\left(\cos\left(bx+a\right)^{2}-1\right)}{d^{5}\cos\left(bx+a\right)^{5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 1)/(d^5*cos(b*x + a)^5), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(bx+a\right)^{2}}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(9/2), x)
```

3.204 $\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx$

Optimal. Leaf size=45

$$\frac{2(d\cos(a+bx))^{7/2}}{7bd^3}-\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

```
[Out] (-2*(d*Cos[a + b*x])^(3/2))/(3*b*d) + (2*(d*Cos[a + b*x])^(7/2))/(7*b*d^3)
```

Rubi [A] time = 0.0421826, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2565, 14}

$$\frac{2(d\cos(a+bx))^{7/2}}{7bd^3}-\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^3,x]
```

```
[Out] (-2*(d*Cos[a + b*x])^(3/2))/(3*b*d) + (2*(d*Cos[a + b*x])^(7/2))/(7*b*d^3)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \sqrt{x}\left(1-\frac{x^2}{d^2}\right)dx, x, d\cos(a+bx)\right)}{bd}\\
&= -\frac{\text{Subst}\left(\int \left(\sqrt{x}-\frac{x^{5/2}}{d^2}\right)dx, x, d\cos(a+bx)\right)}{bd}\\
&= -\frac{2(d\cos(a+bx))^{3/2}}{3bd}+\frac{2(d\cos(a+bx))^{7/2}}{7bd^3}
\end{aligned}$$

Mathematica [A] time = 0.32286, size = 57, normalized size = 1.27

$$-\frac{d\left(3\sin^2(2(a+bx))+16\cos^2(a+bx)-16\sqrt[4]{\cos^2(a+bx)}\right)}{42b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^3,x]
```

```
[Out] -(d*(16*Cos[a + b*x]^2 - 16*(Cos[a + b*x]^2)^(1/4) + 3*Sin[2*(a + b*x)]^2))
/(42*b*Sqrt[d*Cos[a + b*x]])
```

Maple [A] time = 0.043, size = 63, normalized size = 1.4

$$-\frac{8}{21\,b}\sqrt{-2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}d+d}\left(6\,\left(\sin\left(1/2\,bx+a/2\right)\right)^{6}-9\,\left(\sin\left(1/2\,bx+a/2\right)\right)^{4}+\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x)
```

```
[Out] -8/21*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(6*sin(1/2*b*x+1/2*a)^6-9*sin(1/2
*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2+1)/b
```

Maxima [A] time = 0.973501, size = 49, normalized size = 1.09

$$\frac{2\left(3\,\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}-7\,\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}d^{2}\right)}{21\,bd^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2/21*(3*(d*cos(b*x + a))^(7/2) - 7*(d*cos(b*x + a))^(3/2)*d^2)/(b*d^3)
```

Fricas [A] time = 1.8949, size = 88, normalized size = 1.96

$$\frac{2\left(3\,\cos\left(bx+a\right)^{3}-7\,\cos\left(bx+a\right)\right)\sqrt{d\cos\left(bx+a\right)}}{21\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 2/21*(3*cos(b*x + a)^3 - 7*cos(b*x + a))*sqrt(d*cos(b*x + a))/b
```

Sympy [A] time = 11.4533, size = 65, normalized size = 1.44

$$\begin{cases} -\frac{2\sqrt{d}\sin^{2}\left(a+bx\right)\cos^{\frac{3}{2}}\left(a+bx\right)}{3b}-\frac{8\sqrt{d}\cos^{\frac{7}{2}}\left(a+bx\right)}{21b} & \text{for}\ b\neq 0\\ x\sqrt{d\cos\left(a\right)}\sin^{3}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-2*sqrt(d)*sin(a + b*x)**2*cos(a + b*x)**(3/2)/(3*b) - 8*sqrt(d)
*cos(a + b*x)**(7/2)/(21*b), Ne(b, 0)), (x*sqrt(d*cos(a))*sin(a)**3, True))
```

Giac [A] time = 87.9465, size = 49, normalized size = 1.09

$$-\frac{2\left(7\,(d\cos(bx+a))^{\frac{3}{2}}-\frac{3\,(d\cos(bx+a))^{\frac{7}{2}}}{d^2}\right)}{21\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2/21*(7*(d*cos(b*x + a))^(3/2) - 3*(d*cos(b*x + a))^(7/2)/d^2)/(b*d)
```

3.205 $\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=43

$$\frac{2(d\cos(a+bx))^{5/2}}{5bd^3}-\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

```
[Out] (-2*Sqrt[d*Cos[a + b*x]])/(b*d) + (2*(d*Cos[a + b*x])^(5/2))/(5*b*d^3)
```

Rubi [A] time = 0.0462557, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2565, 14}

$$\frac{2(d\cos(a+bx))^{5/2}}{5bd^3}-\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (-2*Sqrt[d*Cos[a + b*x]])/(b*d) + (2*(d*Cos[a + b*x])^(5/2))/(5*b*d^3)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{\sqrt{x}}\,dx, x, d\cos(a+bx)\right)}{bd}\\
&= -\frac{\operatorname{Subst}\left(\int \left(\frac{1}{\sqrt{x}}-\frac{x^{3/2}}{d^2}\right)dx, x, d\cos(a+bx)\right)}{bd}\\
&= -\frac{2\sqrt{d\cos(a+bx)}}{bd}+\frac{2(d\cos(a+bx))^{5/2}}{5bd^3}
\end{aligned}$$

Mathematica [A] time = 0.178724, size = 57, normalized size = 1.33

$$\frac{\cos(a+bx)(\cos(2(a+bx))-9)+8\cos^2(a+bx)^{3/4}\sec(a+bx)}{5b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (Cos[a + b*x]*(-9 + Cos[2*(a + b*x)]) + 8*(Cos[a + b*x]^2)^(3/4)*Sec[a + b*
x])/(5*b*Sqrt[d*Cos[a + b*x]])
```

Maple [B] time = 0.033, size = 92, normalized size = 2.1

$$\frac{1}{5\,db}\left(8\,\sqrt{-2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}d+d}\left(\sin\left(1/2\,bx+a/2\right)\right)^{4}-8\,\sqrt{-2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}d+d}\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] 1/5*(8*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*sin(1/2*b*x+1/2*a)^4-8*(-2*sin(1
/2*b*x+1/2*a)^2*d+d)^(1/2)*sin(1/2*b*x+1/2*a)^2-8*(-2*sin(1/2*b*x+1/2*a)^2*
d+d)^(1/2))/d/b
```

Maxima [A] time = 0.983746, size = 49, normalized size = 1.14

$$-\frac{2\left(5\,\sqrt{d\cos\left(bx+a\right)}-\frac{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}{d^{2}}\right)}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*(5*sqrt(d*cos(b*x + a)) - (d*cos(b*x + a))^(5/2)/d^2)/(b*d)
```

Fricas [A] time = 1.87462, size = 72, normalized size = 1.67

$$\frac{2\,\sqrt{d\cos\left(bx+a\right)}\left(\cos\left(bx+a\right)^{2}-5\right)}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 5)/(b*d)
```

Sympy [A] time = 5.35721, size = 63, normalized size = 1.47

$$\begin{cases} -\frac{2\sin^{2}\left(a+bx\right)\sqrt{\cos\left(a+bx\right)}}{b\sqrt{d}}-\frac{8\cos^{\frac{5}{2}}\left(a+bx\right)}{5b\sqrt{d}} & \text{for}\ b\neq 0\\ \frac{x\sin^{3}\left(a\right)}{\sqrt{d\cos\left(a\right)}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((-2*sin(a + b*x)**2*sqrt(cos(a + b*x))/(b*sqrt(d)) - 8*cos(a + b*
x)**(5/2)/(5*b*sqrt(d)), Ne(b, 0)), (x*sin(a)**3/sqrt(d*cos(a)), True))
```

Giac [A] time = 1.18635, size = 62, normalized size = 1.44

$$\frac{2\left(\sqrt{d\cos\left(bx+a\right)}d^2\cos\left(bx+a\right)^2-5\sqrt{d\cos\left(bx+a\right)}d^2\right)}{5\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(sqrt(d*cos(b*x + a))*d^2*cos(b*x + a)^2 - 5*sqrt(d*cos(b*x + a))*d^2)/
(b*d^3)
```

3.206 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2(d\cos(a+bx))^{3/2}}{3bd^3}+\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] 2/(b*d*Sqrt[d*Cos[a + b*x]]) + (2*(d*Cos[a + b*x])^(3/2))/(3*b*d^3)
```

Rubi [A] time = 0.0513659, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2565, 14}

$$\frac{2(d\cos(a+bx))^{3/2}}{3bd^3}+\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] 2/(b*d*Sqrt[d*Cos[a + b*x]]) + (2*(d*Cos[a + b*x])^(3/2))/(3*b*d^3)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{x^{3/2}}\,dx,x,d\cos(a+bx)\right)}{bd}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{3/2}}-\frac{\sqrt{x}}{d^2}\right)dx,x,d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{bd\sqrt{d\cos(a+bx)}}+\frac{2(d\cos(a+bx))^{3/2}}{3bd^3}\end{aligned}$$

Mathematica [A] time = 0.0747173, size = 46, normalized size = 1.07

$$-\frac{2\left(\sin^2(a+bx)+4\sqrt[4]{\cos^2(a+bx)}-4\right)}{3bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (-2*(-4 + 4*(Cos[a + b*x]^2)^(1/4) + Sin[a + b*x]^2))/(3*b*d*Sqrt[d*Cos[a +
 b*x]])
```

Maple [A] time = 0.1, size = 70, normalized size = 1.6

$$-\frac{8}{3\,d^2b}\sqrt{-2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^2d+d}\left(\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4-\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2+1\right)\left(2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^2-1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] -8/3/d^2*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(sin(1/2*b*x+1/2*a)^4-sin(1/2*
b*x+1/2*a)^2+1)/(2*sin(1/2*b*x+1/2*a)^2-1)/b
```

Maxima [A] time = 0.983387, size = 47, normalized size = 1.09

$$\frac{2\left(\frac{3}{\sqrt{d\cos(bx+a)}}+\frac{(d\cos(bx+a))^{\frac{3}{2}}}{d^2}\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3/sqrt(d*cos(b*x + a)) + (d*cos(b*x + a))^(3/2)/d^2)/(b*d)
```

Fricas [A] time = 1.87155, size = 92, normalized size = 2.14

$$\frac{2\,\sqrt{d\cos\left(bx+a\right)}\left(\cos\left(bx+a\right)^2+3\right)}{3\,bd^2\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 + 3)/(b*d^2*cos(b*x + a))
```

Sympy [A] time = 6.16376, size = 61, normalized size = 1.42

$$\begin{cases} \frac{2\sin^2(a+bx)}{bd^{\frac{3}{2}}\sqrt{\cos(a+bx)}}+\frac{8\cos^{\frac{3}{2}}(a+bx)}{3bd^{\frac{3}{2}}} & \text{for } b\neq 0 \\ \frac{x\sin^3(a)}{(d\cos(a))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((2*sin(a + b*x)**2/(b*d**(3/2)*sqrt(cos(a + b*x))) + 8*cos(a + b*
x)**(3/2)/(3*b*d**(3/2)), Ne(b, 0)), (x*sin(a)**3/(d*cos(a))**(3/2), True))
```

Giac [A] time = 1.20139, size = 57, normalized size = 1.33

$$\frac{2\left(\sqrt{d\cos\left(bx+a\right)}d\cos\left(bx+a\right)+\frac{3\,d^2}{\sqrt{d\cos(bx+a)}}\right)}{3\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(d*cos(b*x + a))*d*cos(b*x + a) + 3*d^2/sqrt(d*cos(b*x + a)))/(b*d
^3)
```

3.207 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{d\cos(a+bx)}}{bd^3}+\frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/(3*b*d*(d*Cos[a + b*x])^(3/2)) + (2*Sqrt[d*Cos[a + b*x]])/(b*d^3)
```

Rubi [A] time = 0.0517832, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2565, 14}

$$\frac{2\sqrt{d\cos(a+bx)}}{bd^3}+\frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] 2/(3*b*d*(d*Cos[a + b*x])^(3/2)) + (2*Sqrt[d*Cos[a + b*x]])/(b*d^3)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{x^{5/2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{5/2}}-\frac{1}{d^2\sqrt{x}}\right)dx, x, d\cos(a+bx)\right)}{bd}\\
&= \frac{2}{3bd(d\cos(a+bx))^{3/2}}+\frac{2\sqrt{d\cos(a+bx)}}{bd^3}
\end{aligned}$$

Mathematica [A] time = 0.111302, size = 48, normalized size = 1.12

$$-\frac{2\left(3\sin^2(a+bx)+4\cos^2(a+bx)^{3/4}-4\right)}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (-2*(-4 + 4*(Cos[a + b*x]^2)^(3/4) + 3*Sin[a + b*x]^2))/(3*b*d*(d*Cos[a + b
*x])^(3/2))
```

Maple [B] time = 0.089, size = 85, normalized size = 2.

$$\frac{8}{3\,d^3b}\sqrt{-2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^2d+d}\left(3\,\left(\sin\left(1/2\,bx+a/2\right)\right)^4-3\,\left(\sin\left(1/2\,bx+a/2\right)\right)^2+1\right)\left(4\,\left(\sin\left(1/2\,bx+a/2\right)\right)^4-4\,\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] 8/3/d^3/(4*sin(1/2*b*x+1/2*a)^4-4*sin(1/2*b*x+1/2*a)^2+1)*(-2*sin(1/2*b*x+1
/2*a)^2*d+d)^(1/2)*(3*sin(1/2*b*x+1/2*a)^4-3*sin(1/2*b*x+1/2*a)^2+1)/b
```

Maxima [A] time = 0.971462, size = 46, normalized size = 1.07

$$\frac{2\left(\frac{1}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}+\frac{3\sqrt{d\cos\left(bx+a\right)}}{d^2}\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(1/(d*cos(b*x + a))^(3/2) + 3*sqrt(d*cos(b*x + a))/d^2)/(b*d)
```

Fricas [A] time = 2.29473, size = 97, normalized size = 2.26

$$\frac{2\sqrt{d\cos\left(bx+a\right)}\left(3\,\cos\left(bx+a\right)^2+1\right)}{3\,bd^3\cos\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(d*cos(b*x + a))*(3*cos(b*x + a)^2 + 1)/(b*d^3*cos(b*x + a)^2)
```

Sympy [A] time = 67.2828, size = 63, normalized size = 1.47

$$\begin{cases}\frac{2\sin^2\left(a+bx\right)}{3bd^{\frac{5}{2}}\cos^{\frac{3}{2}}\left(a+bx\right)}+\frac{8\sqrt{\cos\left(a+bx\right)}}{3bd^{\frac{5}{2}}} & \text{for } b\neq 0\\ \frac{x\sin^3\left(a\right)}{\left(d\cos\left(a\right)\right)^{\frac{5}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((2*sin(a + b*x)**2/(3*b*d**(5/2)*cos(a + b*x)**(3/2)) + 8*sqrt(co
s(a + b*x))/(3*b*d**(5/2)), Ne(b, 0)), (x*sin(a)**3/(d*cos(a))**(5/2), True
))
```

Giac [A] time = 1.18174, size = 55, normalized size = 1.28

$$\frac{2\left(3\sqrt{d\cos\left(bx+a\right)}+\frac{d}{\sqrt{d\cos(bx+a)}\cos(bx+a)}\right)}{3\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(d*cos(b*x + a)) + d/(sqrt(d*cos(b*x + a))*cos(b*x + a)))/(b*d^3
)
```

3.208 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2}{5bd(d\cos(a+bx))^{5/2}}-\frac{2}{bd^3\sqrt{d\cos(a+bx)}}$$

```
[Out] 2/(5*b*d*(d*Cos[a + b*x])^(5/2)) - 2/(b*d^3*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.0508097, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2565, 14}

$$\frac{2}{5bd(d\cos(a+bx))^{5/2}}-\frac{2}{bd^3\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] 2/(5*b*d*(d*Cos[a + b*x])^(5/2)) - 2/(b*d^3*Sqrt[d*Cos[a + b*x]])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{x^{7/2}}\,dx,x,d\cos(a+bx)\right)}{bd}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{7/2}}-\frac{1}{d^2x^{3/2}}\right)dx,x,d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{5bd(d\cos(a+bx))^{5/2}}-\frac{2}{bd^3\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.245879, size = 70, normalized size = 1.63

$$\frac{2\tan^2(a+bx)\left(-4\sqrt[4]{\cos^2(a+bx)}+4\left(\sqrt[4]{\cos^2(a+bx)}-1\right)\csc^2(a+bx)+5\right)}{5bd^3\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*(5 - 4*(Cos[a + b*x]^2)^(1/4) + 4*(-1 + (Cos[a + b*x]^2)^(1/4))*Csc[a +
b*x]^2)*Tan[a + b*x]^2)/(5*b*d^3*Sqrt[d*Cos[a + b*x]])
```

Maple [B] time = 0.103, size = 98, normalized size = 2.3

$$\frac{8}{5\,d^4b}\sqrt{-2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^2d+d}\left(5\,\left(\sin\left(1/2\,bx+a/2\right)\right)^4-5\,\left(\sin\left(1/2\,bx+a/2\right)\right)^2+1\right)\left(8\,\left(\sin\left(1/2\,bx+a/2\right)\right)^6-12\,\left(\mathrm{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] 8/5/d^4/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a
)^2-1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(5*sin(1/2*b*x+1/2*a)^4-5*sin(1/
2*b*x+1/2*a)^2+1)/b
```

Maxima [A] time = 0.962176, size = 50, normalized size = 1.16

$$-\frac{2\left(5\,d^2\cos\left(bx+a\right)^2-d^2\right)}{5\,\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*(5*d^2*cos(b*x + a)^2 - d^2)/((d*cos(b*x + a))^(5/2)*b*d^3)
```

Fricas [A] time = 2.12197, size = 99, normalized size = 2.3

$$-\frac{2\,\sqrt{d\cos\left(bx+a\right)}\left(5\,\cos\left(bx+a\right)^2-1\right)}{5\,bd^4\cos\left(bx+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(d*cos(b*x + a))*(5*cos(b*x + a)^2 - 1)/(b*d^4*cos(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17994, size = 61, normalized size = 1.42

$$-\frac{2\left(5\,d^3\cos\left(bx+a\right)^2-d^3\right)}{5\,\sqrt{d\cos\left(bx+a\right)}bd^6\cos\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] -2/5*(5*d^3*cos(b*x + a)^2 - d^3)/(sqrt(d*cos(b*x + a))*b*d^6*cos(b*x + a)^
2)
```

3.209 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal. Leaf size=45

$$\frac{2}{7bd(d\cos(a+bx))^{7/2}}-\frac{2}{3bd^3(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/(7*b*d*(d*Cos[a + b*x])^(7/2)) - 2/(3*b*d^3*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.0502524, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2565, 14}

$$\frac{2}{7bd(d\cos(a+bx))^{7/2}}-\frac{2}{3bd^3(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] 2/(7*b*d*(d*Cos[a + b*x])^(7/2)) - 2/(3*b*d^3*(d*Cos[a + b*x])^(3/2))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{x^{9/2}}\,dx,x,d\cos(a+bx)\right)}{bd}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{9/2}}-\frac{1}{d^2x^{5/2}}\right)dx,x,d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{7bd(d\cos(a+bx))^{7/2}}-\frac{2}{3bd^3(d\cos(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.279019, size = 70, normalized size = 1.56

$$\frac{2\tan^2(a+bx)\left(-4\cos^2(a+bx)^{3/4}+4\left(\cos^2(a+bx)^{3/4}-1\right)\csc^2(a+bx)+7\right)}{21bd^3(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*(7 - 4*(Cos[a + b*x]^2)^(3/4) + 4*(-1 + (Cos[a + b*x]^2)^(3/4))*Csc[a +
b*x]^2)*Tan[a + b*x]^2)/(21*b*d^3*(d*Cos[a + b*x])^(3/2))
```

Maple [B] time = 0.149, size = 111, normalized size = 2.5

$$-\frac{8}{21\,d^5b}\sqrt{-2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^2d+d}\left(7\,\left(\sin\left(1/2\,bx+a/2\right)\right)^4-7\,\left(\sin\left(1/2\,bx+a/2\right)\right)^2+1\right)\left(16\,\left(\sin\left(1/2\,bx+a/2\right)\right)^8-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x)
```

```
[Out] -8/21/d^5/(16*sin(1/2*b*x+1/2*a)^8-32*sin(1/2*b*x+1/2*a)^6+24*sin(1/2*b*x+1
/2*a)^4-8*sin(1/2*b*x+1/2*a)^2+1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(7*si
n(1/2*b*x+1/2*a)^4-7*sin(1/2*b*x+1/2*a)^2+1)/b
```

Maxima [A] time = 0.972873, size = 50, normalized size = 1.11

$$-\frac{2\left(7\,d^2\cos\left(bx+a\right)^2-3\,d^2\right)}{21\,\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] -2/21*(7*d^2*cos(b*x + a)^2 - 3*d^2)/((d*cos(b*x + a))^(7/2)*b*d^3)
```

Fricas [A] time = 2.06844, size = 100, normalized size = 2.22

$$-\frac{2\sqrt{d\cos\left(bx+a\right)}\left(7\,\cos\left(bx+a\right)^2-3\right)}{21\,bd^5\cos\left(bx+a\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] -2/21*sqrt(d*cos(b*x + a))*(7*cos(b*x + a)^2 - 3)/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18774, size = 61, normalized size = 1.36

$$-\frac{2\left(7\,d^4\cos\left(bx+a\right)^2-3\,d^4\right)}{21\,\sqrt{d\cos\left(bx+a\right)}bd^8\cos\left(bx+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

```
[Out] -2/21*(7*d^4*cos(b*x + a)^2 - 3*d^4)/(sqrt(d*cos(b*x + a))*b*d^8*cos(b*x +
a)^3)
```

3.210 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx$

Optimal. Leaf size=45

$$\frac{2}{9bd(d\cos(a+bx))^{9/2}}-\frac{2}{5bd^3(d\cos(a+bx))^{5/2}}$$

```
[Out] 2/(9*b*d*(d*Cos[a + b*x])^(9/2)) - 2/(5*b*d^3*(d*Cos[a + b*x])^(5/2))
```

Rubi [A] time = 0.0513467, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2565, 14}

$$\frac{2}{9bd(d\cos(a+bx))^{9/2}}-\frac{2}{5bd^3(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] 2/(9*b*d*(d*Cos[a + b*x])^(9/2)) - 2/(5*b*d^3*(d*Cos[a + b*x])^(5/2))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{x^{11/2}}\,dx,x,d\cos(a+bx)\right)}{bd}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{11/2}}-\frac{1}{d^2x^{7/2}}\right)dx,x,d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{9bd(d\cos(a+bx))^{9/2}}-\frac{2}{5bd^3(d\cos(a+bx))^{5/2}}\end{aligned}$$

Mathematica [B] time = 0.547615, size = 94, normalized size = 2.09

$$\frac{2\tan^4(a+bx)\left(4\sqrt[4]{\cos^2(a+bx)}+4\left(\sqrt[4]{\cos^2(a+bx)}-1\right)\csc^4(a+bx)+\left(9-8\sqrt[4]{\cos^2(a+bx)}\right)\csc^2(a+bx)\right)}{45bd^5\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] (2*(4*(Cos[a + b*x]^2)^(1/4) + (9 - 8*(Cos[a + b*x]^2)^(1/4))*Csc[a + b*x]^
2 + 4*(-1 + (Cos[a + b*x]^2)^(1/4))*Csc[a + b*x]^4)*Tan[a + b*x]^4)/(45*b*d
^5*Sqrt[d*Cos[a + b*x]])
```

Maple [B] time = 0.178, size = 124, normalized size = 2.8

$$\frac{8}{45\,d^6b}\sqrt{-2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^2d+d}\left(9\,\left(\sin\left(1/2\,bx+a/2\right)\right)^4-9\,\left(\sin\left(1/2\,bx+a/2\right)\right)^2+1\right)\left(32\,\left(\sin\left(1/2\,bx+a/2\right)\right)^{10}-80$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x)
```

```
[Out] 8/45/d^6/(32*sin(1/2*b*x+1/2*a)^10-80*sin(1/2*b*x+1/2*a)^8+80*sin(1/2*b*x+1
/2*a)^6-40*sin(1/2*b*x+1/2*a)^4+10*sin(1/2*b*x+1/2*a)^2-1)*(-2*sin(1/2*b*x+
1/2*a)^2*d+d)^(1/2)*(9*sin(1/2*b*x+1/2*a)^4-9*sin(1/2*b*x+1/2*a)^2+1)/b
```

Maxima [A] time = 0.977458, size = 50, normalized size = 1.11

$$-\frac{2\left(9\,d^2\cos\left(bx+a\right)^2-5\,d^2\right)}{45\,\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x, algorithm="maxima")
```

```
[Out] -2/45*(9*d^2*cos(b*x + a)^2 - 5*d^2)/((d*cos(b*x + a))^(9/2)*b*d^3)
```

Fricas [A] time = 2.17817, size = 100, normalized size = 2.22

$$-\frac{2\,\sqrt{d\cos\left(bx+a\right)}\left(9\,\cos\left(bx+a\right)^2-5\right)}{45\,bd^6\cos\left(bx+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x, algorithm="fricas")
```

```
[Out] -2/45*sqrt(d*cos(b*x + a))*(9*cos(b*x + a)^2 - 5)/(b*d^6*cos(b*x + a)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21318, size = 61, normalized size = 1.36

$$-\frac{2\left(9\,d^5\cos\left(bx+a\right)^2-5\,d^5\right)}{45\,\sqrt{d\cos\left(bx+a\right)}bd^{10}\cos\left(bx+a\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x, algorithm="giac")
```

```
[Out] -2/45*(9*d^5*cos(b*x + a)^2 - 5*d^5)/(sqrt(d*cos(b*x + a))*b*d^10*cos(b*x +
 a)^4)
```

3.211 $\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx$

Optimal. Leaf size=156

$$\frac{56d^3\sin(a+bx)(d\cos(a+bx))^{3/2}}{3315b}+\frac{56d^4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{1105b\sqrt{\cos(a+bx)}}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}-\frac{12\sin(a}{}$$

```
[Out] (56*d^4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(1105*b*Sqrt[Cos[a
+ b*x]]) + (56*d^3*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(3315*b) + (8*d*(d*
Cos[a + b*x])^(7/2)*Sin[a + b*x])/(663*b) - (12*(d*Cos[a + b*x])^(11/2)*Sin
[a + b*x])/(221*b*d) - (2*(d*Cos[a + b*x])^(11/2)*Sin[a + b*x]^3)/(17*b*d)
```

Rubi [A] time = 0.149011, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2568, 2635, 2640, 2639}

$$\frac{56d^3\sin(a+bx)(d\cos(a+bx))^{3/2}}{3315b}+\frac{56d^4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{1105b\sqrt{\cos(a+bx)}}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}-\frac{12\sin(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^4,x]
```


```
[Out] (56*d^4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(1105*b*Sqrt[Cos[a
+ b*x]]) + (56*d^3*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(3315*b) + (8*d*(d*
Cos[a + b*x])^(7/2)*Sin[a + b*x])/(663*b) - (12*(d*Cos[a + b*x])^(11/2)*Sin
[a + b*x])/(221*b*d) - (2*(d*Cos[a + b*x])^(11/2)*Sin[a + b*x]^3)/(17*b*d)
```


Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{11/2}\sin^3(a+bx)}{17bd} + \frac{6}{17}\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx$$
$$= -\frac{12(d\cos(a+bx))^{11/2}\sin(a+bx)}{221bd} - \frac{2(d\cos(a+bx))^{11/2}\sin^3(a+bx)}{17bd} + \frac{12}{221}\int$$
$$= \frac{8d(d\cos(a+bx))^{7/2}\sin(a+bx)}{663b} - \frac{12(d\cos(a+bx))^{11/2}\sin(a+bx)}{221bd} - 2(d\cos(a$$
$$= \frac{56d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{3315b} + \frac{8d(d\cos(a+bx))^{7/2}\sin(a+bx)}{663b} - 12(d\,co$$
$$= \frac{56d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{3315b} + \frac{8d(d\cos(a+bx))^{7/2}\sin(a+bx)}{663b} - 12(d\,co$$
$$= \frac{56d^4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{1105b\sqrt{\cos(a+bx)}} + \frac{56d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{3315b} + 8d$$

Mathematica [C] time = 0.130511, size = 57, normalized size = 0.37

$$\frac{\sqrt[4]{\cos^2(a+bx)}\tan^5(a+bx)(d\cos(a+bx))^{9/2}\,{}_2F_1\left(-\frac{7}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^4,x]
```

```
[Out] ((d*Cos[a + b*x])^(9/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4, 5/2,
 7/2, Sin[a + b*x]^2]*Tan[a + b*x]^5)/(5*b)
```

Maple [A] time = 0.069, size = 275, normalized size = 1.8

$$-\frac{8\,d^5}{3315\,b}\sqrt{d\left(2\,(\cos(1/2\,bx+a/2))^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(24960\,(\cos(1/2\,bx+a/2))^{19}-124800\,(\cos(1/2\,bx+a/2))^{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x)
```

```
[Out] -8/3315*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^5*(2496
0*cos(1/2*b*x+1/2*a)^19-124800*cos(1/2*b*x+1/2*a)^17+265440*cos(1/2*b*x+1/2
*a)^15-312960*cos(1/2*b*x+1/2*a)^13+222520*cos(1/2*b*x+1/2*a)^11-96360*cos(
1/2*b*x+1/2*a)^9+23866*cos(1/2*b*x+1/2*a)^7-2652*cos(1/2*b*x+1/2*a)^5-35*co
s(1/2*b*x+1/2*a)^3-21*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2
+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))+21*cos(1/2*b*x+1/2*a))/(-d*
(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*
(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{9}{2}}\sin(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^4\cos\left(bx+a\right)^8 - 2\,d^4\cos\left(bx+a\right)^6 + d^4\cos\left(bx+a\right)^4\right)\sqrt{d\cos\left(bx+a\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] integral((d^4*cos(b*x + a)^8 - 2*d^4*cos(b*x + a)^6 + d^4*cos(b*x + a)^4)*s
qrt(d*cos(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(9/2)*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}\sin\left(bx+a\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^4, x)
```

3.212 $\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx$

Optimal. Leaf size=156

$$\frac{8d^3\sin(a+bx)\sqrt{d\cos(a+bx)}}{231b}+\frac{8d^4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{231b\sqrt{d\cos(a+bx)}}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd}-\frac{4\sin(a+bx)(}{5}$$

```
[Out] (8*d^4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(231*b*Sqrt[d*Cos[a + b*x]]) + (8*d^3*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(231*b) + (8*d*(d*Cos[a + b*x])^(5/2)*Sin[a + b*x])/(385*b) - (4*(d*Cos[a + b*x])^(9/2)*Sin[a + b*x])/(55*b*d) - (2*(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^3)/(15*b*d)
```

Rubi [A] time = 0.147477, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2568, 2635, 2642, 2641}

$$\frac{8d^3\sin(a+bx)\sqrt{d\cos(a+bx)}}{231b}+\frac{8d^4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{231b\sqrt{d\cos(a+bx)}}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd}-\frac{4\sin(a+bx)(}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^4,x]
```

```
[Out] (8*d^4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(231*b*Sqrt[d*Cos[a + b*x]]) + (8*d^3*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(231*b) + (8*d*(d*Cos[a + b*x])^(5/2)*Sin[a + b*x])/(385*b) - (4*(d*Cos[a + b*x])^(9/2)*Sin[a + b*x])/(55*b*d) - (2*(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^3)/(15*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1))/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{9/2}\sin^3(a+bx)}{15bd} + \frac{2}{5}\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx$$
$$= -\frac{4(d\cos(a+bx))^{9/2}\sin(a+bx)}{55bd} - \frac{2(d\cos(a+bx))^{9/2}\sin^3(a+bx)}{15bd} + \frac{4}{55}\int (d\cos($$
$$= \frac{8d(d\cos(a+bx))^{5/2}\sin(a+bx)}{385b} - \frac{4(d\cos(a+bx))^{9/2}\sin(a+bx)}{55bd} - \frac{2(d\cos(a+bx}{1}$$
$$= \frac{8d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b} + \frac{8d(d\cos(a+bx))^{5/2}\sin(a+bx)}{385b} - \frac{4(d\cos(a+bx}{5}$$
$$= \frac{8d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b} + \frac{8d(d\cos(a+bx))^{5/2}\sin(a+bx)}{385b} - \frac{4(d\cos(a+bx}{5}$$
$$= \frac{8d^4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{231b\sqrt{d\cos(a+bx)}} + \frac{8d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b} + \frac{8d(d\cos(a+}{}$$

Mathematica [C] time = 0.0970417, size = 57, normalized size = 0.37

$$\frac{\cos^2(a+bx)^{3/4}\tan^5(a+bx)(d\cos(a+bx))^{7/2}\,{}_2F_1\left(-\frac{5}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^4,x]
```

```
[Out] ((d*Cos[a + b*x])^(7/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, 5/2,
 7/2, Sin[a + b*x]^2]*Tan[a + b*x]^5)/(5*b)
```

Maple [A] time = 0.062, size = 262, normalized size = 1.7

$$-\frac{8d^4}{1155b}\sqrt{d\left(2\left(\cos(1/2\,bx+a/2)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(4928\left(\cos(1/2\,bx+a/2)\right)^{17}-22176\left(\cos(1/2\,bx+a/2)\right)^{15}+41\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x)
```

```
[Out] -8/1155*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^4*(4928
*cos(1/2*b*x+1/2*a)^17-22176*cos(1/2*b*x+1/2*a)^15+41216*cos(1/2*b*x+1/2*a)
^13-40768*cos(1/2*b*x+1/2*a)^11+22868*cos(1/2*b*x+1/2*a)^9-6994*cos(1/2*b*x
+1/2*a)^7+926*cos(1/2*b*x+1/2*a)^5+5*cos(1/2*b*x+1/2*a)^3+5*(sin(1/2*b*x+1/
2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a
),2^(1/2))-5*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/
2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{7}{2}}\sin(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^3 \cos\left(bx+a\right)^7 - 2\,d^3 \cos\left(bx+a\right)^5 + d^3 \cos\left(bx+a\right)^3\right)\sqrt{d\cos\left(bx+a\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] integral((d^3*cos(b*x + a)^7 - 2*d^3*cos(b*x + a)^5 + d^3*cos(b*x + a)^3)*s
qrt(d*cos(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(7/2)*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}} \sin\left(bx+a\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^4, x)
```

3.213 $\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx$

Optimal. Leaf size=128

$$\frac{8d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{65b\sqrt{\cos(a+bx)}}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{7/2}}{13bd}-\frac{4\sin(a+bx)(d\cos(a+bx))^{7/2}}{39bd}+\frac{8d\sin(a+bx)(\cdots}{1\cdots}$$

```
[Out] (8*d^2*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(65*b*Sqrt[Cos[a + b
*x]]) + (8*d*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(195*b) - (4*(d*Cos[a + b
*x])^(7/2)*Sin[a + b*x])/(39*b*d) - (2*(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^
3)/(13*b*d)
```

Rubi [A] time = 0.12575, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2568, 2635, 2640, 2639}

$$\frac{8d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{65b\sqrt{\cos(a+bx)}}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{7/2}}{13bd}-\frac{4\sin(a+bx)(d\cos(a+bx))^{7/2}}{39bd}+\frac{8d\sin(a+bx)(\cdots}{1\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^4,x]
```

```
[Out] (8*d^2*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(65*b*Sqrt[Cos[a + b
*x]]) + (8*d*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(195*b) - (4*(d*Cos[a + b
*x])^(7/2)*Sin[a + b*x])/(39*b*d) - (2*(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^
3)/(13*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{7/2}\sin^3(a+bx)}{13bd} + \frac{6}{13}\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx$$
$$= -\frac{4(d\cos(a+bx))^{7/2}\sin(a+bx)}{39bd} - \frac{2(d\cos(a+bx))^{7/2}\sin^3(a+bx)}{13bd} + \frac{4}{39}\int (d\,\mathrm{c}$$
$$= \frac{8d(d\cos(a+bx))^{3/2}\sin(a+bx)}{195b} - \frac{4(d\cos(a+bx))^{7/2}\sin(a+bx)}{39bd} - \frac{2(d\cos(a+$$
$$= \frac{8d(d\cos(a+bx))^{3/2}\sin(a+bx)}{195b} - \frac{4(d\cos(a+bx))^{7/2}\sin(a+bx)}{39bd} - \frac{2(d\cos(a+$$
$$= \frac{8d^2\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{65b\sqrt{\cos(a+bx)}} + \frac{8d(d\cos(a+bx))^{3/2}\sin(a+bx)}{195b} - \frac{4(d\,\mathrm{co}$$

Mathematica [C] time = 0.0735713, size = 65, normalized size = 0.51

$$\frac{\sin^2(a+bx)\sqrt[4]{\cos^2(a+bx)}\tan^3(a+bx)(d\cos(a+bx))^{5/2}\,{}_2F_1\left(-\frac{3}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^4,x]
```

```
[Out] ((d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, 5/2,
 7/2, Sin[a + b*x]^2]*Sin[a + b*x]^2*Tan[a + b*x]^3)/(5*b)
```

Maple [A] time = 0.06, size = 249, normalized size = 2.

$$-\frac{8\,d^3}{195\,b}\sqrt{d\left(2\,(\cos(1/2\,bx+a/2))^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(480\,(\cos(1/2\,bx+a/2))^{15}-1920\,(\cos(1/2\,bx+a/2))^{13}+30\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x)
```

```
[Out] -8/195*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^3*(480*c
os(1/2*b*x+1/2*a)^15-1920*cos(1/2*b*x+1/2*a)^13+3040*cos(1/2*b*x+1/2*a)^11-
2400*cos(1/2*b*x+1/2*a)^9+958*cos(1/2*b*x+1/2*a)^7-156*cos(1/2*b*x+1/2*a)^5
-5*cos(1/2*b*x+1/2*a)^3-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*
a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))+3*cos(1/2*b*x+1/2*a))/(
-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/
(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{5}{2}}\sin(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2\cos\left(bx+a\right)^6-2\,d^2\cos\left(bx+a\right)^4+d^2\cos\left(bx+a\right)^2\right)\sqrt{d\cos\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] integral((d^2*cos(b*x + a)^6 - 2*d^2*cos(b*x + a)^4 + d^2*cos(b*x + a)^2)*s
qrt(d*cos(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(5/2)*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}\sin\left(bx+a\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^4, x)
```

3.214 $\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx$

Optimal. Leaf size=128

$$\frac{8d^2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{77b\sqrt{d\cos(a+bx)}}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{5/2}}{11bd}-\frac{12\sin(a+bx)(d\cos(a+bx))^{5/2}}{77bd}+\frac{8d\sin(a+b}{}$$

```
[Out] (8*d^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(77*b*Sqrt[d*Cos[a + b
*x]]) + (8*d*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(77*b) - (12*(d*Cos[a + b*x
])^(5/2)*Sin[a + b*x])/(77*b*d) - (2*(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^3)
/(11*b*d)
```

Rubi [A] time = 0.133133, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2568, 2635, 2642, 2641}

$$\frac{8d^2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{77b\sqrt{d\cos(a+bx)}}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{5/2}}{11bd}-\frac{12\sin(a+bx)(d\cos(a+bx))^{5/2}}{77bd}+\frac{8d\sin(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^4,x]
```


```
[Out] (8*d^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(77*b*Sqrt[d*Cos[a + b
*x]]) + (8*d*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(77*b) - (12*(d*Cos[a + b*x
])^(5/2)*Sin[a + b*x])/(77*b*d) - (2*(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^3)
/(11*b*d)
```


Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{5/2}\sin^3(a+bx)}{11bd} + \frac{6}{11}\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx$$
$$= -\frac{12(d\cos(a+bx))^{5/2}\sin(a+bx)}{77bd} - \frac{2(d\cos(a+bx))^{5/2}\sin^3(a+bx)}{11bd} + \frac{12}{77}\int (d\cos$$
$$= \frac{8d\sqrt{d\cos(a+bx)}\sin(a+bx)}{77b} - \frac{12(d\cos(a+bx))^{5/2}\sin(a+bx)}{77bd} - \frac{2(d\cos(a+bx)}{11}$$
$$= \frac{8d\sqrt{d\cos(a+bx)}\sin(a+bx)}{77b} - \frac{12(d\cos(a+bx))^{5/2}\sin(a+bx)}{77bd} - \frac{2(d\cos(a+bx)}{11}$$
$$= \frac{8d^2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{77b\sqrt{d\cos(a+bx)}} + \frac{8d\sqrt{d\cos(a+bx)}\sin(a+bx)}{77b} - \frac{12(d\cos(a+}{}$$

Mathematica [C] time = 0.107953, size = 65, normalized size = 0.51

$$\frac{\sin^2(a+bx)\cos^2(a+bx)^{3/4}\tan^3(a+bx)(d\cos(a+bx))^{3/2}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^4,x]
```

```
[Out] ((d*Cos[a + b*x])^(3/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 5/2,
 7/2, Sin[a + b*x]^2]*Sin[a + b*x]^2*Tan[a + b*x]^3)/(5*b)
```

Maple [A] time = 0.068, size = 255, normalized size = 2.

$$-\frac{8\,d^2}{77\,b}\sqrt{d\left(2\,(\cos(1/2\,bx+a/2))^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(112\,\cos(1/2\,bx+a/2)\,(\sin(1/2\,bx+a/2))^{12}-280\,(\sin(1/2\,bx+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x)
```

```
[Out] -8/77*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^2*(112*co
s(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^12-280*sin(1/2*b*x+1/2*a)^10*cos(1/2*b*
x+1/2*a)+228*sin(1/2*b*x+1/2*a)^8*cos(1/2*b*x+1/2*a)-62*sin(1/2*b*x+1/2*a)^
6*cos(1/2*b*x+1/2*a)+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1
)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+sin(1/2*b*x+1/2*a)^2*cos(1/2*
b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/
2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\cos(bx+a)^5 - 2\,d\cos(bx+a)^3 + d\cos(bx+a)\right)\sqrt{d\cos(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] integral((d*cos(b*x + a)^5 - 2*d*cos(b*x + a)^3 + d*cos(b*x + a))*sqrt(d*co
s(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a)^4, x)
```

3.215 $\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx$

Optimal. Leaf size=99

$$-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{3/2}}{9bd}-\frac{4\sin(a+bx)(d\cos(a+bx))^{3/2}}{15bd}+\frac{8E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{15b\sqrt{\cos(a+bx)}}$$

```
[Out] (8*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(15*b*Sqrt[Cos[a + b*x]]
) - (4*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(15*b*d) - (2*(d*Cos[a + b*x])^
(3/2)*Sin[a + b*x]^3)/(9*b*d)
```

Rubi [A] time = 0.0964384, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2568, 2640, 2639}

$$-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{3/2}}{9bd}-\frac{4\sin(a+bx)(d\cos(a+bx))^{3/2}}{15bd}+\frac{8E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{15b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^4,x]
```

```
[Out] (8*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(15*b*Sqrt[Cos[a + b*x]]
) - (4*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(15*b*d) - (2*(d*Cos[a + b*x])^
(3/2)*Sin[a + b*x]^3)/(9*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{3/2}\sin^3(a+bx)}{9bd} + \frac{2}{3}\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx$$
$$= -\frac{4(d\cos(a+bx))^{3/2}\sin(a+bx)}{15bd} - \frac{2(d\cos(a+bx))^{3/2}\sin^3(a+bx)}{9bd} + \frac{4}{15}\int \sqrt{d\,\mathrm{co}$$
$$= -\frac{4(d\cos(a+bx))^{3/2}\sin(a+bx)}{15bd} - \frac{2(d\cos(a+bx))^{3/2}\sin^3(a+bx)}{9bd} + \frac{\left(4\sqrt{d\cos(a}{1}$$
$$= \frac{8\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b\sqrt{\cos(a+bx)}} - \frac{4(d\cos(a+bx))^{3/2}\sin(a+bx)}{15bd} - \frac{2(d\cos(a+}{}$$

Mathematica [C] time = 0.0652732, size = 58, normalized size = 0.59

$$\frac{d\sin^5(a+bx)\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^4,x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 5/2, 7/2, Sin[a + b*x]^2]*
Sin[a + b*x]^5)/(5*b*Sqrt[d*Cos[a + b*x]])
```

Maple [A] time = 0.059, size = 221, normalized size = 2.2

$$-\frac{8\,d}{45\,b}\sqrt{d\left(2\,(\cos(1/2\,bx+a/2))^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(40\,(\cos(1/2\,bx+a/2))^{11}-120\,(\cos(1/2\,bx+a/2))^9+118\,(\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x)
```

```
[Out] -8/45*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d*(40*cos(1
/2*b*x+1/2*a)^11-120*cos(1/2*b*x+1/2*a)^9+118*cos(1/2*b*x+1/2*a)^7-36*cos(1
/2*b*x+1/2*a)^5-5*cos(1/2*b*x+1/2*a)^3-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*c
os(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))+3*cos(1/
2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(
1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cos(bx+a)}\sin(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}+1\right)\sqrt{d\cos\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*cos(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\cos\left(bx+a\right)}\sin\left(bx+a\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^4, x)
```

3.216 $\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=99

$$-\frac{2\sin^3(a+bx)\sqrt{d\cos(a+bx)}}{7bd}-\frac{4\sin(a+bx)\sqrt{d\cos(a+bx)}}{7bd}+\frac{8\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7b\sqrt{d\cos(a+bx)}}$$

```
[Out] (8*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(7*b*Sqrt[d*Cos[a + b*x]])
 - (4*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(7*b*d) - (2*Sqrt[d*Cos[a + b*x]]*
Sin[a + b*x]^3)/(7*b*d)
```

Rubi [A] time = 0.0981128, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2568, 2642, 2641}

$$-\frac{2\sin^3(a+bx)\sqrt{d\cos(a+bx)}}{7bd}-\frac{4\sin(a+bx)\sqrt{d\cos(a+bx)}}{7bd}+\frac{8\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (8*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(7*b*Sqrt[d*Cos[a + b*x]])
 - (4*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(7*b*d) - (2*Sqrt[d*Cos[a + b*x]]*
Sin[a + b*x]^3)/(7*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d\cos(a+bx)}\sin^3(a+bx)}{7bd} + \frac{6}{7}\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$
$$= -\frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{7bd} - \frac{2\sqrt{d\cos(a+bx)}\sin^3(a+bx)}{7bd} + \frac{4}{7}\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx$$
$$= -\frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{7bd} - \frac{2\sqrt{d\cos(a+bx)}\sin^3(a+bx)}{7bd} + \frac{\left(4\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{7\sqrt{d\cos(a+bx)}}$$
$$= \frac{8\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7b\sqrt{d\cos(a+bx)}} - \frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{7bd} - \frac{2\sqrt{d\cos(a+bx)}\sin^3(a+bx)}{7bd}$$

Mathematica [C] time = 0.0695362, size = 58, normalized size = 0.59

$$\frac{d\sin^5(a+bx)\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5b(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 5/2, 7/2, Sin[a + b*x]^2]*
Sin[a + b*x]^5)/(5*b*(d*Cos[a + b*x])^(3/2))
```

Maple [A] time = 0.063, size = 208, normalized size = 2.1

$$-\frac{8}{7b}\sqrt{d\left(2\left(\cos(1/2\,bx+a/2)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(4\left(\sin(1/2\,bx+a/2)\right)^8\cos(1/2\,bx+a/2)-6\left(\sin(1/2\,bx+a/2)\right)^6 c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] -8/7*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(4*sin(1/2*b
*x+1/2*a)^8*cos(1/2*b*x+1/2*a)-6*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)+(s
in(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1
/2*b*x+1/2*a),2^(1/2))+sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/(-d*(2*sin(
1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(
1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^4}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^4/sqrt(d*cos(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}+1\right)\sqrt{d\cos\left(bx+a\right)}}{d\cos\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*cos(b*x + a))/(d*co
s(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(bx+a\right)^{4}}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^4/sqrt(d*cos(b*x + a)), x)
```

3.217 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{12\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd^3}-\frac{24E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^2\sqrt{\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] (-24*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*d^2*Sqrt[Cos[a +
b*x]]) + (12*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(5*b*d^3) + (2*Sin[a + b*
x]^3)/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.104586, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2566, 2568, 2640, 2639}

$$\frac{12\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd^3}-\frac{24E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^2\sqrt{\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (-24*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*d^2*Sqrt[Cos[a +
b*x]]) + (12*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(5*b*d^3) + (2*Sin[a + b*
x]^3)/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{6\int\sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx}{d^2}$$
$$= \frac{12(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd^3} + \frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{12\int\sqrt{d\cos(a+bx)}\,dx}{5d^2}$$
$$= \frac{12(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd^3} + \frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{\left(12\sqrt{d\cos(a+bx)}\right)\int\sqrt{\cos(a+bx)}}{5d^2\sqrt{\cos(a+bx)}}$$
$$= -\frac{24\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^2\sqrt{\cos(a+bx)}} + \frac{12(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd^3} + \frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

Mathematica [C] time = 0.0594919, size = 60, normalized size = 0.6

$$\frac{\sin^5(a+bx)\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{5}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 5/2, 7/2, Sin[a + b*x]^2]*Si
n[a + b*x]^5)/(5*b*d*Sqrt[d*Cos[a + b*x]])
```

Maple [A] time = 0.068, size = 213, normalized size = 2.1

$$-\frac{8}{5\,db}\sqrt{-2\,(\sin(1/2\,bx+a/2))^4\,d+\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2 d}\left(-2\,(\sin(1/2\,bx+a/2))^6\cos(1/2\,bx+a/2)+2\,\cos(1/2\,bx+a/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] -8/5/d*(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(1/2)*(-2*sin(1/2
*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)+2*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+
3*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(c
os(1/2*b*x+1/2*a),2^(1/2))-3*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/(-d*(
2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(
2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}+1\right)\sqrt{d\cos\left(bx+a\right)}}{d^{2}\cos\left(bx+a\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")

[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*cos(b*x + a))/(d^2*cos(b*x + a)^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(bx+a\right)^{4}}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x, algorithm="giac")

[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(3/2), x)

3.218 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=102

$$\frac{4\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd^3}-\frac{8\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

```
[Out] (-8*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a + b
*x]]) + (4*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(3*b*d^3) + (2*Sin[a + b*x]^3
)/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.10562, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2566, 2568, 2642, 2641}

$$\frac{4\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd^3}-\frac{8\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(5/2),x]
```


```
[Out] (-8*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a + b
*x]]) + (4*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(3*b*d^3) + (2*Sin[a + b*x]^3
)/(3*b*d*(d*Cos[a + b*x])^(3/2))
```


Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}} - \frac{2\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx}{d^2}$$

$$= \frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd^3} + \frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}} - \frac{4\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx}{3d^2}$$

$$= \frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd^3} + \frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}} - \frac{\left(4\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{3d^2\sqrt{d\cos(a+bx)}}$$

$$= -\frac{8\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}} + \frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd^3} + \frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

Mathematica [C] time = 0.0688563, size = 60, normalized size = 0.59

$$\frac{\sin^5(a+bx)\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{7}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, 5/2, 7/2, Sin[a + b*x]^2]*Si
n[a + b*x]^5)/(5*b*d*(d*Cos[a + b*x])^(3/2))
```

Maple [B] time = 0.064, size = 286, normalized size = 2.8

$$-\frac{8}{3d^2b}\left(-2\left(\sin\left(1/2\,bx+a/2\right)\right)^6\cos\left(1/2\,bx+a/2\right)+2\sqrt{\left(\sin\left(1/2\,bx+a/2\right)\right)^2}\sqrt{2\left(\sin\left(1/2\,bx+a/2\right)\right)^2-1}\mathrm{EllipticF}\left(\cos(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] -8/3*(-2*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)+2*(sin(1/2*b*x+1/2*a)^2)^(
1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))
*sin(1/2*b*x+1/2*a)^2+2*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4-(sin(1/2*b*
x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/
2*a),2^(1/2))-sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/d^2*(d*(2*cos(1/2*b*
x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)/(-d*(2
*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2
*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}+1\right)\sqrt{d\cos\left(bx+a\right)}}{d^{3}\cos\left(bx+a\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*cos(b*x + a))/(d^3*
cos(b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(bx+a\right)^{4}}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(5/2), x)
```

3.219 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{24E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}$$

```
[Out] (24*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*d^4*Sqrt[Cos[a + b
*x]]) - (12*Sin[a + b*x])/(5*b*d^3*Sqrt[d*Cos[a + b*x]]) + (2*Sin[a + b*x]^
3)/(5*b*d*(d*Cos[a + b*x])^(5/2))
```

Rubi [A] time = 0.11342, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2566, 2640, 2639}

$$-\frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{24E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(7/2),x]
```


```
[Out] (24*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*d^4*Sqrt[Cos[a + b
*x]]) - (12*Sin[a + b*x])/(5*b*d^3*Sqrt[d*Cos[a + b*x]]) + (2*Sin[a + b*x]^
3)/(5*b*d*(d*Cos[a + b*x])^(5/2))
```


Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{6\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx}{5d^2}$$
$$= -\frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}} + \frac{12\int \sqrt{d\cos(a+bx)}\,dx}{5d^4}$$
$$= -\frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}} + \frac{\left(12\sqrt{d\cos(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{5d^4\sqrt{\cos(a+bx)}}$$
$$= \frac{24\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^4\sqrt{\cos(a+bx)}} - \frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}$$

Mathematica [C] time = 0.071335, size = 65, normalized size = 0.64

$$\frac{\sin^5(a+bx)\cos^3(a+bx)\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{9}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5b(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (Cos[a + b*x]^3*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[9/4, 5/2, 7/2, Sin
[a + b*x]^2]*Sin[a + b*x]^5)/(5*b*(d*Cos[a + b*x])^(7/2))
```

Maple [B] time = 0.096, size = 366, normalized size = 3.6

$$-\frac{8}{5\,d^4b}\sqrt{d\left(2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(12\,\sqrt{2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^2-1}\sqrt{\left(\sin\left(1/2\,bx+a/2\right)\right)^2}\mathrm{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] -8/5*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^4/sin(1/2*
b*x+1/2*a)^3/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+
1/2*a)^2-1)*(12*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/
2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^4-14*sin(1/2*b*
x+1/2*a)^6*cos(1/2*b*x+1/2*a)-12*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*
b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a
)^2+14*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+3*(2*sin(1/2*b*x+1/2*a)^2-1)
^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-3
*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))*(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/
2*b*x+1/2*a)^2*d)^(1/2)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(bx+a\right)^4}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos\left(bx+a\right)^4-2\,\cos\left(bx+a\right)^2+1\right)\sqrt{d\cos\left(bx+a\right)}}{d^4\cos\left(bx+a\right)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*cos(b*x + a))/(d^4*
cos(b*x + a)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(bx+a\right)^4}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(7/2), x)
```

3.220 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}}+\frac{8\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7bd^4\sqrt{d\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}$$

```
[Out] (8*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(7*b*d^4*Sqrt[d*Cos[a + b*
x]]) - (4*Sin[a + b*x])/(7*b*d^3*(d*Cos[a + b*x])^(3/2)) + (2*Sin[a + b*x]^
3)/(7*b*d*(d*Cos[a + b*x])^(7/2))
```

Rubi [A] time = 0.111495, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2566, 2642, 2641}

$$-\frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}}+\frac{8\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7bd^4\sqrt{d\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(9/2),x]
```


```
[Out] (8*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(7*b*d^4*Sqrt[d*Cos[a + b*
x]]) - (4*Sin[a + b*x])/(7*b*d^3*(d*Cos[a + b*x])^(3/2)) + (2*Sin[a + b*x]^
3)/(7*b*d*(d*Cos[a + b*x])^(7/2))
```


Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{6\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx}{7d^2}$$
$$= -\frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}} + \frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}} + \frac{4\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx}{7d^4}$$
$$= -\frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}} + \frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}} + \frac{\left(4\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{7d^4\sqrt{d\cos(a+bx)}}$$
$$= \frac{8\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7bd^4\sqrt{d\cos(a+bx)}} - \frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}} + \frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}$$

Mathematica [C] time = 0.0689864, size = 65, normalized size = 0.64

$$\frac{\sin^5(a+bx)\cos^3(a+bx)\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{5}{2},\frac{11}{4};\frac{7}{2};\sin^2(a+bx)\right)}{5b(d\cos(a+bx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (Cos[a + b*x]^3*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[5/2, 11/4, 7/2, Si
n[a + b*x]^2]*Sin[a + b*x]^5)/(5*b*(d*Cos[a + b*x])^(9/2))
```

Maple [B] time = 0.066, size = 398, normalized size = 3.9

$$\frac{8}{7\,d^4b}\left(8\sqrt{(\sin(1/2\,bx+a/2))^2}\sqrt{2\,(\sin(1/2\,bx+a/2))^2-1}\mathrm{EllipticF}\left(\cos(1/2\,bx+a/2),\sqrt{2}\right)(\sin(1/2\,bx+a/2))^6-12\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x)
```

```
[Out] 8/7*(8*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Ellipt
icF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^6-12*(sin(1/2*b*x+1/2*a)
^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(
1/2))*sin(1/2*b*x+1/2*a)^4-6*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)+6*(sin
(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2
*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2+6*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+
1/2*a)^4-(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Elli
pticF(cos(1/2*b*x+1/2*a),2^(1/2))-sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/
d^4*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(2*cos(1/2*b*
x+1/2*a)^2-1)^3/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/si
n(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}+1\right)\sqrt{d\cos\left(bx+a\right)}}{d^{5}\cos\left(bx+a\right)^{5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*cos(b*x + a))/(d^5*
cos(b*x + a)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(bx+a\right)^{4}}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(9/2), x)
```

3.221 $\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=52

$$-\frac{2\cos^{\frac{13}{2}}(a+bx)}{13b}+\frac{4\cos^{\frac{9}{2}}(a+bx)}{9b}-\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] (-2*Cos[a + b*x]^(5/2))/(5*b) + (4*Cos[a + b*x]^(9/2))/(9*b) - (2*Cos[a + b
*x]^(13/2))/(13*b)
```

Rubi [A] time = 0.035061, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 270}

$$-\frac{2\cos^{\frac{13}{2}}(a+bx)}{13b}+\frac{4\cos^{\frac{9}{2}}(a+bx)}{9b}-\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(3/2)*Sin[a + b*x]^5,x]
```

```
[Out] (-2*Cos[a + b*x]^(5/2))/(5*b) + (4*Cos[a + b*x]^(9/2))/(9*b) - (2*Cos[a + b
*x]^(13/2))/(13*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^{3/2}\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(x^{3/2}-2x^{7/2}+x^{11/2}\right)dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\cos^{\frac{9}{2}}(a+bx)}{9b}-\frac{2\cos^{\frac{13}{2}}(a+bx)}{13b}
\end{aligned}$$

Mathematica [B] time = 0.279403, size = 111, normalized size = 2.13

$$\frac{2\sqrt{\cos(a+bx)}\left(-32\sqrt[4]{\cos^2(a+bx)}+45\sin^6(a+bx)\sqrt[4]{\cos^2(a+bx)}-5\sin^4(a+bx)\sqrt[4]{\cos^2(a+bx)}-8\sin^2(a+bx)\sqrt[4]{\cos^2}\right.}{585b\sqrt[4]{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(3/2)*Sin[a + b*x]^5,x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*(32 - 32*(Cos[a + b*x]^2)^(1/4) - 8*(Cos[a + b*x]^2)^
(1/4)*Sin[a + b*x]^2 - 5*(Cos[a + b*x]^2)^(1/4)*Sin[a + b*x]^4 + 45*(Cos[a
+ b*x]^2)^(1/4)*Sin[a + b*x]^6))/(585*b*(Cos[a + b*x]^2)^(1/4))
```

Maple [B] time = 0.088, size = 103, normalized size = 2.

$$-\frac{32}{585\,b}\sqrt{-2\,(\sin(1/2\,bx+a/2))^2+1}\left(180\,(\sin(1/2\,bx+a/2))^{12}-540\,(\sin(1/2\,bx+a/2))^{10}+545\,(\sin(1/2\,bx+a/2))^{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x)
```

```
[Out] -32/585*(-2*sin(1/2*b*x+1/2*a)^2+1)^(1/2)*(180*sin(1/2*b*x+1/2*a)^12-540*si
n(1/2*b*x+1/2*a)^10+545*sin(1/2*b*x+1/2*a)^8-190*sin(1/2*b*x+1/2*a)^6+3*sin
(1/2*b*x+1/2*a)^4+2*sin(1/2*b*x+1/2*a)^2+2)/b
```

Maxima [A] time = 0.972461, size = 49, normalized size = 0.94

$$-\frac{2\left(45\,\cos(bx+a)^{\frac{13}{2}}-130\,\cos(bx+a)^{\frac{9}{2}}+117\,\cos(bx+a)^{\frac{5}{2}}\right)}{585\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -2/585*(45*cos(b*x + a)^(13/2) - 130*cos(b*x + a)^(9/2) + 117*cos(b*x + a)^
(5/2))/b
```

Fricas [A] time = 2.23869, size = 123, normalized size = 2.37

$$-\frac{2\left(45\,\cos(bx+a)^6-130\,\cos(bx+a)^4+117\,\cos(bx+a)^2\right)\sqrt{\cos(bx+a)}}{585\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -2/585*(45*cos(b*x + a)^6 - 130*cos(b*x + a)^4 + 117*cos(b*x + a)^2)*sqrt(c
os(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(3/2)*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^{\frac{3}{2}}\sin(bx+a)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(3/2)*sin(b*x + a)^5, x)
```

3.222 $\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx$

Optimal. Leaf size=100

$$\frac{2d^3(d\cos(a+bx))^{3/2}}{3b}+\frac{d^{9/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d(d\cos(a+bx))^{7/2}}{7b}$$

```
[Out] (d^(9/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b - (d^(9/2)*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/b + (2*d^3*(d*Cos[a + b*x])^(3/2))/(3*b) + (2*d*(d*
Cos[a + b*x])^(7/2))/(7*b)
```

Rubi [A] time = 0.0769487, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 321, 329, 298, 203, 206}

$$\frac{2d^3(d\cos(a+bx))^{3/2}}{3b}+\frac{d^{9/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d(d\cos(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x],x]
```

```
[Out] (d^(9/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b - (d^(9/2)*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/b + (2*d^3*(d*Cos[a + b*x])^(3/2))/(3*b) + (2*d*(d*
Cos[a + b*x])^(7/2))/(7*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^{9/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2d(d\cos(a+bx))^{7/2}}{7b} - \frac{d\operatorname{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{b}$$

$$= \frac{2d^3(d\cos(a+bx))^{3/2}}{3b} + \frac{2d(d\cos(a+bx))^{7/2}}{7b} - \frac{d^3\operatorname{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx\right.}{b}$$

$$= \frac{2d^3(d\cos(a+bx))^{3/2}}{3b} + \frac{2d(d\cos(a+bx))^{7/2}}{7b} - \frac{(2d^3)\operatorname{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a\right.}{b}$$

$$= \frac{2d^3(d\cos(a+bx))^{3/2}}{3b} + \frac{2d(d\cos(a+bx))^{7/2}}{7b} - \frac{d^5\operatorname{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+b\right.}{b}$$

$$= \frac{d^{9/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d^3(d\cos(a+bx))^{3/2}}{3b} + 2d(a$$

Mathematica [A] time = 0.19088, size = 83, normalized size = 0.83

$$\frac{d^4\sqrt{d\cos(a+bx)}\left(2\left(3\cos^2(a+bx)+7\right)\cos^{\frac{3}{2}}(a+bx)+21\tan^{-1}\left(\sqrt{\cos(a+bx)}\right)-21\tanh^{-1}\left(\sqrt{\cos(a+bx)}\right)\right)}{21b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x],x]
```

```
[Out] (d^4*Sqrt[d*Cos[a + b*x]]*(21*ArcTan[Sqrt[Cos[a + b*x]]] - 21*ArcTanh[Sqrt[
Cos[a + b*x]]] + 2*Cos[a + b*x]^(3/2)*(7 + 3*Cos[a + b*x]^2)))/(21*b*Sqrt[C
os[a + b*x]])
```

Maple [B] time = 0.112, size = 318, normalized size = 3.2

$$-\frac{16\,d^4}{7\,b}\sqrt{-2\,(\sin(1/2\,bx+a/2))^2\,d+d}\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^6-\frac{1}{2\,b}d^{\frac{9}{2}}\ln\left(2\,\frac{\sqrt{d}\sqrt{-2\,(\sin(1/2\,bx+a/2))^2\,d+d}-2\,d\cos(1/2\,bx\right.}{\cos(1/2\,bx+a/2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(9/2)*csc(b*x+a),x)
```

```
[Out] -16/7/b*d^4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*sin(1/2*b*x+1/2*a)^6-1/2/b*
d^(9/2)*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^
(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))-1/2/b*d^(9/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*
(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))+24/
7/b*d^4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*sin(1/2*b*x+1/2*a)^4-64/21/b*d^
4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*sin(1/2*b*x+1/2*a)^2-1/(-d)^(1/2)/b*d
^5*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-
d))+20/21/b*d^4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.80212, size = 836, normalized size = 8.36

$$\left[\frac{42\sqrt{-d}d^4\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)+21\sqrt{-d}d^4\log\left(-\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6d\cos(bx+a)+d}{\cos(bx+a)^2+2\cos(bx+a)+1}\right)+8\left(3d^4\right.}{84b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/84*(42*sqrt(-d)*d^4*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x +
a) + d)) + 21*sqrt(-d)*d^4*log(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*
sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos
(b*x + a) + 1)) + 8*(3*d^4*cos(b*x + a)^3 + 7*d^4*cos(b*x + a))*sqrt(d*cos(
b*x + a)))/b, -1/84*(42*d^(9/2)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*co
s(b*x + a) - d)) - 21*d^(9/2)*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a
))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*c
os(b*x + a) + 1)) - 8*(3*d^4*cos(b*x + a)^3 + 7*d^4*cos(b*x + a))*sqrt(d*co
s(b*x + a)))/b]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(9/2)*csc(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{9}{2}} \csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a), x)
```

3.223 $\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx$

Optimal. Leaf size=99

$$\frac{2d^3\sqrt{d\cos(a+bx)}}{b}-\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d(d\cos(a+bx))^{5/2}}{5b}$$

```
[Out] -((d^(7/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b) - (d^(7/2)*ArcTanh[Sqrt
[d*Cos[a + b*x]]/Sqrt[d]])/b + (2*d^3*Sqrt[d*Cos[a + b*x]])/b + (2*d*(d*Cos
[a + b*x])^(5/2))/(5*b)
```

Rubi [A] time = 0.0710352, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 321, 329, 212, 206, 203}

$$\frac{2d^3\sqrt{d\cos(a+bx)}}{b}-\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d(d\cos(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x],x]
```

```
[Out] -((d^(7/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b) - (d^(7/2)*ArcTanh[Sqrt
[d*Cos[a + b*x]]/Sqrt[d]])/b + (2*d^3*Sqrt[d*Cos[a + b*x]])/b + (2*d*(d*Cos
[a + b*x])^(5/2))/(5*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{7/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2d(d\cos(a+bx))^{5/2}}{5b} - \frac{d\,\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{b}$$

$$= \frac{2d^3\sqrt{d\cos(a+bx)}}{b} + \frac{2d(d\cos(a+bx))^{5/2}}{5b} - \frac{d^3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+b\right.}{b}$$

$$= \frac{2d^3\sqrt{d\cos(a+bx)}}{b} + \frac{2d(d\cos(a+bx))^{5/2}}{5b} - \frac{\left(2d^3\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+\right.}{b}$$

$$= \frac{2d^3\sqrt{d\cos(a+bx)}}{b} + \frac{2d(d\cos(a+bx))^{5/2}}{5b} - \frac{d^4\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right.}{b}$$

$$= -\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d^3\sqrt{d\cos(a+bx)}}{b} + \frac{2d(d}{}$$

Mathematica [A] time = 0.180964, size = 80, normalized size = 0.81

$$\frac{d^3\sqrt{d\cos(a+bx)}\left(\sqrt{\cos(a+bx)}(\cos(2(a+bx))+11)-5\tan^{-1}\left(\sqrt{\cos(a+bx)}\right)-5\tanh^{-1}\left(\sqrt{\cos(a+bx)}\right)\right)}{5b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x],x]
```

```
[Out] (d^3*Sqrt[d*Cos[a + b*x]]*(-5*ArcTan[Sqrt[Cos[a + b*x]]] - 5*ArcTanh[Sqrt[C
os[a + b*x]]] + Sqrt[Cos[a + b*x]]*(11 + Cos[2*(a + b*x)])))/(5*b*Sqrt[Cos[
a + b*x]])
```

Maple [B] time = 0.109, size = 280, normalized size = 2.8

$$\frac{8\,d^3}{5\,b}\sqrt{-2\,(\sin(1/2\,bx+a/2))^2\,d+d}\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4 - \frac{1}{2\,b}d^{\frac{7}{2}}\ln\left(2\,\frac{\sqrt{d}\sqrt{-2\,(\sin(1/2\,bx+a/2))^2\,d+d}+2\,d\cos(1/2\,bx+}{\cos(1/2\,bx+a/2)-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(7/2)*csc(b*x+a),x)
```

```
[Out] 8/5/b*d^3*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*sin(1/2*b*x+1/2*a)^4-1/2/b*d^
(7/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1
/2)+2*d*cos(1/2*b*x+1/2*a)-d))-1/2/b*d^(7/2)*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d
^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))-8/5/b
*d^3*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*sin(1/2*b*x+1/2*a)^2+1/(-d)^(1/2)/
b*d^4*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/
2)-d))+12/5/b*d^3*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.82196, size = 791, normalized size = 7.99

$$\left[\frac{10\sqrt{-d}d^3\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)+5\sqrt{-d}d^3\log\left(-\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6d\cos(bx+a)+d}{\cos(bx+a)^2+2\cos(bx+a)+1}\right)+8\left(d^3\cos\right.}{20b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/20*(10*sqrt(-d)*d^3*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x +
a) + d)) + 5*sqrt(-d)*d^3*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*s
qrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(
b*x + a) + 1)) + 8*(d^3*cos(b*x + a)^2 + 5*d^3)*sqrt(d*cos(b*x + a)))/b, 1/
20*(10*d^(7/2)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d))
+ 5*d^(7/2)*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*
x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1))
+ 8*(d^3*cos(b*x + a)^2 + 5*d^3)*sqrt(d*cos(b*x + a)))/b]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(7/2)*csc(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{7}{2}} \csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a), x)
```

3.224 $\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx$

Optimal. Leaf size=78

$$\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d(d\cos(a+bx))^{3/2}}{3b}$$

```
[Out] (d^(5/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b - (d^(5/2)*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/b + (2*d*(d*Cos[a + b*x])^(3/2))/(3*b)
```

Rubi [A] time = 0.0649495, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 321, 329, 298, 203, 206}

$$\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d(d\cos(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x],x]
```

```
[Out] (d^(5/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b - (d^(5/2)*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/b + (2*d*(d*Cos[a + b*x])^(3/2))/(3*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= \frac{2d(d\cos(a+bx))^{3/2}}{3b} - \frac{d\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{b}$$
$$= \frac{2d(d\cos(a+bx))^{3/2}}{3b} - \frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b}$$
$$= \frac{2d(d\cos(a+bx))^{3/2}}{3b} - \frac{d^3\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} + \frac{d^3\,\text{Subst}\left(\int \frac{1}{d+x^2}\right.}{}$$
$$= \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d(d\cos(a+bx))^{3/2}}{3b}$$

Mathematica [A] time = 0.110943, size = 68, normalized size = 0.87

$$\frac{(d\cos(a+bx))^{5/2}\left(2\cos^{\frac{3}{2}}(a+bx) + 3\tan^{-1}\left(\sqrt{\cos(a+bx)}\right) - 3\tanh^{-1}\left(\sqrt{\cos(a+bx)}\right)\right)}{3b\cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x],x]
```

```
[Out] ((d*Cos[a + b*x])^(5/2)*(3*ArcTan[Sqrt[Cos[a + b*x]]] - 3*ArcTanh[Sqrt[Cos[
a + b*x]]] + 2*Cos[a + b*x]^(3/2)))/(3*b*Cos[a + b*x]^(5/2))
```

Maple [B] time = 0.128, size = 244, normalized size = 3.1

$$-\frac{1}{2b}d^{\frac{5}{2}}\ln\left(2\frac{\sqrt{d}\sqrt{-2\left(\sin(1/2\,bx+a/2)\right)^2 d+d} - 2\,d\cos(1/2\,bx+a/2) - d}{\cos(1/2\,bx+a/2)+1}\right) - \frac{1}{2b}d^{\frac{5}{2}}\ln\left(2\frac{\sqrt{d}\sqrt{-2\left(\sin(1/2\,bx+a/2)\right)^2 d}}{\cos(1/2\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(5/2)*csc(b*x+a),x)
```

```
[Out] -1/2/b*d^(5/2)*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))-1/2/b*d^(5/2)*ln(2/(cos(1/2*b*x+1/2
*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-
d))-4/3/b*d^2*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*sin(1/2*b*x+1/2*a)^2-1/(-
d)^(1/2)/b*d^3*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2
*d+d)^(1/2)-d))+2/3/b*d^2*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.71898, size = 757, normalized size = 9.71

$$\left[\frac{6\sqrt{-d}d^2\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)+8\sqrt{d\cos(bx+a)}d^2\cos(bx+a)+3\sqrt{-d}d^2\log\left(-\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos}{\cos(bx+a)^2+2\cos(}\right.}{12b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(-d)*d^2*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x + a
) + d)) + 8*sqrt(d*cos(b*x + a))*d^2*cos(b*x + a) + 3*sqrt(-d)*d^2*log(-(d*
cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*c
os(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)))/b, -1/12*(6*d^(5/2
)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d)) - 8*sqrt(d*co
s(b*x + a))*d^2*cos(b*x + a) - 3*d^(5/2)*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*
cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x +
 a)^2 - 2*cos(b*x + a) + 1)))/b]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(5/2)*csc(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a), x)
```

3.225 $\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx$

Optimal. Leaf size=77

$$-\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d\sqrt{d\cos(a+bx)}}{b}$$

```
[Out] -((d^(3/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b) - (d^(3/2)*ArcTanh[Sqrt
[d*Cos[a + b*x]]/Sqrt[d]])/b + (2*d*Sqrt[d*Cos[a + b*x]])/b
```

Rubi [A] time = 0.0625691, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 321, 329, 212, 206, 203}

$$-\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d\sqrt{d\cos(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x],x]
```

```
[Out] -((d^(3/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b) - (d^(3/2)*ArcTanh[Sqrt
[d*Cos[a + b*x]]/Sqrt[d]])/b + (2*d*Sqrt[d*Cos[a + b*x]])/b
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\
&= \frac{2d\sqrt{d\cos(a+bx)}}{b} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{b}\\
&= \frac{2d\sqrt{d\cos(a+bx)}}{b} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b}\\
&= \frac{2d\sqrt{d\cos(a+bx)}}{b} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{d+x^2}\,dx\right.}{b}\\
&= -\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d\sqrt{d\cos(a+bx)}}{b}
\end{aligned}$$

Mathematica [A] time = 0.0649017, size = 65, normalized size = 0.84

$$\frac{(d\cos(a+bx))^{3/2}\left(2\sqrt{\cos(a+bx)}-\tan^{-1}\left(\sqrt{\cos(a+bx)}\right)-\tanh^{-1}\left(\sqrt{\cos(a+bx)}\right)\right)}{b\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x],x]
```

```
[Out] ((-ArcTan[Sqrt[Cos[a + b*x]]] - ArcTanh[Sqrt[Cos[a + b*x]]] + 2*Sqrt[Cos[a
+ b*x]])*(d*Cos[a + b*x])^(3/2))/(b*Cos[a + b*x]^(3/2))
```

Maple [B] time = 0.141, size = 204, normalized size = 2.7

$$-\frac{1}{2b}d^{\frac{3}{2}}\ln\left(2\frac{\sqrt{d}\sqrt{-2\left(\sin(1/2\,bx+a/2)\right)^2 d+d}+2\,d\cos(1/2\,bx+a/2)-d}{\cos(1/2\,bx+a/2)-1}\right)-\frac{1}{2b}d^{\frac{3}{2}}\ln\left(2\frac{\sqrt{d}\sqrt{-2\left(\sin(1/2\,bx+a/2)\right)^2 d}}{\cos(1/2\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*csc(b*x+a),x)
```

```
[Out] -1/2/b*d^(3/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))-1/2/b*d^(3/2)*ln(2/(cos(1/2*b*x+1/2
*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-
d))+1/(-d)^(1/2)/b*d^2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+
1/2*a)^2*d+d)^(1/2)-d))+2/b*d*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.79855, size = 702, normalized size = 9.12

$$\left[\frac{2\sqrt{-d}d\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)+\sqrt{-d}d\log\left(-\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6d\cos(bx+a)+d}{\cos(bx+a)^2+2\cos(bx+a)+1}\right)+8\sqrt{d\cos(bx}}{4b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-d)*d*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x + a) +
 d)) + sqrt(-d)*d*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*
(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a)
 + 1)) + 8*sqrt(d*cos(b*x + a))*d)/b, 1/4*(2*d^(3/2)*arctan(2*sqrt(d*cos(b*
x + a))*sqrt(d)/(d*cos(b*x + a) - d)) + d^(3/2)*log(-(d*cos(b*x + a)^2 - 4*
sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(co
s(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*d)/b]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*csc(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a), x)
```

3.226 $\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}$$

```
[Out] (Sqrt[d]*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b - (Sqrt[d]*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/b
```

Rubi [A] time = 0.0511264, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2565, 329, 298, 203, 206}

$$\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x],x]
```

```
[Out] (Sqrt[d]*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b - (Sqrt[d]*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/b
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd}$$
$$= -\frac{d\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} + \frac{d\,\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b}$$
$$= \frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}$$

Mathematica [A] time = 0.0364651, size = 51, normalized size = 0.88

$$\frac{\sqrt{d\cos(a+bx)}\left(\tan^{-1}\left(\sqrt{\cos(a+bx)}\right)-\tanh^{-1}\left(\sqrt{\cos(a+bx)}\right)\right)}{b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x],x]
```

```
[Out] ((ArcTan[Sqrt[Cos[a + b*x]]] - ArcTanh[Sqrt[Cos[a + b*x]]])*Sqrt[d*Cos[a +
b*x]])/(b*Sqrt[Cos[a + b*x]])
```

Maple [B] time = 0.112, size = 179, normalized size = 3.1

$$-\frac{1}{2b}\sqrt{d}\ln\left(2\frac{\sqrt{d}\sqrt{-2\left(\sin(1/2\,bx+a/2)\right)^2 d+d}+2\,d\cos(1/2\,bx+a/2)-d}{\cos(1/2\,bx+a/2)-1}\right)-\frac{1}{2b}\sqrt{d}\ln\left(2\frac{\sqrt{d}\sqrt{-2\left(\sin(1/2\,bx+a/2)\right)^2}}{\cos(1/2\,}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*csc(b*x+a),x)
```

```
[Out] -1/2/b*d^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))-1/2/b*d^(1/2)*ln(2/(cos(1/2*b*x+1/2
*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-
d))-1/(-d)^(1/2)/b*d*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/
2*a)^2*d+d)^(1/2)-d))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.63979, size = 664, normalized size = 11.45

$$\left[\frac{2\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)+\sqrt{-d}\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6\,d\cos(bx+a)+d}{\cos(bx+a)^2+2\cos(bx+a)+1}\right)}{4\,b}, \frac{2\sqrt{d}\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) + 1
)/(d*cos(b*x + a))) + sqrt(-d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a
))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*
cos(b*x + a) + 1)))/b, 1/4*(2*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(
b*x + a) - 1)/(sqrt(d)*cos(b*x + a))) + sqrt(d)*log((d*cos(b*x + a)^2 - 4*s
qrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos
(b*x + a)^2 - 2*cos(b*x + a) + 1)))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*csc(b*x+a),x)
```

```
[Out] Integral(sqrt(d*cos(a + b*x))*csc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cos(bx+a)}\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*csc(b*x + a), x)
```

3.227 $\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=59

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}}$$

```
[Out] -(ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*Sqrt[d])) - ArcTanh[Sqrt[d*Cos[a
+ b*x]]/Sqrt[d]]/(b*Sqrt[d])
```

Rubi [A] time = 0.0510998, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2565, 329, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] -(ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*Sqrt[d])) - ArcTanh[Sqrt[d*Cos[a
+ b*x]]/Sqrt[d]]/(b*Sqrt[d])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}}$$

Mathematica [A] time = 0.0399, size = 50, normalized size = 0.85

$$-\frac{\sqrt{\cos(a+bx)}\left(\tan^{-1}\left(\sqrt{\cos(a+bx)}\right)+\tanh^{-1}\left(\sqrt{\cos(a+bx)}\right)\right)}{b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] -(((ArcTan[Sqrt[Cos[a + b*x]]] + ArcTanh[Sqrt[Cos[a + b*x]]])*Sqrt[Cos[a +
b*x]])/(b*Sqrt[d*Cos[a + b*x]]))
```

Maple [B] time = 0.104, size = 177, normalized size = 3.

$$-\frac{1}{2b}\ln\left(2\frac{\sqrt{d}\sqrt{-2\left(\sin\left(1/2\,bx+a/2\right)\right)^2d+d}+2\,d\cos\left(1/2\,bx+a/2\right)-d}{\cos\left(1/2\,bx+a/2\right)-1}\right)\frac{1}{\sqrt{d}}-\frac{1}{2b}\ln\left(2\frac{\sqrt{d}\sqrt{-2\left(\sin\left(1/2\,bx+a/2\right)\right)}}{\cos\left(1/2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] -1/2/d^(1/2)/b*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))-1/2/d^(1/2)/b*ln(2/(cos(1/2*b*x+1/2
*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-
d))+1/(-d)^(1/2)/b*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*
a)^2*d+d)^(1/2)-d))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.5388, size = 676, normalized size = 11.46

$$\left[\frac{2\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2d\cos(bx+a)}\right)-\sqrt{-d}\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6d\cos(bx+a)+d}{\cos(bx+a)^2+2\cos(bx+a)+1}\right)}{4bd}, -\frac{2\sqrt{d}\operatorname{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) + 1
)/(d*cos(b*x + a))) - sqrt(-d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a
))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*
cos(b*x + a) + 1)))/(b*d), -1/4*(2*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*
(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a))) - sqrt(d)*log((d*cos(b*x + a)^2
- 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)
/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)/sqrt(d*cos(a + b*x)), x)
```

Giac [A] time = 1.12228, size = 70, normalized size = 1.19

$$\frac{d\left(\frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}d}-\frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] d*(arctan(sqrt(d*cos(b*x + a))/sqrt(-d))/(sqrt(-d)*d) - arctan(sqrt(d*cos(b
*x + a))/sqrt(d))/d^(3/2))/b
```

3.228 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}}+\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(3/2)) - ArcTanh[Sqrt[d*Cos[a + b
*x]]/Sqrt[d]]/(b*d^(3/2)) + 2/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.0666221, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 325, 329, 298, 203, 206}

$$\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}}+\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(3/2)) - ArcTanh[Sqrt[d*Cos[a + b
*x]]/Sqrt[d]]/(b*d^(3/2)) + 2/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd} \\
&= \frac{2}{bd\sqrt{d\cos(a+bx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd^3} \\
&= \frac{2}{bd\sqrt{d\cos(a+bx)}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^3} \\
&= \frac{2}{bd\sqrt{d\cos(a+bx)}} - \frac{\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd} + \frac{\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+b
\end{aligned}
$$

$$
= \frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}} + \frac{2}{bd\sqrt{d\cos(a+bx)}}
$$

Mathematica [C] time = 0.0551461, size = 36, normalized size = 0.46

$$
\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\cos^2(a+bx)\right)}{bd\sqrt{d\cos(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*Hypergeometric2F1[-1/4, 1, 3/4, Cos[a + b*x]^2])/(b*d*Sqrt[d*Cos[a + b*x
]])
```

Maple [B] time = 0.297, size = 422, normalized size = 5.4

$$
\frac{1}{2b}\left(-\left(4\ln\left(2\frac{\sqrt{-d}\sqrt{-2\left(\sin(1/2\,bx+a/2)\right)^2 d+d}-d}{\cos(1/2\,bx+a/2)}\right)d^{5/2}+2\ln\left(2\frac{\sqrt{d}\sqrt{-2\left(\sin(1/2\,bx+a/2)\right)^2 d+d}+2d\cos(1/2\,b\right.}{\cos(1/2\,bx+a/2)-1}\right.\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] 1/2*(-(4*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^
(1/2)-d))*d^(5/2)+2*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/
2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^2+2*ln(2/(cos(1/2
*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x
+1/2*a)-d))*(-d)^(1/2)*d^2)*sin(1/2*b*x+1/2*a)^2+2*ln(2/cos(1/2*b*x+1/2*a)*
((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(5/2)-4*(-2*sin(1/2*b
*x+1/2*a)^2*d+d)^(1/2)*d^(3/2)*(-d)^(1/2)+ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1
/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/
2)*d^2+ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(
1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^2)/d^(7/2)/(-d)^(1/2)/(2*sin(1
/2*b*x+1/2*a)^2-1)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.72727, size = 853, normalized size = 10.94

$$\left[\frac{2\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)\cos\left(bx+a\right)-\sqrt{-d}\cos\left(bx+a\right)\log\left(\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)\cdot}{\cos(bx+a)^2+2\cos(bx+a)+1}\right.}{4\,bd^2\cos\left(bx+a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) + 1
)/(d*cos(b*x + a)))*cos(b*x + a) - sqrt(-d)*cos(b*x + a)*log((d*cos(b*x + a
)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a)
 + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a)))/(b*d
^2*cos(b*x + a)), 1/4*(2*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x +
 a) - 1)/(sqrt(d)*cos(b*x + a)))*cos(b*x + a) + sqrt(d)*cos(b*x + a)*log((d
*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*c
os(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x
+ a)))/(b*d^2*cos(b*x + a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(a+bx\right)}{\left(d\cos\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)/(d*cos(a + b*x))**(3/2), x)
```

Giac [A] time = 1.12894, size = 89, normalized size = 1.14

$$\frac{d\left(\frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}d^2}+\frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{2}{\sqrt{d\cos(bx+a)}d^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] d*(arctan(sqrt(d*cos(b*x + a))/sqrt(-d))/(sqrt(-d)*d^2) + arctan(sqrt(d*cos
(b*x + a))/sqrt(d))/d^(5/2) + 2/(sqrt(d*cos(b*x + a))*d^2))/b
```

3.229 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}}+\frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

```
[Out] -(ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(5/2))) - ArcTanh[Sqrt[d*Cos[a
+ b*x]]/Sqrt[d]]/(b*d^(5/2)) + 2/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.0642343, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 325, 329, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}}+\frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] -(ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(5/2))) - ArcTanh[Sqrt[d*Cos[a
+ b*x]]/Sqrt[d]]/(b*d^(5/2)) + 2/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd}\\
&= \frac{2}{3bd(d\cos(a+bx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd^3}\\
&= \frac{2}{3bd(d\cos(a+bx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^3}\\
&= \frac{2}{3bd(d\cos(a+bx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^2} - \frac{\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^2}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}} + \frac{2}{3bd(d\cos(a+bx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0578965, size = 38, normalized size = 0.47

$$\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\cos^2(a+bx)\right)}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (2*Hypergeometric2F1[-3/4, 1, 1/4, Cos[a + b*x]^2])/(3*b*d*(d*Cos[a + b*x])
^(3/2))
```

Maple [B] time = 0.226, size = 624, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] 1/6*((24*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^
(1/2)-d))*d^(7/2)-12*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1
```

```
/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3-12*ln(2/(cos(1
/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b
*x+1/2*a)-d))*(-d)^(1/2)*d^3)*sin(1/2*b*x+1/2*a)^4+(-24*ln(2/cos(1/2*b*x+1/
2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(7/2)+12*ln(2/(c
os(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1
/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3+12*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-
2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3
)*sin(1/2*b*x+1/2*a)^2+6*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*
x+1/2*a)^2*d+d)^(1/2)-d))*d^(7/2)+4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(
5/2)*(-d)^(1/2)-3*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*
a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3-3*ln(2/(cos(1/2*b
*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1
/2*a)-d))*(-d)^(1/2)*d^3)/d^(11/2)/(-d)^(1/2)/(4*sin(1/2*b*x+1/2*a)^4-4*sin
(1/2*b*x+1/2*a)^2+1)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.64093, size = 879, normalized size = 10.85

$$\left[\frac{6\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2d\cos(bx+a)}\right)\cos(bx+a)^2 - 3\sqrt{-d}\cos(bx+a)^2\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a}{\cos(bx+a)^2+2\cos(bx+a)}\right.}{12bd^3\cos(bx+a)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) +
1)/(d*cos(b*x + a)))*cos(b*x + a)^2 - 3*sqrt(-d)*cos(b*x + a)^2*log((d*cos(
b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b
*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a)
))/(b*d^3*cos(b*x + a)^2), -1/12*(6*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))
*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a)))*cos(b*x + a)^2 - 3*sqrt(d)*cos(
b*x + a)^2*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x
+ a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) -
8*sqrt(d*cos(b*x + a)))/(b*d^3*cos(b*x + a)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1396, size = 104, normalized size = 1.28

$$\frac{d\left(\frac{3\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}d^3}-\frac{3\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{7}{2}}}+\frac{2}{\sqrt{d\cos(bx+a)}d^3\cos(bx+a)}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*d*(3*arctan(sqrt(d*cos(b*x + a))/sqrt(-d))/(sqrt(-d)*d^3) - 3*arctan(sq
rt(d*cos(b*x + a))/sqrt(d))/d^(7/2) + 2/(sqrt(d*cos(b*x + a))*d^3*cos(b*x +
 a)))/b
```

3.230 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$\frac{2}{bd^3\sqrt{d\cos(a+bx)}}+\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}}+\frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

```
[Out] ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(7/2)) - ArcTanh[Sqrt[d*Cos[a + b
*x]]/Sqrt[d]]/(b*d^(7/2)) + 2/(5*b*d*(d*Cos[a + b*x])^(5/2)) + 2/(b*d^3*Sqr
t[d*Cos[a + b*x]])
```

Rubi [A] time = 0.0778657, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 325, 329, 298, 203, 206}

$$\frac{2}{bd^3\sqrt{d\cos(a+bx)}}+\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}}+\frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(7/2)) - ArcTanh[Sqrt[d*Cos[a + b
*x]]/Sqrt[d]]/(b*d^(7/2)) + 2/(5*b*d*(d*Cos[a + b*x])^(5/2)) + 2/(b*d^3*Sqr
t[d*Cos[a + b*x]])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^{7/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= \frac{2}{5bd(d\cos(a+bx))^{5/2}} - \frac{\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd^3}$$
$$= \frac{2}{5bd(d\cos(a+bx))^{5/2}} + \frac{2}{bd^3\sqrt{d\cos(a+bx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd^5}$$
$$= \frac{2}{5bd(d\cos(a+bx))^{5/2}} + \frac{2}{bd^3\sqrt{d\cos(a+bx)}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^5}$$
$$= \frac{2}{5bd(d\cos(a+bx))^{5/2}} + \frac{2}{bd^3\sqrt{d\cos(a+bx)}} - \frac{\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^3} + \frac{\text{Subst}\left(\right.}{}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}} + \frac{2}{5bd(d\cos(a+bx))^{5/2}} + \frac{2}{bd^3\sqrt{d\cos(a+bx)}}$$

Mathematica [C] time = 0.0682588, size = 38, normalized size = 0.38

$$\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};\cos^2(a+bx)\right)}{5bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*Hypergeometric2F1[-5/4, 1, -1/4, Cos[a + b*x]^2])/(5*b*d*(d*Cos[a + b*x]
)^(5/2))
```

Maple [B] time = 0.22, size = 882, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] 1/10/d^(15/2)/(-d)^(1/2)/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*
sin(1/2*b*x+1/2*a)^2-1)*(10*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2
*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(9/2)-24*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)
*d^(7/2)*(-d)^(1/2)+5*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+
1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4+5*ln(2/(cos(1
/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b
*x+1/2*a)-d))*(-d)^(1/2)*d^4-40*(2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*
sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(9/2)+ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^
(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(
1/2)*d^4+ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)
^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4)*sin(1/2*b*x+1/2*a)^6+20*(
6*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d
))*d^(9/2)-4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(7/2)*(-d)^(1/2)+3*ln(2/
(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos
(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4+3*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(
-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^
4)*sin(1/2*b*x+1/2*a)^4-10*(6*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1
/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(9/2)-8*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2
)*d^(7/2)*(-d)^(1/2)+3*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x
+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4+3*ln(2/(cos(
1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*
b*x+1/2*a)-d))*(-d)^(1/2)*d^4)*sin(1/2*b*x+1/2*a)^2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.70545, size = 942, normalized size = 9.42

$$\left[\frac{10\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2d\cos(bx+a)}\right)\cos(bx+a)^3-5\sqrt{-d}\cos(bx+a)^3\log\left(\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+}{\cos(bx+a)^2+2\cos(bx+}\right.}{20bd^4\cos(bx+a)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] [1/20*(10*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) +
 1)/(d*cos(b*x + a)))*cos(b*x + a)^3 - 5*sqrt(-d)*cos(b*x + a)^3*log((d*cos
(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(
b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a
))*(5*cos(b*x + a)^2 + 1))/(b*d^4*cos(b*x + a)^3), 1/20*(10*sqrt(d)*arctan(
1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a)))*cos(b*x
 + a)^3 + 5*sqrt(d)*cos(b*x + a)^3*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x
 + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 -
 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*(5*cos(b*x + a)^2 + 1))/(b*d
```

```
^4*cos(b*x + a)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13774, size = 127, normalized size = 1.27

$$\frac{d\left(\frac{5\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}d^4}+\frac{5\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{9}{2}}}+\frac{2\left(5d^2\cos(bx+a)^2+d^2\right)}{\sqrt{d\cos(bx+a)}d^6\cos(bx+a)^2}\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] 1/5*d*(5*arctan(sqrt(d*cos(b*x + a))/sqrt(-d))/(sqrt(-d)*d^4) + 5*arctan(sq
rt(d*cos(b*x + a))/sqrt(d))/d^(9/2) + 2*(5*d^2*cos(b*x + a)^2 + d^2)/(sqrt(
d*cos(b*x + a))*d^6*cos(b*x + a)^2))/b
```

3.231 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal. Leaf size=103

$$\frac{2}{3bd^3(d\cos(a+bx))^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}}+\frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

```
[Out] -(ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(9/2))) - ArcTanh[Sqrt[d*Cos[a
+ b*x]]/Sqrt[d]]/(b*d^(9/2)) + 2/(7*b*d*(d*Cos[a + b*x])^(7/2)) + 2/(3*b*d^
3*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.0752345, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 325, 329, 212, 206, 203}

$$\frac{2}{3bd^3(d\cos(a+bx))^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}}+\frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] -(ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(9/2))) - ArcTanh[Sqrt[d*Cos[a
+ b*x]]/Sqrt[d]]/(b*d^(9/2)) + 2/(7*b*d*(d*Cos[a + b*x])^(7/2)) + 2/(3*b*d^
3*(d*Cos[a + b*x])^(3/2))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{9/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd}\\
&= \frac{2}{7bd(d\cos(a+bx))^{7/2}} - \frac{\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd^3}\\
&= \frac{2}{7bd(d\cos(a+bx))^{7/2}} + \frac{2}{3bd^3(d\cos(a+bx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd^5}\\
&= \frac{2}{7bd(d\cos(a+bx))^{7/2}} + \frac{2}{3bd^3(d\cos(a+bx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^5}\\
&= \frac{2}{7bd(d\cos(a+bx))^{7/2}} + \frac{2}{3bd^3(d\cos(a+bx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^4} - \text{Subs}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}} + \frac{2}{7bd(d\cos(a+bx))^{7/2}} + \frac{2}{3bd^3(d\cos(a+bx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0826734, size = 38, normalized size = 0.37

$$\frac{2\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};\cos^2(a+bx)\right)}{7bd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*Hypergeometric2F1[-7/4, 1, -3/4, Cos[a + b*x]^2])/(7*b*d*(d*Cos[a + b*x]
)^(7/2))
```

Maple [B] time = 0.223, size = 1082, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x)
```

```
[Out] 1/42/d^(19/2)/(-d)^(1/2)/(16*sin(1/2*b*x+1/2*a)^8-32*sin(1/2*b*x+1/2*a)^6+2
4*sin(1/2*b*x+1/2*a)^4-8*sin(1/2*b*x+1/2*a)^2+1)*(42*ln(2/cos(1/2*b*x+1/2*a
)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(11/2)+40*(-2*sin(1
/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(9/2)*(-d)^(1/2)-21*ln(2/(cos(1/2*b*x+1/2*a)-1
)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(
-d)^(1/2)*d^5-21*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a
)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^5-336*(-2*ln(2/cos(1
/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(11/2)+
ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*
d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^5+ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2
)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)
*d^5)*sin(1/2*b*x+1/2*a)^8+672*(-2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*
sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(11/2)+ln(2/(cos(1/2*b*x+1/2*a)+1)*(d
^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^
(1/2)*d^5+ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d
)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^5)*sin(1/2*b*x+1/2*a)^6+56*
(-6*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)
-d))*d^(11/2)-2*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(9/2)*(-d)^(1/2)+3*ln
(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*
cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^5+3*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2
)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)
*d^5)*sin(1/2*b*x+1/2*a)^2-56*(-18*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*
sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(11/2)-2*(-2*sin(1/2*b*x+1/2*a)^2*d+d
)^(1/2)*d^(9/2)*(-d)^(1/2)+9*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1
/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^5+9*ln(2
/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*co
s(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^5)*sin(1/2*b*x+1/2*a)^4)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.69262, size = 946, normalized size = 9.18

$$\left[\frac{42\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2d\cos(bx+a)}\right)\cos(bx+a)^4 - 21\sqrt{-d}\cos(bx+a)^4\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(b}{\cos(bx+a)^2+2\cos(bx}\right.}{84\,bd^5\cos(bx+a)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] [1/84*(42*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) +
 1)/(d*cos(b*x + a)))*cos(b*x + a)^4 - 21*sqrt(-d)*cos(b*x + a)^4*log((d*co
s(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos
(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x +
```

```
a))*(7*cos(b*x + a)^2 + 3))/(b*d^5*cos(b*x + a)^4), -1/84*(42*sqrt(d)*arcta
n(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a)))*cos(b
*x + a)^4 - 21*sqrt(d)*cos(b*x + a)^4*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(
b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^
2 - 2*cos(b*x + a) + 1)) - 8*sqrt(d*cos(b*x + a))*(7*cos(b*x + a)^2 + 3))/(
b*d^5*cos(b*x + a)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12811, size = 130, normalized size = 1.26

$$\frac{d\left(\frac{21\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}d^5} - \frac{21\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{11}{2}}} + \frac{2\left(7d^2\cos(bx+a)^2+3d^2\right)}{\sqrt{d\cos(bx+a)}d^7\cos(bx+a)^3}\right)}{21b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

```
[Out] 1/21*d*(21*arctan(sqrt(d*cos(b*x + a))/sqrt(-d))/(sqrt(-d)*d^5) - 21*arctan
(sqrt(d*cos(b*x + a))/sqrt(d))/d^(11/2) + 2*(7*d^2*cos(b*x + a)^2 + 3*d^2)/
(sqrt(d*cos(b*x + a))*d^7*cos(b*x + a)^3))/b
```

3.232 $\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx$

Optimal. Leaf size=124

$$-\frac{15d^5\sin(a+bx)\sqrt{d\cos(a+bx)}}{7b}-\frac{9d^3\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b}-\frac{15d^6\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7b\sqrt{d\cos(a+bx)}}-\frac{d\csc(a+$$

```
[Out] -((d*(d*Cos[a + b*x])^(9/2)*Csc[a + b*x])/b) - (15*d^6*Sqrt[Cos[a + b*x]]*E
llipticF[(a + b*x)/2, 2])/(7*b*Sqrt[d*Cos[a + b*x]]) - (15*d^5*Sqrt[d*Cos[a
 + b*x]]*Sin[a + b*x])/(7*b) - (9*d^3*(d*Cos[a + b*x])^(5/2)*Sin[a + b*x])/
(7*b)
```

Rubi [A] time = 0.101253, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2567, 2635, 2642, 2641}

$$-\frac{15d^5\sin(a+bx)\sqrt{d\cos(a+bx)}}{7b}-\frac{9d^3\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b}-\frac{15d^6\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7b\sqrt{d\cos(a+bx)}}-\frac{d\csc(a+$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^2,x]
```


```
[Out] -((d*(d*Cos[a + b*x])^(9/2)*Csc[a + b*x])/b) - (15*d^6*Sqrt[Cos[a + b*x]]*E
llipticF[(a + b*x)/2, 2])/(7*b*Sqrt[d*Cos[a + b*x]]) - (15*d^5*Sqrt[d*Cos[a
 + b*x]]*Sin[a + b*x])/(7*b) - (9*d^3*(d*Cos[a + b*x])^(5/2)*Sin[a + b*x])/
(7*b)
```


Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{1}{2}\left(9d^2\right)\int (d\cos(a+bx))^{7/2}\,dx$$
$$= -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{9d^3(d\cos(a+bx))^{5/2}\sin(a+bx)}{7b} - \frac{1}{14}\left(45d^4\right)$$
$$= -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{15d^5\sqrt{d\cos(a+bx)}\sin(a+bx)}{7b} - \frac{9d^3(d\cos(a}{}$$
$$= -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{15d^5\sqrt{d\cos(a+bx)}\sin(a+bx)}{7b} - \frac{9d^3(d\cos(a}{}$$
$$= -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{15d^6\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7b\sqrt{d\cos(a+bx)}} - \frac{15d^5\sqrt{d\,c}}{}$$

Mathematica [A] time = 0.35911, size = 89, normalized size = 0.72

$$\frac{d^5\csc(a+bx)\sqrt{d\cos(a+bx)}\left(\sqrt{\cos(a+bx)}(16\cos(2(a+bx))+\cos(4(a+bx))-45)-60\sin(a+bx)F\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{28b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^2,x]
```

```
[Out] (d^5*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]*(Sqrt[Cos[a + b*x]]*(-45 + 16*Cos[2*
(a + b*x)] + Cos[4*(a + b*x)]) - 60*EllipticF[(a + b*x)/2, 2]*Sin[a + b*x])
)/(28*b*Sqrt[Cos[a + b*x]])
```

Maple [A] time = 0.204, size = 242, normalized size = 2.

$$-\frac{d^7}{14\,b}\sqrt{d\left(2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-128\,\left(\sin\left(1/2\,bx+a/2\right)\right)^{12}+384\,\left(\sin\left(1/2\,bx+a/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x)
```

```
[Out] -1/14*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^7/(-2*sin
(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/
2*b*x+1/2*a)*(-128*sin(1/2*b*x+1/2*a)^12+384*sin(1/2*b*x+1/2*a)^10-576*sin(
1/2*b*x+1/2*a)^8+30*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)
^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)+512*sin(1/2
*b*x+1/2*a)^6-204*sin(1/2*b*x+1/2*a)^4+12*sin(1/2*b*x+1/2*a)^2+7)/(d*(2*cos
(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{11}{2}}\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(11/2)*csc(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\cos\left(bx+a\right)}d^{5}\cos\left(bx+a\right)^{5}\csc\left(bx+a\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*d^5*cos(b*x + a)^5*csc(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(11/2)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(bx+a\right)\right)^{\frac{11}{2}}\csc\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(11/2)*csc(b*x + a)^2, x)
```

3.233 $\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx$

Optimal. Leaf size=96

$$-\frac{7d^3\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}-\frac{21d^4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}-\frac{d\csc(a+bx)(d\cos(a+bx))^{7/2}}{b}$$

```
[Out] -((d*(d*Cos[a + b*x])^(7/2)*Csc[a + b*x])/b) - (21*d^4*Sqrt[d*Cos[a + b*x]]
*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]) - (7*d^3*(d*Cos[a + b*
x])^(3/2)*Sin[a + b*x])/(5*b)
```

Rubi [A] time = 0.0824111, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2567, 2635, 2640, 2639}

$$-\frac{7d^3\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}-\frac{21d^4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}-\frac{d\csc(a+bx)(d\cos(a+bx))^{7/2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*(d*Cos[a + b*x])^(7/2)*Csc[a + b*x])/b) - (21*d^4*Sqrt[d*Cos[a + b*x]]
*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]) - (7*d^3*(d*Cos[a + b*
x])^(3/2)*Sin[a + b*x])/(5*b)
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = -\frac{d(d\cos(a+bx))^{7/2}\csc(a+bx)}{b} - \frac{1}{2}\left(7d^2\right)\int (d\cos(a+bx))^{5/2}\,dx$$
$$= -\frac{d(d\cos(a+bx))^{7/2}\csc(a+bx)}{b} - \frac{7d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{5b} - \frac{1}{10}\left(21d^4\right.$$
$$= -\frac{d(d\cos(a+bx))^{7/2}\csc(a+bx)}{b} - \frac{7d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{5b} - \frac{\left(21d^4\sqrt{d}\right.}{}$$
$$= -\frac{d(d\cos(a+bx))^{7/2}\csc(a+bx)}{b} - \frac{21d^4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}} - \frac{7d^3(}{}$$

Mathematica [A] time = 0.240348, size = 74, normalized size = 0.77

$$-\frac{d^4\sqrt{d\cos(a+bx)}\left(21E\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}(\sin(2(a+bx))+5\cot(a+bx))\right)}{5b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^2,x]
```

```
[Out] -(d^4*Sqrt[d*Cos[a + b*x]]*(21*EllipticE[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x
]]*(5*Cot[a + b*x] + Sin[2*(a + b*x)])))/(5*b*Sqrt[Cos[a + b*x]])
```

Maple [B] time = 0.203, size = 229, normalized size = 2.4

$$\frac{d^6}{10\,b}\sqrt{d\left(2\,(\cos(1/2\,bx+a/2))^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-64\,(\sin(1/2\,bx+a/2))^{10}+160\,(\sin(1/2\,bx+a/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x)
```

```
[Out] 1/10*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^6/(-2*sin(
1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/2
*b*x+1/2*a)*(-64*sin(1/2*b*x+1/2*a)^10+160*sin(1/2*b*x+1/2*a)^8+42*(2*sin(1
/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x
+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)-104*sin(1/2*b*x+1/2*a)^6-4*sin(1/2*b*x+
1/2*a)^4+22*sin(1/2*b*x+1/2*a)^2-5)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{9}{2}}\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\cos\left(bx+a\right)}d^4\cos\left(bx+a\right)^4\csc\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*d^4*cos(b*x + a)^4*csc(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(9/2)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}\csc\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a)^2, x)
```

3.234 $\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx$

Optimal. Leaf size=96

$$-\frac{5d^3\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}-\frac{5d^4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{d\cos(a+bx)}}-\frac{d\csc(a+bx)(d\cos(a+bx))^{5/2}}{b}$$

```
[Out] -((d*(d*Cos[a + b*x])^(5/2)*Csc[a + b*x])/b) - (5*d^4*Sqrt[Cos[a + b*x]]*El
lipticF[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]]) - (5*d^3*Sqrt[d*Cos[a +
 b*x]]*Sin[a + b*x])/(3*b)
```

Rubi [A] time = 0.0814523, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2567, 2635, 2642, 2641}

$$-\frac{5d^3\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}-\frac{5d^4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{d\cos(a+bx)}}-\frac{d\csc(a+bx)(d\cos(a+bx))^{5/2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*(d*Cos[a + b*x])^(5/2)*Csc[a + b*x])/b) - (5*d^4*Sqrt[Cos[a + b*x]]*El
lipticF[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]]) - (5*d^3*Sqrt[d*Cos[a +
 b*x]]*Sin[a + b*x])/(3*b)
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = -\frac{d(d\cos(a+bx))^{5/2}\csc(a+bx)}{b} - \frac{1}{2}\left(5d^2\right)\int (d\cos(a+bx))^{3/2}\,dx$$
$$= -\frac{d(d\cos(a+bx))^{5/2}\csc(a+bx)}{b} - \frac{5d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{3b} - \frac{1}{6}\left(5d^4\right)\int \frac{}{\sqrt{d}}$$
$$= -\frac{d(d\cos(a+bx))^{5/2}\csc(a+bx)}{b} - \frac{5d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{3b} - \frac{\left(5d^4\sqrt{\cos(a+}\right.}{6\sqrt{d}}$$
$$= -\frac{d(d\cos(a+bx))^{5/2}\csc(a+bx)}{b} - \frac{5d^4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{d\cos(a+bx)}} - \frac{5d^3\sqrt{d\cos}}{}$$

Mathematica [A] time = 0.221656, size = 73, normalized size = 0.76

$$\frac{d^3\sqrt{d\cos(a+bx)}\left(\sqrt{\cos(a+bx)}(\cos(2(a+bx))-4)\csc(a+bx)-5F\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{3b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^2,x]
```

```
[Out] (d^3*Sqrt[d*Cos[a + b*x]]*(Sqrt[Cos[a + b*x]]*(-4 + Cos[2*(a + b*x)])*Csc[a
 + b*x] - 5*EllipticF[(a + b*x)/2, 2]))/(3*b*Sqrt[Cos[a + b*x]])
```

Maple [A] time = 0.197, size = 216, normalized size = 2.3

$$-\frac{d^5}{6b}\sqrt{d\left(2\left(\cos(1/2\,bx+a/2)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-32\left(\sin(1/2\,bx+a/2)\right)^8+10\left(2\left(\sin(1/2\,bx+a/2)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x)
```

```
[Out] -1/6*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^5/(-2*sin(
1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/2
*b*x+1/2*a)*(-32*sin(1/2*b*x+1/2*a)^8+10*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*(
sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b
*x+1/2*a)+64*sin(1/2*b*x+1/2*a)^6-28*sin(1/2*b*x+1/2*a)^4-4*sin(1/2*b*x+1/2
*a)^2+3)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{7}{2}}\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\cos\left(bx+a\right)}d^3\cos\left(bx+a\right)^3\csc\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*d^3*cos(b*x + a)^3*csc(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(7/2)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}\csc\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a)^2, x)
```

3.235 $\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{3d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}-\frac{d\csc(a+bx)(d\cos(a+bx))^{3/2}}{b}$$

```
[Out] -((d*(d*Cos[a + b*x])^(3/2)*Csc[a + b*x])/b) - (3*d^2*Sqrt[d*Cos[a + b*x]]*
EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

Rubi [A] time = 0.0627215, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2567, 2640, 2639}

$$-\frac{3d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}-\frac{d\csc(a+bx)(d\cos(a+bx))^{3/2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*(d*Cos[a + b*x])^(3/2)*Csc[a + b*x])/b) - (3*d^2*Sqrt[d*Cos[a + b*x]]*
EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx &= -\frac{d(d\cos(a+bx))^{3/2}\csc(a+bx)}{b}-\frac{1}{2}\left(3d^2\right)\int\sqrt{d\cos(a+bx)}\,dx\\ &= -\frac{d(d\cos(a+bx))^{3/2}\csc(a+bx)}{b}-\frac{\left(3d^2\sqrt{d\cos(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,dx}{2\sqrt{\cos(a+bx)}}\\ &= -\frac{d(d\cos(a+bx))^{3/2}\csc(a+bx)}{b}-\frac{3d^2\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.14493, size = 58, normalized size = 0.88

$$-\frac{(d\cos(a+bx))^{5/2}\left(3E\left(\frac{1}{2}(a+bx)\middle|2\right)+\cos^{\frac{3}{2}}(a+bx)\csc(a+bx)\right)}{b\cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^2,x]
```

```
[Out] -(((d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^(3/2)*Csc[a + b*x] + 3*EllipticE[(a
 + b*x)/2, 2]))/(b*Cos[a + b*x]^(5/2)))
```

Maple [B] time = 0.396, size = 203, normalized size = 3.1

$$\frac{d^4}{2b}\sqrt{d\left(2\left(\cos(1/2\,bx+a/2)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(6\left(2\left(\sin(1/2\,bx+a/2)\right)^2-1\right)^{3/2}\sqrt{(\sin(1/2\,bx+a/}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x)
```

```
[Out] 1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^4/(-2*sin(1
/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/2*
b*x+1/2*a)*(6*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)
*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)+8*sin(1/2*b*x+1/2
*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)/(d*(2*cos(1/2*b*x+1
/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\cos(bx+a)}d^2\cos(bx+a)^2\csc(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*d^2*cos(b*x + a)^2*csc(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(5/2)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a)^2, x)
```

3.236 $\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{d^2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{d\cos(a+bx)}}-\frac{d\csc(a+bx)\sqrt{d\cos(a+bx)}}{b}$$

```
[Out] -((d*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x])/b) - (d^2*Sqrt[Cos[a + b*x]]*Ellipt
icF[(a + b*x)/2, 2])/(b*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.0637282, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2567, 2642, 2641}

$$-\frac{d^2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{d\cos(a+bx)}}-\frac{d\csc(a+bx)\sqrt{d\cos(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x])/b) - (d^2*Sqrt[Cos[a + b*x]]*Ellipt
icF[(a + b*x)/2, 2])/(b*Sqrt[d*Cos[a + b*x]])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx &= -\frac{d\sqrt{d\cos(a+bx)}\csc(a+bx)}{b}-\frac{1}{2}d^2\int\frac{1}{\sqrt{d\cos(a+bx)}}\,dx\\ &= -\frac{d\sqrt{d\cos(a+bx)}\csc(a+bx)}{b}-\frac{\left(d^2\sqrt{\cos(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx}{2\sqrt{d\cos(a+bx)}}\\ &= -\frac{d\sqrt{d\cos(a+bx)}\csc(a+bx)}{b}-\frac{d^2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0975214, size = 56, normalized size = 0.85

$$-\frac{(d\cos(a+bx))^{3/2}\left(F\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}\csc(a+bx)\right)}{b\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^2,x]
```

```
[Out] -(((d*Cos[a + b*x])^(3/2)*(Sqrt[Cos[a + b*x]]*Csc[a + b*x] + EllipticF[(a +
 b*x)/2, 2]))/(b*Cos[a + b*x]^(3/2)))
```

Maple [B] time = 0.266, size = 190, normalized size = 2.9

$$-\frac{d^3}{2b}\sqrt{d\left(2\,(\cos(1/2\,bx+a/2))^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\,\left(2\,(\sin(1/2\,bx+a/2))^2-1\right)^{3/2}\sqrt{(\sin(1/2\,bx+a/2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x)
```

```
[Out] -1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^3/(-2*sin(
1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/2
*b*x+1/2*a)*(2*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2
)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)+4*sin(1/2*b*x+1/
2*a)^4-4*sin(1/2*b*x+1/2*a)^2+1)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\cos(bx+a)}d\cos(bx+a)\csc(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*d*cos(b*x + a)*csc(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^2, x)
```

3.237 $\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx$

Optimal. Leaf size=65

$$-\frac{\csc(a+bx)(d\cos(a+bx))^{3/2}}{bd}-\frac{E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

```
[Out] -(((d*Cos[a + b*x])^(3/2)*Csc[a + b*x])/(b*d)) - (Sqrt[d*Cos[a + b*x]]*Elli
pticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

Rubi [A] time = 0.0572064, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2570, 2640, 2639}

$$-\frac{\csc(a+bx)(d\cos(a+bx))^{3/2}}{bd}-\frac{E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^2,x]
```

```
[Out] -(((d*Cos[a + b*x])^(3/2)*Csc[a + b*x])/(b*d)) - (Sqrt[d*Cos[a + b*x]]*Elli
pticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx &= -\frac{(d\cos(a+bx))^{3/2}\csc(a+bx)}{bd}-\frac{1}{2}\int \sqrt{d\cos(a+bx)}\,dx\\ &= -\frac{(d\cos(a+bx))^{3/2}\csc(a+bx)}{bd}-\frac{\sqrt{d\cos(a+bx)}\int \sqrt{\cos(a+bx)}\,dx}{2\sqrt{\cos(a+bx)}}\\ &= -\frac{(d\cos(a+bx))^{3/2}\csc(a+bx)}{bd}-\frac{\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0814993, size = 56, normalized size = 0.86

$$-\frac{\sqrt{d\cos(a+bx)}\left(E\left(\frac{1}{2}(a+bx)\middle|2\right)+\cos^{\frac{3}{2}}(a+bx)\csc(a+bx)\right)}{b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^2,x]
```

```
[Out] -((Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^(3/2)*Csc[a + b*x] + EllipticE[(a + b
*x)/2, 2]))/(b*Sqrt[Cos[a + b*x]]))
```

Maple [B] time = 0.323, size = 203, normalized size = 3.1

$$\frac{d^2}{2b}\sqrt{d\left(2\left(\cos\left(1/2\,bx+a/2\right)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\left(2\left(\sin\left(1/2\,bx+a/2\right)\right)^2-1\right)^{3/2}\sqrt{\left(\sin\left(1/2\,bx+a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x)
```

```
[Out] 1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^2/cos(1/2*b
*x+1/2*a)/(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)*sin(1/2*
b*x+1/2*a)*(2*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)
*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)+8*sin(1/2*b*x+1/2
*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)/(d*(2*cos(1/2*b*x+1
/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\cos\left(bx+a\right)}\csc\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*csc(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\cos\left(bx+a\right)}\csc\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cos{(a+bx)}}\csc^2{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*csc(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(d*cos(a + b*x))*csc(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cos{(bx+a)}}\csc{(bx+a)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*csc(b*x + a)^2, x)
```

3.238 $\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{d\cos(a+bx)}}-\frac{\csc(a+bx)\sqrt{d\cos(a+bx)}}{bd}$$

```
[Out] -((Sqrt[d*Cos[a + b*x]]*Csc[a + b*x])/(b*d)) + (Sqrt[Cos[a + b*x]]*Elliptic
F[(a + b*x)/2, 2])/(b*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.0578789, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2570, 2642, 2641}

$$\frac{\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{d\cos(a+bx)}}-\frac{\csc(a+bx)\sqrt{d\cos(a+bx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] -((Sqrt[d*Cos[a + b*x]]*Csc[a + b*x])/(b*d)) + (Sqrt[Cos[a + b*x]]*Elliptic
F[(a + b*x)/2, 2])/(b*Sqrt[d*Cos[a + b*x]])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx &= -\frac{\sqrt{d\cos(a+bx)}\csc(a+bx)}{bd}+\frac{1}{2}\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx\\
&= -\frac{\sqrt{d\cos(a+bx)}\csc(a+bx)}{bd}+\frac{\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{2\sqrt{d\cos(a+bx)}}\\
&= -\frac{\sqrt{d\cos(a+bx)}\csc(a+bx)}{bd}+\frac{\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{d\cos(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.0836877, size = 47, normalized size = 0.73

$$\frac{\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)-\cot(a+bx)}{b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (-Cot[a + b*x] + Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(b*Sqrt[d*Co
s[a + b*x]])
```

Maple [B] time = 0.27, size = 188, normalized size = 2.9

$$\frac{d}{2b}\sqrt{d\left(2\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{2}}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\left(2\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}-1\right)^{3/2}\sqrt{\left(\sin\left(1/2\,bx+a/2\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] 1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1
/2*a)/(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)*d*sin(1/2*b*
x+1/2*a)*(2*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*E
llipticF(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)-4*sin(1/2*b*x+1/2*a
)^4+4*sin(1/2*b*x+1/2*a)^2-1)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(bx+a\right)^{2}}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/sqrt(d*cos(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\cos\left(bx+a\right)}\csc\left(bx+a\right)^{2}}{d\cos\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^2/(d*cos(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)**2/sqrt(d*cos(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/sqrt(d*cos(b*x + a)), x)
```

3.239 $\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{3E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\cos(a+bx)}}+\frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] -(Csc[a + b*x]/(b*d*Sqrt[d*Cos[a + b*x]])) - (3*Sqrt[d*Cos[a + b*x]]*Ellipt
icE[(a + b*x)/2, 2])/(b*d^2*Sqrt[Cos[a + b*x]]) + (3*Sin[a + b*x])/(b*d*Sqr
t[d*Cos[a + b*x]])
```

Rubi [A] time = 0.0804821, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2570, 2636, 2640, 2639}

$$-\frac{3E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\cos(a+bx)}}+\frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] -(Csc[a + b*x]/(b*d*Sqrt[d*Cos[a + b*x]])) - (3*Sqrt[d*Cos[a + b*x]]*Ellipt
icE[(a + b*x)/2, 2])/(b*d^2*Sqrt[Cos[a + b*x]]) + (3*Sin[a + b*x])/(b*d*Sqr
t[d*Cos[a + b*x]])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = -\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}} + \frac{3}{2}\int \frac{1}{(d\cos(a+bx))^{3/2}}\,dx$$
$$= -\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}} + \frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{3\int \sqrt{d\cos(a+bx)}\,dx}{2d^2}$$
$$= -\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}} + \frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{\left(3\sqrt{d\cos(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{2d^2\sqrt{\cos(a+bx)}}$$
$$= -\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{3\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{bd^2\sqrt{\cos(a+bx)}} + \frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

Mathematica [A] time = 0.171588, size = 65, normalized size = 0.69

$$\frac{2\sin(a+bx) - \cos(a+bx)\cot(a+bx) - 3\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (-(Cos[a + b*x]*Cot[a + b*x]) - 3*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2,
 2] + 2*Sin[a + b*x])/(b*d*Sqrt[d*Cos[a + b*x]])
```

Maple [A] time = 0.319, size = 209, normalized size = 2.2

$$-\frac{1}{2\,d^3b}\sqrt{d\left(2\,(\cos(1/2\,bx+a/2))^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(-2\,(\sin(1/2\,bx+a/2))^4\,d+\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2 d\right)^{\frac{3}{2}}\left(6\sqrt{2}\,(\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] -1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^3/cos(1/2*
b*x+1/2*a)/sin(1/2*b*x+1/2*a)^5/(2*sin(1/2*b*x+1/2*a)^2-1)^2*(-2*sin(1/2*b*
x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)*(6*(2*sin(1/2*b*x+1/2*a)^2-1)^(1
/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*cos(
1/2*b*x+1/2*a)+12*sin(1/2*b*x+1/2*a)^4-12*sin(1/2*b*x+1/2*a)^2+1)/(d*(2*cos
(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\cos\left(bx+a\right)}\csc\left(bx+a\right)^2}{d^2\cos\left(bx+a\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^2/(d^2*cos(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2\left(a+bx\right)}{\left(d\cos\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)**2/(d*cos(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(bx+a\right)^2}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

3.240 $\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=98

$$\frac{5\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}+\frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}$$

```
[Out] -(Csc[a + b*x]/(b*d*(d*Cos[a + b*x])^(3/2))) + (5*Sqrt[Cos[a + b*x]]*Ellipt
icF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]) + (5*Sin[a + b*x])/(3*b
*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.0829941, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2570, 2636, 2642, 2641}

$$\frac{5\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}+\frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] -(Csc[a + b*x]/(b*d*(d*Cos[a + b*x])^(3/2))) + (5*Sqrt[Cos[a + b*x]]*Ellipt
icF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]) + (5*Sin[a + b*x])/(3*b
*d*(d*Cos[a + b*x])^(3/2))
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}} + \frac{5}{2}\int \frac{1}{(d\cos(a+bx))^{5/2}}\,dx$$

$$= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}} + \frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}} + \frac{5\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx}{6d^2}$$

$$= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}} + \frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}} + \frac{\left(5\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{6d^2\sqrt{d\cos(a+bx)}}$$

$$= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}} + \frac{5\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}} + \frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

Mathematica [A] time = 0.140831, size = 62, normalized size = 0.63

$$\frac{2\tan(a+bx) - 3\cot(a+bx) + 5\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (-3*Cot[a + b*x] + 5*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 2*Tan[a
 + b*x])/(3*b*d^2*Sqrt[d*Cos[a + b*x]])
```

Maple [A] time = 0.235, size = 190, normalized size = 1.9

$$\frac{1}{6\,db}\sqrt{d\left(2\,(\cos(1/2\,bx+a/2))^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(10\,\left(2\,(\sin(1/2\,bx+a/2))^2-1\right)^{3/2}\sqrt{(\sin(1/2\,bx+a/2))^2}\text{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] 1/6*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d/(-2*sin(1/2
*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*(10*(2*sin
(1/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cos(1/2*b
*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)-20*sin(1/2*b*x+1/2*a)^4+20*sin(1/2*b*
x+1/2*a)^2-3)*sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\cos(bx+a)}\csc(bx+a)^2}{d^3\cos(bx+a)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^2/(d^3*cos(b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

3.241 $\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=126

$$\frac{21\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}-\frac{21E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\cos(a+bx)}}+\frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}$$

```
[Out] -(Csc[a + b*x]/(b*d*(d*Cos[a + b*x])^(5/2))) - (21*Sqrt[d*Cos[a + b*x]]*Ell
ipticE[(a + b*x)/2, 2])/(5*b*d^4*Sqrt[Cos[a + b*x]]) + (7*Sin[a + b*x])/(5*
b*d*(d*Cos[a + b*x])^(5/2)) + (21*Sin[a + b*x])/(5*b*d^3*Sqrt[d*Cos[a + b*x
]])
```

Rubi [A] time = 0.102205, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2570, 2636, 2640, 2639}

$$\frac{21\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}-\frac{21E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\cos(a+bx)}}+\frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] -(Csc[a + b*x]/(b*d*(d*Cos[a + b*x])^(5/2))) - (21*Sqrt[d*Cos[a + b*x]]*Ell
ipticE[(a + b*x)/2, 2])/(5*b*d^4*Sqrt[Cos[a + b*x]]) + (7*Sin[a + b*x])/(5*
b*d*(d*Cos[a + b*x])^(5/2)) + (21*Sin[a + b*x])/(5*b*d^3*Sqrt[d*Cos[a + b*x
]])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}} + \frac{7}{2}\int \frac{1}{(d\cos(a+bx))^{7/2}}\,dx$$

$$= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}} + \frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} + \frac{21\int \frac{1}{(d\cos(a+bx))^{3/2}}\,dx}{10d^2}$$

$$= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}} + \frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} + \frac{21\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} - \frac{21\int \sqrt{d\cos(a+bx}}{10d^4}$$

$$= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}} + \frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} + \frac{21\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} - \frac{\left(21\sqrt{d\cos(a+bx)}\right)}{10d^4\sqrt{\mathrm{co}}}$$

$$= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}} - \frac{21\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^4\sqrt{\cos(a+bx)}} + \frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} + \frac{21}{5bd^3}$$

Mathematica [A] time = 0.167683, size = 82, normalized size = 0.65

$$\frac{16\sin(a+bx) - 5\cos(a+bx)\cot(a+bx) - 21\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right) + 2\tan(a+bx)\sec(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (-5*Cos[a + b*x]*Cot[a + b*x] - 21*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2
, 2] + 16*Sin[a + b*x] + 2*Sec[a + b*x]*Tan[a + b*x])/(5*b*d^3*Sqrt[d*Cos[a
 + b*x]])
```

Maple [B] time = 0.299, size = 408, normalized size = 3.2

$$-\frac{1}{10\,d^5b}\sqrt{d\left(2\,(\cos(1/2\,bx+a/2))^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(168\,\sqrt{2\,(\sin(1/2\,bx+a/2))^2-1}\sqrt{(\sin(1/2\,bx+a/2))^2}\mathrm{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] -1/10*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^5/(2*sin(
1/2*b*x+1/2*a)^2-1)/cos(1/2*b*x+1/2*a)/sin(1/2*b*x+1/2*a)^5/(8*sin(1/2*b*x+
1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)*(168*(2*sin(1/2*
b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/
2*a),2^(1/2))*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+336*sin(1/2*b*x+1/2*a
)^8-168*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*Ellip
ticE(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^2-67
2*sin(1/2*b*x+1/2*a)^6+42*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2
*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)+448*s
in(1/2*b*x+1/2*a)^4-112*sin(1/2*b*x+1/2*a)^2+5)*(-2*sin(1/2*b*x+1/2*a)^4*d+
sin(1/2*b*x+1/2*a)^2*d)^(3/2)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx + a)^2}{(d\cos (bx + a))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\cos (bx + a)}\csc (bx + a)^2}{d^4\cos (bx + a)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^2/(d^4*cos(b*x + a)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx + a)^2}{(d\cos (bx + a))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

3.242 $\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx$

Optimal. Leaf size=135

$$-\frac{9d^5\sqrt{d\cos(a+bx)}}{2b}-\frac{9d^3(d\cos(a+bx))^{5/2}}{10b}+\frac{9d^{11/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{9d^{11/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+$$

```
[Out] (9*d^(11/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (9*d^(11/2)*ArcTa
nh[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (9*d^5*Sqrt[d*Cos[a + b*x]])/(2*b
) - (9*d^3*(d*Cos[a + b*x])^(5/2))/(10*b) - (d*(d*Cos[a + b*x])^(9/2)*Csc[a
 + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0894689, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2565, 288, 321, 329, 212, 206, 203}

$$-\frac{9d^5\sqrt{d\cos(a+bx)}}{2b}-\frac{9d^3(d\cos(a+bx))^{5/2}}{10b}+\frac{9d^{11/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{9d^{11/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^3,x]
```

```
[Out] (9*d^(11/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (9*d^(11/2)*ArcTa
nh[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (9*d^5*Sqrt[d*Cos[a + b*x]])/(2*b
) - (9*d^3*(d*Cos[a + b*x])^(5/2))/(10*b) - (d*(d*Cos[a + b*x])^(9/2)*Csc[a
 + b*x]^2)/(2*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{11/2}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{d(d\cos(a+bx))^{9/2}\csc^2(a+bx)}{2b} + \frac{(9d)\,\text{Subst}\left(\int \frac{x^{7/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4b}$$

$$= -\frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d(d\cos(a+bx))^{9/2}\csc^2(a+bx)}{2b} + \frac{\left(9d^3\right)\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{d^2}}\right.}{4b}$$

$$= -\frac{9d^5\sqrt{d\cos(a+bx)}}{2b} - \frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d(d\cos(a+bx))^{9/2}\csc^2(a+bx)}{2b} +$$

$$= -\frac{9d^5\sqrt{d\cos(a+bx)}}{2b} - \frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d(d\cos(a+bx))^{9/2}\csc^2(a+bx)}{2b} +$$

$$= -\frac{9d^5\sqrt{d\cos(a+bx)}}{2b} - \frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d(d\cos(a+bx))^{9/2}\csc^2(a+bx)}{2b} +$$

$$= \frac{9d^{11/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{9d^{11/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{9d^5\sqrt{d\cos(a+bx)}}{2b} -$$

Mathematica [A] time = 2.07647, size = 137, normalized size = 1.01

$$\frac{d(d\cos(a+bx))^{9/2}\left(-\frac{21}{2}\left(8\sqrt{\cos(a+bx)}+\log\left(1-\sqrt{\cos(a+bx)}\right)-\log\left(\sqrt{\cos(a+bx)}+1\right)\right)+45\tan^{-1}\left(\sqrt{\cos(a+bx)}\right)\right.}{20b\cos^{\frac{9}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^3,x]
```

```
[Out] (d*(d*Cos[a + b*x])^(9/2)*(45*ArcTan[Sqrt[Cos[a + b*x]]] + 24*ArcTanh[Sqrt[
Cos[a + b*x]]] - 2*Sqrt[Cos[a + b*x]]*(2*Cos[2*(a + b*x)] + 5*Csc[a + b*x]^
2) - (21*(8*Sqrt[Cos[a + b*x]] + Log[1 - Sqrt[Cos[a + b*x]]] - Log[1 + Sqrt
[Cos[a + b*x]]]))/2))/(20*b*Cos[a + b*x]^(9/2))
```

Maple [B] time = 0.221, size = 433, normalized size = 3.2

$$-\frac{8\,d^5}{5\,b}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4\sqrt{2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^2d-d}+\frac{8\,d^5}{5\,b}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2\sqrt{2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^2d-d}+\frac{8\,d^5}{5\,b}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x)
```

```
[Out] -8/5/b*d^5*cos(1/2*b*x+1/2*a)^4*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)+8/5/b*d^
5*cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)+8/5/b*d^5*(2*cos(
1/2*b*x+1/2*a)^2*d-d)^(1/2)-6/b*d^5*(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)+9/
8/b*d^(11/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*
d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))+1/16/b*d^5/(cos(1/2*b*x+1/2*a)-1)*(
-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+9/8/b*d^(11/2)*ln((-4*d*cos(1/2*b*x+1/2*
a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1
))-9/4/b*d^6/(-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^
(1/2))/cos(1/2*b*x+1/2*a))-1/16/b*d^5/(cos(1/2*b*x+1/2*a)+1)*(-2*sin(1/2*b*
x+1/2*a)^2*d+d)^(1/2)-1/8/b*d^5/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^
2*d-d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.56949, size = 1061, normalized size = 7.86

$$\left[-\frac{90\left(d^5\cos\left(bx+a\right)^2-d^5\right)\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)-45\left(d^5\cos\left(bx+a\right)^2-d^5\right)\sqrt{-d}\log\left(-\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx}}{\cos(bx}\right.}{80\left(b\cos\left(bx+a\right)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/80*(90*(d^5*cos(b*x + a)^2 - d^5)*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a)
)*sqrt(-d)/(d*cos(b*x + a) + d)) - 45*(d^5*cos(b*x + a)^2 - d^5)*sqrt(-d)*l
```

```
og(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1)
- 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*(4*d^5*c
os(b*x + a)^4 + 36*d^5*cos(b*x + a)^2 - 45*d^5)*sqrt(d*cos(b*x + a)))/(b*co
s(b*x + a)^2 - b), -1/80*(90*(d^5*cos(b*x + a)^2 - d^5)*sqrt(d)*arctan(2*sq
rt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d)) - 45*(d^5*cos(b*x + a)^2 -
 d^5)*sqrt(d)*log(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(
b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)
) + 8*(4*d^5*cos(b*x + a)^4 + 36*d^5*cos(b*x + a)^2 - 45*d^5)*sqrt(d*cos(b*
x + a)))/(b*cos(b*x + a)^2 - b)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(11/2)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{11}{2}}\csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(11/2)*csc(b*x + a)^3, x)
```

3.243 $\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx$

Optimal. Leaf size=113

$$-\frac{7d^3(d\cos(a+bx))^{3/2}}{6b}-\frac{7d^{9/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{7d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+bx)(d\cos(a+bx))^{7/2}}{2b}$$

```
[Out] (-7*d^(9/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (7*d^(9/2)*ArcTan
h[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (7*d^3*(d*Cos[a + b*x])^(3/2))/(6*
b) - (d*(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.081668, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2565, 288, 321, 329, 298, 203, 206}

$$-\frac{7d^3(d\cos(a+bx))^{3/2}}{6b}-\frac{7d^{9/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{7d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+bx)(d\cos(a+bx))^{7/2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^3,x]
```

```
[Out] (-7*d^(9/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (7*d^(9/2)*ArcTan
h[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (7*d^3*(d*Cos[a + b*x])^(3/2))/(6*
b) - (d*(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^2)/(2*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{9/2}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{d(d\cos(a+bx))^{7/2}\csc^2(a+bx)}{2b} + \frac{(7d)\,\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4b}$$

$$= -\frac{7d^3(d\cos(a+bx))^{3/2}}{6b} - \frac{d(d\cos(a+bx))^{7/2}\csc^2(a+bx)}{2b} + \frac{\left(7d^3\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,d\cdots\right.}{4b}$$

$$= -\frac{7d^3(d\cos(a+bx))^{3/2}}{6b} - \frac{d(d\cos(a+bx))^{7/2}\csc^2(a+bx)}{2b} + \frac{\left(7d^3\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,d\cdots\right.}{2\cdots}$$

$$= -\frac{7d^3(d\cos(a+bx))^{3/2}}{6b} - \frac{d(d\cos(a+bx))^{7/2}\csc^2(a+bx)}{2b} + \frac{\left(7d^5\right)\text{Subst}\left(\int \frac{1}{d-x^2}\,d\cdots\right.}{4\cdots}$$

$$= -\frac{7d^{9/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{7d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{7d^3(d\cos(a+bx))^{3/2}}{6b} - \cdots$$

Mathematica [C] time = 0.635051, size = 78, normalized size = 0.69

$$\frac{d^5\left(21\sqrt[4]{-\cot^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\csc^2(a+bx)\right)+(2\cos(2(a+bx))-5)\cot^2(a+bx)\right)}{6b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^3,x]
```

```
[Out] (d^5*((-5 + 2*Cos[2*(a + b*x)])*Cot[a + b*x]^2 + 21*(-Cot[a + b*x]^2)^(1/4)
*Hypergeometric2F1[1/4, 1/4, 5/4, Csc[a + b*x]^2]))/(6*b*Sqrt[d*Cos[a + b*x
]])
```

Maple [B] time = 0.204, size = 394, normalized size = 3.5

$$-\frac{4d^4}{3b}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2\sqrt{2\left(\cos\left(1/2\,bx+a/2\right)\right)^2d-d}-\frac{4d^4}{3b}\sqrt{2\left(\cos\left(1/2\,bx+a/2\right)\right)^2d-d}+2\frac{d^4\sqrt{d\left(2\left(\cos\left(1/2\,bx+\right.\right.\right.}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x)
```

```
[Out] -4/3/b*d^4*cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)-4/3/b*d^
4*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)+2/b*d^4*(d*(2*cos(1/2*b*x+1/2*a)^2-1))
^(1/2)+7/8/b*d^(9/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1
/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))+1/16/b*d^4/(cos(1/2*b*x+1/2
*a)-1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+7/8/b*d^(9/2)*ln((-4*d*cos(1/2*b
*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1
/2*a)+1))+7/4/b*d^5/(-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^
2*d-d)^(1/2))/cos(1/2*b*x+1/2*a))-1/16/b*d^4/(cos(1/2*b*x+1/2*a)+1)*(-2*sin
(1/2*b*x+1/2*a)^2*d+d)^(1/2)+1/8/b*d^4/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+
1/2*a)^2*d-d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.4935, size = 1027, normalized size = 9.09

$$\left[-\frac{42\left(d^4\cos\left(bx+a\right)^2-d^4\right)\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)-21\left(d^4\cos\left(bx+a\right)^2-d^4\right)\sqrt{-d}\log\left(-\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx}}{\cos(bx}\right.}{48\left(b\cos\left(bx+a\right)^2-b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(42*(d^4*cos(b*x + a)^2 - d^4)*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a)
)*sqrt(-d)/(d*cos(b*x + a) + d)) - 21*(d^4*cos(b*x + a)^2 - d^4)*sqrt(-d)*l
og(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1)
- 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*(4*d^4*c
os(b*x + a)^3 - 7*d^4*cos(b*x + a))*sqrt(d*cos(b*x + a)))/(b*cos(b*x + a)^2
 - b), 1/48*(42*(d^4*cos(b*x + a)^2 - d^4)*sqrt(d)*arctan(2*sqrt(d*cos(b*x
+ a))*sqrt(d)/(d*cos(b*x + a) - d)) + 21*(d^4*cos(b*x + a)^2 - d^4)*sqrt(d)
*log(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1)
 + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) - 8*(4*d^4*
cos(b*x + a)^3 - 7*d^4*cos(b*x + a))*sqrt(d*cos(b*x + a)))/(b*cos(b*x + a)^
```

```
2 - b)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(9/2)*csc(b*x+a)**3,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{9}{2}}\csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a)^3, x)
```

3.244 $\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx$

Optimal. Leaf size=113

$$-\frac{5d^3\sqrt{d\cos(a+bx)}}{2b}+\frac{5d^{7/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{5d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+bx)(d\cos(a+bx))^{5/2}}{2b}$$

```
[Out] (5*d^(7/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (5*d^(7/2)*ArcTanh
[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (5*d^3*Sqrt[d*Cos[a + b*x]])/(2*b)
- (d*(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0801313, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2565, 288, 321, 329, 212, 206, 203}

$$-\frac{5d^3\sqrt{d\cos(a+bx)}}{2b}+\frac{5d^{7/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{5d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+bx)(d\cos(a+bx))^{5/2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^3,x]
```

```
[Out] (5*d^(7/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (5*d^(7/2)*ArcTanh
[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (5*d^3*Sqrt[d*Cos[a + b*x]])/(2*b)
- (d*(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^2)/(2*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{7/2}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{d(d\cos(a+bx))^{5/2}\csc^2(a+bx)}{2b} + \frac{(5d)\,\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4b}$$

$$= -\frac{5d^3\sqrt{d\cos(a+bx)}}{2b} - \frac{d(d\cos(a+bx))^{5/2}\csc^2(a+bx)}{2b} + \frac{\left(5d^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\right.}{4b}$$

$$= -\frac{5d^3\sqrt{d\cos(a+bx)}}{2b} - \frac{d(d\cos(a+bx))^{5/2}\csc^2(a+bx)}{2b} + \frac{\left(5d^3\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx,\right.}{2b}$$

$$= -\frac{5d^3\sqrt{d\cos(a+bx)}}{2b} - \frac{d(d\cos(a+bx))^{5/2}\csc^2(a+bx)}{2b} + \frac{\left(5d^4\right)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx,\right.}{4b}$$

$$= \frac{5d^{7/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{5d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{5d^3\sqrt{d\cos(a+bx)}}{2b} - \frac{d(d}{}$$

Mathematica [A] time = 1.20431, size = 118, normalized size = 1.04

$$\frac{(d\cos(a+bx))^{7/2}\left(-8\sqrt{\cos(a+bx)} - \log\left(1-\sqrt{\cos(a+bx)}\right) + \log\left(\sqrt{\cos(a+bx)}+1\right) + 5\tan^{-1}\left(\sqrt{\cos(a+bx)}\right) - 2\sqrt{\text{co}}\right.}{4b\cos^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^3,x]
```

```
[Out] ((d*Cos[a + b*x])^(7/2)*(5*ArcTan[Sqrt[Cos[a + b*x]]] + 3*ArcTanh[Sqrt[Cos[
a + b*x]]] - 8*Sqrt[Cos[a + b*x]] - 2*Sqrt[Cos[a + b*x]]*Csc[a + b*x]^2 - L
og[1 - Sqrt[Cos[a + b*x]]] + Log[1 + Sqrt[Cos[a + b*x]]]))/(4*b*Cos[a + b*x
```

```
]^(7/2))
```

Maple [B] time = 0.207, size = 327, normalized size = 2.9

$$-2\frac{d^3\sqrt{d\left(2\left(\cos\left(1/2\,bx+a/2\right)\right)^2-1\right)}}{b}+\frac{5}{8\,b}d^{\frac{7}{2}}\ln\left(\left(4\,d\cos\left(1/2\,bx+a/2\right)+2\,\sqrt{d}\sqrt{-2\left(\sin\left(1/2\,bx+a/2\right)\right)^2d+d}-2\,d\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x)
```

```
[Out] -2/b*d^3*(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)+5/8/b*d^(7/2)*ln((4*d*cos(1/2
*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x
+1/2*a)-1))+1/16/b*d^3/(cos(1/2*b*x+1/2*a)-1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)
^(1/2)+5/8/b*d^(7/2)*ln((-4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+
1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1))-5/4/b*d^4/(-d)^(1/2)*ln((-
2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2))/cos(1/2*b*x+1/2*a))-1/
16/b*d^3/(cos(1/2*b*x+1/2*a)+1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-1/8/b*d
^3/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.51047, size = 991, normalized size = 8.77

$$\left[-\frac{10\left(d^3\cos\left(bx+a\right)^2-d^3\right)\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)-5\left(d^3\cos\left(bx+a\right)^2-d^3\right)\sqrt{-d}\log\left(-\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+}}{\cos(bx+}\right.}{16\left(b\cos\left(bx+a\right)^2-b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(10*(d^3*cos(b*x + a)^2 - d^3)*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a)
)*sqrt(-d)/(d*cos(b*x + a) + d)) - 5*(d^3*cos(b*x + a)^2 - d^3)*sqrt(-d)*lo
g(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) -
 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*(4*d^3*co
s(b*x + a)^2 - 5*d^3)*sqrt(d*cos(b*x + a)))/(b*cos(b*x + a)^2 - b), -1/16*(
10*(d^3*cos(b*x + a)^2 - d^3)*sqrt(d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)
/(d*cos(b*x + a) - d)) - 5*(d^3*cos(b*x + a)^2 - d^3)*sqrt(d)*log(-(d*cos(b
*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x
 + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*(4*d^3*cos(b*x + a)^2
 - 5*d^3)*sqrt(d*cos(b*x + a)))/(b*cos(b*x + a)^2 - b)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(7/2)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{7}{2}}\csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a)^3, x)
```

3.245 $\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx$

Optimal. Leaf size=91

$$-\frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{3d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+bx)(d\cos(a+bx))^{3/2}}{2b}$$

```
[Out] (-3*d^(5/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (3*d^(5/2)*ArcTan
h[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (d*(d*Cos[a + b*x])^(3/2)*Csc[a +
b*x]^2)/(2*b)
```

Rubi [A] time = 0.0734711, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2565, 288, 329, 298, 203, 206}

$$-\frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{3d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+bx)(d\cos(a+bx))^{3/2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^3,x]
```

```
[Out] (-3*d^(5/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (3*d^(5/2)*ArcTan
h[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (d*(d*Cos[a + b*x])^(3/2)*Csc[a +
b*x]^2)/(2*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{5/2}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{d(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2b} + \frac{(3d)\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4b}$$
$$= -\frac{d(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2b} + \frac{(3d)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2b}$$
$$= -\frac{d(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2b} + \frac{\left(3d^3\right)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b} - $$
$$= -\frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{3d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{d(d\cos(a+bx))^{3/2}\csc^2}{2b}$$

Mathematica [C] time = 0.291163, size = 65, normalized size = 0.71

$$-\frac{d^3\left(\cot^2(a+bx)-3\sqrt[4]{-\cot^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\csc^2(a+bx)\right)\right)}{2b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^3,x]
```

```
[Out] -(d^3*(Cot[a + b*x]^2 - 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/
4, 5/4, Csc[a + b*x]^2]))/(2*b*Sqrt[d*Cos[a + b*x]])
```

Maple [B] time = 0.291, size = 300, normalized size = 3.3

$$\frac{3}{8b}d^{\frac{5}{2}}\ln\left(\left(4\,d\cos(1/2\,bx+a/2)+2\,\sqrt{d}\sqrt{-2\,(\sin(1/2\,bx+a/2))^2\,d+d}-2\,d\right)\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{-1}\right)+\frac{d^2}{16\,b}\sqrt{-2\,(\sin(1/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x)
```

```
[Out] 3/8/b*d^(5/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2
*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))+1/16/b*d^2/(cos(1/2*b*x+1/2*a)-1)*
(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+3/8/b*d^(5/2)*ln((-4*d*cos(1/2*b*x+1/2*
a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1
))+3/4/b*d^3/(-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^
(1/2))/cos(1/2*b*x+1/2*a))-1/16/b*d^2/(cos(1/2*b*x+1/2*a)+1)*(-2*sin(1/2*b*
x+1/2*a)^2*d+d)^(1/2)+1/8/b*d^2/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^
2*d-d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.45011, size = 988, normalized size = 10.86

$$\left[\frac{8\sqrt{d\cos(bx+a)}d^2\cos(bx+a) - 6\left(d^2\cos(bx+a)^2 - d^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2d\cos(bx+a)}\right) + 3\left(d^2\cos(bx+\right.}{16\left(b\cos(bx+a)^2 - b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(d*cos(b*x + a))*d^2*cos(b*x + a) - 6*(d^2*cos(b*x + a)^2 - d^
2)*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) + 1)/(d*
cos(b*x + a))) + 3*(d^2*cos(b*x + a)^2 - d^2)*sqrt(-d)*log((d*cos(b*x + a)^
2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) +
 d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)))/(b*cos(b*x + a)^2 - b), 1/16*(8
*sqrt(d*cos(b*x + a))*d^2*cos(b*x + a) - 6*(d^2*cos(b*x + a)^2 - d^2)*sqrt(
d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a)
)) + 3*(d^2*cos(b*x + a)^2 - d^2)*sqrt(d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*
cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x +
 a)^2 - 2*cos(b*x + a) + 1)))/(b*cos(b*x + a)^2 - b)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(5/2)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a)^3, x)
```

3.246 $\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx$

Optimal. Leaf size=91

$$\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+bx)\sqrt{d\cos(a+bx)}}{2b}$$

```
[Out] (d^(3/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (d^(3/2)*ArcTanh[Sqr
t[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (d*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^2)
/(2*b)
```

Rubi [A] time = 0.0705803, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2565, 288, 329, 212, 206, 203}

$$\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+bx)\sqrt{d\cos(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^3,x]
```

```
[Out] (d^(3/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (d^(3/2)*ArcTanh[Sqr
t[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (d*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^2)
/(2*b)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{3/2}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{d\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2b} + \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4b}$$

$$= -\frac{d\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2b} + \frac{d\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2b}$$

$$= -\frac{d\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2b} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b} + \frac{d^2\,\text{Su}}{}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{d\sqrt{d\cos(a+bx)}\csc^2(a+b}{2b}$$

Mathematica [C] time = 0.185491, size = 76, normalized size = 0.84

$$\frac{\left(-\cot^2(a+bx)\right)^{3/4}\sec^3(a+bx)(d\cos(a+bx))^{3/2}\left({}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\csc^2(a+bx)\right)+3\sqrt[4]{-\cot^2(a+bx)}\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^3,x]
```

```
[Out] ((d*Cos[a + b*x])^(3/2)*(-Cot[a + b*x]^2)^(3/4)*(3*(-Cot[a + b*x]^2)^(1/4)
+ Hypergeometric2F1[3/4, 3/4, 7/4, Csc[a + b*x]^2])*Sec[a + b*x]^3)/(6*b)
```

Maple [B] time = 0.2, size = 294, normalized size = 3.2

$$\frac{1}{8b}d^{\frac{3}{2}}\ln\left(\left(4d\cos(1/2bx+a/2)+2\sqrt{d}\sqrt{-2(\sin(1/2bx+a/2))^2d+d}-2d\right)\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{-1}\right)+\frac{d}{16b}\sqrt{-2(\sin(1/}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x)
```

```
[Out] 1/8/b*d^(3/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2
*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))+1/16/b*d/(cos(1/2*b*x+1/2*a)-1)*(-
2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+1/8/b*d^(3/2)*ln((-4*d*cos(1/2*b*x+1/2*a)
+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1))
-1/4/b*d^2/(-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1
/2))/cos(1/2*b*x+1/2*a))-1/16/b*d/(cos(1/2*b*x+1/2*a)+1)*(-2*sin(1/2*b*x+1/
2*a)^2*d+d)^(1/2)-1/8/b*d/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)
^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66192, size = 922, normalized size = 10.13

$$\left[-\frac{2\left(d\cos(bx+a)^2-d\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-\left(d\cos(bx+a)^2-d\right)\sqrt{-d}\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(b}}{\cos(b}\right.}{16\left(b\cos(bx+a)^2-b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(d*cos(b*x + a)^2 - d)*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*s
qrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - (d*cos(b*x + a)^2 - d)*sqrt(
-d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) -
 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) - 8*sqrt
(d*cos(b*x + a))*d)/(b*cos(b*x + a)^2 - b), 1/16*(2*(d*cos(b*x + a)^2 - d)*
sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x
 + a))) + (d*cos(b*x + a)^2 - d)*sqrt(d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*c
os(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x +
a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*d)/(b*cos(b*x + a)^2 -
 b)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^3, x)
```

3.247 $\int \sqrt{d \cos(a+bx)} \csc^3(a+bx)\, dx$

Optimal. Leaf size=93

$$\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{\csc^2(a+bx)(d\cos(a+bx))^{3/2}}{2bd}-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}$$

```
[Out] (Sqrt[d]*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (Sqrt[d]*ArcTanh[Sqr
t[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - ((d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^2)
/(2*b*d)
```

Rubi [A] time = 0.065534, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2565, 290, 329, 298, 203, 206}

$$\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{\csc^2(a+bx)(d\cos(a+bx))^{3/2}}{2bd}-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^3,x]
```

```
[Out] (Sqrt[d]*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (Sqrt[d]*ArcTanh[Sqr
t[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - ((d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^2)
/(2*b*d)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2bd} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4bd}$$

$$= -\frac{(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2bd} - \frac{\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2bd}$$

$$= -\frac{(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2bd} - \frac{d\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b} + \frac{d\,\text{Subs}}{}$$

$$= \frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2bd}$$

Mathematica [C] time = 0.250396, size = 62, normalized size = 0.67

$$-\frac{d\left(\sqrt[4]{-\cot^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\csc^2(a+bx)\right)+\cot^2(a+bx)\right)}{2b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^3,x]
```

```
[Out] -(d*(Cot[a + b*x]^2 + (-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5
/4, Csc[a + b*x]^2]))/(2*b*Sqrt[d*Cos[a + b*x]])
```

Maple [B] time = 0.256, size = 289, normalized size = 3.1

$$\frac{1}{16\,b}\sqrt{-2\,(\sin(1/2\,bx+a/2))^2\,d+d}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{-1} - \frac{1}{8\,b}\sqrt{d}\ln\left(\left(4\,d\cos(1/2\,bx+a/2)+2\,\sqrt{d}\sqrt{-2\,(\sin(1/2\,bx+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x)
```

```
[Out] 1/16/b/(cos(1/2*b*x+1/2*a)-1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-1/8/b*d^(
1/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/
2)-2*d)/(cos(1/2*b*x+1/2*a)-1))-1/16/b/(cos(1/2*b*x+1/2*a)+1)*(-2*sin(1/2*b
*x+1/2*a)^2*d+d)^(1/2)-1/8/b*d^(1/2)*ln((-4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*
(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1))+1/8/b/cos(
1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)-1/4/b*d/(-d)^(1/2)*ln((
-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2))/cos(1/2*b*x+1/2*a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.7607, size = 940, normalized size = 10.11

$$\left[\frac{2\left(\cos(bx+a)^2-1\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)+\left(\cos(bx+a)^2-1\right)\sqrt{-d}\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\sqrt{-}}{\cos(bx+a)^2+}\right.}{16\left(b\cos(bx+a)^2-b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(cos(b*x + a)^2 - 1)*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt
(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) + (cos(b*x + a)^2 - 1)*sqrt(-d)*l
og((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) -
 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*co
s(b*x + a))*cos(b*x + a))/(b*cos(b*x + a)^2 - b), 1/16*(2*(cos(b*x + a)^2 -
 1)*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos
(b*x + a))) + (cos(b*x + a)^2 - 1)*sqrt(d)*log((d*cos(b*x + a)^2 - 4*sqrt(d
*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x
+ a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*cos(b*x + a))/(b*cos
(b*x + a)^2 - b)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cos(bx+a)}\csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*csc(b*x + a)^3, x)
```

3.248 $\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=93

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}}-\frac{\csc^2(a+bx)\sqrt{d\cos(a+bx)}}{2bd}-\frac{3\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}}$$

```
[Out] (-3*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*Sqrt[d]) - (3*ArcTanh[Sqrt[d
*Cos[a + b*x]]/Sqrt[d]])/(4*b*Sqrt[d]) - (Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]
^2)/(2*b*d)
```

Rubi [A] time = 0.0654275, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2565, 290, 329, 212, 206, 203}

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}}-\frac{\csc^2(a+bx)\sqrt{d\cos(a+bx)}}{2bd}-\frac{3\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (-3*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*Sqrt[d]) - (3*ArcTanh[Sqrt[d
*Cos[a + b*x]]/Sqrt[d]])/(4*b*Sqrt[d]) - (Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]
^2)/(2*b*d)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2bd} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd}$$

$$= -\frac{\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2bd} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2bd}$$

$$= -\frac{\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2bd} - \frac{3\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x,\right.}{4b}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}} - \frac{3\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}} - \frac{\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2bd}$$

Mathematica [C] time = 0.22831, size = 69, normalized size = 0.74

$$\frac{d\left(-\cot^2(a+bx)\right)^{3/4}\left(\sqrt[4]{-\cot^2(a+bx)} - {}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\csc^2(a+bx)\right)\right)}{2b(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (d*(-Cot[a + b*x]^2)^(3/4)*((-Cot[a + b*x]^2)^(1/4) - Hypergeometric2F1[3/4
, 3/4, 7/4, Csc[a + b*x]^2]))/(2*b*(d*Cos[a + b*x])^(3/2))
```

Maple [B] time = 0.312, size = 297, normalized size = 3.2

$$-\frac{3}{8b}\ln\left(\left(4d\cos(1/2\,bx+a/2)+2\sqrt{d}\sqrt{-2\,(\sin(1/2\,bx+a/2))^2 d+d}-2d\right)\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{-1}\right)\frac{1}{\sqrt{d}}+\frac{1}{16\,bd}\sqrt{-2\,(\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] -3/8/b/d^(1/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))+1/16/b/d/(cos(1/2*b*x+1/2*a)-1)*(
-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-3/8/b/d^(1/2)*ln((-4*d*cos(1/2*b*x+1/2*a
)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1)
)+3/4/b/(-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)
)/cos(1/2*b*x+1/2*a))-1/16/b/d/(cos(1/2*b*x+1/2*a)+1)*(-2*sin(1/2*b*x+1/2*a
)^2*d+d)^(1/2)-1/8/b/d/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66232, size = 922, normalized size = 9.91

$$\left[\frac{6\left(\cos(bx+a)^2-1\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2d\cos(bx+a)}\right)-3\left(\cos(bx+a)^2-1\right)\sqrt{-d}\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}}{\cos(bx+a)^2}\right.}{16\left(bd\cos(bx+a)^2-bd\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(6*(cos(b*x + a)^2 - 1)*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt
(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - 3*(cos(b*x + a)^2 - 1)*sqrt(-d)
*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1)
 - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*
cos(b*x + a)))/(b*d*cos(b*x + a)^2 - b*d), -1/16*(6*(cos(b*x + a)^2 - 1)*sq
rt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x +
 a))) - 3*(cos(b*x + a)^2 - 1)*sqrt(d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos
(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)
^2 - 2*cos(b*x + a) + 1)) - 8*sqrt(d*cos(b*x + a)))/(b*d*cos(b*x + a)^2 - b
*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)**3/sqrt(d*cos(a + b*x)), x)
```

Giac [A] time = 1.13803, size = 123, normalized size = 1.32

$$\frac{d^3\left(\frac{2\sqrt{d\cos(bx+a)}}{\left(d^2\cos(bx+a)^2-d^2\right)d^2}+\frac{3\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}d^3}-\frac{3\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{7}{2}}}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*d^3*(2*sqrt(d*cos(b*x + a))/((d^2*cos(b*x + a)^2 - d^2)*d^2) + 3*arctan
(sqrt(d*cos(b*x + a))/sqrt(-d))/(sqrt(-d)*d^3) - 3*arctan(sqrt(d*cos(b*x +
a))/sqrt(d))/d^(7/2))/b
```

3.249 $\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=115

$$\frac{5\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}}+\frac{5}{2bd\sqrt{d\cos(a+bx)}}-\frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}}$$

```
[Out] (5*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(3/2)) - (5*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(3/2)) + 5/(2*b*d*Sqrt[d*Cos[a + b*x]]) - Cs
c[a + b*x]^2/(2*b*d*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.0825338, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2565, 290, 325, 329, 298, 203, 206}

$$\frac{5\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}}+\frac{5}{2bd\sqrt{d\cos(a+bx)}}-\frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (5*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(3/2)) - (5*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(3/2)) + 5/(2*b*d*Sqrt[d*Cos[a + b*x]]) - Cs
c[a + b*x]^2/(2*b*d*Sqrt[d*Cos[a + b*x]])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}\\
&= -\frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd}\\
&= \frac{5}{2bd\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}} - \frac{5\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4bd^3}\\
&= \frac{5}{2bd\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}} - \frac{5\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2bd^3}\\
&= \frac{5}{2bd\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}} - \frac{5\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4bd} + \frac{5\,\text{Subst}}{}\\
&= \frac{5\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}} - \frac{5\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}} + \frac{5}{2bd\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}}
\end{aligned}
$$

Mathematica [C] time = 0.245834, size = 91, normalized size = 0.79

$$
\frac{5\cot^2(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\csc^2(a+bx)\right) - \left(-\cot^2(a+bx)\right)^{3/4}\left(\cot^2(a+bx)-4\right)}{2bd\left(-\cot^2(a+bx)\right)^{3/4}\sqrt{d\cos(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (-((-Cot[a + b*x]^2)^(3/4)*(-4 + Cot[a + b*x]^2)) + 5*Cot[a + b*x]^2*Hyperg
eometric2F1[1/4, 1/4, 5/4, Csc[a + b*x]^2])/(2*b*d*Sqrt[d*Cos[a + b*x]]*(-C
ot[a + b*x]^2)^(3/4))
```

Maple [B] time = 0.338, size = 705, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] 1/8/d^(7/2)/(-d)^(1/2)/sin(1/2*b*x+1/2*a)^2/(2*sin(1/2*b*x+1/2*a)^4-3*sin(1
/2*b*x+1/2*a)^2+1)*(-(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(3/2)*(-d)^(1/2)
-sin(1/2*b*x+1/2*a)^6*(20*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b
*x+1/2*a)^2*d+d)^(1/2)-d))*d^(5/2)+10*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*
(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d
^2+10*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1
/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^2)+5*(6*ln(2/cos(1/2*b*x+1/2*a)
*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(5/2)-4*(-2*sin(1/2*
b*x+1/2*a)^2*d+d)^(1/2)*d^(3/2)*(-d)^(1/2)+3*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d
^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^
(1/2)*d^2+3*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d
+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^2)*sin(1/2*b*x+1/2*a)^4-5
*(2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)
-d))*d^(5/2)-4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(3/2)*(-d)^(1/2)+ln(2/
(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos
(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^2+ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2
*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^2)
*sin(1/2*b*x+1/2*a)^2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.71421, size = 1091, normalized size = 9.49

$$\left[\frac{10\left(\cos\left(bx+a\right)^{3}-\cos\left(bx+a\right)\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos\left(bx+a\right)}\sqrt{-d}\left(\cos\left(bx+a\right)+1\right)}{2\,d\cos\left(bx+a\right)}\right)-5\left(\cos\left(bx+a\right)^{3}-\cos\left(bx+a\right)\right)\sqrt{-d}\log\left(\right.}{16\left(bd^{2}\cos\left(bx+a\right)^{3}-bd^{2}\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(10*(cos(b*x + a)^3 - cos(b*x + a))*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*
x + a))*sqrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - 5*(cos(b*x + a)^3 -
 cos(b*x + a))*sqrt(-d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt
(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x
 + a) + 1)) + 8*sqrt(d*cos(b*x + a))*(5*cos(b*x + a)^2 - 4))/(b*d^2*cos(b*x
 + a)^3 - b*d^2*cos(b*x + a)), 1/16*(10*(cos(b*x + a)^3 - cos(b*x + a))*sqr
t(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x +
a))) + 5*(cos(b*x + a)^3 - cos(b*x + a))*sqrt(d)*log((d*cos(b*x + a)^2 - 4*
sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(co
s(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*(5*cos(b*x + a
)^2 - 4))/(b*d^2*cos(b*x + a)^3 - b*d^2*cos(b*x + a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)**3/(d*cos(a + b*x))**(3/2), x)
```

Giac [A] time = 1.14128, size = 162, normalized size = 1.41

$$\frac{d^3\left(\frac{5\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}d^4}+\frac{5\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{9}{2}}}+\frac{2\left(5d^2\cos(bx+a)^2-4d^2\right)}{\left(\sqrt{d\cos(bx+a)}d^2\cos(bx+a)^2-\sqrt{d\cos(bx+a)}d^2\right)d^4}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*d^3*(5*arctan(sqrt(d*cos(b*x + a))/sqrt(-d))/(sqrt(-d)*d^4) + 5*arctan(
sqrt(d*cos(b*x + a))/sqrt(d))/d^(9/2) + 2*(5*d^2*cos(b*x + a)^2 - 4*d^2)/((
sqrt(d*cos(b*x + a))*d^2*cos(b*x + a)^2 - sqrt(d*cos(b*x + a))*d^2)*d^4))/b
```

3.250 $\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}}+\frac{7}{6bd(d\cos(a+bx))^{3/2}}-\frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}}$$

```
[Out] (-7*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(5/2)) - (7*ArcTanh[Sqrt[d
*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(5/2)) + 7/(6*b*d*(d*Cos[a + b*x])^(3/2)) -
 Csc[a + b*x]^2/(2*b*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.0833959, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2565, 290, 325, 329, 212, 206, 203}

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}}+\frac{7}{6bd(d\cos(a+bx))^{3/2}}-\frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
```


```
[Out] (-7*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(5/2)) - (7*ArcTanh[Sqrt[d
*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(5/2)) + 7/(6*b*d*(d*Cos[a + b*x])^(3/2)) -
 Csc[a + b*x]^2/(2*b*d*(d*Cos[a + b*x])^(3/2))
```


Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}} - \frac{7\,\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd}$$

$$= \frac{7}{6bd(d\cos(a+bx))^{3/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}} - \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd^3}$$

$$= \frac{7}{6bd(d\cos(a+bx))^{3/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}} - \frac{7\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2bd^3}$$

$$= \frac{7}{6bd(d\cos(a+bx))^{3/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}} - \frac{7\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4bd^2} - \frac{7\,\text{Su}}{}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}} - \frac{7\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}} + \frac{7}{6bd(d\cos(a+bx))^{3/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))}$$

Mathematica [C] time = 0.366873, size = 92, normalized size = 0.8

$$\frac{7\cot^2(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\csc^2(a+bx)\right)+\sqrt[4]{-\cot^2(a+bx)}\left(4-3\cot^2(a+bx)\right)}{6bd\sqrt[4]{-\cot^2(a+bx)}(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((-Cot[a + b*x]^2)^(1/4)*(4 - 3*Cot[a + b*x]^2) + 7*Cot[a + b*x]^2*Hypergeo
metric2F1[3/4, 3/4, 7/4, Csc[a + b*x]^2])/(6*b*d*(d*Cos[a + b*x])^(3/2)*(-C
ot[a + b*x]^2)^(1/4))
```

Maple [B] time = 0.345, size = 909, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] 1/24/d^(11/2)/(-d)^(1/2)/sin(1/2*b*x+1/2*a)^2/(4*sin(1/2*b*x+1/2*a)^6-8*sin
(1/2*b*x+1/2*a)^4+5*sin(1/2*b*x+1/2*a)^2-1)*(3*(-2*sin(1/2*b*x+1/2*a)^2*d+d
)^(1/2)*d^(5/2)*(-d)^(1/2)+84*(2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*si
n(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(7/2)-ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1
/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/
2)*d^3-ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(
1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3)*sin(1/2*b*x+1/2*a)^8-168*(2
*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d)
)*d^(7/2)-ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d
)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3-ln(2/(cos(1/2*b*x+1/2*a)-
1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*
(-d)^(1/2)*d^3)*sin(1/2*b*x+1/2*a)^6-7*(6*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/
2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(7/2)+4*(-2*sin(1/2*b*x+1/2*a)
^2*d+d)^(1/2)*d^(5/2)*(-d)^(1/2)-3*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2
*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3-
3*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+
2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3)*sin(1/2*b*x+1/2*a)^2+7*(30*ln(2/
cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(7
/2)+4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(5/2)*(-d)^(1/2)-15*ln(2/(cos(1
/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b
*x+1/2*a)-d))*(-d)^(1/2)*d^3-15*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*si
n(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3)*si
n(1/2*b*x+1/2*a)^4)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.80907, size = 1111, normalized size = 9.66

$$\left[\frac{42\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-21\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}\right)\sqrt{-d}\,\mathrm{lo}}{48\left(bd^{3}\cos\left(bx+a\right)^{4}-bd^{3}\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(42*(cos(b*x + a)^4 - cos(b*x + a)^2)*sqrt(-d)*arctan(1/2*sqrt(d*cos(
b*x + a))*sqrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - 21*(cos(b*x + a)^
4 - cos(b*x + a)^2)*sqrt(-d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))
*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*co
s(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*(7*cos(b*x + a)^2 - 4))/(b*d^3*co
s(b*x + a)^4 - b*d^3*cos(b*x + a)^2), -1/48*(42*(cos(b*x + a)^4 - cos(b*x +
 a)^2)*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*
cos(b*x + a))) - 21*(cos(b*x + a)^4 - cos(b*x + a)^2)*sqrt(d)*log((d*cos(b*
x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x
+ a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) - 8*sqrt(d*cos(b*x + a))*(
7*cos(b*x + a)^2 - 4))/(b*d^3*cos(b*x + a)^4 - b*d^3*cos(b*x + a)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14238, size = 154, normalized size = 1.34

$$\frac{d^3\left(\frac{6\sqrt{d\cos(bx+a)}}{\left(d^2\cos(bx+a)^2-d^2\right)d^4}+\frac{21\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}d^5}-\frac{21\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{11}{2}}}+\frac{8}{\sqrt{d\cos(bx+a)}d^5\cos(bx+a)}\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*d^3*(6*sqrt(d*cos(b*x + a))/((d^2*cos(b*x + a)^2 - d^2)*d^4) + 21*arct
an(sqrt(d*cos(b*x + a))/sqrt(-d))/(sqrt(-d)*d^5) - 21*arctan(sqrt(d*cos(b*x
 + a))/sqrt(d))/d^(11/2) + 8/(sqrt(d*cos(b*x + a))*d^5*cos(b*x + a)))/b
```

3.251 $\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=137

$$\frac{9}{2bd^3\sqrt{d\cos(a+bx)}}+\frac{9\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}}+\frac{9}{10bd(d\cos(a+bx))^{5/2}}-\frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}}$$

```
[Out] (9*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(7/2)) - (9*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(7/2)) + 9/(10*b*d*(d*Cos[a + b*x])^(5/2)) +
 9/(2*b*d^3*Sqrt[d*Cos[a + b*x]]) - Csc[a + b*x]^2/(2*b*d*(d*Cos[a + b*x])^
(5/2))
```

Rubi [A] time = 0.0921429, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2565, 290, 325, 329, 298, 203, 206}

$$\frac{9}{2bd^3\sqrt{d\cos(a+bx)}}+\frac{9\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}}+\frac{9}{10bd(d\cos(a+bx))^{5/2}}-\frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (9*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(7/2)) - (9*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(7/2)) + 9/(10*b*d*(d*Cos[a + b*x])^(5/2)) +
 9/(2*b*d^3*Sqrt[d*Cos[a + b*x]]) - Csc[a + b*x]^2/(2*b*d*(d*Cos[a + b*x])^
(5/2))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^{7/2}\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}} - \frac{9\,\text{Subst}\left(\int \frac{1}{x^{7/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd}$$

$$= \frac{9}{10bd(d\cos(a+bx))^{5/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}} - \frac{9\,\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd^3}$$

$$= \frac{9}{10bd(d\cos(a+bx))^{5/2}} + \frac{9}{2bd^3\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}} - \frac{9\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x\right.}{4bd^5}$$

$$= \frac{9}{10bd(d\cos(a+bx))^{5/2}} + \frac{9}{2bd^3\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}} - \frac{9\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x\right.}{2bd^5}$$

$$= \frac{9}{10bd(d\cos(a+bx))^{5/2}} + \frac{9}{2bd^3\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}} - \frac{9\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x,\right.}{4bd^3}$$

$$= \frac{9\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}} - \frac{9\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}} + \frac{9}{10bd(d\cos(a+bx))^{5/2}} + \frac{9}{2bd^3\sqrt{d\cos(a+bx}}$$

Mathematica [C] time = 0.454599, size = 102, normalized size = 0.74

$$\frac{45\cot^2(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\csc^2(a+bx)\right) + \left(-\cot^2(a+bx)\right)^{3/4}\left(-5\cot^2(a+bx)+4\sec^2(a+bx)+40\right)}{10bd^3\left(-\cot^2(a+bx)\right)^{3/4}\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (45*Cot[a + b*x]^2*Hypergeometric2F1[1/4, 1/4, 5/4, Csc[a + b*x]^2] + (-Cot
[a + b*x]^2)^(3/4)*(40 - 5*Cot[a + b*x]^2 + 4*Sec[a + b*x]^2))/(10*b*d^3*Sq
rt[d*Cos[a + b*x]]*(-Cot[a + b*x]^2)^(3/4))
```

Maple [B] time = 0.381, size = 1165, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] 1/40/d^(15/2)/(-d)^(1/2)/sin(1/2*b*x+1/2*a)^2/(8*sin(1/2*b*x+1/2*a)^8-20*si
n(1/2*b*x+1/2*a)^6+18*sin(1/2*b*x+1/2*a)^4-7*sin(1/2*b*x+1/2*a)^2+1)*(-5*(-
2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(7/2)*(-d)^(1/2)-360*(2*ln(2/cos(1/2*b*
x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(9/2)+ln(2/(
cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(
1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4+ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*
sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4)*
sin(1/2*b*x+1/2*a)^10-180*(-10*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(
1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(9/2)+4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/
2)*d^(7/2)*(-d)^(1/2)-5*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*
x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4-5*ln(2/(cos
(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2
*b*x+1/2*a)-d))*(-d)^(1/2)*d^4)*sin(1/2*b*x+1/2*a)^8+90*(-18*ln(2/cos(1/2*b
*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(9/2)+16*(-
2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(7/2)*(-d)^(1/2)-9*ln(2/(cos(1/2*b*x+1/
2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)
-d))*(-d)^(1/2)*d^4-9*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+
1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4)*sin(1/2*b*x+
1/2*a)^6-9*(-70*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)-d))*d^(9/2)+104*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(7/2)*(-
d)^(1/2)-35*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d
+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4-35*ln(2/(cos(1/2*b*x+1/
2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)
-d))*(-d)^(1/2)*d^4)*sin(1/2*b*x+1/2*a)^4+9*(-10*ln(2/cos(1/2*b*x+1/2*a)*((
-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(9/2)+24*(-2*sin(1/2*b*
x+1/2*a)^2*d+d)^(1/2)*d^(7/2)*(-d)^(1/2)-5*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(
1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1
/2)*d^4-5*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d
)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4)*sin(1/2*b*x+1/2*a)^2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.87237, size = 1166, normalized size = 8.51

$$\left[\frac{90\left(\cos(bx+a)^5-\cos(bx+a)^3\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-45\left(\cos(bx+a)^5-\cos(bx+a)^3\right)\sqrt{-d}\log\left(\right.}{80\left(bd^4\cos(bx+a)^5-b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] [1/80*(90*(cos(b*x + a)^5 - cos(b*x + a)^3)*sqrt(-d)*arctan(1/2*sqrt(d*cos(
b*x + a))*sqrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - 45*(cos(b*x + a)^
5 - cos(b*x + a)^3)*sqrt(-d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))
*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*co
s(b*x + a) + 1)) + 8*(45*cos(b*x + a)^4 - 36*cos(b*x + a)^2 - 4)*sqrt(d*cos
(b*x + a)))/(b*d^4*cos(b*x + a)^5 - b*d^4*cos(b*x + a)^3), 1/80*(90*(cos(b*
x + a)^5 - cos(b*x + a)^3)*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x
 + a) - 1)/(sqrt(d)*cos(b*x + a))) + 45*(cos(b*x + a)^5 - cos(b*x + a)^3)*s
qrt(d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a)
 + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*(4
5*cos(b*x + a)^4 - 36*cos(b*x + a)^2 - 4)*sqrt(d*cos(b*x + a)))/(b*d^4*cos(
b*x + a)^5 - b*d^4*cos(b*x + a)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16341, size = 185, normalized size = 1.35

$$\frac{d^3\left(\frac{10\sqrt{d\cos(bx+a)}\cos(bx+a)}{\left(d^2\cos(bx+a)^2-d^2\right)d^5}+\frac{45\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}d^6}+\frac{45\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{13}{2}}}+\frac{8\left(10\,d^2\cos(bx+a)^2+d^2\right)}{\sqrt{d\cos(bx+a)}d^8\cos(bx+a)^2}\right)}{20\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] 1/20*d^3*(10*sqrt(d*cos(b*x + a))*cos(b*x + a)/((d^2*cos(b*x + a)^2 - d^2)*
d^5) + 45*arctan(sqrt(d*cos(b*x + a))/sqrt(-d))/(sqrt(-d)*d^6) + 45*arctan(
sqrt(d*cos(b*x + a))/sqrt(d))/d^(13/2) + 8*(10*d^2*cos(b*x + a)^2 + d^2)/(s
qrt(d*cos(b*x + a))*d^8*cos(b*x + a)^2))/b
```

3.252 $\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx$

Optimal. Leaf size=22

$$-\frac{5(d\cos(a+bx))^{6/5}}{6bd}$$

```
[Out] (-5*(d*Cos[a + b*x])^(6/5))/(6*b*d)
```

Rubi [A] time = 0.0222108, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$-\frac{5(d\cos(a+bx))^{6/5}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(1/5)*Sin[a + b*x],x]

[Out] (-5*(d*Cos[a + b*x])^(6/5))/(6*b*d)
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \sqrt[5]{x}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{5(d\cos(a+bx))^{6/5}}{6bd}\end{aligned}$$

Mathematica [A] time = 0.0201343, size = 22, normalized size = 1.

$$-\frac{5(d\cos(a+bx))^{6/5}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(1/5)*Sin[a + b*x],x]

[Out] (-5*(d*Cos[a + b*x])^(6/5))/(6*b*d)
```

Maple [A] time = 0.004, size = 19, normalized size = 0.9

$$-\frac{5}{6bd}(d\cos(bx+a))^{\frac{6}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/5)*sin(b*x+a),x)
```

```
[Out] -5/6*(d*cos(b*x+a))^(6/5)/b/d
```

Maxima [A] time = 0.977107, size = 24, normalized size = 1.09

$$-\frac{5\,(d\cos(bx+a))^{\frac{6}{5}}}{6\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/5)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -5/6*(d*cos(b*x + a))^(6/5)/(b*d)
```

Fricas [A] time = 2.20784, size = 59, normalized size = 2.68

$$-\frac{5\,(d\cos(bx+a))^{\frac{1}{5}}\cos(bx+a)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/5)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -5/6*(d*cos(b*x + a))^(1/5)*cos(b*x + a)/b
```

Sympy [A] time = 33.7158, size = 34, normalized size = 1.55

$$\begin{cases} -\frac{5\sqrt[5]{d}\cos^{\frac{6}{5}}(a+bx)}{6b} & \text{for } b \neq 0 \\ x\sqrt[5]{d\cos(a)}\sin(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/5)*sin(b*x+a),x)
```

```
[Out] Piecewise((-5*d**(1/5)*cos(a + b*x)**(6/5)/(6*b), Ne(b, 0)), (x*(d*cos(a))*
*(1/5)*sin(a), True))
```

Giac [A] time = 1.23505, size = 28, normalized size = 1.27

$$-\frac{5\,(d\cos(bx+a))^{\frac{1}{5}}\cos(bx+a)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/5)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -5/6*(d*cos(b*x + a))^(1/5)*cos(b*x + a)/b
```

3.253 $\int \cos^3(x)\sqrt{\sin(x)}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{7}\sin^{\frac{7}{2}}(x)$$

```
[Out] (2*Sin[x]^(3/2))/3 - (2*Sin[x]^(7/2))/7
```

Rubi [A] time = 0.0240266, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2564, 14}

$$\frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{7}\sin^{\frac{7}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Sqrt[Sin[x]],x]
```

```
[Out] (2*Sin[x]^(3/2))/3 - (2*Sin[x]^(7/2))/7
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\sqrt{\sin(x)}\,dx &= \text{Subst}\left(\int \sqrt{x}\left(1-x^2\right)\,dx,x,\sin(x)\right)\\ &= \text{Subst}\left(\int \left(\sqrt{x}-x^{5/2}\right)\,dx,x,\sin(x)\right)\\ &= \frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{7}\sin^{\frac{7}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.0129075, size = 18, normalized size = 0.86

$$\frac{1}{21}\sin^{\frac{3}{2}}(x)(3\cos(2x)+11)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Sqrt[Sin[x]],x]
```

```
[Out] ((11 + 3*Cos[2*x])*Sin[x]^(3/2))/21
```

Maple [A] time = 0.042, size = 14, normalized size = 0.7

$$\frac{2}{3}(\sin(x))^{\frac{3}{2}} - \frac{2}{7}(\sin(x))^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^(1/2),x)
```

```
[Out] 2/3*sin(x)^(3/2)-2/7*sin(x)^(7/2)
```

Maxima [A] time = 0.978358, size = 18, normalized size = 0.86

$$-\frac{2}{7}\sin(x)^{\frac{7}{2}} + \frac{2}{3}\sin(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

Fricas [A] time = 2.14321, size = 49, normalized size = 2.33

$$\frac{2}{21}\left(3\cos(x)^2 + 4\right)\sin(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*cos(x)^2 + 4)*sin(x)^(3/2)
```

Sympy [B] time = 45.7995, size = 167, normalized size = 7.95

$$\frac{28\sqrt{2}\sqrt{\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^{\frac{11}{2}}\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right) + 63\tan^4\left(\frac{x}{2}\right) + 63\tan^2\left(\frac{x}{2}\right) + 21} + \frac{8\sqrt{2}\sqrt{\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^{\frac{7}{2}}\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right) + 63\tan^4\left(\frac{x}{2}\right) + 63\tan^2\left(\frac{x}{2}\right) + 21} + \frac{28\sqrt{2}\sqrt{\frac{}{\tan}}}{21\tan^6\left(\frac{x}{2}\right) + 63\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**(1/2),x)
```

```
[Out] 28*sqrt(2)*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(11/2)/(21*tan(x/2)**6 + 63*
tan(x/2)**4 + 63*tan(x/2)**2 + 21) + 8*sqrt(2)*sqrt(1/(tan(x/2)**2 + 1))*ta
n(x/2)**(7/2)/(21*tan(x/2)**6 + 63*tan(x/2)**4 + 63*tan(x/2)**2 + 21) + 28*
sqrt(2)*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(3/2)/(21*tan(x/2)**6 + 63*tan(
x/2)**4 + 63*tan(x/2)**2 + 21)
```

Giac [A] time = 1.10135, size = 18, normalized size = 0.86

$$-\frac{2}{7} \sin(x)^{\frac{7}{2}} + \frac{2}{3} \sin(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

3.254 $\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{5}\sin^{\frac{5}{2}}(x)-\frac{2}{9}\sin^{\frac{9}{2}}(x)$$

```
[Out] (2*Sin[x]^(5/2))/5 - (2*Sin[x]^(9/2))/9
```

Rubi [A] time = 0.0243724, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2564, 14}

$$\frac{2}{5}\sin^{\frac{5}{2}}(x)-\frac{2}{9}\sin^{\frac{9}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Sin[x]^(3/2),x]
```

```
[Out] (2*Sin[x]^(5/2))/5 - (2*Sin[x]^(9/2))/9
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx &= \text{Subst}\left(\int x^{3/2}\left(1-x^2\right)\,dx,x,\sin(x)\right)\\ &= \text{Subst}\left(\int \left(x^{3/2}-x^{7/2}\right)\,dx,x,\sin(x)\right)\\ &= \frac{2}{5}\sin^{\frac{5}{2}}(x)-\frac{2}{9}\sin^{\frac{9}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.0126511, size = 18, normalized size = 0.86

$$\frac{1}{45}\sin^{\frac{5}{2}}(x)(5\cos(2x)+13)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Sin[x]^(3/2),x]
```

```
[Out] ((13 + 5*Cos[2*x])*Sin[x]^(5/2))/45
```

Maple [A] time = 0.04, size = 14, normalized size = 0.7

$$\frac{2}{5}(\sin(x))^{\frac{5}{2}} - \frac{2}{9}(\sin(x))^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^(3/2),x)
```

```
[Out] 2/5*sin(x)^(5/2)-2/9*sin(x)^(9/2)
```

Maxima [A] time = 0.964323, size = 18, normalized size = 0.86

$$-\frac{2}{9}\sin(x)^{\frac{9}{2}} + \frac{2}{5}\sin(x)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/9*sin(x)^(9/2) + 2/5*sin(x)^(5/2)
```

Fricas [A] time = 2.22841, size = 65, normalized size = 3.1

$$-\frac{2}{45}\left(5\cos(x)^4 - \cos(x)^2 - 4\right)\sqrt{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/45*(5*cos(x)^4 - cos(x)^2 - 4)*sqrt(sin(x))
```

Sympy [A] time = 68.453, size = 24, normalized size = 1.14

$$\frac{8\sin^{\frac{9}{2}}(x)}{45} + \frac{2\sin^{\frac{5}{2}}(x)\cos^2(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**(3/2),x)
```

```
[Out] 8*sin(x)**(9/2)/45 + 2*sin(x)**(5/2)*cos(x)**2/5
```

Giac [A] time = 1.10614, size = 18, normalized size = 0.86

$$-\frac{2}{9}\sin(x)^{\frac{9}{2}} + \frac{2}{5}\sin(x)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/9*sin(x)^(9/2) + 2/5*sin(x)^(5/2)
```

3.255 $\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{7}\sin^{\frac{7}{2}}(x)-\frac{2}{11}\sin^{\frac{11}{2}}(x)$$

```
[Out] (2*Sin[x]^(7/2))/7 - (2*Sin[x]^(11/2))/11
```

Rubi [A] time = 0.0249217, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2564, 14}

$$\frac{2}{7}\sin^{\frac{7}{2}}(x)-\frac{2}{11}\sin^{\frac{11}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Sin[x]^(5/2),x]
```

```
[Out] (2*Sin[x]^(7/2))/7 - (2*Sin[x]^(11/2))/11
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx &= \text{Subst}\left(\int x^{5/2}\left(1-x^2\right)\,dx,x,\sin(x)\right)\\ &= \text{Subst}\left(\int \left(x^{5/2}-x^{9/2}\right)\,dx,x,\sin(x)\right)\\ &= \frac{2}{7}\sin^{\frac{7}{2}}(x)-\frac{2}{11}\sin^{\frac{11}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.0115088, size = 18, normalized size = 0.86

$$\frac{1}{77}\sin^{\frac{7}{2}}(x)(7\cos(2x)+15)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Sin[x]^(5/2),x]
```

```
[Out] ((15 + 7*Cos[2*x])*Sin[x]^(7/2))/77
```

Maple [A] time = 0.036, size = 14, normalized size = 0.7

$$\frac{2}{7}(\sin(x))^{\frac{7}{2}} - \frac{2}{11}(\sin(x))^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^(5/2),x)
```

```
[Out] 2/7*sin(x)^(7/2)-2/11*sin(x)^(11/2)
```

Maxima [A] time = 0.963436, size = 18, normalized size = 0.86

$$-\frac{2}{11}\sin(x)^{\frac{11}{2}} + \frac{2}{7}\sin(x)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/11*sin(x)^(11/2) + 2/7*sin(x)^(7/2)
```

Fricas [A] time = 2.34417, size = 68, normalized size = 3.24

$$-\frac{2}{77}\left(7\cos(x)^4 - 3\cos(x)^2 - 4\right)\sin(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/77*(7*cos(x)^4 - 3*cos(x)^2 - 4)*sin(x)^(3/2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09783, size = 18, normalized size = 0.86

$$-\frac{2}{11}\sin(x)^{\frac{11}{2}} + \frac{2}{7}\sin(x)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/11*sin(x)^(11/2) + 2/7*sin(x)^(7/2)
```

3.256 $\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx$

Optimal. Leaf size=19

$$2\sqrt{\sin(x)}-\frac{2}{5}\sin^{\frac{5}{2}}(x)$$

```
[Out] 2*Sqrt[Sin[x]] - (2*Sin[x]^(5/2))/5
```

Rubi [A] time = 0.0232385, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2564, 14}

$$2\sqrt{\sin(x)}-\frac{2}{5}\sin^{\frac{5}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/Sqrt[Sin[x]],x]
```

```
[Out] 2*Sqrt[Sin[x]] - (2*Sin[x]^(5/2))/5
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx &= \text{Subst}\left(\int \frac{1-x^2}{\sqrt{x}}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{\sqrt{x}}-x^{3/2}\right)dx, x, \sin(x)\right)\\ &= 2\sqrt{\sin(x)}-\frac{2}{5}\sin^{\frac{5}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.0085595, size = 16, normalized size = 0.84

$$\frac{1}{5}\sqrt{\sin(x)}(\cos(2x)+9)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/Sqrt[Sin[x]],x]
```

```
[Out] ((9 + Cos[2*x])*Sqrt[Sin[x]])/5
```

Maple [A] time = 0.039, size = 14, normalized size = 0.7

$$-\frac{2}{5}(\sin(x))^{\frac{5}{2}}+2\sqrt{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/sin(x)^(1/2),x)
```

```
[Out] -2/5*sin(x)^(5/2)+2*sin(x)^(1/2)
```

Maxima [A] time = 1.00441, size = 18, normalized size = 0.95

$$-\frac{2}{5}\sin(x)^{\frac{5}{2}}+2\sqrt{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*sin(x)^(5/2) + 2*sqrt(sin(x))
```

Fricas [A] time = 2.17845, size = 45, normalized size = 2.37

$$\frac{2}{5}\left(\cos(x)^2+4\right)\sqrt{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(cos(x)^2 + 4)*sqrt(sin(x))
```

Sympy [B] time = 51.1132, size = 323, normalized size = 17.

$$\frac{10\sqrt{2}\tan^{5}\left(\frac{x}{2}\right)}{5\sqrt{\frac{1}{\tan^{2}\left(\frac{x}{2}\right)+1}}\tan^{\frac{13}{2}}\left(\frac{x}{2}\right)+15\sqrt{\frac{1}{\tan^{2}\left(\frac{x}{2}\right)+1}}\tan^{\frac{9}{2}}\left(\frac{x}{2}\right)+15\sqrt{\frac{1}{\tan^{2}\left(\frac{x}{2}\right)+1}}\tan^{\frac{5}{2}}\left(\frac{x}{2}\right)+5\sqrt{\frac{1}{\tan^{2}\left(\frac{x}{2}\right)+1}}\sqrt{\tan\left(\frac{x}{2}\right)}}+\frac{}{5\sqrt{\frac{1}{\tan^{2}\left(\frac{x}{2}\right)+1}}\ \mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/sin(x)**(1/2),x)
```

```
[Out] 10*sqrt(2)*tan(x/2)**5/(5*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(13/2) + 15*s
qrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(9/2) + 15*sqrt(1/(tan(x/2)**2 + 1))*tan
(x/2)**(5/2) + 5*sqrt(1/(tan(x/2)**2 + 1))*sqrt(tan(x/2))) + 12*sqrt(2)*tan
(x/2)**3/(5*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(13/2) + 15*sqrt(1/(tan(x/2
)**2 + 1))*tan(x/2)**(9/2) + 15*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(5/2) +
```

```
 5*sqrt(1/(tan(x/2)**2 + 1))*sqrt(tan(x/2))) + 10*sqrt(2)*tan(x/2)/(5*sqrt(
1/(tan(x/2)**2 + 1))*tan(x/2)**(13/2) + 15*sqrt(1/(tan(x/2)**2 + 1))*tan(x/
2)**(9/2) + 15*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(5/2) + 5*sqrt(1/(tan(x/
2)**2 + 1))*sqrt(tan(x/2)))
```

Giac [A] time = 1.11444, size = 18, normalized size = 0.95

$$-\frac{2}{5}\sin(x)^{\frac{5}{2}}+2\sqrt{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/5*sin(x)^(5/2) + 2*sqrt(sin(x))
```

3.257 $\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx$

Optimal. Leaf size=132

$$\frac{7d^3(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{30bc}+\frac{7d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{20b\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a$$

```
[Out] (7*d^3*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(30*b*c) + (d*(d*Cos[
a + b*x])^(7/2)*(c*Sin[a + b*x])^(3/2))/(5*b*c) + (7*d^4*Sqrt[d*Cos[a + b*x
]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(20*b*Sqrt[Sin[2*a +
2*b*x]])
```

Rubi [A] time = 0.159601, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2569, 2572, 2639}

$$\frac{7d^3(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{30bc}+\frac{7d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{20b\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (7*d^3*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(30*b*c) + (d*(d*Cos[
a + b*x])^(7/2)*(c*Sin[a + b*x])^(3/2))/(5*b*c) + (7*d^4*Sqrt[d*Cos[a + b*x
]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(20*b*Sqrt[Sin[2*a +
2*b*x]])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \frac{d(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bc} + \frac{1}{10}\left(7d^2\right)\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+\cdots$$
$$= \frac{7d^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{30bc} + \frac{d(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bc}$$
$$= \frac{7d^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{30bc} + \frac{d(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bc}$$
$$= \frac{7d^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{30bc} + \frac{d(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bc}$$

Mathematica [C] time = 0.102623, size = 70, normalized size = 0.53

$$\frac{2d^4\sqrt[4]{\cos^2(a+bx)}\tan(a+bx)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};\frac{7}{4};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*d^4*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4,
3/4, 7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b)
```

Maple [B] time = 0.224, size = 540, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/120/b*2^(1/2)*(c*sin(b*x+a))^(1/2)*(d*cos(b*x+a))^(9/2)*(12*cos(b*x+a)^6
*2^(1/2)+2*cos(b*x+a)^4*2^(1/2)+42*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/
sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*
x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^
(1/2),1/2*2^(1/2))-21*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(
1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x
+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(
1/2))+42*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(
-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-21*(-(-1+cos(b*x+a
)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a)
)/sin(b*x+a))^(1/2),1/2*2^(1/2))+7*cos(b*x+a)^2*2^(1/2)-21*cos(b*x+a)*2^(1/
2))/cos(b*x+a)^5/sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{9}{2}}\sqrt{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*sqrt(c*sin(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}d^4\cos\left(bx+a\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d^4*cos(b*x + a)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(9/2)*(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.258 $\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx$

Optimal. Leaf size=95

$$\frac{d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}$$

```
[Out] (d*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(3*b*c) + (d^2*Sqrt[d*Cos
[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(2*b*Sqrt[Sin
[2*a + 2*b*x]])
```

Rubi [A] time = 0.103153, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2569, 2572, 2639}

$$\frac{d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (d*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(3*b*c) + (d^2*Sqrt[d*Cos
[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(2*b*Sqrt[Sin
[2*a + 2*b*x]])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx=\frac{d(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bc}+\frac{1}{2}d^2\int\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,d$$

$$=\frac{d(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bc}+\frac{\left(d^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int\sqrt{}}{2\sqrt{\sin(2a+2bx)}}$$

$$=\frac{d(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bc}+\frac{d^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c}}{2b\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] time = 0.0893808, size = 70, normalized size = 0.74

$$\frac{2d^2\sqrt[4]{\cos^2(a+bx)}\tan(a+bx)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{7}{4};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*d^2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4,
3/4, 7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b)
```

Maple [B] time = 0.147, size = 518, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/12/b*2^(1/2)*(c*sin(b*x+a))^(1/2)*(d*cos(b*x+a))^(5/2)*(2*cos(b*x+a)^4*2
^(1/2)-3*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b
*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elli
pticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*cos(b*x+a
)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*((1-cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*
x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2),1/2*2^(1/2))+6*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellip
ticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2
*2^(1/2)-3*cos(b*x+a)*2^(1/2))/sin(b*x+a)/cos(b*x+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{5}{2}}\sqrt{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}d^2\cos(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d^2*cos(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(5/2)*(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{5}{2}}\sqrt{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a)), x)
```

3.259 $\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx$

Optimal. Leaf size=53

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{b\sqrt{\sin(2a+2bx)}}$$

```
[Out] (Sqrt[d*Cos[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b
*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0489386, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2572, 2639}

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (Sqrt[d*Cos[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b
*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx &= \frac{\left(\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{\sin(2a+2bx)}}\\ &= \frac{\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [C] time = 0.064663, size = 67, normalized size = 1.26

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\tan(a+bx)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 3/4,
7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b)
```

Maple [B] time = 0.107, size = 505, normalized size = 9.5

$$-\frac{\sqrt{2}}{2\,b\sin(bx+a)\cos(bx+a)}\left(2\,\cos(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/2/b*2^(1/2)*(2*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*(
(-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(
1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-co
s(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*((1-cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1
+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x
+a))^(1/2),1/2*2^(1/2))-((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*
EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x
+a)^2*2^(1/2)-cos(b*x+a)*2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)
/sin(b*x+a)/cos(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(c*sin(a + b*x))*sqrt(d*cos(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)), x)
```

3.260 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(b*c*d*Sqrt[d*Cos[a + b*x]]) - (2*Sqrt[d*Cos[a +
 b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b*d^2*Sqrt[Sin[2
*a + 2*b*x]])
```

Rubi [A] time = 0.106754, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2571, 2572, 2639}

$$\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(b*c*d*Sqrt[d*Cos[a + b*x]]) - (2*Sqrt[d*Cos[a +
 b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b*d^2*Sqrt[Sin[2
*a + 2*b*x]])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx &= \frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx}{d^2}\\ &= \frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{\left(2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int \sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}}\\ &= \frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [C] time = 0.107327, size = 70, normalized size = 0.75

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\tan(a+bx)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};\sin^2(a+bx)\right)}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[3/4, 5/4,
7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*d^2)
```

Maple [B] time = 0.134, size = 493, normalized size = 5.3

$$\frac{\cos(bx+a)\sqrt{2}}{b\sin(bx+a)}\left(2\cos(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\mathrm{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] 1/b*2^(1/2)*(2*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1
+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2
)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b
*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+co
s(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2*2^(1/2))-((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ell
ipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)
*2^(1/2)+2^(1/2))*(c*sin(b*x+a))^(1/2)*cos(b*x+a)/(d*cos(b*x+a))^(3/2)/sin(
b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sin(bx+a)}}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{d^2\cos(bx+a)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(d^2*cos(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sin{(a+bx)}}}{(d\cos{(a+bx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Integral(sqrt(c*sin(a + b*x))/(d*cos(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sin{(bx+a)}}}{(d\cos{(bx+a)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(3/2), x)
```

3.261 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=134

$$\frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}}-\frac{4E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(5*b*c*d*(d*Cos[a + b*x])^(5/2)) + (4*(c*Sin[a +
 b*x])^(3/2))/(5*b*c*d^3*Sqrt[d*Cos[a + b*x]]) - (4*Sqrt[d*Cos[a + b*x]]*El
lipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(5*b*d^4*Sqrt[Sin[2*a + 2*
b*x]])
```

Rubi [A] time = 0.162821, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2571, 2572, 2639}

$$\frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}}-\frac{4E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(5*b*c*d*(d*Cos[a + b*x])^(5/2)) + (4*(c*Sin[a +
 b*x])^(3/2))/(5*b*c*d^3*Sqrt[d*Cos[a + b*x]]) - (4*Sqrt[d*Cos[a + b*x]]*El
lipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(5*b*d^4*Sqrt[Sin[2*a + 2*
b*x]])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}} + \frac{2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx}{5d^2}$$
$$= \frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}} + \frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}} - \frac{4\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx}{5d^4}$$
$$= \frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}} + \frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}} - \frac{\left(4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int \sqrt{\sin(}}{5d^4\sqrt{\sin(2a+2bx)}}$$
$$= \frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}} + \frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}} - \frac{4\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(}}{5bd^4\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] time = 0.149874, size = 70, normalized size = 0.52

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\tan(a+bx)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};\sin^2(a+bx)\right)}{3bd^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[3/4, 9/4,
7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*d^4)
```

Maple [B] time = 0.144, size = 528, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] 1/5/b*2^(1/2)*(4*cos(b*x+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*
((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^
(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2
*cos(b*x+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(
((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+4*cos(b*x+a)^2*((
1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^2*((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b
*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^3*2^(1/2)+cos(b*x+a)^2*2^(1/2)+2^(1/
2))*(c*sin(b*x+a))^(1/2)*cos(b*x+a)/(d*cos(b*x+a))^(7/2)/sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sin(bx+a)}}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}{d^4\cos\left(bx+a\right)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(d^4*cos(b*x + a)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(7/2), x)
```

3.262 $\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx$

Optimal. Leaf size=320

$$-\frac{\sqrt{c}d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b}+\frac{\sqrt{c}d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{4\sqrt{2}b}+\frac{\sqrt{c}d^{3/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a\right.}{8\sqrt{2}b}$$

```
[Out] -(Sqrt[c]*d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c
]*Sqrt[d*Cos[a + b*x]])])/(4*Sqrt[2]*b) + (Sqrt[c]*d^(3/2)*ArcTan[1 + (Sqrt
[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])])/(4*Sqrt[
2]*b) + (Sqrt[c]*d^(3/2)*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]
])/Sqrt[d*Cos[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b) - (Sqrt[c]*d
^(3/2)*Log[Sqrt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a +
b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b) + (d*Sqrt[d*Cos[a + b*x]]*(c*S
in[a + b*x])^(3/2))/(2*b*c)
```

Rubi [A] time = 0.295161, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2569, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt{c}d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b}+\frac{\sqrt{c}d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{4\sqrt{2}b}+\frac{\sqrt{c}d^{3/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a\right.}{8\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -(Sqrt[c]*d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c
]*Sqrt[d*Cos[a + b*x]])])/(4*Sqrt[2]*b) + (Sqrt[c]*d^(3/2)*ArcTan[1 + (Sqrt
[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])])/(4*Sqrt[
2]*b) + (Sqrt[c]*d^(3/2)*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]
])/Sqrt[d*Cos[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b) - (Sqrt[c]*d
^(3/2)*Log[Sqrt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a +
b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b) + (d*Sqrt[d*Cos[a + b*x]]*(c*S
in[a + b*x])^(3/2))/(2*b*c)
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx = \frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bc} + \frac{1}{4}d^2\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx$$

$$= \frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bc} + \frac{\left(cd^3\right)\text{Subst}\left(\int \frac{x^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2b}$$

$$= \frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bc} - \frac{\left(cd^2\right)\text{Subst}\left(\int \frac{c-dx^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{4b} +$$

$$= \frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bc} + \frac{(cd)\text{Subst}\left(\int \frac{1}{\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{8b}$$

$$= \frac{\sqrt{c}d^{3/2}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b} - \frac{\sqrt{c}d^{3/2}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\,\mathrm{s}}}{\sqrt{d\cos(}}\right.}{8\sqrt{}}$$

$$= -\frac{\sqrt{c}d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b} + \frac{\sqrt{c}d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b} + \frac{\sqrt{c}}{}$$

Mathematica [C] time = 0.119344, size = 70, normalized size = 0.22

$$\frac{2d^2\cos^2(a+bx)^{3/4}\tan(a+bx)\sqrt{c\sin(a+bx)}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\sin^2(a+bx)\right)}{3b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*d^2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, Sin[a + b*x
]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*Sqrt[d*Cos[a + b*x]])
```

Maple [C] time = 0.079, size = 514, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x)
```

```
[Out] 1/8/b*2^(1/2)*(I*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellipti
cPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-I*(
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin
(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+((1-cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+
cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x
+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+
a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I
,1/2*2^(1/2))+2*cos(b*x+a)^2*2^(1/2)-2*cos(b*x+a)*2^(1/2))*(d*cos(b*x+a))^(
3/2)*(c*sin(b*x+a))^(1/2)*sin(b*x+a)/(-1+cos(b*x+a))/cos(b*x+a)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{3}{2}}\sqrt{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*sqrt(c*sin(b*x + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.263 $\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=280

$$-\frac{\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}b\sqrt{d}}+\frac{\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)+\sqrt{c}\cdots\right.}{2\sqrt{2}b\sqrt{d}}\cdots$$

```
[Out] -((Sqrt[c]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[
d*Cos[a + b*x]])]])/(Sqrt[2]*b*Sqrt[d])) + (Sqrt[c]*ArcTan[1 + (Sqrt[2]*Sqrt
[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])]])/(Sqrt[2]*b*Sqrt[
d]) + (Sqrt[c]*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*
Cos[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(2*Sqrt[2]*b*Sqrt[d]) - (Sqrt[c]*Log
[Sqrt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a + b*x]] + Sq
rt[c]*Tan[a + b*x]])/(2*Sqrt[2]*b*Sqrt[d])
```

Rubi [A] time = 0.188675, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}b\sqrt{d}}+\frac{\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)+\sqrt{c}\cdots\right.}{2\sqrt{2}b\sqrt{d}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]]/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] -((Sqrt[c]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[
d*Cos[a + b*x]])]])/(Sqrt[2]*b*Sqrt[d])) + (Sqrt[c]*ArcTan[1 + (Sqrt[2]*Sqrt
[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])]])/(Sqrt[2]*b*Sqrt[
d]) + (Sqrt[c]*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*
Cos[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(2*Sqrt[2]*b*Sqrt[d]) - (Sqrt[c]*Log
[Sqrt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a + b*x]] + Sq
rt[c]*Tan[a + b*x]])/(2*Sqrt[2]*b*Sqrt[d])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx &= \frac{(2cd)\,\mathrm{Subst}\left(\int \frac{x^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{b} \\
&= -\frac{c\,\mathrm{Subst}\left(\int \frac{c-dx^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{b} + \frac{c\,\mathrm{Subst}\left(\int \frac{c+dx^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{b} \\
&= \frac{c\,\mathrm{Subst}\left(\int \frac{1}{\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2bd} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{\frac{c}{d}+\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2bd} + \frac{\sqrt{c}\,\mathrm{Subs}}{} \\
&= \frac{\sqrt{c}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}b\sqrt{d}} - \frac{\sqrt{c}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+b\right.}{2\sqrt{2}b\sqrt{d}} \\
&= -\frac{\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}} + \frac{\sqrt{c}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}} + \frac{\sqrt{c}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+b}}{\sqrt{d\cos(a+bx)}}\right.}{2\sqrt{2}b\sqrt{}}
\end{aligned}$$

Mathematica [C] time = 0.0623537, size = 67, normalized size = 0.24

$$\frac{2\cos^2(a+bx)^{3/4}\tan(a+bx)\sqrt{c\sin(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\sin^2(a+bx)\right)}{3b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, Sin[a + b*x]^2]*
Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*Sqrt[d*Cos[a + b*x]])
```

Maple [C] time = 0.083, size = 271, normalized size = 1.

$$-\frac{\sqrt{2}\sin(bx+a)}{2b(-1+\cos(bx+a))}\sqrt{c\sin(bx+a)}\left(i\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\mathrm{EllipticPi}\left(\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] -1/2/b*2^(1/2)*(c*sin(b*x+a))^(1/2)*(I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos(b*x+a))/
sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)/(-1+cos(b*x+a))/(d*cos(b*x+a))
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sin(bx+a)}}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/sqrt(d*cos(b*x + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(c*sin(a + b*x))/sqrt(d*cos(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sin(bx+a)}}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/sqrt(d*cos(b*x + a)), x)
```

3.264 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2(c\sin(a+bx))^{3/2}}{3bcd(d\cos(a+bx))^{3/2}}$$

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(3*b*c*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.0529633, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2563}

$$\frac{2(c\sin(a+bx))^{3/2}}{3bcd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(5/2),x]

[Out] (2*(c*Sin[a + b*x])^(3/2))/(3*b*c*d*(d*Cos[a + b*x])^(3/2))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{3bcd(d\cos(a+bx))^{3/2}}$$

Mathematica [A] time = 0.0826124, size = 37, normalized size = 1.

$$\frac{2(c\sin(a+bx))^{3/2}}{3bcd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(5/2),x]

[Out] (2*(c*Sin[a + b*x])^(3/2))/(3*b*c*d*(d*Cos[a + b*x])^(3/2))
```

Maple [A] time = 0.108, size = 38, normalized size = 1.

$$\frac{2\cos(bx+a)\sin(bx+a)}{3b}\sqrt{c\sin(bx+a)}\,(d\cos(bx+a))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] 2/3/b*sin(b*x+a)*cos(b*x+a)*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sin(bx+a)}}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(5/2), x)
```

Fricas [A] time = 3.2955, size = 112, normalized size = 3.03

$$\frac{2\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\sin(bx+a)}{3\,bd^3\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sin(b*x + a)/(b*d^3*cos(b*x +
 a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sin(bx+a)}}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(5/2), x)
```

3.265 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal. Leaf size=75

$$\frac{8(c\sin(a+bx))^{3/2}}{21bcd^3(d\cos(a+bx))^{3/2}}+\frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}}$$

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(7*b*c*d*(d*Cos[a + b*x])^(7/2)) + (8*(c*Sin[a +
 b*x])^(3/2))/(21*b*c*d^3*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.112531, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2571, 2563}

$$\frac{8(c\sin(a+bx))^{3/2}}{21bcd^3(d\cos(a+bx))^{3/2}}+\frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(7*b*c*d*(d*Cos[a + b*x])^(7/2)) + (8*(c*Sin[a +
 b*x])^(3/2))/(21*b*c*d^3*(d*Cos[a + b*x])^(3/2))
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx&=\frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}}+\frac{4\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx}{7d^2}\\&=\frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}}+\frac{8(c\sin(a+bx))^{3/2}}{21bcd^3(d\cos(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.242327, size = 57, normalized size = 0.76

$$\frac{2(2\cos(2(a+bx))+5)\sec^4(a+bx)(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{21bcd^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(5 + 2*Cos[2*(a + b*x)])*Sec[a + b*x]^4*(c*Sin[a +
b*x])^(3/2))/(21*b*c*d^5)
```

Maple [A] time = 0.107, size = 50, normalized size = 0.7

$$\frac{\left(8\,\left(\cos\left(bx+a\right)\right)^{2}+6\right)\cos\left(bx+a\right)\sin\left(bx+a\right)}{21\,b}\sqrt{c\sin\left(bx+a\right)}\left(d\cos\left(bx+a\right)\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x)
```

```
[Out] 2/21/b*(4*cos(b*x+a)^2+3)*(c*sin(b*x+a))^(1/2)*cos(b*x+a)*sin(b*x+a)/(d*cos
(b*x+a))^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(9/2), x)
```

Fricas [A] time = 3.70182, size = 144, normalized size = 1.92

$$\frac{2\,\sqrt{d\cos\left(bx+a\right)}\left(4\,\cos\left(bx+a\right)^{2}+3\right)\sqrt{c\sin\left(bx+a\right)}\sin\left(bx+a\right)}{21\,bd^{5}\cos\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] 2/21*sqrt(d*cos(b*x + a))*(4*cos(b*x + a)^2 + 3)*sqrt(c*sin(b*x + a))*sin(b
*x + a)/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(9/2), x)
```

3.266 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx$

Optimal. Leaf size=112

$$\frac{64(c\sin(a+bx))^{3/2}}{231bcd^5(d\cos(a+bx))^{3/2}}+\frac{16(c\sin(a+bx))^{3/2}}{77bcd^3(d\cos(a+bx))^{7/2}}+\frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}$$

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(11*b*c*d*(d*Cos[a + b*x])^(11/2)) + (16*(c*Sin[
a + b*x])^(3/2))/(77*b*c*d^3*(d*Cos[a + b*x])^(7/2)) + (64*(c*Sin[a + b*x])
^(3/2))/(231*b*c*d^5*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.171186, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2571, 2563}

$$\frac{64(c\sin(a+bx))^{3/2}}{231bcd^5(d\cos(a+bx))^{3/2}}+\frac{16(c\sin(a+bx))^{3/2}}{77bcd^3(d\cos(a+bx))^{7/2}}+\frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(13/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(11*b*c*d*(d*Cos[a + b*x])^(11/2)) + (16*(c*Sin[
a + b*x])^(3/2))/(77*b*c*d^3*(d*Cos[a + b*x])^(7/2)) + (64*(c*Sin[a + b*x])
^(3/2))/(231*b*c*d^5*(d*Cos[a + b*x])^(3/2))
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx &= \frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}+\frac{8\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx}{11d^2}\\ &= \frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}+\frac{16(c\sin(a+bx))^{3/2}}{77bcd^3(d\cos(a+bx))^{7/2}}+\frac{32\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx}{77d^4}\\ &= \frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}+\frac{16(c\sin(a+bx))^{3/2}}{77bcd^3(d\cos(a+bx))^{7/2}}+\frac{64(c\sin(a+bx))^{3/2}}{231bcd^5(d\cos(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.234605, size = 67, normalized size = 0.6

$$\frac{2(28\cos(2(a+bx))+4\cos(4(a+bx))+45)\sec^6(a+bx)(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{231bcd^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(13/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(45 + 28*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Sec
[a + b*x]^6*(c*Sin[a + b*x])^(3/2))/(231*b*c*d^7)
```

Maple [A] time = 0.144, size = 60, normalized size = 0.5

$$\frac{\left(64 \left(\cos \left(bx+a\right)\right)^{4}+48 \left(\cos \left(bx+a\right)\right)^{2}+42\right)\cos \left(bx+a\right)\sin \left(bx+a\right)}{231\,b}\sqrt{c\sin \left(bx+a\right)}\left(d\cos \left(bx+a\right)\right)^{-\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x)
```

```
[Out] 2/231/b*(32*cos(b*x+a)^4+24*cos(b*x+a)^2+21)*(c*sin(b*x+a))^(1/2)*cos(b*x+a
)*sin(b*x+a)/(d*cos(b*x+a))^(13/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sin \left(bx+a\right)}}{\left(d\cos \left(bx+a\right)\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(13/2), x)
```

Fricas [A] time = 4.00519, size = 176, normalized size = 1.57

$$\frac{2\left(32\,\cos \left(bx+a\right)^{4}+24\,\cos \left(bx+a\right)^{2}+21\right)\sqrt{d\cos \left(bx+a\right)}\sqrt{c\sin \left(bx+a\right)}\sin \left(bx+a\right)}{231\,bd^{7}\cos \left(bx+a\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x, algorithm="fricas")
```

```
[Out] 2/231*(32*cos(b*x + a)^4 + 24*cos(b*x + a)^2 + 21)*sqrt(d*cos(b*x + a))*sqr
t(c*sin(b*x + a))*sin(b*x + a)/(b*d^7*cos(b*x + a)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sin(bx+a)}}{(d\cos(bx+a))^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(13/2), x)
```

3.267 $\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx$

Optimal. Leaf size=131

$$\frac{c^2d^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{12b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bd}+\frac{cd\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{6b}$$

```
[Out] (c*d*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(6*b) - (c*(d*Cos[a + b*x])
^(5/2)*Sqrt[c*Sin[a + b*x]])/(3*b*d) + (c^2*d^2*EllipticF[a - Pi/4 + b*x, 2
]*Sqrt[Sin[2*a + 2*b*x]])/(12*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])
```

Rubi [A] time = 0.178903, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2568, 2569, 2573, 2641}

$$\frac{c^2d^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{12b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bd}+\frac{cd\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (c*d*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(6*b) - (c*(d*Cos[a + b*x])
^(5/2)*Sqrt[c*Sin[a + b*x]])/(3*b*d) + (c^2*d^2*EllipticF[a - Pi/4 + b*x, 2
]*Sqrt[Sin[2*a + 2*b*x]])/(12*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = -\frac{c(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bd} + \frac{1}{6}c^2\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx$$
$$= \frac{cd\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6b} - \frac{c(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bd} + \frac{1}{12}\left(c^2 d\right.$$
$$= \frac{cd\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6b} - \frac{c(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bd} + \frac{\left(c^2d^2\sqrt{\right.}}{12\sqrt{}}$$
$$= \frac{cd\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6b} - \frac{c(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bd} + \frac{c^2d^2F\left(\right.}{12b\sqrt{}}$$

Mathematica [C] time = 0.120046, size = 71, normalized size = 0.54

$$\frac{2cd\cos^2(a+bx)^{3/4}\tan^2(a+bx)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{9}{4};\sin^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (2*c*d*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4,
5/4, 9/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x]^2)/(5*b)
```

Maple [A] time = 0.132, size = 216, normalized size = 1.7

$$-\frac{\sqrt{2}}{12\,b\sin(bx+a)\,(-1+\cos(bx+a))\,(\cos(bx+a))^2}\left(\sin(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+}{\sin(bx+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x)
```

```
[Out] -1/12/b*2^(1/2)*(sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*c
os(b*x+a)^4*2^(1/2)-2*cos(b*x+a)^3*2^(1/2)-cos(b*x+a)^2*2^(1/2)+cos(b*x+a)*
2^(1/2))*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/sin(b*x+a)/(-1+cos(b*x+a
))/cos(b*x+a)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{3}{2}}\,(c\sin(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*(c*sin(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}cd\cos\left(bx+a\right)\sin\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c*d*cos(b*x + a)*sin(b*x
 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*(c*sin(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.268 $\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=93

$$\frac{c^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}-\frac{c\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd}$$

```
[Out] -((c*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(b*d)) + (c^2*EllipticF[a -
 Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Si
n[a + b*x]])
```

Rubi [A] time = 0.116628, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2568, 2573, 2641}

$$\frac{c^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}-\frac{c\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(3/2)/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] -((c*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(b*d)) + (c^2*EllipticF[a -
 Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Si
n[a + b*x]])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{c\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bd} + \frac{1}{2}c^2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx$$
$$= -\frac{c\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bd} + \frac{\left(c^2\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$
$$= -\frac{c\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bd} + \frac{c^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

Mathematica [C] time = 0.0806252, size = 67, normalized size = 0.72

$$\frac{2\cos^2(a+bx)^{3/4}\tan(a+bx)(c\sin(a+bx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{9}{4};\sin^2(a+bx)\right)}{5b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 5/4, 9/4, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(3/2)*Tan[a + b*x])/(5*b*Sqrt[d*Cos[a + b*x]])
```

Maple [A] time = 0.118, size = 182, normalized size = 2.

$$-\frac{\sqrt{2}}{2\,b\left(-1+\cos(bx+a)\right)\sin(bx+a)}\left(\sin(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] -1/2/b*2^(1/2)*(sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-
1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/
2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(
b*x+a)^2*2^(1/2)-cos(b*x+a)*2^(1/2))*(c*sin(b*x+a))^(3/2)/(-1+cos(b*x+a))/(
d*cos(b*x+a))^(1/2)/sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/sqrt(d*cos(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}c\sin(bx+a)}{d\cos(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c*sin(b*x + a)/(d*cos(b*
x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(c\sin(bx+a))^{\frac{3}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/sqrt(d*cos(b*x + a)), x)
```

3.269 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=98

$$\frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3bd^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}$$

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(3*b*d*(d*Cos[a + b*x])^(3/2)) - (c^2*EllipticF[
a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]*Sq
rt[c*Sin[a + b*x]])
```

Rubi [A] time = 0.122679, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2566, 2573, 2641}

$$\frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3bd^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(3*b*d*(d*Cos[a + b*x])^(3/2)) - (c^2*EllipticF[
a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]*Sq
rt[c*Sin[a + b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx &= \frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}} - \frac{c^2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx}{3d^2}\\ &= \frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}} - \frac{\left(c^2\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{3d^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\\ &= \frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}} - \frac{c^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\end{aligned}$$

Mathematica [C] time = 0.16754, size = 67, normalized size = 0.68

$$\frac{2\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{9}{4};\sin^2(a+bx)\right)}{5bcd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[5/4, 7/4, 9/4, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(5/2))/(5*b*c*d*(d*Cos[a + b*x])^(3/2))
```

Maple [A] time = 0.093, size = 186, normalized size = 1.9

$$\frac{\cos(bx+a)\sqrt{2}}{3b(-1+\cos(bx+a))\sin(bx+a)}\left(\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] 1/3/b*2^(1/2)*(EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2
^(1/2))*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)*cos(b
*x+a)+cos(b*x+a)*2^(1/2)-2^(1/2))*(c*sin(b*x+a))^(3/2)*cos(b*x+a)/(-1+cos(b
*x+a))/(d*cos(b*x+a))^(5/2)/sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}c\sin\left(bx+a\right)}{d^3\cos\left(bx+a\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c*sin(b*x + a)/(d^3*cos(
b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(5/2), x)
```

3.270 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{2c^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{21bd^4\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}-\frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}}+\frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}$$

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(7*b*d*(d*Cos[a + b*x])^(7/2)) - (2*c*Sqrt[c*Sin
[a + b*x]])/(21*b*d^3*(d*Cos[a + b*x])^(3/2)) - (2*c^2*EllipticF[a - Pi/4 +
 b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(21*b*d^4*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[
a + b*x]])
```

Rubi [A] time = 0.186147, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2566, 2571, 2573, 2641}

$$-\frac{2c^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{21bd^4\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}-\frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}}+\frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(7*b*d*(d*Cos[a + b*x])^(7/2)) - (2*c*Sqrt[c*Sin
[a + b*x]])/(21*b*d^3*(d*Cos[a + b*x])^(3/2)) - (2*c^2*EllipticF[a - Pi/4 +
 b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(21*b*d^4*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[
a + b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}} - \frac{c^2\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx}{7d^2}$$
$$= \frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}} - \frac{\left(2c^2\right)\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx}{21d^4}$$
$$= \frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}} - \frac{\left(2c^2\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{21d^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$
$$= \frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}} - \frac{2c^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}}{21bd^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

Mathematica [C] time = 0.155713, size = 70, normalized size = 0.53

$$\frac{2\cos^2(a+bx)^{7/4}\cot(a+bx)(c\sin(a+bx))^{7/2}\,{}_2F_1\left(\frac{5}{4},\frac{11}{4};\frac{9}{4};\sin^2(a+bx)\right)}{5bc^2(d\cos(a+bx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(7/4)*Cot[a + b*x]*Hypergeometric2F1[5/4, 11/4, 9/4, Si
n[a + b*x]^2]*(c*Sin[a + b*x])^(7/2))/(5*b*c^2*(d*Cos[a + b*x])^(9/2))
```

Maple [A] time = 0.103, size = 215, normalized size = 1.6

$$\frac{\cos(bx+a)\sqrt{2}}{21\,b\,(-1+\cos(bx+a))\sin(bx+a)}\left(2\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos}{\sin(b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x)
```

```
[Out] 1/21/b*2^(1/2)*(2*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellipt
icF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*sin(b*x+a)*co
s(b*x+a)^3-cos(b*x+a)^3*2^(1/2)+cos(b*x+a)^2*2^(1/2)+3*cos(b*x+a)*2^(1/2)-3
*2^(1/2))*(c*sin(b*x+a))^(3/2)*cos(b*x+a)/(-1+cos(b*x+a))/(d*cos(b*x+a))^(9
/2)/sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}c\sin\left(bx+a\right)}{d^5\cos\left(bx+a\right)^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c*sin(b*x + a)/(d^5*cos(
b*x + a)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.271 $\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx$

Optimal. Leaf size=320

$$\frac{c^{3/2}\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}b}-\frac{c^{3/2}\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{4\sqrt{2}b}-\frac{c^{3/2}\sqrt{d}\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+$$

```
[Out] (c^(3/2)*Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]
*Sqrt[c*Sin[a + b*x]])])/(4*Sqrt[2]*b) - (c^(3/2)*Sqrt[d]*ArcTan[1 + (Sqrt[
2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*Sin[a + b*x]])])/(4*Sqrt[2
]*b) - (c^(3/2)*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] - (Sqrt[2]*Sqrt[
c]*Sqrt[d*Cos[a + b*x]])/Sqrt[c*Sin[a + b*x]]])/(8*Sqrt[2]*b) + (c^(3/2)*Sq
rt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] + (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a +
b*x]])/Sqrt[c*Sin[a + b*x]]])/(8*Sqrt[2]*b) - (c*(d*Cos[a + b*x])^(3/2)*Sqr
t[c*Sin[a + b*x]])/(2*b*d)
```

Rubi [A] time = 0.280711, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2568, 2575, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^{3/2}\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}b}-\frac{c^{3/2}\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{4\sqrt{2}b}-\frac{c^{3/2}\sqrt{d}\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (c^(3/2)*Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]
*Sqrt[c*Sin[a + b*x]])])/(4*Sqrt[2]*b) - (c^(3/2)*Sqrt[d]*ArcTan[1 + (Sqrt[
2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*Sin[a + b*x]])])/(4*Sqrt[2
]*b) - (c^(3/2)*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] - (Sqrt[2]*Sqrt[
c]*Sqrt[d*Cos[a + b*x]])/Sqrt[c*Sin[a + b*x]]])/(8*Sqrt[2]*b) + (c^(3/2)*Sq
rt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] + (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a +
b*x]])/Sqrt[c*Sin[a + b*x]]])/(8*Sqrt[2]*b) - (c*(d*Cos[a + b*x])^(3/2)*Sqr
t[c*Sin[a + b*x]])/(2*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx &= -\frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd}+\frac{1}{4}c^2\int\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx\\
&= -\frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd}-\frac{\left(c^3d\right)\text{Subst}\left(\int\frac{x^2}{d^2+c^2x^4}\,dx,x,\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2b}\\
&= -\frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd}+\frac{\left(c^2d\right)\text{Subst}\left(\int\frac{d-cx^2}{d^2+c^2x^4}\,dx,x,\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{4b}-\\
&= -\frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd}-\frac{\left(c^{3/2}\sqrt{d}\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{c}}+2x}{-\frac{d}{c}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}-x^2}\,dx,x,\frac{\sqrt{d\cos}}{\sqrt{c\sin}}\right.}{8\sqrt{2}b}\\
&= -\frac{c^{3/2}\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{8\sqrt{2}b}+\frac{c^{3/2}\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\,\text{co}\right.}{}\\
&= \frac{c^{3/2}\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}b}-\frac{c^{3/2}\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}b}-\frac{c^{3/2}\sqrt{}}{}
\end{aligned}
$$

Mathematica [C] time = 0.0699715, size = 67, normalized size = 0.21

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\tan(a+bx)(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\sin^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 5/4,
9/4, Sin[a + b*x]^2]*(c*Sin[a + b*x])^(3/2)*Tan[a + b*x])/(5*b)
```

Maple [C] time = 0.078, size = 654, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x)
```

```
[Out] -1/8/b*2^(1/2)*(c*sin(b*x+a))^(3/2)*(d*cos(b*x+a))^(1/2)*(I*((1-cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/s
in(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(b*x+a)-I*((1-cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+c
os(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+
a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(b*x+a)-2*sin(b*x+a)*((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b
*x+a))^(1/2),1/2*2^(1/2))+((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1
+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2
)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(
1/2))*sin(b*x+a)+((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellipti
cPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin
(b*x+a)+2*cos(b*x+a)^3*2^(1/2)-2*cos(b*x+a)^2*2^(1/2))/sin(b*x+a)/cos(b*x+a
)/(-1+cos(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cos(bx+a)}\,(c\sin(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.272 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=313

$$-\frac{c^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}bd^{3/2}}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}bd^{3/2}}+\frac{c^{3/2}\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+bx)+\right.}{2\sqrt{2}bd^{3/2}}$$

```
[Out] -((c^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[
c*Sin[a + b*x]])]))/(Sqrt[2]*b*d^(3/2))) + (c^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*Sin[a + b*x]])]))/(Sqrt[2]*b*d^(3/
2)) + (c^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] - (Sqrt[2]*Sqrt[c]*Sqrt[d
*Cos[a + b*x]])/Sqrt[c*Sin[a + b*x]]])/(2*Sqrt[2]*b*d^(3/2)) - (c^(3/2)*Log
[Sqrt[d] + Sqrt[d]*Cot[a + b*x] + (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/Sq
rt[c*Sin[a + b*x]]])/(2*Sqrt[2]*b*d^(3/2)) + (2*c*Sqrt[c*Sin[a + b*x]])/(b*
d*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.275586, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2566, 2575, 297, 1162, 617, 204, 1165, 628}

$$-\frac{c^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}bd^{3/2}}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}bd^{3/2}}+\frac{c^{3/2}\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+bx)+\right.}{2\sqrt{2}bd^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] -((c^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[
c*Sin[a + b*x]])]))/(Sqrt[2]*b*d^(3/2))) + (c^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*Sin[a + b*x]])]))/(Sqrt[2]*b*d^(3/
2)) + (c^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] - (Sqrt[2]*Sqrt[c]*Sqrt[d
*Cos[a + b*x]])/Sqrt[c*Sin[a + b*x]]])/(2*Sqrt[2]*b*d^(3/2)) - (c^(3/2)*Log
[Sqrt[d] + Sqrt[d]*Cot[a + b*x] + (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/Sq
rt[c*Sin[a + b*x]]])/(2*Sqrt[2]*b*d^(3/2)) + (2*c*Sqrt[c*Sin[a + b*x]])/(b*
d*Sqrt[d*Cos[a + b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}} - \frac{c^2\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx}{d^2}$$

$$= \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}} + \frac{(2c^3)\,\text{Subst}\left(\int \frac{x^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{bd}$$

$$= \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}} - \frac{c^2\,\text{Subst}\left(\int \frac{d-cx^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{bd} + \frac{c^2\,\text{Subst}\left(\int \frac{d+cx^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\,c}}{\sqrt{c\,s}}\right.}{bd}$$

$$= \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}} + \frac{c^{3/2}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{c}}+2x}{-\frac{d}{c}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}-x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}bd^{3/2}} + \frac{c^{3/2}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{c}}-2}{-\frac{d}{c}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}}\right.}{2\sqrt{2}b}$$

$$= \frac{c^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}bd^{3/2}} - \frac{c^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)+\frac{\sqrt{2}\sqrt{c}\sqrt{}}{\sqrt{c\,\text{si}}}\right.}{2\sqrt{2}bd^{3/2}}$$

$$= -\frac{c^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}bd^{3/2}} + \frac{c^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}bd^{3/2}} + \frac{c^{3/2}\log\left(\sqrt{d}+\sqrt{d}\,\text{c}\right.}{}$$

Mathematica [C] time = 0.163983, size = 67, normalized size = 0.21

$$\frac{2\sqrt[4]{\cos^2(a+bx)}(c\sin(a+bx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\sin^2(a+bx)\right)}{5bcd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(5/2))/(5*b*c*d*Sqrt[d*Cos[a + b*x]])
```

Maple [C] time = 0.092, size = 642, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] 1/2/b*2^(1/2)*(I*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellipti
cPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin
(b*x+a)-I*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1
-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(b*x+a)
+((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/s
in(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(b*x+a)+((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(b*x+a)-2*sin(b*x+a)*((1-
```

```
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*cos(b*x+a)*2^(1/2)-2*2^(1/2))*(c*
sin(b*x+a))^(3/2)*cos(b*x+a)/(-1+cos(b*x+a))/(d*cos(b*x+a))^(3/2)/sin(b*x+a
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(3/2), x)
```

3.273 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2(c\sin(a+bx))^{5/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

```
[Out] (2*(c*Sin[a + b*x])^(5/2))/(5*b*c*d*(d*Cos[a + b*x])^(5/2))
```

Rubi [A] time = 0.0597498, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2563}

$$\frac{2(c\sin(a+bx))^{5/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(5/2))/(5*b*c*d*(d*Cos[a + b*x])^(5/2))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2(c\sin(a+bx))^{5/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

Mathematica [A] time = 0.123286, size = 40, normalized size = 1.08

$$\frac{2\cot(a+bx)(c\sin(a+bx))^{7/2}}{5bc^2(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*Cot[a + b*x]*(c*Sin[a + b*x])^(7/2))/(5*b*c^2*(d*Cos[a + b*x])^(7/2))
```

Maple [A] time = 0.066, size = 38, normalized size = 1.

$$\frac{2\cos(bx+a)\sin(bx+a)}{5b}(c\sin(bx+a))^{\frac{3}{2}}(d\cos(bx+a))^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] 2/5/b*sin(b*x+a)*cos(b*x+a)*(c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(7/2), x)
```

Fricas [A] time = 3.54889, size = 127, normalized size = 3.43

$$-\frac{2\left(c\cos(bx+a)^2-c\right)\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{5\,bd^4\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(c*cos(b*x + a)^2 - c)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(b*d^
4*cos(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(7/2), x)
```

3.274 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{8c\sqrt{c\sin(a+bx)}}{45bd^5\sqrt{d\cos(a+bx)}}-\frac{2c\sqrt{c\sin(a+bx)}}{45bd^3(d\cos(a+bx))^{5/2}}+\frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}}$$

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(9*b*d*(d*Cos[a + b*x])^(9/2)) - (2*c*Sqrt[c*Sin
[a + b*x]])/(45*b*d^3*(d*Cos[a + b*x])^(5/2)) - (8*c*Sqrt[c*Sin[a + b*x]])/
(45*b*d^5*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.18419, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2566, 2571, 2563}

$$-\frac{8c\sqrt{c\sin(a+bx)}}{45bd^5\sqrt{d\cos(a+bx)}}-\frac{2c\sqrt{c\sin(a+bx)}}{45bd^3(d\cos(a+bx))^{5/2}}+\frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(9*b*d*(d*Cos[a + b*x])^(9/2)) - (2*c*Sqrt[c*Sin
[a + b*x]])/(45*b*d^3*(d*Cos[a + b*x])^(5/2)) - (8*c*Sqrt[c*Sin[a + b*x]])/
(45*b*d^5*Sqrt[d*Cos[a + b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = \frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}} - \frac{c^2\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx}{9d^2}$$
$$= \frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{45bd^3(d\cos(a+bx))^{5/2}} - \frac{(4c^2)\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx}{45d^4}$$
$$= \frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{45bd^3(d\cos(a+bx))^{5/2}} - \frac{8c\sqrt{c\sin(a+bx)}}{45bd^5\sqrt{d\cos(a+bx)}}$$

Mathematica [A] time = 0.298347, size = 57, normalized size = 0.54

$$\frac{2(2\cos(2(a+bx))+7)\sec^5(a+bx)(c\sin(a+bx))^{5/2}\sqrt{d\cos(a+bx)}}{45bcd^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(7 + 2*Cos[2*(a + b*x)])*Sec[a + b*x]^5*(c*Sin[a +
b*x])^(5/2))/(45*b*c*d^6)
```

Maple [A] time = 0.072, size = 50, normalized size = 0.5

$$\frac{\left(8\,(\cos(bx+a))^2+10\right)\cos(bx+a)\sin(bx+a)}{45\,b}(c\sin(bx+a))^{\frac{3}{2}}(d\cos(bx+a))^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x)
```

```
[Out] 2/45/b*(4*cos(b*x+a)^2+5)*(c*sin(b*x+a))^(3/2)*cos(b*x+a)*sin(b*x+a)/(d*cos
(b*x+a))^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(11/2), x)
```

Fricas [A] time = 4.18239, size = 159, normalized size = 1.5

$$-\frac{2\left(4\,c\cos(bx+a)^4+c\cos(bx+a)^2-5\,c\right)\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{45\,bd^6\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x, algorithm="fricas")
```

```
[Out] -2/45*(4*c*cos(b*x + a)^4 + c*cos(b*x + a)^2 - 5*c)*sqrt(d*cos(b*x + a))*sq
rt(c*sin(b*x + a))/(b*d^6*cos(b*x + a)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.275 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{64c\sqrt{c\sin(a+bx)}}{585bd^7\sqrt{d\cos(a+bx)}}-\frac{16c\sqrt{c\sin(a+bx)}}{585bd^5(d\cos(a+bx))^{5/2}}-\frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}}+\frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}}$$

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(13*b*d*(d*Cos[a + b*x])^(13/2)) - (2*c*Sqrt[c*S
in[a + b*x]])/(117*b*d^3*(d*Cos[a + b*x])^(9/2)) - (16*c*Sqrt[c*Sin[a + b*x
]])/(585*b*d^5*(d*Cos[a + b*x])^(5/2)) - (64*c*Sqrt[c*Sin[a + b*x]])/(585*b
*d^7*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.240282, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2566, 2571, 2563}

$$-\frac{64c\sqrt{c\sin(a+bx)}}{585bd^7\sqrt{d\cos(a+bx)}}-\frac{16c\sqrt{c\sin(a+bx)}}{585bd^5(d\cos(a+bx))^{5/2}}-\frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}}+\frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(15/2),x]
```

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(13*b*d*(d*Cos[a + b*x])^(13/2)) - (2*c*Sqrt[c*S
in[a + b*x]])/(117*b*d^3*(d*Cos[a + b*x])^(9/2)) - (16*c*Sqrt[c*Sin[a + b*x
]])/(585*b*d^5*(d*Cos[a + b*x])^(5/2)) - (64*c*Sqrt[c*Sin[a + b*x]])/(585*b
*d^7*Sqrt[d*Cos[a + b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}dx = \frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}} - \frac{c^2\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}dx}{13d^2}$$
$$= \frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}} - \frac{(8c^2)\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}dx}{117d^4}$$
$$= \frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}} - \frac{16c\sqrt{c\sin(a+bx)}}{585bd^5(d\cos(a+bx))^{5/2}} - \frac{(32c^2)\int}{}$$
$$= \frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}} - \frac{16c\sqrt{c\sin(a+bx)}}{585bd^5(d\cos(a+bx))^{5/2}} - \frac{64c\sqrt{c}}{585bd^7\sqrt{}}$$

Mathematica [A] time = 0.310305, size = 67, normalized size = 0.48

$$\frac{2(36\cos(2(a+bx))+4\cos(4(a+bx))+77)\sec^7(a+bx)(c\sin(a+bx))^{5/2}\sqrt{d\cos(a+bx)}}{585bcd^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(15/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(77 + 36*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Sec
[a + b*x]^7*(c*Sin[a + b*x])^(5/2))/(585*b*c*d^8)
```

Maple [A] time = 0.109, size = 60, normalized size = 0.4

$$\frac{\left(64\,(\cos(bx+a))^4+80\,(\cos(bx+a))^2+90\right)\cos(bx+a)\sin(bx+a)}{585\,b}(c\sin(bx+a))^{\frac{3}{2}}(d\cos(bx+a))^{-\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x)
```

```
[Out] 2/585/b*(32*cos(b*x+a)^4+40*cos(b*x+a)^2+45)*(c*sin(b*x+a))^(3/2)*cos(b*x+a
)*sin(b*x+a)/(d*cos(b*x+a))^(15/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(15/2), x)
```

Fricas [A] time = 5.36217, size = 194, normalized size = 1.38

$$-\frac{2\left(32\,c\cos\left(bx+a\right)^{6}+8\,c\cos\left(bx+a\right)^{4}+5\,c\cos\left(bx+a\right)^{2}-45\,c\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}{585\,bd^{8}\cos\left(bx+a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x, algorithm="fricas")
```

```
[Out] -2/585*(32*c*cos(b*x + a)^6 + 8*c*cos(b*x + a)^4 + 5*c*cos(b*x + a)^2 - 45*
c)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(b*d^8*cos(b*x + a)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(15/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.276 $\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx$

Optimal. Leaf size=166

$$\frac{3c^2d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{40b\sqrt{\sin(2a+2bx)}}+\frac{cd^3(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{20b}-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{11/2}}{7bd}$$

```
[Out] (c*d^3*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(20*b) + (3*c*d*(d*Co
s[a + b*x])^(7/2)*(c*Sin[a + b*x])^(3/2))/(70*b) - (c*(d*Cos[a + b*x])^(11/
2)*(c*Sin[a + b*x])^(3/2))/(7*b*d) + (3*c^2*d^4*Sqrt[d*Cos[a + b*x]]*Ellipt
icE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(40*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.235413, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2568, 2569, 2572, 2639}

$$\frac{3c^2d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{40b\sqrt{\sin(2a+2bx)}}+\frac{cd^3(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{20b}-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{11/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(9/2)*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (c*d^3*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(20*b) + (3*c*d*(d*Co
s[a + b*x])^(7/2)*(c*Sin[a + b*x])^(3/2))/(70*b) - (c*(d*Cos[a + b*x])^(11/
2)*(c*Sin[a + b*x])^(3/2))/(7*b*d) + (3*c^2*d^4*Sqrt[d*Cos[a + b*x]]*Ellipt
icE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(40*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx &= -\frac{c(d\cos(a+bx))^{11/2}(c\sin(a+bx))^{3/2}}{7bd}+\frac{1}{14}\left(3c^2\right)\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a\\ &= \frac{3cd(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{70b}-\frac{c(d\cos(a+bx))^{11/2}(c\sin(a+bx))^{3/2}}{7bd}\\ &= \frac{cd^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{20b}+\frac{3cd(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{70b}\\ &= \frac{cd^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{20b}+\frac{3cd(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{70b}\\ &= \frac{cd^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{20b}+\frac{3cd(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{70b}\end{aligned}$$

Mathematica [C] time = 0.172968, size = 72, normalized size = 0.43

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\sec^5(a+bx)(c\sin(a+bx))^{7/2}(d\cos(a+bx))^{9/2}\,{}_2F_1\left(-\frac{7}{4},\frac{7}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (2*(d*Cos[a + b*x])^(9/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4, 7/
4, 11/4, Sin[a + b*x]^2]*Sec[a + b*x]^5*(c*Sin[a + b*x])^(7/2))/(7*b*c)
```

Maple [B] time = 0.148, size = 545, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x)
```

```
[Out] 1/560/b*2^(1/2)*(40*cos(b*x+a)^8*2^(1/2)-52*cos(b*x+a)^6*2^(1/2)-2*cos(b*x+
a)^4*2^(1/2)-42*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-
1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/
2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+21*c
os(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-42*((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b
*x+a))^(1/2),1/2*2^(1/2))+21*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*(
(-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(
1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-7*
cos(b*x+a)^2*2^(1/2)+21*cos(b*x+a)*2^(1/2))*(d*cos(b*x+a))^(9/2)*(c*sin(b*x
+a))^(5/2)/sin(b*x+a)^3/cos(b*x+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{9}{2}}(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*(c*sin(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(c^2d^4\cos\left(bx+a\right)^6-c^2d^4\cos\left(bx+a\right)^4\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*d^4*cos(b*x + a)^6 - c^2*d^4*cos(b*x + a)^4)*sqrt(d*cos(b*x
+ a))*sqrt(c*sin(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(9/2)*(c*sin(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.277 $\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx$

Optimal. Leaf size=131

$$\frac{3c^2d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{20b\sqrt{\sin(2a+2bx)}}-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bd}+\frac{cd(c\sin(a+bx))^{3/2}(d\cos(}{10b}$$

```
[Out] (c*d*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(10*b) - (c*(d*Cos[a +
b*x])^(7/2)*(c*Sin[a + b*x])^(3/2))/(5*b*d) + (3*c^2*d^2*Sqrt[d*Cos[a + b*x
]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(20*b*Sqrt[Sin[2*a +
2*b*x]])
```

Rubi [A] time = 0.175756, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2568, 2569, 2572, 2639}

$$\frac{3c^2d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{20b\sqrt{\sin(2a+2bx)}}-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bd}+\frac{cd(c\sin(a+bx))^{3/2}(d\cos(}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(5/2)*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (c*d*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(10*b) - (c*(d*Cos[a +
b*x])^(7/2)*(c*Sin[a + b*x])^(3/2))/(5*b*d) + (3*c^2*d^2*Sqrt[d*Cos[a + b*x
]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(20*b*Sqrt[Sin[2*a +
2*b*x]])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = -\frac{c(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bd} + \frac{1}{10}\left(3c^2\right)\int (d\cos(a+bx))^{5/2}\sqrt{c\sin}$$
$$= \frac{cd(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{10b} - \frac{c(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bd}$$
$$= \frac{cd(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{10b} - \frac{c(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bd}$$
$$= \frac{cd(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{10b} - \frac{c(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bd}$$

Mathematica [C] time = 0.177644, size = 70, normalized size = 0.53

$$\frac{2d^2\sqrt[4]{\cos^2(a+bx)}\tan(a+bx)(c\sin(a+bx))^{5/2}\sqrt{d\cos(a+bx)}\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (2*d^2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4,
7/4, 11/4, Sin[a + b*x]^2]*(c*Sin[a + b*x])^(5/2)*Tan[a + b*x])/(7*b)
```

Maple [B] time = 0.08, size = 532, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x)
```

```
[Out] 1/40/b*2^(1/2)*(4*cos(b*x+a)^6*2^(1/2)-6*cos(b*x+a)^4*2^(1/2)-6*cos(b*x+a)*
((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+3*cos(b*x+a)*((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b
*x+a))^(1/2),1/2*2^(1/2))-6*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+3*(
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin
(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)^2*2^(1/2)+3*cos(b*x+a
)*2^(1/2))*(d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2)/sin(b*x+a)^3/cos(b*x+a
)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{5}{2}}(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*(c*sin(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\left(c^2d^2\cos\left(bx+a\right)^4-c^2d^2\cos\left(bx+a\right)^2\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*d^2*cos(b*x + a)^4 - c^2*d^2*cos(b*x + a)^2)*sqrt(d*cos(b*x
+ a))*sqrt(c*sin(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(5/2)*(c*sin(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}\left(c\sin\left(bx+a\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*(c*sin(b*x + a))^(5/2), x)
```

3.278 $\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx$

Optimal. Leaf size=95

$$\frac{c^2 E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bd}$$

```
[Out] -(c*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(3*b*d) + (c^2*Sqrt[d*Co
s[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(2*b*Sqrt[Si
n[2*a + 2*b*x]])
```

Rubi [A] time = 0.108401, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2568, 2572, 2639}

$$\frac{c^2 E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] -(c*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(3*b*d) + (c^2*Sqrt[d*Co
s[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(2*b*Sqrt[Si
n[2*a + 2*b*x]])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx=-\frac{c(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bd}+\frac{1}{2}c^2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}$$

$$=-\frac{c(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bd}+\frac{\left(c^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int}{2\sqrt{\sin(2a+2bx)}}$$

$$=-\frac{c(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bd}+\frac{c^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{}}{2b\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] time = 0.095088, size = 67, normalized size = 0.71

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\tan(a+bx)(c\sin(a+bx))^{5/2}\sqrt{d\cos(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 7/4,
11/4, Sin[a + b*x]^2]*(c*Sin[a + b*x])^(5/2)*Tan[a + b*x])/(7*b)
```

Maple [B] time = 0.121, size = 519, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x)
```

```
[Out] -1/12/b*2^(1/2)*(6*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*
((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^
(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3
*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+si
n(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((
1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^4*2^(1
/2)+6*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+co
s(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2*2^(1/2))+5*cos(b*x+a)^2*2^(1/2)-3*cos(b*x+a)*2^(1/2))*(d*cos(b*
x+a))^(1/2)*(c*sin(b*x+a))^(5/2)/sin(b*x+a)^3/cos(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cos(bx+a)}\,(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(c^2\cos(bx+a)^2-c^2\right)\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cos(bx+a)}\,(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^(5/2), x)
```

3.279 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}}-\frac{3c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(b*d*Sqrt[d*Cos[a + b*x]]) - (3*c^2*Sqrt[d*Cos
[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b*d^2*Sqrt[S
in[2*a + 2*b*x]])
```

Rubi [A] time = 0.12023, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2566, 2572, 2639}

$$\frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}}-\frac{3c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(b*d*Sqrt[d*Cos[a + b*x]]) - (3*c^2*Sqrt[d*Cos
[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b*d^2*Sqrt[S
in[2*a + 2*b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx &= \frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}}-\frac{\left(3c^2\right)\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx}{d^2}\\
&= \frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}}-\frac{\left(3c^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int \sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}}\\
&= \frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}}-\frac{3c^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

Mathematica [C] time = 0.120092, size = 67, normalized size = 0.71

$$\frac{2\sqrt[4]{\cos^2(a+bx)}(c\sin(a+bx))^{7/2}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7bcd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 7/4, 11/4, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(7/2))/(7*b*c*d*Sqrt[d*Cos[a + b*x]])
```

Maple [B] time = 0.093, size = 508, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] 1/2/b*2^(1/2)*(6*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*c
os(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*((1-cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-
1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2),1/2*2^(1/2))-3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-
1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/
2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(
b*x+a)^2*2^(1/2)-3*cos(b*x+a)*2^(1/2)+2*2^(1/2))*(c*sin(b*x+a))^(5/2)*cos(b
*x+a)/sin(b*x+a)^3/(d*cos(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(c^2\cos(bx+a)^2-c^2\right)\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{d^2\cos(bx+a)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
a))/(d^2*cos(b*x + a)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sin(bx+a))^{\frac{5}{2}}}{(d \cos(bx+a))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(3/2), x)
```

3.280 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=133

$$\frac{6c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}-\frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}}$$

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(5*b*d*(d*Cos[a + b*x])^(5/2)) - (6*c*(c*Sin[a
 + b*x])^(3/2))/(5*b*d^3*Sqrt[d*Cos[a + b*x]]) + (6*c^2*Sqrt[d*Cos[a + b*x]
]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(5*b*d^4*Sqrt[Sin[2*a
+ 2*b*x]])
```

Rubi [A] time = 0.173949, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2566, 2571, 2572, 2639}

$$\frac{6c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}-\frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(5*b*d*(d*Cos[a + b*x])^(5/2)) - (6*c*(c*Sin[a
 + b*x])^(3/2))/(5*b*d^3*Sqrt[d*Cos[a + b*x]]) + (6*c^2*Sqrt[d*Cos[a + b*x]
]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(5*b*d^4*Sqrt[Sin[2*a
+ 2*b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}} - \frac{\left(3c^2\right)\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx}{5d^2}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{\left(6c^2\right)\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx}{5d^4}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{\left(6c^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int \sqrt{\sin(2a}}{5d^4\sqrt{\sin(2a+2bx)}}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{6c^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+}}{5bd^4\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] time = 0.181862, size = 70, normalized size = 0.53

$$\frac{2\cos^2(a+bx)^{5/4}\cot(a+bx)(c\sin(a+bx))^{9/2}\,{}_2F_1\left(\frac{7}{4},\frac{9}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7bc^2(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(5/4)*Cot[a + b*x]*Hypergeometric2F1[7/4, 9/4, 11/4, Si
n[a + b*x]^2]*(c*Sin[a + b*x])^(9/2))/(7*b*c^2*(d*Cos[a + b*x])^(7/2))
```

Maple [B] time = 0.105, size = 531, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] -1/5/b*2^(1/2)*(6*cos(b*x+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))
^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-
3*cos(b*x+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF
(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*cos(b*x+a)^2*(
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin
(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*cos(b*x+a)^2*((1-cos(b*x+a)+si
n(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*(
(-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2),1/2*2^(1/2))-3*cos(b*x+a)^3*2^(1/2)+4*cos(b*x+a)^2*2^(1/2)-2^
(1/2))*(c*sin(b*x+a))^(5/2)*cos(b*x+a)/sin(b*x+a)^3/(d*cos(b*x+a))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c^2\cos\left(bx+a\right)^2-c^2\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}{d^4\cos\left(bx+a\right)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
a))/(d^4*cos(b*x + a)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.281 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx$

Optimal. Leaf size=168

$$\frac{4c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{15bd^6\sqrt{\sin(2a+2bx)}}-\frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}}-\frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}}+\frac{2c(c\sin(a+bx)}{9bd(d\cos(a+bx}$$

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(9*b*d*(d*Cos[a + b*x])^(9/2)) - (2*c*(c*Sin[a
 + b*x])^(3/2))/(15*b*d^3*(d*Cos[a + b*x])^(5/2)) - (4*c*(c*Sin[a + b*x])^(
3/2))/(15*b*d^5*Sqrt[d*Cos[a + b*x]]) + (4*c^2*Sqrt[d*Cos[a + b*x]]*Ellipti
cE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(15*b*d^6*Sqrt[Sin[2*a + 2*b*x]
])
```

Rubi [A] time = 0.239653, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2566, 2571, 2572, 2639}

$$\frac{4c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{15bd^6\sqrt{\sin(2a+2bx)}}-\frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}}-\frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}}+\frac{2c(c\sin(a+bx)}{9bd(d\cos(a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(9*b*d*(d*Cos[a + b*x])^(9/2)) - (2*c*(c*Sin[a
 + b*x])^(3/2))/(15*b*d^3*(d*Cos[a + b*x])^(5/2)) - (4*c*(c*Sin[a + b*x])^(
3/2))/(15*b*d^5*Sqrt[d*Cos[a + b*x]]) + (4*c^2*Sqrt[d*Cos[a + b*x]]*Ellipti
cE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(15*b*d^6*Sqrt[Sin[2*a + 2*b*x]
])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}dx = \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{c^2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}dx}{3d^2}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}} - \frac{\left(2c^2\right)\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}dx}{15d^4}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}} - \frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}} + \frac{\left(4c^2\right)\int \sqrt{d\cos(}}{}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}} - \frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}} + \frac{\left(4c^2\sqrt{d\cos(a+}\right.}{}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}} - \frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}} + \frac{4c^2\sqrt{d\cos(a+}}{1}$$

Mathematica [C] time = 0.182339, size = 72, normalized size = 0.43

$$\frac{2\cos^5(a+bx)\sqrt[4]{\cos^2(a+bx)}(c\sin(a+bx))^{7/2}\,{}_2F_1\left(\frac{7}{4},\frac{13}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7bc(d\cos(a+bx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] (2*Cos[a + b*x]^5*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[7/4, 13/4, 11/4,
 Sin[a + b*x]^2]*(c*Sin[a + b*x])^(7/2))/(7*b*c*(d*Cos[a + b*x])^(11/2))
```

Maple [B] time = 0.086, size = 544, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x)
```

```
[Out] 1/45/b*2^(1/2)*(6*cos(b*x+a)^5*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))
^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-
12*cos(b*x+a)^5*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elliptic
E(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*cos(b*x+a)^4*
((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-12*cos(b*x+a)^4*((1-cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2),1/2*2^(1/2))+6*2^(1/2)*cos(b*x+a)^5-3*cos(b*x+a)^4*2^(1/2)-
8*cos(b*x+a)^2*2^(1/2)+5*2^(1/2))*(c*sin(b*x+a))^(5/2)*cos(b*x+a)/sin(b*x+a
```

```
)^3/(d*cos(b*x+a))^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c^2\cos(bx+a)^2-c^2\right)\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{d^6\cos(bx+a)^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
a))/(d^6*cos(b*x + a)^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.282 $\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=320

$$-\frac{3c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b\sqrt{d}}+\frac{3c^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{4\sqrt{2}b\sqrt{d}}+\frac{3c^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b\sqrt{d}}$$

```
[Out] (-3*c^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt
[d*Cos[a + b*x]])])/(4*Sqrt[2]*b*Sqrt[d]) + (3*c^(5/2)*ArcTan[1 + (Sqrt[2]*
Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])])/(4*Sqrt[2]*b
*Sqrt[d]) + (3*c^(5/2)*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])
/Sqrt[d*Cos[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b*Sqrt[d]) - (3*c
^(5/2)*Log[Sqrt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a +
b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b*Sqrt[d]) - (c*Sqrt[d*Cos[a + b*
x]]*(c*Sin[a + b*x])^(3/2))/(2*b*d)
```

Rubi [A] time = 0.257508, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2568, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b\sqrt{d}}+\frac{3c^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{4\sqrt{2}b\sqrt{d}}+\frac{3c^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (-3*c^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt
[d*Cos[a + b*x]])])/(4*Sqrt[2]*b*Sqrt[d]) + (3*c^(5/2)*ArcTan[1 + (Sqrt[2]*
Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])])/(4*Sqrt[2]*b
*Sqrt[d]) + (3*c^(5/2)*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])
/Sqrt[d*Cos[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b*Sqrt[d]) - (3*c
^(5/2)*Log[Sqrt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a +
b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b*Sqrt[d]) - (c*Sqrt[d*Cos[a + b*
x]]*(c*Sin[a + b*x])^(3/2))/(2*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{c\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bd} + \frac{1}{4}\left(3c^2\right)\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx$$

$$= -\frac{c\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bd} + \frac{\left(3c^3d\right)\text{Subst}\left(\int \frac{x^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2b}$$

$$= -\frac{c\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bd} - \frac{\left(3c^3\right)\text{Subst}\left(\int \frac{c-dx^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{4b} + \frac{\left(3c^3\right)\text{Sub}}{}$$

$$= -\frac{c\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bd} + \frac{\left(3c^3\right)\text{Subst}\left(\int \frac{1}{\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{8bd} + \frac{\left(3c^3\right)}{}$$

$$= \frac{3c^{5/2}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b\sqrt{d}} - \frac{3c^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\,\text{ta}\right.}{8\sqrt{2}b\sqrt{d}}$$

$$= -\frac{3c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b\sqrt{d}} + \frac{3c^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b\sqrt{d}} + \frac{3c^{5/2}\log\left(\sqrt{c}-\frac{\sqrt{2}}{}\right.}{}$$

Mathematica [C] time = 0.125682, size = 67, normalized size = 0.21

$$\frac{2\cos^2(a+bx)^{3/4}\tan(a+bx)(c\sin(a+bx))^{5/2}\,{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 7/4, 11/4, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(5/2)*Tan[a + b*x])/(7*b*Sqrt[d*Cos[a + b*x]])
```

Maple [C] time = 0.118, size = 510, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] -1/8/b*2^(1/2)*(3*I*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b
*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elli
pticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-
3*I*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b
*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*((1-cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2
)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*((1-cos(b*x+a)+sin(b*x+a))/sin(b
*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))
/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),
1/2+1/2*I,1/2*2^(1/2))+2*cos(b*x+a)^2*2^(1/2)-2*cos(b*x+a)*2^(1/2))*(c*sin(
b*x+a))^(5/2)/(-1+cos(b*x+a))/(d*cos(b*x+a))^(1/2)/sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/sqrt(d*cos(b*x + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/sqrt(d*cos(b*x + a)), x)
```

3.283 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=315

$$\frac{c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}bd^{5/2}}-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}bd^{5/2}}-\frac{c^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)+\sqrt{c}\right.}{2\sqrt{2}bd^{5/2}}$$

```
[Out] (c^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*
Cos[a + b*x]])])/(Sqrt[2]*b*d^(5/2)) - (c^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]
*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])])/(Sqrt[2]*b*d^(5/2))
 - (c^(5/2)*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos
[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(2*Sqrt[2]*b*d^(5/2)) + (c^(5/2)*Log[Sq
rt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a + b*x]] + Sqrt[
c]*Tan[a + b*x]])/(2*Sqrt[2]*b*d^(5/2)) + (2*c*(c*Sin[a + b*x])^(3/2))/(3*b
*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.2657, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2566, 2574, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}bd^{5/2}}-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}bd^{5/2}}-\frac{c^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)+\sqrt{c}\right.}{2\sqrt{2}bd^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (c^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*
Cos[a + b*x]])])/(Sqrt[2]*b*d^(5/2)) - (c^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]
*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])])/(Sqrt[2]*b*d^(5/2))
 - (c^(5/2)*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos
[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(2*Sqrt[2]*b*d^(5/2)) + (c^(5/2)*Log[Sq
rt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a + b*x]] + Sqrt[
c]*Tan[a + b*x]])/(2*Sqrt[2]*b*d^(5/2)) + (2*c*(c*Sin[a + b*x])^(3/2))/(3*b
*d*(d*Cos[a + b*x])^(3/2))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}} - \frac{c^2\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}dx}{d^2}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}} - \frac{(2c^3)\,\text{Subst}\left(\int \frac{x^2}{c^2+d^2x^4}dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{bd}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}} + \frac{c^3\,\text{Subst}\left(\int \frac{c-dx^2}{c^2+d^2x^4}dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{bd^2} - \frac{c^3\,\text{Subst}\left(\int \frac{c+dx^2}{c^2+d^2x^4}dx, x,\right.}{bd^2}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}} - \frac{c^3\,\text{Subst}\left(\int \frac{1}{\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2bd^3} - \frac{c^3\,\text{Subst}\left(\int \frac{1}{\frac{c}{d}+\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x\right.}{2bd}$$

$$= -\frac{c^{5/2}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}bd^{5/2}} + \frac{c^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\,\text{ta}\right.}{2\sqrt{2}bd^{5/2}}$$

$$= \frac{c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}bd^{5/2}} - \frac{c^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}bd^{5/2}} - \frac{c^{5/2}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\,\text{s}}}{\sqrt{d\cos}}\right.}{2\sqrt{}}$$

Mathematica [C] time = 0.125588, size = 67, normalized size = 0.21

$$\frac{2\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{7/2}\,{}_2F_1\left(\frac{7}{4},\frac{7}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7bcd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, 7/4, 11/4, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(7/2))/(7*b*c*d*(d*Cos[a + b*x])^(3/2))
```

Maple [C] time = 0.093, size = 532, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] 1/6/b*2^(1/2)*(3*I*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellip
ticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*c
os(b*x+a)-3*I*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi
(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*
x+a)-3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-co
s(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)-3*
((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)+2*cos(b*x
+a)*2^(1/2)-2*2^(1/2))*(c*sin(b*x+a))^(5/2)*cos(b*x+a)/(-1+cos(b*x+a))/(d*c
```

```
os(b*x+a))^(5/2)/sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(5/2), x)
```

3.284 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2(c\sin(a+bx))^{7/2}}{7bcd(d\cos(a+bx))^{7/2}}$$

```
[Out] (2*(c*Sin[a + b*x])^(7/2))/(7*b*c*d*(d*Cos[a + b*x])^(7/2))
```

Rubi [A] time = 0.059941, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2563}

$$\frac{2(c\sin(a+bx))^{7/2}}{7bcd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(7/2))/(7*b*c*d*(d*Cos[a + b*x])^(7/2))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2(c\sin(a+bx))^{7/2}}{7bcd(d\cos(a+bx))^{7/2}}$$

Mathematica [A] time = 0.16591, size = 40, normalized size = 1.08

$$\frac{2\cot(a+bx)(c\sin(a+bx))^{9/2}}{7bc^2(d\cos(a+bx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*Cot[a + b*x]*(c*Sin[a + b*x])^(9/2))/(7*b*c^2*(d*Cos[a + b*x])^(9/2))
```

Maple [A] time = 0.066, size = 38, normalized size = 1.

$$\frac{2\cos(bx+a)\sin(bx+a)}{7b}(c\sin(bx+a))^{\frac{5}{2}}(d\cos(bx+a))^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x)
```

```
[Out] 2/7/b*sin(b*x+a)*cos(b*x+a)*(c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(9/2), x)
```

Fricas [A] time = 3.21275, size = 150, normalized size = 4.05

$$-\frac{2\left(c^2\cos(bx+a)^2-c^2\right)\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\sin(bx+a)}{7\,bd^5\cos(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] -2/7*(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*s
in(b*x + a)/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.285 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{8c(c\sin(a+bx))^{3/2}}{77bd^5(d\cos(a+bx))^{3/2}}-\frac{6c(c\sin(a+bx))^{3/2}}{77bd^3(d\cos(a+bx))^{7/2}}+\frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}}$$

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(11*b*d*(d*Cos[a + b*x])^(11/2)) - (6*c*(c*Sin
[a + b*x])^(3/2))/(77*b*d^3*(d*Cos[a + b*x])^(7/2)) - (8*c*(c*Sin[a + b*x])
^(3/2))/(77*b*d^5*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.174937, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2566, 2571, 2563}

$$-\frac{8c(c\sin(a+bx))^{3/2}}{77bd^5(d\cos(a+bx))^{3/2}}-\frac{6c(c\sin(a+bx))^{3/2}}{77bd^3(d\cos(a+bx))^{7/2}}+\frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(13/2),x]
```

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(11*b*d*(d*Cos[a + b*x])^(11/2)) - (6*c*(c*Sin
[a + b*x])^(3/2))/(77*b*d^3*(d*Cos[a + b*x])^(7/2)) - (8*c*(c*Sin[a + b*x])
^(3/2))/(77*b*d^5*(d*Cos[a + b*x])^(3/2))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}dx = \frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}} - \frac{(3c^2)\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}dx}{11d^2}$$
$$= \frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{77bd^3(d\cos(a+bx))^{7/2}} - \frac{(12c^2)\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}dx}{77d^4}$$
$$= \frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{77bd^3(d\cos(a+bx))^{7/2}} - \frac{8c(c\sin(a+bx))^{3/2}}{77bd^5(d\cos(a+bx))^{3/2}}$$

Mathematica [A] time = 0.304794, size = 57, normalized size = 0.54

$$\frac{2c^4(2\cos(2(a+bx))+9)\tan^5(a+bx)}{77bd^6(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(13/2),x]
```

```
[Out] (2*c^4*(9 + 2*Cos[2*(a + b*x)])*Tan[a + b*x]^5)/(77*b*d^6*Sqrt[d*Cos[a + b*
x]]*(c*Sin[a + b*x])^(3/2))
```

Maple [A] time = 0.069, size = 50, normalized size = 0.5

$$\frac{\left(8\,(\cos(bx+a))^2+14\right)\cos(bx+a)\sin(bx+a)}{77\,b}(c\sin(bx+a))^{\frac{5}{2}}(d\cos(bx+a))^{-\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x)
```

```
[Out] 2/77/b*(4*cos(b*x+a)^2+7)*(c*sin(b*x+a))^(5/2)*cos(b*x+a)*sin(b*x+a)/(d*cos
(b*x+a))^(13/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(13/2), x)
```

Fricas [A] time = 4.38668, size = 188, normalized size = 1.77

$$-\frac{2\left(4\,c^2\cos(bx+a)^4+3\,c^2\cos(bx+a)^2-7\,c^2\right)\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\sin(bx+a)}{77\,bd^7\cos(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*c^2*cos(b*x + a)^4 + 3*c^2*cos(b*x + a)^2 - 7*c^2)*sqrt(d*cos(b*x
+ a))*sqrt(c*sin(b*x + a))*sin(b*x + a)/(b*d^7*cos(b*x + a)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(13/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.286 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{64c(c\sin(a+bx))^{3/2}}{1155bd^7(d\cos(a+bx))^{3/2}}-\frac{16c(c\sin(a+bx))^{3/2}}{385bd^5(d\cos(a+bx))^{7/2}}-\frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}}+\frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}}$$

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(15*b*d*(d*Cos[a + b*x])^(15/2)) - (2*c*(c*Sin
[a + b*x])^(3/2))/(55*b*d^3*(d*Cos[a + b*x])^(11/2)) - (16*c*(c*Sin[a + b*x
])^(3/2))/(385*b*d^5*(d*Cos[a + b*x])^(7/2)) - (64*c*(c*Sin[a + b*x])^(3/2)
)/(1155*b*d^7*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.234812, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2566, 2571, 2563}

$$-\frac{64c(c\sin(a+bx))^{3/2}}{1155bd^7(d\cos(a+bx))^{3/2}}-\frac{16c(c\sin(a+bx))^{3/2}}{385bd^5(d\cos(a+bx))^{7/2}}-\frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}}+\frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(17/2),x]
```


```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(15*b*d*(d*Cos[a + b*x])^(15/2)) - (2*c*(c*Sin
[a + b*x])^(3/2))/(55*b*d^3*(d*Cos[a + b*x])^(11/2)) - (16*c*(c*Sin[a + b*x
])^(3/2))/(385*b*d^5*(d*Cos[a + b*x])^(7/2)) - (64*c*(c*Sin[a + b*x])^(3/2)
)/(1155*b*d^7*(d*Cos[a + b*x])^(3/2))
```


Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}} - \frac{c^2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx}{5d^2}$$
$$= \frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}} - \frac{\left(8c^2\right)\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx}{55d^4}$$
$$= \frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}} - \frac{16c(c\sin(a+bx))^{3/2}}{385bd^5(d\cos(a+bx))^{7/2}} - \frac{\left(32c^2\right)\int}{}$$
$$= \frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}} - \frac{16c(c\sin(a+bx))^{3/2}}{385bd^5(d\cos(a+bx))^{7/2}} - \frac{64c(c\,s}{1155bd^7(}$$

Mathematica [A] time = 0.47729, size = 67, normalized size = 0.48

$$\frac{2(44\cos(2(a+bx))+4\cos(4(a+bx))+117)\sec^8(a+bx)(c\sin(a+bx))^{7/2}\sqrt{d\cos(a+bx)}}{1155bcd^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(17/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(117 + 44*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Se
c[a + b*x]^8*(c*Sin[a + b*x])^(7/2))/(1155*b*c*d^9)
```

Maple [A] time = 0.116, size = 60, normalized size = 0.4

$$\frac{\left(64\,(\cos(bx+a))^4+112\,(\cos(bx+a))^2+154\right)\cos(bx+a)\sin(bx+a)}{1155\,b}(c\sin(bx+a))^{\frac{5}{2}}(d\cos(bx+a))^{-\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x)
```

```
[Out] 2/1155/b*(32*cos(b*x+a)^4+56*cos(b*x+a)^2+77)*(c*sin(b*x+a))^(5/2)*cos(b*x+
a)*sin(b*x+a)/(d*cos(b*x+a))^(17/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(17/2), x)
```

Fricas [A] time = 5.81589, size = 227, normalized size = 1.61

$$-\frac{2\left(32c^2\cos(bx+a)^6+24c^2\cos(bx+a)^4+21c^2\cos(bx+a)^2-77c^2\right)\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\sin(bx+a)}{1155\,bd^9\cos(bx+a)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(32*c^2*cos(b*x + a)^6 + 24*c^2*cos(b*x + a)^4 + 21*c^2*cos(b*x + a
)^2 - 77*c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sin(b*x + a)/(b*d^9
*cos(b*x + a)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(17/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.287 $\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=226

$$\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}b}-\frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\cdots\right)}{2\sqrt{2}b}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - A
rcTan[1 + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - Lo
g[1 + Cot[a + b*x] - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(2*Sq
rt[2]*b) + Log[1 + Cot[a + b*x] + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a +
 b*x]]]/(2*Sqrt[2]*b) - (2*Sqrt[Sin[a + b*x]])/(b*Sqrt[Cos[a + b*x]]) + (2*
Sin[a + b*x]^(5/2))/(5*b*Cos[a + b*x]^(5/2))
```

Rubi [A] time = 0.149362, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2566, 2575, 297, 1162, 617, 204, 1165, 628}

$$\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}b}-\frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\cdots\right)}{2\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(7/2)/Cos[a + b*x]^(7/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - A
rcTan[1 + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - Lo
g[1 + Cot[a + b*x] - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(2*Sq
rt[2]*b) + Log[1 + Cot[a + b*x] + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a +
 b*x]]]/(2*Sqrt[2]*b) - (2*Sqrt[Sin[a + b*x]])/(b*Sqrt[Cos[a + b*x]]) + (2*
Sin[a + b*x]^(5/2))/(5*b*Cos[a + b*x]^(5/2))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} - \int \frac{\sin^{\frac{3}{2}}(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$
$$= -\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} + \int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx$$
$$= -\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}$$
$$= -\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \right.}{b}$$
$$= -\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\right.}{2}$$
$$= -\frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} + \frac{\log\left(1+\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} - \frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}$$
$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} + \cdots$$

Mathematica [C] time = 0.0563914, size = 57, normalized size = 0.25

$$\frac{2\sin^{\frac{9}{2}}(a+bx)\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{9}{4},\frac{9}{4};\frac{13}{4};\sin^2(a+bx)\right)}{9b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(7/2)/Cos[a + b*x]^(7/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[9/4, 9/4, 13/4, Sin[a + b*x]^2]
*Sin[a + b*x]^(9/2))/(9*b*Sqrt[Cos[a + b*x]])
```

Maple [C] time = 0.178, size = 702, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x)
```

```
[Out] -1/10/b*2^(1/2)*(5*I*cos(b*x+a)^2*sin(b*x+a)*EllipticPi((-(-1+cos(b*x+a)-si
n(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(b*x+a)-sin(b*x
+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+c
os(b*x+a))/sin(b*x+a))^(1/2)-5*I*cos(b*x+a)^2*sin(b*x+a)*(-(-1+cos(b*x+a)-s
in(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*
((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/
sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+5*cos(b*x+a)^2*sin(b*x+a)*Elliptic
Pi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-
(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/si
```

```
n(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)+5*cos(b*x+a)^2*sin(b*x+a
)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos
(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-10*cos(b*x+a)^
2*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF
((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+12*cos(b*x+a)^
3*2^(1/2)-12*cos(b*x+a)^2*2^(1/2)-2*cos(b*x+a)*2^(1/2)+2*2^(1/2))*sin(b*x+a
)^(1/2)/(-1+cos(b*x+a))/cos(b*x+a)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^{\frac{7}{2}}}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(7/2)/cos(b*x + a)^(7/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(7/2)/cos(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.288 $\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx$

Optimal. Leaf size=16

$$\frac{2\sin^{\frac{5}{2}}(x)}{5\cos^{\frac{5}{2}}(x)}$$

[Out] (2*Sin[x]^(5/2))/(5*Cos[x]^(5/2))

Rubi [A] time = 0.0226398, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2563}

$$\frac{2\sin^{\frac{5}{2}}(x)}{5\cos^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^(3/2)/Cos[x]^(7/2),x]

[Out] (2*Sin[x]^(5/2))/(5*Cos[x]^(5/2))

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = \frac{2\sin^{\frac{5}{2}}(x)}{5\cos^{\frac{5}{2}}(x)}$$

Mathematica [A] time = 0.0184556, size = 16, normalized size = 1.

$$\frac{2\sin^{\frac{5}{2}}(x)}{5\cos^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^(3/2)/Cos[x]^(7/2),x]

[Out] (2*Sin[x]^(5/2))/(5*Cos[x]^(5/2))

Maple [B] time = 0.048, size = 33, normalized size = 2.1

$$\frac{-\left(\sin \left(x\right)\right)^{2}+\left(\cos \left(x\right)\right)^{2}-2 \cos \left(x\right)+1}{-5+5 \cos \left(x\right)} \left(\sin \left(x\right)\right)^{\frac{5}{2}} \left(\cos \left(x\right)\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^(3/2)/cos(x)^(7/2),x)
```

```
[Out] 1/5*(-sin(x)^2+cos(x)^2-2*cos(x)+1)*sin(x)^(5/2)/(-1+cos(x))/cos(x)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin \left(x\right)^{\frac{3}{2}}}{\cos \left(x\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(3/2)/cos(x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^(3/2)/cos(x)^(7/2), x)
```

Fricas [A] time = 2.69549, size = 63, normalized size = 3.94

$$-\frac{2 \left(\cos \left(x\right)^{2}-1\right) \sqrt{\sin \left(x\right)}}{5 \cos \left(x\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(3/2)/cos(x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(cos(x)^2 - 1)*sqrt(sin(x))/cos(x)^(5/2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**(3/2)/cos(x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin \left(x\right)^{\frac{3}{2}}}{\cos \left(x\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(3/2)/cos(x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(x)^(3/2)/cos(x)^(7/2), x)
```

3.289 $\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx$

Optimal. Leaf size=122

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqrt[2]) + ArcTan[1 + (Sq
rt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqrt[2] + Log[1 - (Sqrt[2]*Sqrt[Sin[x]])/
Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Co
s[x]] + Tan[x]]/(2*Sqrt[2])
```

Rubi [A] time = 0.0816404, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[x]]/Sqrt[Cos[x]],x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqrt[2]) + ArcTan[1 + (Sq
rt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqrt[2] + Log[1 - (Sqrt[2]*Sqrt[Sin[x]])/
Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Co
s[x]] + Tan[x]]/(2*Sqrt[2])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx &= 2\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)\\
&= -\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right) + \mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right) + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right) + \frac{\mathrm{Subst}\big(}{}\\
&= \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{2\sqrt{2}} + \frac{\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}} -\\
&= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}} + \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right.}{2\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.0123281, size = 38, normalized size = 0.31

$$\frac{2\sin^{\frac{3}{2}}(x)\cos^2(x)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\sin^2(x)\right)}{3\cos^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sin[x]]/Sqrt[Cos[x]],x]
```

```
[Out] (2*(Cos[x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, Sin[x]^2]*Sin[x]^(3/2)
)/(3*Cos[x]^(3/2))
```

Maple [C] time = 0.055, size = 171, normalized size = 1.4

$$-\frac{\sqrt{2}}{-2+2\cos(x)}(\sin(x))^{\frac{3}{2}}\left(i\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(x)-\sin(x)}{\sin(x)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(x)-\sin}{\sin(x)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^(1/2)/cos(x)^(1/2),x)
```

```
[Out] -1/2*2^(1/2)*sin(x)^(3/2)*(I*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),
1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2+
1/2*I,1/2*2^(1/2))-EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2-1/2*I,
1/2*2^(1/2))-EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^
(1/2)))*((-1+cos(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*(-(-1+
cos(x)-sin(x))/sin(x))^(1/2)/(-1+cos(x))/cos(x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(1/2)/cos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(x))/sqrt(cos(x)), x)
```

Fricas [B] time = 5.7764, size = 1613, normalized size = 13.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(1/2)/cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*(2*cos(x)^3 - 2*cos(x)^2*sin(x) + sqrt(2)*sqrt(2*(sq
rt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*sin(x)
+ 1)*sqrt(cos(x))*sqrt(sin(x)) - sqrt(2)*sqrt(cos(x))*sqrt(sin(x)) - 2*cos(
x))/(cos(x)^3 + cos(x)^2*sin(x) - cos(x))) + 1/4*sqrt(2)*arctan(-1/2*(2*cos
(x)^3 - 2*cos(x)^2*sin(x) - sqrt(2)*sqrt(-2*(sqrt(2)*cos(x) + sqrt(2)*sin(x
))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*sin(x) + 1)*sqrt(cos(x))*sqrt(sin(x
)) + sqrt(2)*sqrt(cos(x))*sqrt(sin(x)) - 2*cos(x))/(cos(x)^3 + cos(x)^2*sin
(x) - cos(x))) - 1/4*sqrt(2)*arctan(-(sqrt(-2*(sqrt(2)*cos(x) + sqrt(2)*sin
(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*sin(x) + 1)*(sqrt(2)*sqrt(cos(x))
*sqrt(sin(x)) + cos(x) + sin(x)) + sqrt(2)*sqrt(cos(x))*sqrt(sin(x)))/(cos(
x) - sin(x))) - 1/4*sqrt(2)*arctan(-(sqrt(2*(sqrt(2)*cos(x) + sqrt(2)*sin(x
))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*sin(x) + 1)*(sqrt(2)*sqrt(cos(x))*s
qrt(sin(x)) - cos(x) - sin(x)) + sqrt(2)*sqrt(cos(x))*sqrt(sin(x)))/(cos(x)
 - sin(x))) - 1/8*sqrt(2)*log(2*(sqrt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(
x))*sqrt(sin(x)) + 4*cos(x)*sin(x) + 1) + 1/8*sqrt(2)*log(-2*(sqrt(2)*cos(x
) + sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**(1/2)/cos(x)**(1/2),x)
```

```
[Out] Integral(sqrt(sin(x))/sqrt(cos(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(1/2)/cos(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(x))/sqrt(cos(x)), x)
```

3.290 $\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx$

Optimal. Leaf size=143

$$-\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{4\sqrt{2}}+\frac{3\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{8\sqrt{2}}-\frac{3\log\left(\tan(x)\right.}{8}$$

```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]])/(4*Sqrt[2]) + (3*ArcTa
n[1 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]])/(4*Sqrt[2]) + (3*Log[1 - (Sqrt[
2]*Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]])/(8*Sqrt[2]) - (3*Log[1 + (Sqrt[2]*
Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]])/(8*Sqrt[2]) - (Sqrt[Cos[x]]*Sin[x]^(3
/2))/2
```

Rubi [A] time = 0.113751, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2568, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{4\sqrt{2}}+\frac{3\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{8\sqrt{2}}-\frac{3\log\left(\tan(x)\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^(5/2)/Sqrt[Cos[x]],x]
```

```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]])/(4*Sqrt[2]) + (3*ArcTa
n[1 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]])/(4*Sqrt[2]) + (3*Log[1 - (Sqrt[
2]*Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]])/(8*Sqrt[2]) - (3*Log[1 + (Sqrt[2]*
Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]])/(8*Sqrt[2]) - (Sqrt[Cos[x]]*Sin[x]^(3
/2))/2
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx &= -\frac{1}{2}\sqrt{\cos(x)}\sin^{\frac{3}{2}}(x) + \frac{3}{4}\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx\\
&= -\frac{1}{2}\sqrt{\cos(x)}\sin^{\frac{3}{2}}(x) + \frac{3}{2}\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)\\
&= -\frac{1}{2}\sqrt{\cos(x)}\sin^{\frac{3}{2}}(x) - \frac{3}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right) + \frac{3}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)\\
&= -\frac{1}{2}\sqrt{\cos(x)}\sin^{\frac{3}{2}}(x) + \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right) + \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x\right.\\
&= \frac{3\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{8\sqrt{2}} - \frac{3\log\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{8\sqrt{2}} - \frac{1}{2}\sqrt{\cos(x)}\sin^{\frac{3}{2}}(x) + \frac{3\,\text{Subst}\left(\int -\right.}{}\\
&= -\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}} + \frac{3\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{8\sqrt{2}} - \frac{3\log\left(1+\frac{\sqrt{2}\right.}{8}
\end{aligned}$$

Mathematica [C] time = 0.0110258, size = 38, normalized size = 0.27

$$\frac{2\sin^{\frac{7}{2}}(x)\cos^2(x)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\sin^2(x)\right)}{7\cos^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^(5/2)/Sqrt[Cos[x]],x]
```

```
[Out] (2*(Cos[x]^2)^(3/4)*Hypergeometric2F1[3/4, 7/4, 11/4, Sin[x]^2]*Sin[x]^(7/2
))/(7*Cos[x]^(3/2))
```

Maple [C] time = 0.171, size = 2667, normalized size = 18.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^(5/2)/cos(x)^(1/2),x)
```

```
[Out] -1/32*2^(1/2)*sin(x)^(3/2)*(12*cos(x)*sin(x)^2*(-(-1+cos(x)-sin(x))/sin(x))
^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*Ellipti
cPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))+4*2^(1/2)-3*(
-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+co
s(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2-1/2*I
,1/2*2^(1/2))-3*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(
x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))-16*cos(x)^3*2^(1/2)-4*sin(x)^2*2^(1/2)+4*cos
(x)^4*2^(1/2)+24*cos(x)^2*2^(1/2)-16*cos(x)*2^(1/2)+8*cos(x)*sin(x)^2*2^(1/
2)-4*cos(x)^2*sin(x)^2*2^(1/2)+12*cos(x)*sin(x)^2*(-(-1+cos(x)-sin(x))/sin(
x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*Elli
pticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*sin(x)
^4*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-
1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2-1
/2*I,1/2*2^(1/2))-3*I*sin(x)^4*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(
x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)
-sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*cos(x)^4*(-(-1+cos(x)-sin
(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(
1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3
*I*cos(x)^4*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^
(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1
/2),1/2+1/2*I,1/2*2^(1/2))+6*I*sin(x)^2*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*
((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(
-1+cos(x)-sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-6*I*sin(x)^2*(-(-1+c
os(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/
sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2
^(1/2))-12*I*cos(x)^3*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x)
)/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/
sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))+12*I*cos(x)^3*(-(-1+cos(x)-sin(x))/sin
(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*Ell
ipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+18*I*cos(
x)^2*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*(
(-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2
-1/2*I,1/2*2^(1/2))-18*I*cos(x)^2*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+c
os(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos
(x)-sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))-12*I*cos(x)*(-(-1+cos(x)-s
in(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))
^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))
+12*I*cos(x)*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))
^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(
1/2),1/2+1/2*I,1/2*2^(1/2))-6*cos(x)^2*sin(x)^2*(-(-1+cos(x)-sin(x))/sin(x)
)^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*Ellipt
```

```
icPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-6*cos(x)^2*s
in(x)^2*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2
)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),
1/2+1/2*I,1/2*2^(1/2))+6*I*cos(x)^2*sin(x)^2*(-(-1+cos(x)-sin(x))/sin(x))^(
1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticP
i((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-6*I*cos(x)^2*si
n(x)^2*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)
*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1
/2+1/2*I,1/2*2^(1/2))-12*I*cos(x)*sin(x)^2*(-(-1+cos(x)-sin(x))/sin(x))^(1/
2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(
(-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))+12*I*cos(x)*sin(x
)^2*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((
-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2+
1/2*I,1/2*2^(1/2))-3*sin(x)^4*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x
)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-
sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*sin(x)^4*(-(-1+cos(x)-sin(x)
)/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2
)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*co
s(x)^4*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)
*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1
/2-1/2*I,1/2*2^(1/2))-3*cos(x)^4*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+co
s(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(
x)-sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*sin(x)^2*(-(-1+cos(x)-sin
(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(
1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-6
*sin(x)^2*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1
/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2
),1/2+1/2*I,1/2*2^(1/2))+12*cos(x)^3*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-
1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+
cos(x)-sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))+12*cos(x)^3*(-(-1+cos(x
)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(
x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/
2))-18*cos(x)^2*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(
x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))-18*cos(x)^2*(-(-1+cos(x)-sin(x))/sin(x))^(1/
2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(
(-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+12*cos(x)*(-(-1+c
os(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/
sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2
^(1/2))+12*cos(x)*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/si
n(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+cos(x)-sin(x))/sin(
x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-
1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi((-(-1+
cos(x)-sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*(-(-1+cos(x)-sin(x)
)/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2
)*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2)))/(-1
+cos(x))^3/cos(x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x)^{\frac{5}{2}}}{\sqrt{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(5/2)/cos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^(5/2)/sqrt(cos(x)), x)
```

Fricas [B] time = 5.43189, size = 1666, normalized size = 11.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(5/2)/cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(cos(x))*sin(x)^(3/2) + 3/16*sqrt(2)*arctan(1/2*(2*cos(x)^3 - 2*co
s(x)^2*sin(x) + sqrt(2)*sqrt(2*(sqrt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x
))*sqrt(sin(x)) + 4*cos(x)*sin(x) + 1)*sqrt(cos(x))*sqrt(sin(x)) - sqrt(2)*
sqrt(cos(x))*sqrt(sin(x)) - 2*cos(x))/(cos(x)^3 + cos(x)^2*sin(x) - cos(x))
) + 3/16*sqrt(2)*arctan(-1/2*(2*cos(x)^3 - 2*cos(x)^2*sin(x) - sqrt(2)*sqrt
(-2*(sqrt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*
sin(x) + 1)*sqrt(cos(x))*sqrt(sin(x)) + sqrt(2)*sqrt(cos(x))*sqrt(sin(x)) -
 2*cos(x))/(cos(x)^3 + cos(x)^2*sin(x) - cos(x))) - 3/16*sqrt(2)*arctan(-(s
qrt(-2*(sqrt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(
x)*sin(x) + 1)*(sqrt(2)*sqrt(cos(x))*sqrt(sin(x)) + cos(x) + sin(x)) + sqrt
(2)*sqrt(cos(x))*sqrt(sin(x)))/(cos(x) - sin(x))) - 3/16*sqrt(2)*arctan(-(s
qrt(2*(sqrt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x
)*sin(x) + 1)*(sqrt(2)*sqrt(cos(x))*sqrt(sin(x)) - cos(x) - sin(x)) + sqrt(
2)*sqrt(cos(x))*sqrt(sin(x)))/(cos(x) - sin(x))) - 3/32*sqrt(2)*log(2*(sqrt
(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*sin(x) +
1) + 3/32*sqrt(2)*log(-2*(sqrt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x))*sqr
t(sin(x)) + 4*cos(x)*sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**(5/2)/cos(x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x)^{\frac{5}{2}}}{\sqrt{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(5/2)/cos(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(x)^(5/2)/sqrt(cos(x)), x)
```

3.291 $\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=132

$$\frac{5d^3\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{6bc}+\frac{5d^4\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{12b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bc}$$

```
[Out] (5*d^3*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(6*b*c) + (d*(d*Cos[a + b
*x])^(5/2)*Sqrt[c*Sin[a + b*x]])/(3*b*c) + (5*d^4*EllipticF[a - Pi/4 + b*x,
 2]*Sqrt[Sin[2*a + 2*b*x]])/(12*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]]
)
```

Rubi [A] time = 0.178004, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2569, 2573, 2641}

$$\frac{5d^3\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{6bc}+\frac{5d^4\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{12b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(7/2)/Sqrt[c*Sin[a + b*x]],x]
```


```
[Out] (5*d^3*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(6*b*c) + (d*(d*Cos[a + b
*x])^(5/2)*Sqrt[c*Sin[a + b*x]])/(3*b*c) + (5*d^4*EllipticF[a - Pi/4 + b*x,
 2]*Sqrt[Sin[2*a + 2*b*x]])/(12*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]]
)
```


Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{d(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bc} + \frac{1}{6}\left(5d^2\right)\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx$$
$$= \frac{5d^3\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6bc} + \frac{d(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bc} + \frac{1}{12}\left(5d^4\right)\int \frac{}{\sqrt{d\cos(a}}$$
$$= \frac{5d^3\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6bc} + \frac{d(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bc} + \frac{\left(5d^4\sqrt{\sin(2a+2bx)}\right)}{12\sqrt{d\cos(a+bx)}}$$
$$= \frac{5d^3\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6bc} + \frac{d(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bc} + \frac{5d^4F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{12b\sqrt{d\cos(a+bx)}}$$

Mathematica [C] time = 0.111582, size = 70, normalized size = 0.53

$$\frac{2\cos^2(a+bx)^{3/4}\sec^5(a+bx)\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{7/2}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};\frac{5}{4};\sin^2(a+bx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*(d*Cos[a + b*x])^(7/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, 1/
4, 5/4, Sin[a + b*x]^2]*Sec[a + b*x]^5*Sqrt[c*Sin[a + b*x]])/(b*c)
```

Maple [A] time = 0.112, size = 216, normalized size = 1.6

$$-\frac{\sqrt{2}\sin(bx+a)}{12\,b\,(-1+\cos(bx+a))\,(\cos(bx+a))^4}\left(5\sin(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+}{\sin(bx+a)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/12/b*2^(1/2)*(5*sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*
((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^
(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2
*cos(b*x+a)^4*2^(1/2)+2*cos(b*x+a)^3*2^(1/2)-5*cos(b*x+a)^2*2^(1/2)+5*cos(b
*x+a)*2^(1/2))*(d*cos(b*x+a))^(7/2)*sin(b*x+a)/(-1+cos(b*x+a))/cos(b*x+a)^4
/(c*sin(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\cos(bx+a))^{\frac{7}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)/sqrt(c*sin(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}d^3\cos(bx+a)^3}{c\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d^3*cos(b*x + a)^3/(c*si
n(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(7/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\cos(bx+a))^{\frac{7}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)/sqrt(c*sin(b*x + a)), x)
```

3.292 $\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=92

$$\frac{d^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}$$

```
[Out] (d*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(b*c) + (d^2*EllipticF[a - Pi
/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a
 + b*x]])
```

Rubi [A] time = 0.112895, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2569, 2573, 2641}

$$\frac{d^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (d*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(b*c) + (d^2*EllipticF[a - Pi
/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a
 + b*x]])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{d\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bc} + \frac{1}{2}d^2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx$$
$$= \frac{d\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bc} + \frac{\left(d^2\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$
$$= \frac{d\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bc} + \frac{d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

Mathematica [C] time = 0.107491, size = 68, normalized size = 0.74

$$\frac{2d^2\cos^2(a+bx)^{3/4}\tan(a+bx)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};\sin^2(a+bx)\right)}{b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*d^2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, Sin[a + b*x
]^2]*Tan[a + b*x])/(b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])
```

Maple [A] time = 0.099, size = 188, normalized size = 2.

$$-\frac{\sqrt{2}\sin(bx+a)}{2b(-1+\cos(bx+a))(\cos(bx+a))^2}\left(\sin(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+}{\sin(bx+a)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/2/b*2^(1/2)*(sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-
1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/
2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(
b*x+a)^2*2^(1/2)+cos(b*x+a)*2^(1/2))*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/(-1+co
s(b*x+a))/cos(b*x+a)^2/(c*sin(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\cos(bx+a))^{\frac{3}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)/sqrt(c*sin(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}d\cos(bx+a)}{c\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d*cos(b*x + a)/(c*sin(b*
x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\cos(bx+a))^{\frac{3}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)/sqrt(c*sin(b*x + a)), x)
```

3.293 $\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}$$

```
[Out] (EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(b*Sqrt[d*Cos[a + b*x
]]*Sqrt[c*Sin[a + b*x]])
```

Rubi [A] time = 0.0554808, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2573, 2641}

$$\frac{\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(b*Sqrt[d*Cos[a + b*x
]]*Sqrt[c*Sin[a + b*x]])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx &= \frac{\sqrt{\sin(2a+2bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\\ &= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}}{b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\end{aligned}$$

Mathematica [C] time = 0.0573399, size = 65, normalized size = 1.23

$$\frac{2\cos^2(a+bx)^{3/4}\tan(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\sin^2(a+bx)\right)}{b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, Sin[a + b*x]^2]*
Tan[a + b*x])/(b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])
```

Maple [B] time = 0.089, size = 151, normalized size = 2.9

$$-\frac{\sqrt{2}\left(\sin\left(bx+a\right)\right)^{2}}{b\left(-1+\cos\left(bx+a\right)\right)}\sqrt{\frac{1-\cos\left(bx+a\right)+\sin\left(bx+a\right)}{\sin\left(bx+a\right)}}\sqrt{\frac{-1+\cos\left(bx+a\right)+\sin\left(bx+a\right)}{\sin\left(bx+a\right)}}\sqrt{\frac{-1+\cos\left(bx+a\right)}{\sin\left(bx+a\right)}}\mathrm{EllipticF}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/b*2^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(
1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)^2/(c*si
n(b*x+a))^(1/2)/(-1+cos(b*x+a))/(d*cos(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}{cd\cos\left(bx+a\right)\sin\left(bx+a\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(c*d*cos(b*x + a)*sin(b*
x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\sin\left(a+bx\right)}\sqrt{d\cos\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))**(1/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*sin(a + b*x))*sqrt(d*cos(a + b*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))), x)
```

3.294 $\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=97

$$\frac{2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3bd^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}$$

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(3*b*c*d*(d*Cos[a + b*x])^(3/2)) + (2*EllipticF[a
- Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]*Sqrt
[c*Sin[a + b*x]])
```

Rubi [A] time = 0.113712, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2571, 2573, 2641}

$$\frac{2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3bd^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(3*b*c*d*(d*Cos[a + b*x])^(3/2)) + (2*EllipticF[a
- Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]*Sqrt
[c*Sin[a + b*x]])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}} + \frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx}{3d^2}$$
$$= \frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}} + \frac{\left(2\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{3d^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$
$$= \frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}} + \frac{2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

Mathematica [C] time = 0.106237, size = 65, normalized size = 0.67

$$\frac{2\cos^2(a+bx)^{3/4}\sqrt{c\sin(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{5}{4};\sin^2(a+bx)\right)}{bcd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/4, 7/4, 5/4, Sin[a + b*x]^2]*
Sqrt[c*Sin[a + b*x]])/(b*c*d*(d*Cos[a + b*x])^(3/2))
```

Maple [A] time = 0.112, size = 184, normalized size = 1.9

$$-\frac{\sqrt{2}\sin(bx+a)\cos(bx+a)}{3\,b\,(-1+\cos(bx+a))}\left(2\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},1/2\,\sqrt{2}\right)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/3/b*2^(1/2)*(2*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/
2*2^(1/2))*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)*co
s(b*x+a)-cos(b*x+a)*2^(1/2)+2^(1/2))*sin(b*x+a)*cos(b*x+a)/(-1+cos(b*x+a))/
(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\cos(bx+a))^{\frac{5}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}{cd^{3}\cos\left(bx+a\right)^{3}\sin\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(c*d^3*cos(b*x + a)^3*si
n(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))**(5/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}\sqrt{c\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a))), x)
```

3.295 $\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=134

$$\frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}}+\frac{4\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{7bd^4\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}$$

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(7*b*c*d*(d*Cos[a + b*x])^(7/2)) + (4*Sqrt[c*Sin[a
 + b*x]])/(7*b*c*d^3*(d*Cos[a + b*x])^(3/2)) + (4*EllipticF[a - Pi/4 + b*x,
 2]*Sqrt[Sin[2*a + 2*b*x]])/(7*b*d^4*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*
x]])
```

Rubi [A] time = 0.172269, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2571, 2573, 2641}

$$\frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}}+\frac{4\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{7bd^4\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]]),x]
```


```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(7*b*c*d*(d*Cos[a + b*x])^(7/2)) + (4*Sqrt[c*Sin[a
 + b*x]])/(7*b*c*d^3*(d*Cos[a + b*x])^(3/2)) + (4*EllipticF[a - Pi/4 + b*x,
 2]*Sqrt[Sin[2*a + 2*b*x]])/(7*b*d^4*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*
x]])
```


Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}dx = \frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{6\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}dx}{7d^2}$$
$$= \frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}} + \frac{4\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx}{7d^4}$$
$$= \frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}} + \frac{\left(4\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a}}}{7d^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(}}$$
$$= \frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}} + \frac{4F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a}}{7bd^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(}}$$

Mathematica [C] time = 0.132659, size = 70, normalized size = 0.52

$$\frac{2\cos^3(a+bx)\cos^2(a+bx)^{3/4}\sqrt{c\sin(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{11}{4};\frac{5}{4};\sin^2(a+bx)\right)}{bc(d\cos(a+bx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Cos[a + b*x]^3*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/4, 11/4, 5/4,
Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]])/(b*c*(d*Cos[a + b*x])^(9/2))
```

Maple [A] time = 0.125, size = 212, normalized size = 1.6

$$-\frac{\sqrt{2}\sin(bx+a)\cos(bx+a)}{7b(-1+\cos(bx+a))}\left(4\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/7/b*2^(1/2)*(4*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellipt
icF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*sin(b*x+a)*co
s(b*x+a)^3-2*cos(b*x+a)^3*2^(1/2)+2*cos(b*x+a)^2*2^(1/2)-cos(b*x+a)*2^(1/2)
+2^(1/2))*sin(b*x+a)*cos(b*x+a)/(-1+cos(b*x+a))/(d*cos(b*x+a))^(9/2)/(c*sin
(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\cos(bx+a))^{\frac{9}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*cos(b*x + a))^(9/2)*sqrt(c*sin(b*x + a))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{cd^5\cos(bx+a)^5\sin(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(c*d^5*cos(b*x + a)^5*si
n(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))**(9/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\cos(bx+a))^{\frac{9}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*cos(b*x + a))^(9/2)*sqrt(c*sin(b*x + a))), x)
```

3.296 $\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=280

$$\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}b\sqrt{c}}-\frac{\sqrt{d}\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+bx)+\sqrt{d}\right)}{2\sqrt{2}b\sqrt{c}}+$$

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*
Sin[a + b*x]])])/(Sqrt[2]*b*Sqrt[c]) - (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[c]
*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*Sin[a + b*x]])])/(Sqrt[2]*b*Sqrt[c])
 - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] - (Sqrt[2]*Sqrt[c]*Sqrt[d*Co
s[a + b*x]])/Sqrt[c*Sin[a + b*x]]])/(2*Sqrt[2]*b*Sqrt[c]) + (Sqrt[d]*Log[Sq
rt[d] + Sqrt[d]*Cot[a + b*x] + (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/Sqrt[
c*Sin[a + b*x]]])/(2*Sqrt[2]*b*Sqrt[c])
```

Rubi [A] time = 0.176786, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2575, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}b\sqrt{c}}-\frac{\sqrt{d}\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+bx)+\sqrt{d}\right)}{2\sqrt{2}b\sqrt{c}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]/Sqrt[c*Sin[a + b*x]],x]
```


```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*
Sin[a + b*x]])])/(Sqrt[2]*b*Sqrt[c]) - (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[c]
*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*Sin[a + b*x]])])/(Sqrt[2]*b*Sqrt[c])
 - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] - (Sqrt[2]*Sqrt[c]*Sqrt[d*Co
s[a + b*x]])/Sqrt[c*Sin[a + b*x]]])/(2*Sqrt[2]*b*Sqrt[c]) + (Sqrt[d]*Log[Sq
rt[d] + Sqrt[d]*Cot[a + b*x] + (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/Sqrt[
c*Sin[a + b*x]]])/(2*Sqrt[2]*b*Sqrt[c])
```


Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = -\frac{(2cd)\,\mathrm{Subst}\left(\int \frac{x^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{b}$$

$$= \frac{d\,\mathrm{Subst}\left(\int \frac{d-cx^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{b} - \frac{d\,\mathrm{Subst}\left(\int \frac{d+cx^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{b}$$

$$= -\frac{\sqrt{d}\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{c}}+2x}{-\frac{d}{c}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}-x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{c}}-2x}{-\frac{d}{c}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}-x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}b\sqrt{c}}$$

$$= -\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}b\sqrt{c}} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cdots}}{\sqrt{c\sin\cdots}}\right.}{2\sqrt{2}b\sqrt{c}}$$

$$= \frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a\cdots\right.}{2\sqrt{2}\cdots}$$

Mathematica [C] time = 0.0609574, size = 65, normalized size = 0.23

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\tan(a+bx)\sqrt{d\cos(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\sin^2(a+bx)\right)}{b\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/4,
5/4, Sin[a + b*x]^2]*Tan[a + b*x])/(b*Sqrt[c*Sin[a + b*x]])
```

Maple [C] time = 0.096, size = 312, normalized size = 1.1

$$-\frac{\sqrt{2}\left(\sin\left(bx+a\right)\right)^{2}}{2\,b\cos\left(bx+a\right)\left(-1+\cos\left(bx+a\right)\right)}\sqrt{d\cos\left(bx+a\right)}\sqrt{\frac{1-\cos\left(bx+a\right)+\sin\left(bx+a\right)}{\sin\left(bx+a\right)}}\sqrt{\frac{-1+\cos\left(bx+a\right)+\sin\left(bx+a\right)}{\sin\left(bx+a\right)}}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/2/b*2^(1/2)*(d*cos(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*
x+a))^(1/2)*(I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-
1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2
),1/2+1/2*I,1/2*2^(1/2))+EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+
a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin
(b*x+a))^(1/2),1/2*2^(1/2)))*sin(b*x+a)^2/(c*sin(b*x+a))^(1/2)/cos(b*x+a)/(
-1+cos(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\cos\left(bx+a\right)}}{\sqrt{c\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))/sqrt(c*sin(b*x + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\cos\left(a+bx\right)}}{\sqrt{c\sin\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(d*cos(a + b*x))/sqrt(c*sin(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\cos(bx+a)}}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))/sqrt(c*sin(b*x + a)), x)
```

3.297 $\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=35

$$\frac{2\sqrt{c\sin(a+bx)}}{bcd\sqrt{d\cos(a+bx)}}$$

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(b*c*d*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.0539481, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2563}

$$\frac{2\sqrt{c\sin(a+bx)}}{bcd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(b*c*d*Sqrt[d*Cos[a + b*x]])
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx=\frac{2\sqrt{c\sin(a+bx)}}{bcd\sqrt{d\cos(a+bx)}}$$

Mathematica [A] time = 0.0621568, size = 36, normalized size = 1.03

$$\frac{\sin(2(a+bx))}{b\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] Sin[2*(a + b*x)]/(b*(d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]])
```

Maple [A] time = 0.069, size = 38, normalized size = 1.1

$$2\,\frac{\cos(bx+a)\sin(bx+a)}{b\left(d\cos(bx+a)\right)^{3/2}\sqrt{c\sin(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] 2/b*sin(b*x+a)*cos(b*x+a)/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\cos(bx+a))^{\frac{3}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*cos(b*x + a))^(3/2)*sqrt(c*sin(b*x + a))), x)
```

Fricas [A] time = 2.61902, size = 92, normalized size = 2.63

$$\frac{2\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{bcd^2\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(b*c*d^2*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))**(3/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\cos(bx+a))^{\frac{3}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*cos(b*x + a))^(3/2)*sqrt(c*sin(b*x + a))), x)
```

3.298 $\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=75

$$\frac{8\sqrt{c\sin(a+bx)}}{5bcd^3\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}$$

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(5*b*c*d*(d*Cos[a + b*x])^(5/2)) + (8*Sqrt[c*Sin[a
 + b*x]])/(5*b*c*d^3*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.11123, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2571, 2563}

$$\frac{8\sqrt{c\sin(a+bx)}}{5bcd^3\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Cos[a + b*x])^(7/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(5*b*c*d*(d*Cos[a + b*x])^(5/2)) + (8*Sqrt[c*Sin[a
 + b*x]])/(5*b*c*d^3*Sqrt[d*Cos[a + b*x]])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx &= \frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}+\frac{4\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx}{5d^2}\\ &= \frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}+\frac{8\sqrt{c\sin(a+bx)}}{5bcd^3\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.172872, size = 52, normalized size = 0.69

$$\frac{2(2\cos(2(a+bx))+3)\tan(a+bx)}{5bd^2\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Cos[a + b*x])^(7/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*(3 + 2*Cos[2*(a + b*x)])*Tan[a + b*x])/(5*b*d^2*(d*Cos[a + b*x])^(3/2)*S
qrt[c*Sin[a + b*x]])
```

Maple [A] time = 0.079, size = 50, normalized size = 0.7

$$\frac{\left(8\,\left(\cos\left(bx+a\right)\right)^{2}+2\right)\sin\left(bx+a\right)\cos\left(bx+a\right)}{5\,b}\left(d\cos\left(bx+a\right)\right)^{-\frac{7}{2}}\frac{1}{\sqrt{c\sin\left(bx+a\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] 2/5/b*(4*cos(b*x+a)^2+1)*sin(b*x+a)*cos(b*x+a)/(d*cos(b*x+a))^(7/2)/(c*sin(
b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}\sqrt{c\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*cos(b*x + a))^(7/2)*sqrt(c*sin(b*x + a))), x)
```

Fricas [A] time = 2.68717, size = 128, normalized size = 1.71

$$\frac{2\,\sqrt{d\cos\left(bx+a\right)}\left(4\,\cos\left(bx+a\right)^{2}+1\right)\sqrt{c\sin\left(bx+a\right)}}{5\,bcd^{4}\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/5*sqrt(d*cos(b*x + a))*(4*cos(b*x + a)^2 + 1)*sqrt(c*sin(b*x + a))/(b*c*d
^4*cos(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))**(7/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}\sqrt{c\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*cos(b*x + a))^(7/2)*sqrt(c*sin(b*x + a))), x)
```

3.299 $\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=112

$$\frac{64\sqrt{c\sin(a+bx)}}{45bcd^5\sqrt{d\cos(a+bx)}}+\frac{16\sqrt{c\sin(a+bx)}}{45bcd^3(d\cos(a+bx))^{5/2}}+\frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}$$

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(9*b*c*d*(d*Cos[a + b*x])^(9/2)) + (16*Sqrt[c*Sin[
a + b*x]])/(45*b*c*d^3*(d*Cos[a + b*x])^(5/2)) + (64*Sqrt[c*Sin[a + b*x]])/
(45*b*c*d^5*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.168754, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2571, 2563}

$$\frac{64\sqrt{c\sin(a+bx)}}{45bcd^5\sqrt{d\cos(a+bx)}}+\frac{16\sqrt{c\sin(a+bx)}}{45bcd^3(d\cos(a+bx))^{5/2}}+\frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Cos[a + b*x])^(11/2)*Sqrt[c*Sin[a + b*x]]),x]
```


```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(9*b*c*d*(d*Cos[a + b*x])^(9/2)) + (16*Sqrt[c*Sin[
a + b*x]])/(45*b*c*d^3*(d*Cos[a + b*x])^(5/2)) + (64*Sqrt[c*Sin[a + b*x]])/
(45*b*c*d^5*Sqrt[d*Cos[a + b*x]])
```


Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx &= \frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}+\frac{8\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx}{9d^2}\\ &= \frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}+\frac{16\sqrt{c\sin(a+bx)}}{45bcd^3(d\cos(a+bx))^{5/2}}+\frac{32\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin}}}{45d^4}\\ &= \frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}+\frac{16\sqrt{c\sin(a+bx)}}{45bcd^3(d\cos(a+bx))^{5/2}}+\frac{64\sqrt{c\sin(a+bx)}}{45bcd^5\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.244228, size = 67, normalized size = 0.6

$$\frac{2(20\cos(2(a+bx))+4\cos(4(a+bx))+21)\sec^5(a+bx)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{45bcd^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Cos[a + b*x])^(11/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(21 + 20*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Sec
[a + b*x]^5*Sqrt[c*Sin[a + b*x]])/(45*b*c*d^6)
```

Maple [A] time = 0.088, size = 60, normalized size = 0.5

$$\frac{\left(64\,(\cos(bx+a))^4+16\,(\cos(bx+a))^2+10\right)\sin(bx+a)\cos(bx+a)}{45\,b}(d\cos(bx+a))^{-\frac{11}{2}}\frac{1}{\sqrt{c\sin(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] 2/45/b*(32*cos(b*x+a)^4+8*cos(b*x+a)^2+5)*sin(b*x+a)*cos(b*x+a)/(d*cos(b*x+
a))^(11/2)/(c*sin(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(d\cos(bx+a))^{\frac{11}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/((d*cos(b*x + a))^(11/2)*sqrt(c*sin(b*x + a))), x)
```

Fricas [A] time = 3.36594, size = 157, normalized size = 1.4

$$\frac{2\left(32\,\cos(bx+a)^4+8\,\cos(bx+a)^2+5\right)\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{45\,bcd^6\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas
")
```

```
[Out] 2/45*(32*cos(b*x + a)^4 + 8*cos(b*x + a)^2 + 5)*sqrt(d*cos(b*x + a))*sqrt(c
*sin(b*x + a))/(b*c*d^6*cos(b*x + a)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))**(11/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\cos(bx+a))^{\frac{11}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*cos(b*x + a))^(11/2)*sqrt(c*sin(b*x + a))), x)
```

3.300 $\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}} dx$

Optimal. Leaf size=174

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}b}-\frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b}+\frac{\log\left(\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - A
rcTan[1 + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - Lo
g[1 + Cot[a + b*x] - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(2*Sq
rt[2]*b) + Log[1 + Cot[a + b*x] + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a +
 b*x]]]/(2*Sqrt[2]*b)
```

Rubi [A] time = 0.0889395, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2575, 297, 1162, 617, 204, 1165, 628}

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}b}-\frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b}+\frac{\log\left(\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[a + b*x]]/Sqrt[Sin[a + b*x]],x]
```


```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - A
rcTan[1 + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - Lo
g[1 + Cot[a + b*x] - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(2*Sq
rt[2]*b) + Log[1 + Cot[a + b*x] + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a +
 b*x]]]/(2*Sqrt[2]*b)
```


Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+}{-1-\sqrt{2}}\right.}{}\\
&= -\frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} + \frac{\log\left(1+\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\right.}{}\\
&= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} + \frac{l}{}
\end{aligned}
$$

Mathematica [C] time = 0.025568, size = 55, normalized size = 0.32

$$
\frac{2\sqrt{\sin(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\sin^2(a+bx)\right)}{b\sqrt{\cos(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[a + b*x]]/Sqrt[Sin[a + b*x]],x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, Sin[a + b*x]^2]*
Sqrt[Sin[a + b*x]])/(b*Sqrt[Cos[a + b*x]])
```

Maple [C] time = 0.086, size = 298, normalized size = 1.7

$$-\frac{\sqrt{2}}{2\,b\,(-1+\cos(bx+a))}\sqrt{-\frac{-1+\cos(bx+a)-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\Big(i\mathrm{Ell}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x)
```

```
[Out] -1/2/b*2^(1/2)/cos(b*x+a)^(1/2)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1
/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+
a))^(1/2)*(I*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-
1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1
/2),1/2+1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+
a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/si
n(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*EllipticF((-(-1+cos(b*x+a)-sin(b*x
+a))/sin(b*x+a))^(1/2),1/2*2^(1/2)))*sin(b*x+a)^(3/2)/(-1+cos(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(bx+a)}}{\sqrt{\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(b*x + a))/sqrt(sin(b*x + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(1/2)/sin(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(cos(a + b*x))/sqrt(sin(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(bx+a)}}{\sqrt{\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(b*x + a))/sqrt(sin(b*x + a)), x)
```

3.301 $\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=199

$$-\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}b}-\frac{\log\left(\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{2\sqrt{2}b}+\frac{\log\left(\tan\right.}{}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b) - A
rcTan[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b) - Lo
g[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan[a + b*x]]/(2*Sq
rt[2]*b) + Log[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan[a
+ b*x]]/(2*Sqrt[2]*b) - (2*Sqrt[Cos[a + b*x]])/(b*Sqrt[Sin[a + b*x]])
```

Rubi [A] time = 0.11384, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2567, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}b}-\frac{\log\left(\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{2\sqrt{2}b}+\frac{\log\left(\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(3/2)/Sin[a + b*x]^(3/2),x]
```


```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b) - A
rcTan[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b) - Lo
g[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan[a + b*x]]/(2*Sq
rt[2]*b) + Log[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan[a
+ b*x]]/(2*Sqrt[2]*b) - (2*Sqrt[Cos[a + b*x]])/(b*Sqrt[Sin[a + b*x]])
```


Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} - \int \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\,dx$$

$$= -\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b}$$

$$= -\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b}$$

$$= -\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{2b} \cdots$$

$$= -\frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} + \frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} - \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} \cdots$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} - \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} + \cdots$$

Mathematica [C] time = 0.0352818, size = 55, normalized size = 0.28

$$-\frac{2\cos^2(a+bx)^{3/4}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\sin^2(a+bx)\right)}{b\sqrt{\sin(a+bx)}\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(3/2)/Sin[a + b*x]^(3/2),x]
```

```
[Out] (-2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, Sin[a + b*x]^
2])/(b*Cos[a + b*x]^(3/2)*Sqrt[Sin[a + b*x]])
```

Maple [C] time = 0.079, size = 953, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x)
```

```
[Out] -1/2/b*2^(1/2)*(I*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b
*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elli
pticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*cos(b*x+a)-I*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellipti
cPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*c
os(b*x+a)+I*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi
((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*(-
(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x
+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-cos(b*x+a)*(-(-1+c
os(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x
+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-s
in(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-cos(b*x+a)*(-(-1+cos(b*
x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*
x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-(-(-1+cos(b*x+a)-sin(b*x+a))
/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b
*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))-(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/
2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a
))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*
I,1/2*2^(1/2))+2*cos(b*x+a)*2^(1/2))/sin(b*x+a)^(1/2)/cos(b*x+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^{\frac{3}{2}}}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(3/2)/sin(b*x + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(3/2)/sin(b*x+a)**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)**(3/2)/sin(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^{\frac{3}{2}}}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(3/2)/sin(b*x + a)^(3/2), x)
```

3.302 $\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=201

$$-\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}b}+\frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b}-\frac{\log\left(\mathrm{co}\right.}{}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b))
+ ArcTan[1 + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) +
 Log[1 + Cot[a + b*x] - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(2
*Sqrt[2]*b) - Log[1 + Cot[a + b*x] + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[
a + b*x]]]/(2*Sqrt[2]*b) - (2*Cos[a + b*x]^(3/2))/(3*b*Sin[a + b*x]^(3/2))
```

Rubi [A] time = 0.115262, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2567, 2575, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}b}+\frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b}-\frac{\log\left(\mathrm{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(5/2)/Sin[a + b*x]^(5/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b))
+ ArcTan[1 + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) +
 Log[1 + Cot[a + b*x] - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(2
*Sqrt[2]*b) - Log[1 + Cot[a + b*x] + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[
a + b*x]]]/(2*Sqrt[2]*b) - (2*Cos[a + b*x]^(3/2))/(3*b*Sin[a + b*x]^(3/2))
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx &= -\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)} - \int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx \\
&= -\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)} + \frac{2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b} \\
&= -\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b} \\
&= -\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2b} \\
&= \frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} - \frac{\log\left(1+\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} - \frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)} \\
&= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} + \frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} -
\end{aligned}$$

Mathematica [C] time = 0.034081, size = 57, normalized size = 0.28

$$-\frac{2\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};\sin^2(a+bx)\right)}{3b\sin^{\frac{3}{2}}(a+bx)\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(5/2)/Sin[a + b*x]^(5/2),x]
```

```
[Out] (-2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, Sin[a + b*x]^
2])/(3*b*Sqrt[Cos[a + b*x]]*Sin[a + b*x]^(3/2))
```

Maple [C] time = 0.095, size = 1281, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x)
```

```
[Out] -4/3/b*2^(1/2)*cos(b*x+a)^(5/2)*(-1+cos(b*x+a))^3*(3*I*(-(-1+cos(b*x+a)-sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/si
n(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(b*x+a)*cos(b*x+a)-3*I*(-(-1+cos(
b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(
b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(b*x+a)*cos(b*x+a)+3*I*
(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b
*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(b*x+a)-3*I*(
-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/s
in(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*
x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(b*x+a)+3*(-(-
1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a
)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(b*x+a)*cos(b*x+a
)+3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+c
os(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(b*x+a)*c
os(b*x+a)-6*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(
(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*sin(b*x+a)*cos(
b*x+a)+3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-
(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(b*x
+a)+3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1
+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(b*x+a)
-6*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a
))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos
(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*sin(b*x+a)+2*cos(b*x+a)^
2*2^(1/2))/sin(b*x+a)^(3/2)/(-1+cos(b*x+a)+sin(b*x+a))^3/(-1+cos(b*x+a)-sin
(b*x+a))^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^{\frac{5}{2}}}{\sin(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(5/2)/sin(b*x + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(5/2)/sin(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^{\frac{5}{2}}}{\sin(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(5/2)/sin(b*x + a)^(5/2), x)
```

3.303 $\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=226

$$-\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}+\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}b}+\frac{\log\left(\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+}}{\sqrt{\cos(a+b}}\right.}{2\sqrt{2}b}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b))
+ ArcTan[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b) +
 Log[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan[a + b*x]]/(2
*Sqrt[2]*b) - Log[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan
[a + b*x]]/(2*Sqrt[2]*b) - (2*Cos[a + b*x]^(5/2))/(5*b*Sin[a + b*x]^(5/2))
+ (2*Sqrt[Cos[a + b*x]])/(b*Sqrt[Sin[a + b*x]])
```

Rubi [A] time = 0.138994, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2567, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}+\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}b}+\frac{\log\left(\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+}}{\sqrt{\cos(a+b}}\right.}{2\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(7/2)/Sin[a + b*x]^(7/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b))
+ ArcTan[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b) +
 Log[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan[a + b*x]]/(2
*Sqrt[2]*b) - Log[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan
[a + b*x]]/(2*Sqrt[2]*b) - (2*Cos[a + b*x]^(5/2))/(5*b*Sin[a + b*x]^(5/2))
+ (2*Sqrt[Cos[a + b*x]])/(b*Sqrt[Sin[a + b*x]])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} - \int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$

$$= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} + \int \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\,dx$$

$$= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} + \frac{2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b}$$

$$= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(}{\sqrt{\cos(}\right.}{b}$$

$$= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,d\right.}{2b}$$

$$= \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} - \frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} - \frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2}{b}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} + \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} - \log$$

Mathematica [C] time = 0.0394258, size = 57, normalized size = 0.25

$$-\frac{2\cos^2(a+bx)^{3/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{5}{4};-\frac{1}{4};\sin^2(a+bx)\right)}{5b\sin^{\frac{5}{2}}(a+bx)\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(7/2)/Sin[a + b*x]^(7/2),x]
```

```
[Out] (-2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, -5/4, -1/4, Sin[a + b*x]
^2])/(5*b*Cos[a + b*x]^(3/2)*Sin[a + b*x]^(5/2))
```

Maple [C] time = 0.113, size = 1966, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x)
```

```
[Out] -8/5/b*2^(1/2)*cos(b*x+a)^(7/2)*(-1+cos(b*x+a))^4*(5*I*(-(-1+cos(b*x+a)-sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/si
n(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*I*(-(-1+cos(b*x+a)-sin(b*x+a))/sin
(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a
))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1
/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)^2+5*I*(-(-1+cos(b*x+a)-sin(b*x+a))/si
n(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(
```

```
1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)^2+5*I*(-(-1+cos(b*x+a)-sin(b*x+a))/s
in(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x
+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^
(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)-5*cos(b*x+a)^3*(-(-1+cos(b*x+a)-sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/si
n(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*cos(b*x+a)^3*(-(-1+cos(b*x+a)-sin(
b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-
1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin
(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*I*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
)/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/
2),1/2-1/2*I,1/2*2^(1/2))+5*I*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2
)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a)
)^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I
,1/2*2^(1/2))*cos(b*x+a)^3-5*cos(b*x+a)^2*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
)/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/
2),1/2-1/2*I,1/2*2^(1/2))-5*cos(b*x+a)^2*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b
*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))
/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2
),1/2+1/2*I,1/2*2^(1/2))-5*I*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)
*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))
^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,
1/2*2^(1/2))*cos(b*x+a)^3-5*I*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2
)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a)
)^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I
,1/2*2^(1/2))*cos(b*x+a)+5*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+
a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1
/2-1/2*I,1/2*2^(1/2))+5*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b
*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+
1/2*I,1/2*2^(1/2))+5*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+co
s(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*E
llipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1
/2))+5*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-
1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+12*cos(b*
x+a)^3*2^(1/2)-10*cos(b*x+a)*2^(1/2))/sin(b*x+a)^(5/2)/(-1+cos(b*x+a)+sin(b
*x+a))^4/(-1+cos(b*x+a)-sin(b*x+a))^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^{\frac{7}{2}}}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(7/2)/sin(b*x + a)^(7/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(7/2)/sin(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.304 $\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-3/2, 2/3, 5/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(4/3))/(4*b*f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.0393083, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-3/2, 2/3, 5/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(4/3))/(4*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}}{4bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0552097, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)\sqrt[3]{b\sin(e+fx)}\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, 2/3, 5/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int \left(\cos \left(fx+e\right)\right)^{4} \sqrt[3]{b\sin \left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin \left(fx+e\right)\right)^{\frac{1}{3}} \cos \left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sin \left(fx+e\right)\right)^{\frac{1}{3}} \cos \left(fx+e\right)^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3)*cos(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(b*sin(f*x+e))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin \left(fx+e\right)\right)^{\frac{1}{3}} \cos \left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="giac")

[Out] integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^4, x)
```

3.305 $\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-1/2, 2/3, 5/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(4/3))/(4*b*f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.0382235, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-1/2, 2/3, 5/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(4/3))/(4*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}}{4bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0442239, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)\sqrt[3]{b\sin(e+fx)}\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, 2/3, 5/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{2} \sqrt[3]{b \sin\left(fx+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin\left(fx+e\right)\right)^{\frac{1}{3}} \cos\left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sin\left(fx+e\right)\right)^{\frac{1}{3}} \cos\left(fx+e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3)*cos(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \sin\left(e+fx\right)} \cos^{2}\left(e+fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(b*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral((b*sin(e + f*x))**(1/3)*cos(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin\left(fx+e\right)\right)^{\frac{1}{3}} \cos\left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^2, x)
```

3.306 $\int \sqrt[3]{b \sin(e+fx)}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[e + f*x]^2]*(b*Sin[e +
 f*x])^(4/3))/(4*b*f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.01495, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[e + f*x]^2]*(b*Sin[e +
 f*x])^(4/3))/(4*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}}{4bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.036219, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)\sqrt[3]{b\sin(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \sqrt[3]{b\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(1/3),x)
```

```
[Out] int((b*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral((b*sin(e + f*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3),x, algorithm="giac")

[Out] integrate((b*sin(f*x + e))^(1/3), x)
```

3.307 $\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal. Leaf size=58

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};\sin^2(e+fx)\right)}{4bf}$$

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 3/2, 5/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(4/3))/(4*b*f)
```

Rubi [A] time = 0.0376715, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};\sin^2(e+fx)\right)}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 3/2, 5/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(4/3))/(4*b*f)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{4/3}}{4bf}$$

Mathematica [A] time = 0.0403223, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)\sqrt[3]{b\sin(e+fx)}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};\sin^2(e+fx)\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 3/2, 5/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{2} \sqrt[3]{b \sin \left(fx+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx+e\right)\right)^{\frac{1}{3}} \sec \left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sin \left(fx+e\right)\right)^{\frac{1}{3}} \sec \left(fx+e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3)*sec(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(b*sin(f*x+e))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx+e\right)\right)^{\frac{1}{3}} \sec \left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^2, x)
```

3.308 $\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal. Leaf size=58

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};\sin^2(e+fx)\right)}{4bf}$$

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 5/2, 5/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(4/3))/(4*b*f)
```

Rubi [A] time = 0.0378301, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};\sin^2(e+fx)\right)}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 5/2, 5/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(4/3))/(4*b*f)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{4/3}}{4bf}$$

Mathematica [A] time = 0.0377969, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)\sqrt[3]{b\sin(e+fx)}\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};\sin^2(e+fx)\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 5/2, 5/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int \left(\sec \left(f x+e\right)\right)^{4} \sqrt[3]{b \sin \left(f x+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(f x+e\right)\right)^{\frac{1}{3}} \sec \left(f x+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sin \left(f x+e\right)\right)^{\frac{1}{3}} \sec \left(f x+e\right)^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3)*sec(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(b*sin(f*x+e))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(f x+e\right)\right)^{\frac{1}{3}} \sec \left(f x+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^4, x)
```

3.309 $\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(-\frac{3}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-3/2, 4/3, 7/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(8/3))/(8*b*f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.0433625, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(-\frac{3}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-3/2, 4/3, 7/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(8/3))/(8*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx=\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{8/3}}{8bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0551033, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)(b\sin(e+fx))^{5/3}\,{}_2F_1\left(-\frac{3}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, 4/3, 7/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int \left(\cos \left(fx+e\right)\right)^{4} \left(b\sin \left(fx+e\right)\right)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin \left(fx+e\right)\right)^{\frac{5}{3}} \cos \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sin \left(fx+e\right)\right)^{\frac{2}{3}} b\cos \left(fx+e\right)^{4} \sin \left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*b*cos(f*x + e)^4*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(b*sin(f*x+e))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin \left(fx+e\right)\right)^{\frac{5}{3}} \cos \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^4, x)
```

3.310 $\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(-\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-1/2, 4/3, 7/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(8/3))/(8*b*f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.0450176, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(-\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-1/2, 4/3, 7/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(8/3))/(8*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx=\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{8/3}}{8bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0509162, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)(b\sin(e+fx))^{5/3}\,{}_2F_1\left(-\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, 4/3, 7/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \left(\cos \left(f x + e\right)\right)^{2} \left(b \sin \left(f x + e\right)\right)^{\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)

[Out] int(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(f x + e\right)\right)^{\frac{5}{3}} \cos \left(f x + e\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")

[Out] integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sin \left(f x + e\right)\right)^{\frac{2}{3}} b \cos \left(f x + e\right)^{2} \sin \left(f x + e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")

[Out] integral((b*sin(f*x + e))^(2/3)*b*cos(f*x + e)^2*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(b*sin(f*x+e))**(5/3),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(f x + e\right)\right)^{\frac{5}{3}} \cos \left(f x + e\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^2, x)
```

3.311 $\int (b\sin(e+fx))^{5/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Sin[e + f*x]^2]*(b*Sin[e +
 f*x])^(8/3))/(8*b*f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.0141124, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Sin[e + f*x]^2]*(b*Sin[e +
 f*x])^(8/3))/(8*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sin(e+fx))^{5/3}\,dx = \frac{3\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{8/3}}{8bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.041237, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)(b\sin(e+fx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, 4/3, 7/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(5/3),x)
```

```
[Out] int((b*sin(f*x+e))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{2}{3}}b\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*b*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3), x)
```

3.312 $\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{7}{3};\sin^2(e+fx)\right)}{8bf}$$

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 3/2, 7/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(8/3))/(8*b*f)
```

Rubi [A] time = 0.0440728, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{7}{3};\sin^2(e+fx)\right)}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 3/2, 7/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(8/3))/(8*b*f)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{7}{3};\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{8/3}}{8bf}$$

Mathematica [A] time = 0.0508624, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)(b\sin(e+fx))^{5/3}\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{7}{3};\sin^2(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 3/2, 7/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{2} \left(b\sin \left(fx+e\right)\right)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin \left(fx+e\right)\right)^{\frac{5}{3}} \sec \left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3)*sec(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sin \left(fx+e\right)\right)^{\frac{2}{3}} b\sec \left(fx+e\right)^{2} \sin \left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*b*sec(f*x + e)^2*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(b*sin(f*x+e))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin \left(fx+e\right)\right)^{\frac{5}{3}} \sec \left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3)*sec(f*x + e)^2, x)
```

3.313 $\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(\frac{4}{3},\frac{5}{2};\frac{7}{3};\sin^2(e+fx)\right)}{8bf}$$

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 5/2, 7/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(8/3))/(8*b*f)
```

Rubi [A] time = 0.0444031, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(\frac{4}{3},\frac{5}{2};\frac{7}{3};\sin^2(e+fx)\right)}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 5/2, 7/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(8/3))/(8*b*f)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx=\frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(\frac{4}{3},\frac{5}{2};\frac{7}{3};\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{8/3}}{8bf}$$

Mathematica [A] time = 0.0466806, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)(b\sin(e+fx))^{5/3}\,{}_2F_1\left(\frac{4}{3},\frac{5}{2};\frac{7}{3};\sin^2(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 5/2, 7/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{4} \left(b \sin \left(fx+e\right)\right)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx+e\right)\right)^{\frac{5}{3}} \sec \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3)*sec(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b \sin \left(fx+e\right)\right)^{\frac{2}{3}} b \sec \left(fx+e\right)^{4} \sin \left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*b*sec(f*x + e)^4*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(b*sin(f*x+e))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx+e\right)\right)^{\frac{5}{3}} \sec \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3)*sec(f*x + e)^4, x)
```

3.314 $\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-3/2, 1/3, 4/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(2/3))/(2*b*f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.0436346, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-3/2, 1/3, 4/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(2/3))/(2*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{2/3}}{2bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0522626, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)}{2f\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, 1/3, 4/3, Sin[e + f*x]^2]*T
an[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \left(\cos \left(f x+e\right)\right)^{4} \frac{1}{\sqrt[3]{b \sin \left(f x+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos \left(f x+e\right)^{4}}{\left(b \sin \left(f x+e\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b \sin \left(f x+e\right)\right)^{\frac{2}{3}} \cos \left(f x+e\right)^{4}}{b \sin \left(f x+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*cos(f*x + e)^4/(b*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(b*sin(f*x+e))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(1/3), x)
```

3.315 $\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-1/2, 1/3, 4/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(2/3))/(2*b*f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.0392388, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-1/2, 1/3, 4/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(2/3))/(2*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{2/3}}{2bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0433031, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)}{2f\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, 1/3, 4/3, Sin[e + f*x]^2]*T
an[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \left(\cos \left(f x+e\right)\right)^{2} \frac{1}{\sqrt[3]{b \sin \left(f x+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos \left(f x+e\right)^{2}}{\left(b \sin \left(f x+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b \sin \left(f x+e\right)\right)^{\frac{2}{3}} \cos \left(f x+e\right)^{2}}{b \sin \left(f x+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*cos(f*x + e)^2/(b*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(b*sin(f*x+e))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

3.316 $\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[e + f*x]^2]*(b*Sin[e +
 f*x])^(2/3))/(2*b*f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.0146014, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(-1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[e + f*x]^2]*(b*Sin[e +
 f*x])^(2/3))/(2*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\cos(e+fx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{2/3}}{2bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0352756, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2f\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(-1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[e + f*x]^2]*Ta
n[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(1/(b*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{2}{3}}}{b\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)/(b*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral((b*sin(e + f*x))**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sin\left(fx+e\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(-1/3), x)
```

3.317 $\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal. Leaf size=58

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{3}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2bf}$$

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 3/2, 4/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(2/3))/(2*b*f)
```

Rubi [A] time = 0.0397975, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{3}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 3/2, 4/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(2/3))/(2*b*f)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{3}{2};\frac{4}{3};\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{2/3}}{2bf}$$

Mathematica [A] time = 0.0417054, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)\,{}_2F_1\left(\frac{1}{3},\frac{3}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2f\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 3/2, 4/3, Sin[e + f*x]^2]*Ta
n[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{2} \frac{1}{\sqrt[3]{b \sin \left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec \left(fx+e\right)^{2}}{\left(b \sin \left(fx+e\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b \sin \left(fx+e\right)\right)^{\frac{2}{3}} \sec \left(fx+e\right)^{2}}{b \sin \left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*sec(f*x + e)^2/(b*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{2}\left(e+fx\right)}{\sqrt[3]{b \sin \left(e+fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(b*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral(sec(e + f*x)**2/(b*sin(e + f*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

3.318 $\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal. Leaf size=58

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{5}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2bf}$$

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 5/2, 4/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(2/3))/(2*b*f)
```

Rubi [A] time = 0.0385146, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{5}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 5/2, 4/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(2/3))/(2*b*f)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{5}{2};\frac{4}{3};\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{2/3}}{2bf}$$

Mathematica [A] time = 0.0385799, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)\,{}_2F_1\left(\frac{1}{3},\frac{5}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2f\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 5/2, 4/3, Sin[e + f*x]^2]*Ta
n[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{4} \frac{1}{\sqrt[3]{b \sin \left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b \sin \left(fx+e\right)\right)^{\frac{2}{3}} \sec \left(fx+e\right)^{4}}{b \sin \left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*sec(f*x + e)^4/(b*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(b*sin(f*x+e))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec \left(fx+e\right)^{4}}{\left(b \sin \left(fx+e\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^4/(b*sin(f*x + e))^(1/3), x)
```

3.319 $\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

```
[Out] (-3*Cos[e + f*x]*Hypergeometric2F1[-3/2, -1/3, 2/3, Sin[e + f*x]^2])/(2*b*f
*Sqrt[Cos[e + f*x]^2]*(b*Sin[e + f*x])^(2/3))
```

Rubi [A] time = 0.0462221, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Cos[e + f*x]*Hypergeometric2F1[-3/2, -1/3, 2/3, Sin[e + f*x]^2])/(2*b*f
*Sqrt[Cos[e + f*x]^2]*(b*Sin[e + f*x])^(2/3))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx=-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

Mathematica [A] time = 0.0466599, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2f(b\sin(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, -1/3, 2/3, Sin[e + f*x]^2]
*Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{4} \left(b\sin\left(fx+e\right)\right)^{-\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\cos\left(fx+e\right)^{4}}{b^{2}\cos\left(fx+e\right)^{2}-b^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(1/3)*cos(f*x + e)^4/(b^2*cos(f*x + e)^2 - b^2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(b*sin(f*x+e))**(5/3),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(5/3), x)
```

3.320 $\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

```
[Out] (-3*Cos[e + f*x]*Hypergeometric2F1[-1/2, -1/3, 2/3, Sin[e + f*x]^2])/(2*b*f
*Sqrt[Cos[e + f*x]^2]*(b*Sin[e + f*x])^(2/3))
```

Rubi [A] time = 0.0456165, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Cos[e + f*x]*Hypergeometric2F1[-1/2, -1/3, 2/3, Sin[e + f*x]^2])/(2*b*f
*Sqrt[Cos[e + f*x]^2]*(b*Sin[e + f*x])^(2/3))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx=-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

Mathematica [A] time = 0.0448701, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2f(b\sin(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, -1/3, 2/3, Sin[e + f*x]^2]
*Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{2} \left(b\sin\left(fx+e\right)\right)^{-\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\cos\left(fx+e\right)^{2}}{b^{2}\cos\left(fx+e\right)^{2}-b^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(1/3)*cos(f*x + e)^2/(b^2*cos(f*x + e)^2 - b^2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(b*sin(f*x+e))**(5/3),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(5/3), x)
```

3.321 $\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

```
[Out] (-3*Cos[e + f*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sin[e + f*x]^2])/(2*b*f*
Sqrt[Cos[e + f*x]^2]*(b*Sin[e + f*x])^(2/3))
```

Rubi [A] time = 0.0144688, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(-5/3),x]
```

```
[Out] (-3*Cos[e + f*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sin[e + f*x]^2])/(2*b*f*
Sqrt[Cos[e + f*x]^2]*(b*Sin[e + f*x])^(2/3))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx=-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

Mathematica [A] time = 0.0337511, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2f(b\sin(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(-5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sin[e + f*x]^2]*
Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{-\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(1/(b*sin(f*x+e))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(-5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}{b^2\cos\left(fx+e\right)^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(1/3)/(b^2*cos(f*x + e)^2 - b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(e+fx\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))**(5/3),x)
```

```
[Out] Integral((b*sin(e + f*x))**(-5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(-5/3), x)
```

3.322 $\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2bf(b\sin(e+fx))^{2/3}}$$

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 3/2, 2/3, Sin[e + f*x]^2]*
Sec[e + f*x])/(2*b*f*(b*Sin[e + f*x])^(2/3))
```

Rubi [A] time = 0.0455026, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$-\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2bf(b\sin(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 3/2, 2/3, Sin[e + f*x]^2]*
Sec[e + f*x])/(2*b*f*(b*Sin[e + f*x])^(2/3))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{2}{3};\sin^2(e+fx)\right)\sec(e+fx)}{2bf(b\sin(e+fx))^{2/3}}$$

Mathematica [A] time = 0.0424613, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2f(b\sin(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 3/2, 2/3, Sin[e + f*x]^2]*
Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{2} \left(b \sin \left(fx+e\right)\right)^{-\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b \sin \left(fx+e\right)\right)^{\frac{1}{3}} \sec \left(fx+e\right)^{2}}{b^{2} \cos \left(fx+e\right)^{2}-b^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(1/3)*sec(f*x + e)^2/(b^2*cos(f*x + e)^2 - b^2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(b*sin(f*x+e))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec \left(fx+e\right)^{2}}{\left(b \sin \left(fx+e\right)\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e))^(5/3), x)
```

3.323 $\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{5}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2bf(b\sin(e+fx))^{2/3}}$$

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 5/2, 2/3, Sin[e + f*x]^2]*
Sec[e + f*x])/(2*b*f*(b*Sin[e + f*x])^(2/3))
```

Rubi [A] time = 0.0464785, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$-\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{5}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2bf(b\sin(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 5/2, 2/3, Sin[e + f*x]^2]*
Sec[e + f*x])/(2*b*f*(b*Sin[e + f*x])^(2/3))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{2};\frac{2}{3};\sin^2(e+fx)\right)\sec(e+fx)}{2bf(b\sin(e+fx))^{2/3}}$$

Mathematica [A] time = 0.0367019, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\cos^2(e+fx)}\tan(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{5}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2f(b\sin(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 5/2, 2/3, Sin[e + f*x]^2]*
Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^{4} \left(b\sin\left(fx+e\right)\right)^{-\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sec\left(fx+e\right)^{4}}{b^{2}\cos\left(fx+e\right)^{2}-b^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(1/3)*sec(f*x + e)^4/(b^2*cos(f*x + e)^2 - b^2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(b*sin(f*x+e))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^4/(b*sin(f*x + e))^(5/3), x)
```

3.324 $\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx$

Optimal. Leaf size=128

$$-\frac{\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b}+\frac{\log\left(\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*Sin[a + b*x]^(2/3))/Cos[a + b*x]^(2/3))/Sqrt[3]])/
(2*b) - Log[1 + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)]/(2*b) + Log[1 - Sin[
a + b*x]^(2/3)/Cos[a + b*x]^(2/3) + Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3)]/
(4*b)
```

Rubi [A] time = 0.15127, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2574, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b}+\frac{\log\left(\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(1/3)/Cos[a + b*x]^(1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*Sin[a + b*x]^(2/3))/Cos[a + b*x]^(2/3))/Sqrt[3]])/
(2*b) - Log[1 + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)]/(2*b) + Log[1 - Sin[
a + b*x]^(2/3)/Cos[a + b*x]^(2/3) + Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3)]/
(4*b)
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx &= \frac{3\,\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}\\
&= \frac{3\,\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}\\
&= -\frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{3\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b}\\
&= -\frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}\\
&= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b}
\end{aligned}
$$

Mathematica [C] time = 0.040584, size = 57, normalized size = 0.45

$$\frac{3\sin^{\frac{4}{3}}(a+bx)\cos^2(a+bx)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\sin^2(a+bx)\right)}{4b\cos^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(1/3)/Cos[a + b*x]^(1/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, Sin[a + b*x]^2]*
Sin[a + b*x]^(4/3))/(4*b*Cos[a + b*x]^(4/3))
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \sqrt[3]{\sin(bx+a)}\frac{1}{\sqrt[3]{\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x)
```

```
[Out] int(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^{\frac{1}{3}}}{\cos(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(1/3)/cos(b*x + a)^(1/3), x)
```

Fricas [A] time = 2.24578, size = 424, normalized size = 3.31

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cos(bx+a)-2\sqrt{3}\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}}{3\cos(bx+a)}\right)-2\log\left(\frac{\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)}{\cos(bx+a)}\right)+\log\left(\frac{\cos(bx+a)^2-\cos(bx+a)^{\frac{4}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{4}{3}}}{\cos(bx+a)^2}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*cos(b*x + a) - 2*sqrt(3)*cos(b*x + a)^(
1/3)*sin(b*x + a)^(2/3))/cos(b*x + a)) - 2*log((cos(b*x + a)^(1/3)*sin(b*x
+ a)^(2/3) + cos(b*x + a))/cos(b*x + a)) + log((cos(b*x + a)^2 - cos(b*x +
a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3))/cos(b*
x + a)^2))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{\sin (a+bx)}}{\sqrt[3]{\cos (a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(1/3)/cos(b*x+a)**(1/3),x)
```

```
[Out] Integral(sin(a + b*x)**(1/3)/cos(a + b*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (bx+a)^{\frac{1}{3}}}{\cos (bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(1/3)/cos(b*x + a)^(1/3), x)
```

3.325 $\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx$

Optimal. Leaf size=224

$$\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}-\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b}+\frac{\tan^{-1}\left(\right.}{}$$

```
[Out] -ArcTan[Sqrt[3] - (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/(2*b) + ArcTan
[Sqrt[3] + (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/(2*b) + ArcTan[Sin[a
+ b*x]^(1/3)/Cos[a + b*x]^(1/3)]/b + (Sqrt[3]*Log[1 - (Sqrt[3]*Sin[a + b*x]
^(1/3))/Cos[a + b*x]^(1/3) + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b)
- (Sqrt[3]*Log[1 + (Sqrt[3]*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3) + Sin[a
+ b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b)
```

Rubi [A] time = 0.329462, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2574, 295, 634, 618, 204, 628, 203}

$$\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}-\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b}+\frac{\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3),x]
```

```
[Out] -ArcTan[Sqrt[3] - (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/(2*b) + ArcTan
[Sqrt[3] + (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/(2*b) + ArcTan[Sin[a
+ b*x]^(1/3)/Cos[a + b*x]^(1/3)]/b + (Sqrt[3]*Log[1 - (Sqrt[3]*Sin[a + b*x]
^(1/3))/Cos[a + b*x]^(1/3) + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b)
- (Sqrt[3]*Log[1 + (Sqrt[3]*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3) + Sin[a
+ b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b)
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx = \frac{3\,\mathrm{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\mathrm{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\mathrm{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx, x\right.}{b}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{4b} + \frac{\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{4b}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\sqrt{3}\log\left(1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{\sqrt{3}\log\left(1+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right.}{4b}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} + \frac{\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\sqrt{3}\log\left(1-\frac{\sqrt{3}\sqrt[3]{\sin}}{\sqrt[3]{\cos(}}\right.}{4}$$

Mathematica [C] time = 0.0405218, size = 57, normalized size = 0.25

$$\frac{3\sin^{\frac{5}{3}}(a+bx)\cos^2(a+bx)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{5}{6};\frac{11}{6};\sin^2(a+bx)\right)}{5b\cos^{\frac{5}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11/6, Sin[a + b*x]^2]
*Sin[a + b*x]^(5/3))/(5*b*Cos[a + b*x]^(5/3))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int (\sin(bx+a))^{\frac{2}{3}} (\cos(bx+a))^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x)
```

```
[Out] int(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^{\frac{2}{3}}}{\cos(bx+a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(2/3)/cos(b*x + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(2/3)/cos(b*x+a)**(2/3),x)
```

```
[Out] Integral(sin(a + b*x)**(2/3)/cos(a + b*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^{\frac{2}{3}}}{\cos(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(2/3)/cos(b*x + a)^(2/3), x)
```

3.326 $\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx$

Optimal. Leaf size=249

$$\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}+\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}-\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right.}{2b}$$

```
[Out] -ArcTan[Sqrt[3] - (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/(2*b) + ArcTan
[Sqrt[3] + (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/(2*b) + ArcTan[Cos[a
+ b*x]^(1/3)/Sin[a + b*x]^(1/3)]/b + (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Si
n[a + b*x]^(2/3) - (Sqrt[3]*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b)
- (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3) + (Sqrt[3]*Cos[a +
 b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b) + (3*Sin[a + b*x]^(1/3))/(b*Cos[a +
 b*x]^(1/3))
```

Rubi [A] time = 0.347817, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2566, 2575, 295, 634, 618, 204, 628, 203}

$$\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}+\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}-\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3),x]
```

```
[Out] -ArcTan[Sqrt[3] - (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/(2*b) + ArcTan
[Sqrt[3] + (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/(2*b) + ArcTan[Cos[a
+ b*x]^(1/3)/Sin[a + b*x]^(1/3)]/b + (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Si
n[a + b*x]^(2/3) - (Sqrt[3]*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b)
- (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3) + (Sqrt[3]*Cos[a +
 b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b) + (3*Sin[a + b*x]^(1/3))/(b*Cos[a +
 b*x]^(1/3))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}} - \int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx$$

$$= \frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}} + \frac{3\,\mathrm{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= \frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}} + \frac{\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} + \frac{\mathrm{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} + \text{Su}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} + \frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} + \frac{\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{3}x}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} + \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)} - \frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} - \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)} + \frac{\sqrt{3}\sqrt[3]{\cos(a}}{\sqrt[3]{\sin(a+}}\right.}{4b}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} + \frac{\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} + \frac{\sqrt{3}\log\left(1+\frac{\cos}{\sin}\right.}{}$$

Mathematica [C] time = 0.052391, size = 57, normalized size = 0.23

$$\frac{3\sin^{\frac{7}{3}}(a+bx)\sqrt[6]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{7}{6},\frac{7}{6};\frac{13}{6};\sin^2(a+bx)\right)}{7b\sqrt[3]{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(1/6)*Hypergeometric2F1[7/6, 7/6, 13/6, Sin[a + b*x]^2]
*Sin[a + b*x]^(7/3))/(7*b*Cos[a + b*x]^(1/3))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int (\sin(bx+a))^{\frac{4}{3}} (\cos(bx+a))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x)
```

```
[Out] int(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^{\frac{4}{3}}}{\cos(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(4/3)/cos(b*x + a)^(4/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(4/3)/cos(b*x+a)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^{\frac{4}{3}}}{\cos(bx+a)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(4/3)/cos(b*x + a)^(4/3), x)
```

3.327 $\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx$

Optimal. Leaf size=155

$$\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}+\frac{\log\left(\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*Cos[a + b*x]^(2/3))/Sin[a + b*x]^(2/3))/Sqrt[3]])/
(2*b) + Log[1 + Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3) - Cos[a + b*x]^(2/3)/
Sin[a + b*x]^(2/3)]/(4*b) - Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3)]/
(2*b) + (3*Sin[a + b*x]^(2/3))/(2*b*Cos[a + b*x]^(2/3))
```

Rubi [A] time = 0.174671, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2566, 2575, 275, 292, 31, 634, 618, 204, 628}

$$\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}+\frac{\log\left(\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(5/3)/Cos[a + b*x]^(5/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*Cos[a + b*x]^(2/3))/Sin[a + b*x]^(2/3))/Sqrt[3]])/
(2*b) + Log[1 + Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3) - Cos[a + b*x]^(2/3)/
Sin[a + b*x]^(2/3)]/(4*b) - Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3)]/
(2*b) + (3*Sin[a + b*x]^(2/3))/(2*b*Cos[a + b*x]^(2/3))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} - \int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx$$

$$= \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} + \frac{3\,\mathrm{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} + \frac{3\,\mathrm{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} - \frac{\mathrm{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\mathrm{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} + \frac{\mathrm{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{1-x\cdots}\right.}{\cdots}$$

$$= \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,d\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{3\sin^{\frac{2}{3}}(a\cdots}{2b\cos^{\frac{2}{3}}(\cdots}$$

Mathematica [C] time = 0.0558447, size = 57, normalized size = 0.37

$$\frac{3\sin^{\frac{8}{3}}(a+bx)\sqrt[3]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\sin^2(a+bx)\right)}{8b\cos^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(5/3)/Cos[a + b*x]^(5/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, Sin[a + b*x]^2]*
Sin[a + b*x]^(8/3))/(8*b*Cos[a + b*x]^(2/3))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int (\sin(bx+a))^{\frac{5}{3}}(\cos(bx+a))^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x)
```

```
[Out] int(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^{\frac{5}{3}}}{\cos(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(5/3)/cos(b*x + a)^(5/3), x)
```

Fricas [A] time = 3.23847, size = 572, normalized size = 3.69

$$\frac{2\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}-\sqrt{3}\sin(bx+a)}{3\sin(bx+a)}\right)\cos(bx+a)+\cos(bx+a)\log\left(\frac{4\left(\cos(bx+a)^2-\cos(bx+a)^{\frac{4}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)\right.}{\cos(bx+a)^2-1}\right.}{4\,b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3)
- sqrt(3)*sin(b*x + a))/sin(b*x + a))*cos(b*x + a) + cos(b*x + a)*log(4*(co
s(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)*s
in(b*x + a)^(4/3) - 1)/(cos(b*x + a)^2 - 1)) - 2*cos(b*x + a)*log(-2*(cos(b
*x + a)^(2/3)*sin(b*x + a)^(1/3) + sin(b*x + a))/sin(b*x + a)) + 6*cos(b*x
+ a)^(1/3)*sin(b*x + a)^(2/3))/(b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(5/3)/cos(b*x+a)**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^{\frac{5}{3}}}{\cos(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(5/3)/cos(b*x + a)^(5/3), x)
```

3.328 $\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx$

Optimal. Leaf size=155

$$\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}+\frac{\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\log\left(\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sin[a + b*x]^(2/3))/Cos[a + b*x]^(2/3))/Sqrt[3]])/(
2*b) + Log[1 + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)]/(2*b) - Log[1 - Sin[a
 + b*x]^(2/3)/Cos[a + b*x]^(2/3) + Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3)]/(
4*b) + (3*Sin[a + b*x]^(4/3))/(4*b*Cos[a + b*x]^(4/3))
```

Rubi [A] time = 0.110121, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2566, 2574, 275, 292, 31, 634, 618, 204, 628}

$$\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}+\frac{\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\log\left(\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(7/3)/Cos[a + b*x]^(7/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sin[a + b*x]^(2/3))/Cos[a + b*x]^(2/3))/Sqrt[3]])/(
2*b) + Log[1 + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)]/(2*b) - Log[1 - Sin[a
 + b*x]^(2/3)/Cos[a + b*x]^(2/3) + Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3)]/(
4*b) + (3*Sin[a + b*x]^(4/3))/(4*b*Cos[a + b*x]^(4/3))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} - \int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx$$

$$= \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}$$

$$= \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-x+x}\right.}{4}$$

$$= \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} + \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} + \frac{3\sin^{\frac{4}{3}}(a+}{4b\cos^{\frac{4}{3}}(a}$$

Mathematica [C] time = 0.0525653, size = 57, normalized size = 0.37

$$\frac{3\sin^{\frac{10}{3}}(a+bx)\cos^2(a+bx)^{2/3}\,{}_2F_1\left(\frac{5}{3},\frac{5}{3};\frac{8}{3};\sin^2(a+bx)\right)}{10b\cos^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(7/3)/Cos[a + b*x]^(7/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(2/3)*Hypergeometric2F1[5/3, 5/3, 8/3, Sin[a + b*x]^2]*
Sin[a + b*x]^(10/3))/(10*b*Cos[a + b*x]^(4/3))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int (\sin(bx+a))^{\frac{7}{3}}(\cos(bx+a))^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x)
```

```
[Out] int(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^{\frac{7}{3}}}{\cos(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(7/3)/cos(b*x + a)^(7/3), x)
```

Fricas [A] time = 2.91365, size = 566, normalized size = 3.65

$$-\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cos(bx+a)-2\sqrt{3}\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}}{3\cos(bx+a)}\right)\cos(bx+a)^2-2\cos(bx+a)^2\log\left(\frac{\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)}{\cos(bx+a)}\right)+c}{4\,b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*cos(b*x + a) - 2*sqrt(3)*cos(b*x + a)^
(1/3)*sin(b*x + a)^(2/3))/cos(b*x + a))*cos(b*x + a)^2 - 2*cos(b*x + a)^2*l
og((cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3) + cos(b*x + a))/cos(b*x + a)) + c
os(b*x + a)^2*log((cos(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3) +
 cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3))/cos(b*x + a)^2) - 3*cos(b*x + a)^(2
/3)*sin(b*x + a)^(4/3))/(b*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(7/3)/cos(b*x+a)**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^{\frac{7}{3}}}{\cos(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(7/3)/cos(b*x + a)^(7/3), x)
```

3.329 $\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx$

Optimal. Leaf size=128

$$-\frac{\log\left(\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{4b}+\frac{\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{2b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Cos[a + b*x]^(2/3))/Sin[a + b*x]^(2/3))/Sqrt[3]])/(
2*b) - Log[1 + Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3) - Cos[a + b*x]^(2/3)/S
in[a + b*x]^(2/3)]/(4*b) + Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3)]/(
2*b)
```

Rubi [A] time = 0.0822267, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2575, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{\log\left(\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{4b}+\frac{\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{2b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(1/3)/Sin[a + b*x]^(1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Cos[a + b*x]^(2/3))/Sin[a + b*x]^(2/3))/Sqrt[3]])/(
2*b) - Log[1 + Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3) - Cos[a + b*x]^(2/3)/S
in[a + b*x]^(2/3)]/(4*b) + Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3)]/(
2*b)
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx &= -\frac{3\,\mathrm{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} \\
&= -\frac{3\,\mathrm{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} \\
&= \frac{\mathrm{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\mathrm{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} \\
&= \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\mathrm{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} \\
&= -\frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} \\
&= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}
\end{aligned}
$$

Mathematica [C] time = 0.0256764, size = 57, normalized size = 0.45

$$\frac{3\sin^{\frac{2}{3}}(a+bx)\sqrt[3]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\sin^2(a+bx)\right)}{2b\cos^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(1/3)/Sin[a + b*x]^(1/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, Sin[a + b*x]^2]*
Sin[a + b*x]^(2/3))/(2*b*Cos[a + b*x]^(2/3))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \sqrt[3]{\cos(bx+a)}\frac{1}{\sqrt[3]{\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x)
```

```
[Out] int(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^{\frac{1}{3}}}{\sin(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(1/3)/sin(b*x + a)^(1/3), x)
```

Fricas [A] time = 2.88885, size = 444, normalized size = 3.47

$$-\frac{2\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}-\sqrt{3}\sin(bx+a)}{3\sin(bx+a)}\right)+\log\left(\frac{4\left(\cos(bx+a)^2-\cos(bx+a)^{\frac{4}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{4}{3}}-1\right)}{\cos(bx+a)^2-1}\right)-2\,\mathrm{l}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3)
 - sqrt(3)*sin(b*x + a))/sin(b*x + a)) + log(4*(cos(b*x + a)^2 - cos(b*x +
a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3) - 1)/(c
os(b*x + a)^2 - 1)) - 2*log(-2*(cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3) + sin
```

```
(b*x + a))/sin(b*x + a)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{\cos{(a+bx)}}}{\sqrt[3]{\sin{(a+bx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(1/3)/sin(b*x+a)**(1/3),x)
```

```
[Out] Integral(cos(a + b*x)**(1/3)/sin(a + b*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos{(bx+a)}^{\frac{1}{3}}}{\sin{(bx+a)}^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(1/3)/sin(b*x + a)^(1/3), x)
```

3.330 $\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx$

Optimal. Leaf size=225

$$-\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}+\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b}-\tan^{-1}$$

```
[Out] ArcTan[Sqrt[3] - (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/(2*b) - ArcTan[
Sqrt[3] + (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/(2*b) - ArcTan[Cos[a +
 b*x]^(1/3)/Sin[a + b*x]^(1/3)]/b - (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Sin
[a + b*x]^(2/3) - (Sqrt[3]*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b) +
 (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3) + (Sqrt[3]*Cos[a +
b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b)
```

Rubi [A] time = 0.300267, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2575, 295, 634, 618, 204, 628, 203}

$$-\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}+\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b}-\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3),x]
```

```
[Out] ArcTan[Sqrt[3] - (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/(2*b) - ArcTan[
Sqrt[3] + (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/(2*b) - ArcTan[Cos[a +
 b*x]^(1/3)/Sin[a + b*x]^(1/3)]/b - (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Sin
[a + b*x]^(2/3) - (Sqrt[3]*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b) +
 (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3) + (Sqrt[3]*Cos[a +
b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b)
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = -\frac{3\,\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx,\right.}{b}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} + \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+}}{\sqrt[3]{\sin(a+b}}\right.}{4b}$$

$$= \frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} - \frac{\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+b}{\sin^{\frac{2}{3}}(a+b}\right.}{4b}$$

Mathematica [C] time = 0.0279857, size = 55, normalized size = 0.24

$$\frac{3\sqrt[3]{\sin(a+bx)}\sqrt[6]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{6};\frac{7}{6};\sin^2(a+bx)\right)}{b\sqrt[3]{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/6, 7/6, Sin[a + b*x]^2]*
Sin[a + b*x]^(1/3))/(b*Cos[a + b*x]^(1/3))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int (\cos (bx + a))^{\frac{2}{3}} (\sin (bx + a))^{-\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x)
```

```
[Out] int(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (bx + a)^{\frac{2}{3}}}{\sin (bx + a)^{\frac{2}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(2/3)/sin(b*x + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{2}{3}} (a + bx)}{\sin^{\frac{2}{3}} (a + bx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(2/3)/sin(b*x+a)**(2/3),x)
```

```
[Out] Integral(cos(a + b*x)**(2/3)/sin(a + b*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^{\frac{2}{3}}}{\sin(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(2/3)/sin(b*x + a)^(2/3), x)
```

3.331 $\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx$

Optimal. Leaf size=250

$$-\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}-\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}+\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\cdots}}{\sqrt[3]{c\cdots}}\right.}{2b}$$

```
[Out] ArcTan[Sqrt[3] - (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/(2*b) - ArcTan[
Sqrt[3] + (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/(2*b) - ArcTan[Sin[a +
 b*x]^(1/3)/Cos[a + b*x]^(1/3)]/b - (Sqrt[3]*Log[1 - (Sqrt[3]*Sin[a + b*x]^
(1/3))/Cos[a + b*x]^(1/3) + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b) +
 (Sqrt[3]*Log[1 + (Sqrt[3]*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3) + Sin[a +
 b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b) - (3*Cos[a + b*x]^(1/3))/(b*Sin[a +
b*x]^(1/3))
```

Rubi [A] time = 0.325825, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2567, 2574, 295, 634, 618, 204, 628, 203}

$$-\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}-\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}+\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\cdots}}{\sqrt[3]{c\cdots}}\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3),x]
```

```
[Out] ArcTan[Sqrt[3] - (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/(2*b) - ArcTan[
Sqrt[3] + (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/(2*b) - ArcTan[Sin[a +
 b*x]^(1/3)/Cos[a + b*x]^(1/3)]/b - (Sqrt[3]*Log[1 - (Sqrt[3]*Sin[a + b*x]^
(1/3))/Cos[a + b*x]^(1/3) + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b) +
 (Sqrt[3]*Log[1 + (Sqrt[3]*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3) + Sin[a +
 b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b) - (3*Cos[a + b*x]^(1/3))/(b*Sin[a +
b*x]^(1/3))
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = -\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}} - \int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx$$

$$= -\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}} - \frac{3\,\mathrm{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}$$

$$= -\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}} - \frac{\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} - \ldots$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} - \frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}} - \frac{\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{4b} - \frac{\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{3}\ldots}\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} - \frac{\sqrt{3}\log\left(1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\sqrt{3}\log\left(1+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}}{\cos^{\frac{2}{3}}}\ldots\right.}{4b}$$

$$= \frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} - \frac{\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} - \frac{\sqrt{3}\log\left(1-\frac{\sqrt{3}\sqrt[3]{\sin\ldots}}{\sqrt[3]{\cos\ldots}}\right.}{}$$

Mathematica [C] time = 0.0337652, size = 55, normalized size = 0.22

$$-\frac{3\cos^2(a+bx)^{5/6}\,{}_2F_1\left(-\frac{1}{6},-\frac{1}{6};\frac{5}{6};\sin^2(a+bx)\right)}{b\sqrt[3]{\sin(a+bx)}\cos^{\frac{5}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3),x]
```

```
[Out] (-3*(Cos[a + b*x]^2)^(5/6)*Hypergeometric2F1[-1/6, -1/6, 5/6, Sin[a + b*x]^
2])/(b*Cos[a + b*x]^(5/3)*Sin[a + b*x]^(1/3))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int (\cos(bx+a))^{\frac{4}{3}}(\sin(bx+a))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x)
```

```
[Out] int(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^{\frac{4}{3}}}{\sin(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(4/3)/sin(b*x + a)^(4/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(4/3)/sin(b*x+a)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(bx+a\right)^{\frac{4}{3}}}{\sin\left(bx+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(4/3)/sin(b*x + a)^(4/3), x)
```

3.332 $\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx$

Optimal. Leaf size=155

$$-\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}+\frac{\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\log\left(\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sin[a + b*x]^(2/3))/Cos[a + b*x]^(2/3))/Sqrt[3]])/(
2*b) + Log[1 + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)]/(2*b) - Log[1 - Sin[a
 + b*x]^(2/3)/Cos[a + b*x]^(2/3) + Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3)]/(
4*b) - (3*Cos[a + b*x]^(2/3))/(2*b*Sin[a + b*x]^(2/3))
```

Rubi [A] time = 0.111131, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2567, 2574, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}+\frac{\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\log\left(\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(5/3)/Sin[a + b*x]^(5/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sin[a + b*x]^(2/3))/Cos[a + b*x]^(2/3))/Sqrt[3]])/(
2*b) + Log[1 + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)]/(2*b) - Log[1 - Sin[a
 + b*x]^(2/3)/Cos[a + b*x]^(2/3) + Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3)]/(
4*b) - (3*Cos[a + b*x]^(2/3))/(2*b*Sin[a + b*x]^(2/3))
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = -\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} - \int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx$$

$$= -\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}$$

$$= -\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\right.}{4}$$

$$= \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\cos^{\frac{2}{3}}(a+}{2b\sin^{\frac{2}{3}}(a+}$$

Mathematica [C] time = 0.0353784, size = 57, normalized size = 0.37

$$-\frac{3\cos^2(a+bx)^{2/3}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};\sin^2(a+bx)\right)}{2b\sin^{\frac{2}{3}}(a+bx)\cos^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(5/3)/Sin[a + b*x]^(5/3),x]
```

```
[Out] (-3*(Cos[a + b*x]^2)^(2/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, Sin[a + b*x]^
2])/(2*b*Cos[a + b*x]^(4/3)*Sin[a + b*x]^(2/3))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int (\cos(bx+a))^{\frac{5}{3}} (\sin(bx+a))^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x)
```

```
[Out] int(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos{(bx+a)}^{\frac{5}{3}}}{\sin{(bx+a)}^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(5/3)/sin(b*x + a)^(5/3), x)
```

Fricas [A] time = 3.19399, size = 555, normalized size = 3.58

$$-\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cos(bx+a)-2\sqrt{3}\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}}{3\cos(bx+a)}\right)\sin{(bx+a)}-2\log\left(\frac{\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)}{\cos(bx+a)}\right)\sin{(bx+a)}+\log}{4\,b\sin{(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*cos(b*x + a) - 2*sqrt(3)*cos(b*x + a)^
(1/3)*sin(b*x + a)^(2/3))/cos(b*x + a))*sin(b*x + a) - 2*log((cos(b*x + a)^
(1/3)*sin(b*x + a)^(2/3) + cos(b*x + a))/cos(b*x + a))*sin(b*x + a) + log((
cos(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)
*sin(b*x + a)^(4/3))/cos(b*x + a)^2)*sin(b*x + a) + 6*cos(b*x + a)^(2/3)*si
n(b*x + a)^(1/3))/(b*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(5/3)/sin(b*x+a)**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos{(bx+a)}^{\frac{5}{3}}}{\sin{(bx+a)}^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(5/3)/sin(b*x + a)^(5/3), x)
```

3.333 $\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx$

Optimal. Leaf size=155

$$-\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}+\frac{\log\left(\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*Cos[a + b*x]^(2/3))/Sin[a + b*x]^(2/3))/Sqrt[3]])/
(2*b) + Log[1 + Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3) - Cos[a + b*x]^(2/3)/
Sin[a + b*x]^(2/3)]/(4*b) - Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3)]/
(2*b) - (3*Cos[a + b*x]^(4/3))/(4*b*Sin[a + b*x]^(4/3))
```

Rubi [A] time = 0.13426, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2567, 2575, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}+\frac{\log\left(\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(7/3)/Sin[a + b*x]^(7/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*Cos[a + b*x]^(2/3))/Sin[a + b*x]^(2/3))/Sqrt[3]])/
(2*b) + Log[1 + Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3) - Cos[a + b*x]^(2/3)/
Sin[a + b*x]^(2/3)]/(4*b) - Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3)]/
(2*b) - (3*Cos[a + b*x]^(4/3))/(4*b*Sin[a + b*x]^(4/3))
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = -\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} - \int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx$$

$$= -\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= -\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{3\,\text{Subst}\left(\int \frac{1}{1-x+}\right.}{}$$

$$= \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)} - \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)} - \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cos^{\frac{4}{3}}(a}{4b\sin^{\frac{4}{3}}(a}$$

Mathematica [C] time = 0.0344197, size = 57, normalized size = 0.37

$$-\frac{3\sqrt[3]{\cos^2(a+bx)}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};\sin^2(a+bx)\right)}{4b\sin^{\frac{4}{3}}(a+bx)\cos^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(7/3)/Sin[a + b*x]^(7/3),x]
```

```
[Out] (-3*(Cos[a + b*x]^2)^(1/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, Sin[a + b*x]^
2])/(4*b*Cos[a + b*x]^(2/3)*Sin[a + b*x]^(4/3))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int (\cos(bx+a))^{\frac{7}{3}} (\sin(bx+a))^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x)
```

```
[Out] int(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^{\frac{7}{3}}}{\sin(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(7/3)/sin(b*x + a)^(7/3), x)
```

Fricas [A] time = 2.94877, size = 621, normalized size = 4.01

$$\frac{2\left(\sqrt{3}\cos(bx+a)^2-\sqrt{3}\right)\arctan\left(\frac{2\sqrt{3}\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}-\sqrt{3}\sin(bx+a)}{3\sin(bx+a)}\right)+\left(\cos(bx+a)^2-1\right)\log\left(\frac{4\left(\cos(bx+a)^2-\cos(bx+a)^{\frac{4}{3}}\sin\cdots\right.}{\cos\cdots}\right.}{4\left(b\cos(b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(3)*cos(b*x + a)^2 - sqrt(3))*arctan(1/3*(2*sqrt(3)*cos(b*x + a
)^(2/3)*sin(b*x + a)^(1/3) - sqrt(3)*sin(b*x + a))/sin(b*x + a)) + (cos(b*x
 + a)^2 - 1)*log(4*(cos(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3)
+ cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3) - 1)/(cos(b*x + a)^2 - 1)) - 2*(cos
(b*x + a)^2 - 1)*log(-2*(cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3) + sin(b*x +
a))/sin(b*x + a)) + 3*cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3))/(b*cos(b*x + a
)^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(7/3)/sin(b*x+a)**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^{\frac{7}{3}}}{\sin(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(7/3)/sin(b*x + a)^(7/3), x)
```

3.334 $\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx$

Optimal. Leaf size=16

$$-\frac{3\cos^{\frac{5}{3}}(x)}{5\sin^{\frac{5}{3}}(x)}$$

```
[Out] (-3*Cos[x]^(5/3))/(5*Sin[x]^(5/3))
```

Rubi [A] time = 0.0220673, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2563}

$$-\frac{3\cos^{\frac{5}{3}}(x)}{5\sin^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^(2/3)/Sin[x]^(8/3),x]
```

```
[Out] (-3*Cos[x]^(5/3))/(5*Sin[x]^(5/3))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx = -\frac{3\cos^{\frac{5}{3}}(x)}{5\sin^{\frac{5}{3}}(x)}$$

Mathematica [A] time = 0.010515, size = 16, normalized size = 1.

$$-\frac{3\cos^{\frac{5}{3}}(x)}{5\sin^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^(2/3)/Sin[x]^(8/3),x]
```

```
[Out] (-3*Cos[x]^(5/3))/(5*Sin[x]^(5/3))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int (\cos(x))^{\frac{2}{3}} (\sin(x))^{-\frac{8}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^(2/3)/sin(x)^(8/3),x)
```

```
[Out] int(cos(x)^(2/3)/sin(x)^(8/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)^{\frac{2}{3}}}{\sin(x)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(2/3)/sin(x)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)^(2/3)/sin(x)^(8/3), x)
```

Fricas [A] time = 2.45454, size = 62, normalized size = 3.88

$$\frac{3 \cos(x)^{\frac{5}{3}} \sin(x)^{\frac{1}{3}}}{5 \left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(2/3)/sin(x)^(8/3),x, algorithm="fricas")
```

```
[Out] 3/5*cos(x)^(5/3)*sin(x)^(1/3)/(cos(x)^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**(2/3)/sin(x)**(8/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)^{\frac{2}{3}}}{\sin(x)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(2/3)/sin(x)^(8/3),x, algorithm="giac")
```

```
[Out] integrate(cos(x)^(2/3)/sin(x)^(8/3), x)
```

3.335 $\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx$

Optimal. Leaf size=16

$$\frac{3\sin^{\frac{5}{3}}(x)}{5\cos^{\frac{5}{3}}(x)}$$

```
[Out] (3*Sin[x]^(5/3))/(5*Cos[x]^(5/3))
```

Rubi [A] time = 0.0231872, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2563}

$$\frac{3\sin^{\frac{5}{3}}(x)}{5\cos^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^(2/3)/Cos[x]^(8/3),x]

[Out] (3*Sin[x]^(5/3))/(5*Cos[x]^(5/3))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx = \frac{3\sin^{\frac{5}{3}}(x)}{5\cos^{\frac{5}{3}}(x)}$$

Mathematica [A] time = 0.0152747, size = 16, normalized size = 1.

$$\frac{3\sin^{\frac{5}{3}}(x)}{5\cos^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^(2/3)/Cos[x]^(8/3),x]

[Out] (3*Sin[x]^(5/3))/(5*Cos[x]^(5/3))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int (\sin (x))^{\frac{2}{3}} (\cos (x))^{-\frac{8}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^(2/3)/cos(x)^(8/3),x)
```

```
[Out] int(sin(x)^(2/3)/cos(x)^(8/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (x)^{\frac{2}{3}}}{\cos (x)^{\frac{8}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(2/3)/cos(x)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^(2/3)/cos(x)^(8/3), x)
```

Fricas [A] time = 2.38153, size = 42, normalized size = 2.62

$$\frac{3 \sin (x)^{\frac{5}{3}}}{5 \cos (x)^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(2/3)/cos(x)^(8/3),x, algorithm="fricas")
```

```
[Out] 3/5*sin(x)^(5/3)/cos(x)^(5/3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**(2/3)/cos(x)**(8/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (x)^{\frac{2}{3}}}{\cos (x)^{\frac{8}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(2/3)/cos(x)^(8/3),x, algorithm="giac")
```

```
[Out] integrate(sin(x)^(2/3)/cos(x)^(8/3), x)
```

3.336 $\int \cos^n(e+fx)\sin^m(e+fx)\,dx$

Optimal. Leaf size=80

$$-\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}\cos^{n+1}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{f(n+1)}$$

```
[Out] -((Cos[e + f*x]^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 + n)/2,
Cos[e + f*x]^2]*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(f*(1 +
 n)))
```

Rubi [A] time = 0.0419246, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2576}

$$-\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}\cos^{n+1}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^n*Sin[e + f*x]^m,x]
```

```
[Out] -((Cos[e + f*x]^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 + n)/2,
Cos[e + f*x]^2]*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(f*(1 +
 n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx=-\frac{\cos^{1+n}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)\sin^{-1+m}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}}{f(1+n)}$$

Mathematica [A] time = 0.108206, size = 79, normalized size = 0.99

$$\frac{\sin^{m+1}(e+fx)\cos^{n-1}(e+fx)\cos^2(e+fx)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^n*Sin[e + f*x]^m,x]
```

```
[Out] (Cos[e + f*x]^(-1 + n)*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F1[(1 +
m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x]^(1 + m))/(f*(1 + m
```

```
))
```

Maple [F] time = 0.562, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{n} \left(\sin\left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^n*sin(f*x+e)^m,x)
```

```
[Out] int(cos(f*x+e)^n*sin(f*x+e)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)^{n} \sin\left(fx+e\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^n*sin(f*x+e)^m,x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^n*sin(f*x + e)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cos\left(fx+e\right)^{n} \sin\left(fx+e\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^n*sin(f*x+e)^m,x, algorithm="fricas")
```

```
[Out] integral(cos(f*x + e)^n*sin(f*x + e)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^{m}\left(e+fx\right)\cos^{n}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**n*sin(f*x+e)**m,x)
```

```
[Out] Integral(sin(e + f*x)**m*cos(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)^{n} \sin\left(fx+e\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^n*sin(f*x+e)^m,x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^n*sin(f*x + e)^m, x)
```

3.337 $\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx$

Optimal. Leaf size=85

$$-\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)}$$

```
[Out] -(((d*Cos[e + f*x])^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 + n)
/2, Cos[e + f*x]^2]*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(d*
f*(1 + n)))
```

Rubi [A] time = 0.0445437, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2576}

$$-\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*Sin[e + f*x]^m,x]
```

```
[Out] -(((d*Cos[e + f*x])^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 + n)
/2, Cos[e + f*x]^2]*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(d*
f*(1 + n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx = -\frac{(d\cos(e+fx))^{1+n}\,{}_2F_1\left(\frac{1-m}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)\sin^{-1+m}(e+fx)\sin^2(e+fx)^{1}}{df(1+n)}$$

Mathematica [A] time = 0.116539, size = 82, normalized size = 0.96

$$\frac{d\sin^{m+1}(e+fx)\cos^2(e+fx)^{\frac{1-n}{2}}(d\cos(e+fx))^{n-1}\,{}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*Sin[e + f*x]^m,x]
```

```
[Out] (d*(d*Cos[e + f*x])^(-1 + n)*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F1
[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x]^(1 + m))/(f*
```

```
(1 + m))
```

Maple [F] time = 0.573, size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^{n}\left(\sin\left(fx+e\right)\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*sin(f*x+e)^m,x)
```

```
[Out] int((d*cos(f*x+e))^n*sin(f*x+e)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*sin(f*x+e)^m,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^n*sin(f*x + e)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d\cos\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*sin(f*x+e)^m,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^n*sin(f*x + e)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(e+fx\right)\right)^{n}\sin^{m}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*sin(f*x+e)**m,x)
```

```
[Out] Integral((d*cos(e + f*x))**n*sin(e + f*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*sin(f*x+e)^m,x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^n*sin(f*x + e)^m, x)
```

3.338 $\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx$

Optimal. Leaf size=83

$$-\frac{b\sin^2(e+fx)^{\frac{1-m}{2}}\cos^{n+1}(e+fx)(b\sin(e+fx))^{m-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{f(n+1)}$$

```
[Out] -((b*Cos[e + f*x]^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 + n)/2
, Cos[e + f*x]^2]*(b*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(
f*(1 + n)))
```

Rubi [A] time = 0.0432349, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2576}

$$-\frac{b\sin^2(e+fx)^{\frac{1-m}{2}}\cos^{n+1}(e+fx)(b\sin(e+fx))^{m-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^n*(b*Sin[e + f*x])^m,x]
```

```
[Out] -((b*Cos[e + f*x]^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 + n)/2
, Cos[e + f*x]^2]*(b*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(
f*(1 + n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx = -\frac{b\cos^{1+n}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)(b\sin(e+fx))^{-1+m}\sin^2(e+}{f(1+n)}$$

Mathematica [A] time = 0.0763074, size = 85, normalized size = 1.02

$$\frac{\sin(e+fx)\cos^{n-1}(e+fx)\cos^2(e+fx)^{\frac{1-n}{2}}(b\sin(e+fx))^m\,{}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^n*(b*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]^(-1 + n)*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F1[(1 +
m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*Sin[e + f*x])^m
```

```
)/(f*(1 + m))
```

Maple [F] time = 0.399, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{n} \left(b\sin\left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^n*(b*sin(f*x+e))^m,x)
```

```
[Out] int(cos(f*x+e)^n*(b*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{m} \cos\left(fx+e\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^n*(b*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^m*cos(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sin\left(fx+e\right)\right)^{m}\cos\left(fx+e\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^n*(b*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^m*cos(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(e+fx\right)\right)^{m} \cos^{n}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**n*(b*sin(f*x+e))**m,x)
```

```
[Out] Integral((b*sin(e + f*x))**m*cos(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{m} \cos\left(fx+e\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^n*(b*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^m*cos(f*x + e)^n, x)
```

3.339 $\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx$

Optimal. Leaf size=88

$$-\frac{b\sin^2(e+fx)^{\frac{1-m}{2}}(b\sin(e+fx))^{m-1}(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)}$$

```
[Out] -((b*(d*Cos[e + f*x])^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 +
n)/2, Cos[e + f*x]^2]*(b*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2
))/(d*f*(1 + n)))
```

Rubi [A] time = 0.0480973, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2576}

$$-\frac{b\sin^2(e+fx)^{\frac{1-m}{2}}(b\sin(e+fx))^{m-1}(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(b*Sin[e + f*x])^m,x]
```

```
[Out] -((b*(d*Cos[e + f*x])^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 +
n)/2, Cos[e + f*x]^2]*(b*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2
))/(d*f*(1 + n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx = -\frac{b(d\cos(e+fx))^{1+n}\,{}_2F_1\left(\frac{1-m}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)(b\sin(e+fx))^{-1+m}\sin^2(e}{df(1+n)}$$

Mathematica [A] time = 0.094801, size = 85, normalized size = 0.97

$$\frac{\tan(e+fx)\cos^2(e+fx)^{\frac{1-n}{2}}(b\sin(e+fx))^m(d\cos(e+fx))^n\,{}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*(b*Sin[e + f*x])^m,x]
```

```
[Out] ((d*Cos[e + f*x])^n*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F1[(1 + m)/
2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*(b*Sin[e + f*x])^m*Tan[e + f*x])/(
```

```
f*(1 + m))
```

Maple [F] time = 0.432, size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^{n} \left(b\sin\left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x)
```

```
[Out] int((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^{n} \left(b\sin\left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^n*(b*sin(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d\cos\left(fx+e\right)\right)^{n} \left(b\sin\left(fx+e\right)\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^n*(b*sin(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(e+fx\right)\right)^{m} \left(d\cos\left(e+fx\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(b*sin(f*x+e))**m,x)
```

```
[Out] Integral((b*sin(e + f*x))**m*(d*cos(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^{n} \left(b\sin\left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^n*(b*sin(f*x + e))^m, x)
```

3.340 $\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=74

$$-\frac{2(c\sin(a+bx))^{m+3}}{bc^3(m+3)}+\frac{(c\sin(a+bx))^{m+5}}{bc^5(m+5)}+\frac{(c\sin(a+bx))^{m+1}}{bc(m+1)}$$

```
[Out] (c*Sin[a + b*x])^(1 + m)/(b*c*(1 + m)) - (2*(c*Sin[a + b*x])^(3 + m))/(b*c^
3*(3 + m)) + (c*Sin[a + b*x])^(5 + m)/(b*c^5*(5 + m))
```

Rubi [A] time = 0.0697001, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2564, 270}

$$-\frac{2(c\sin(a+bx))^{m+3}}{bc^3(m+3)}+\frac{(c\sin(a+bx))^{m+5}}{bc^5(m+5)}+\frac{(c\sin(a+bx))^{m+1}}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*(c*Sin[a + b*x])^m,x]
```

```
[Out] (c*Sin[a + b*x])^(1 + m)/(b*c*(1 + m)) - (2*(c*Sin[a + b*x])^(3 + m))/(b*c^
3*(3 + m)) + (c*Sin[a + b*x])^(5 + m)/(b*c^5*(5 + m))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx &= \frac{\text{Subst}\left(\int x^m\left(1-\frac{x^2}{c^2}\right)^2\,dx,x,c\sin(a+bx)\right)}{bc}\\ &= \frac{\text{Subst}\left(\int \left(x^m-\frac{2x^{2+m}}{c^2}+\frac{x^{4+m}}{c^4}\right)dx,x,c\sin(a+bx)\right)}{bc}\\ &= \frac{(c\sin(a+bx))^{1+m}}{bc(1+m)}-\frac{2(c\sin(a+bx))^{3+m}}{bc^3(3+m)}+\frac{(c\sin(a+bx))^{5+m}}{bc^5(5+m)}\end{aligned}$$

Mathematica [A] time = 0.315901, size = 55, normalized size = 0.74

$$\frac{\sin(a+bx)\left(\frac{\sin^4(a+bx)}{m+5}-\frac{2\sin^2(a+bx)}{m+3}+\frac{1}{m+1}\right)(c\sin(a+bx))^m}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sin[a + b*x]*(c*Sin[a + b*x])^m*((1 + m)^(-1) - (2*Sin[a + b*x]^2)/(3 + m)
 + Sin[a + b*x]^4/(5 + m)))/b
```

Maple [F] time = 1.115, size = 0, normalized size = 0.

$$\int (\cos (bx + a))^5 (c \sin (bx + a))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5*(c*sin(b*x+a))^m,x)
```

```
[Out] int(cos(b*x+a)^5*(c*sin(b*x+a))^m,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.60802, size = 177, normalized size = 2.39

$$\frac{\left(\left(m^2 + 4\,m + 3\right)\cos (bx + a)^4 + 4\,(m + 1)\cos (bx + a)^2 + 8\right)(c \sin (bx + a))^m \sin (bx + a)}{bm^3 + 9\,bm^2 + 23\,bm + 15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] ((m^2 + 4*m + 3)*cos(b*x + a)^4 + 4*(m + 1)*cos(b*x + a)^2 + 8)*(c*sin(b*x
+ a))^m*sin(b*x + a)/(b*m^3 + 9*b*m^2 + 23*b*m + 15*b)
```

Sympy [A] time = 79.3416, size = 2050, normalized size = 27.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5*(c*sin(b*x+a))**m,x)
```

```
[Out] Piecewise((x*(c*sin(a))**m*cos(a)**5, Eq(b, 0)), ((log(sin(a + b*x))/b + co
s(a + b*x)**2/(2*b*sin(a + b*x)**2) - cos(a + b*x)**4/(4*b*sin(a + b*x)**4)
)/c**5, Eq(m, -5)), ((16*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(
```

```
8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**
2) + 32*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x
/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 16*log(tan(a
/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan
(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 16*log(tan(a/2 + b*x/2))*tan(
a/2 + b*x/2)**6/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*t
an(a/2 + b*x/2)**2) - 32*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(8*b*tan
(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 16
*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(8*b*tan(a/2 + b*x/2)**6 + 16*b*
tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - tan(a/2 + b*x/2)**8/(8*b*t
an(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) +
18*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4
+ 8*b*tan(a/2 + b*x/2)**2) - 1/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*
x/2)**4 + 8*b*tan(a/2 + b*x/2)**2))/c**3, Eq(m, -3)), ((-log(tan(a/2 + b*x/
2)**2 + 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2
)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a
/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/
2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 6*
log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4
*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2
+ b) - 4*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/
2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b
*x/2)**2 + b) - log(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**8 + 4*b*t
an(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b)
 + log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*t
an(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b)
 + 4*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b
*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 +
b) + 6*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4
*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2
+ b) + 4*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 +
 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**
2 + b) + log(tan(a/2 + b*x/2))/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2
)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*tan(a/2 +
 b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 +
 b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*tan(a/2 + b*x/2)**4/(b*tan(a/
2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan
(a/2 + b*x/2)**2 + b) - 4*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*
tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b
))/c, Eq(m, -1)), (c**m*m**2*sin(a + b*x)*sin(a + b*x)**m*cos(a + b*x)**4/(
b*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 4*c**m*m*sin(a + b*x)**3*sin(a + b*x)*
*m*cos(a + b*x)**2/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 8*c**m*m*sin(a + b
*x)*sin(a + b*x)**m*cos(a + b*x)**4/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 8
*c**m*sin(a + b*x)**5*sin(a + b*x)**m/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) +
 20*c**m*sin(a + b*x)**3*sin(a + b*x)**m*cos(a + b*x)**2/(b*m**3 + 9*b*m**2
 + 23*b*m + 15*b) + 15*c**m*sin(a + b*x)*sin(a + b*x)**m*cos(a + b*x)**4/(b
*m**3 + 9*b*m**2 + 23*b*m + 15*b), True))
```

Giac [B] time = 1.12502, size = 335, normalized size = 4.53

$$\frac{(c\sin(bx+a))^m c^5 m^2 \sin(bx+a)^5 + 4\,(c\sin(bx+a))^m c^5 m \sin(bx+a)^5 - 2\,(c\sin(bx+a))^m c^5 m^2 \sin(bx+a)^3 + 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] ((c*sin(b*x + a))^m*c^5*m^2*sin(b*x + a)^5 + 4*(c*sin(b*x + a))^m*c^5*m*sin
(b*x + a)^5 - 2*(c*sin(b*x + a))^m*c^5*m^2*sin(b*x + a)^3 + 3*(c*sin(b*x +
a))^m*c^5*sin(b*x + a)^5 - 12*(c*sin(b*x + a))^m*c^5*m*sin(b*x + a)^3 + (c*
sin(b*x + a))^m*c^5*m^2*sin(b*x + a) - 10*(c*sin(b*x + a))^m*c^5*sin(b*x +
a)^3 + 8*(c*sin(b*x + a))^m*c^5*m*sin(b*x + a) + 15*(c*sin(b*x + a))^m*c^5*
sin(b*x + a))/((c^4*m^3 + 9*c^4*m^2 + 23*c^4*m + 15*c^4)*b*c)
```

3.341 $\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=50

$$\frac{(c\sin(a+bx))^{m+1}}{bc(m+1)}-\frac{(c\sin(a+bx))^{m+3}}{bc^3(m+3)}$$

```
[Out] (c*Sin[a + b*x])^(1 + m)/(b*c*(1 + m)) - (c*Sin[a + b*x])^(3 + m)/(b*c^3*(3
 + m))
```

Rubi [A] time = 0.0506607, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2564, 14}

$$\frac{(c\sin(a+bx))^{m+1}}{bc(m+1)}-\frac{(c\sin(a+bx))^{m+3}}{bc^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*(c*Sin[a + b*x])^m,x]
```

```
[Out] (c*Sin[a + b*x])^(1 + m)/(b*c*(1 + m)) - (c*Sin[a + b*x])^(3 + m)/(b*c^3*(3
 + m))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx &= \frac{\text{Subst}\left(\int x^m\left(1-\frac{x^2}{c^2}\right)dx,x,c\sin(a+bx)\right)}{bc}\\ &= \frac{\text{Subst}\left(\int\left(x^m-\frac{x^{2+m}}{c^2}\right)dx,x,c\sin(a+bx)\right)}{bc}\\ &= \frac{(c\sin(a+bx))^{1+m}}{bc(1+m)}-\frac{(c\sin(a+bx))^{3+m}}{bc^3(3+m)}\end{aligned}$$

Mathematica [A] time = 0.0845519, size = 48, normalized size = 0.96

$$\frac{\sin(a+bx)((m+1)\cos(2(a+bx))+m+5)(c\sin(a+bx))^m}{2b(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*(c*Sin[a + b*x])^m,x]
```

```
[Out] ((5 + m + (1 + m)*Cos[2*(a + b*x)])*Sin[a + b*x]*(c*Sin[a + b*x])^m)/(2*b*(
1 + m)*(3 + m))
```

Maple [F] time = 0.801, size = 0, normalized size = 0.

$$\int (\cos (bx + a))^3 (c \sin (bx + a))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*(c*sin(b*x+a))^m,x)
```

```
[Out] int(cos(b*x+a)^3*(c*sin(b*x+a))^m,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.45199, size = 113, normalized size = 2.26

$$\frac{\left((m+1) \cos (bx + a)^2 + 2\right) (c \sin (bx + a))^m \sin (bx + a)}{bm^2 + 4\, bm + 3\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] ((m + 1)*cos(b*x + a)^2 + 2)*(c*sin(b*x + a))^m*sin(b*x + a)/(b*m^2 + 4*b*m
 + 3*b)
```

Sympy [A] time = 13.1929, size = 530, normalized size = 10.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*(c*sin(b*x+a))**m,x)
```

```
[Out] Piecewise((x*(c*sin(a))**m*cos(a)**3, Eq(b, 0)), ((-log(sin(a + b*x))/b - c
os(a + b*x)**2/(2*b*sin(a + b*x)**2))/c**3, Eq(m, -3)), ((-log(tan(a/2 + b*
x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x
```

```
/2)**2 + b) - 2*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2
 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2)**2 + 1)/
(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2
))*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b
) + 2*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4 + 2*
b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2))/(b*tan(a/2 + b*x/2)**4 +
 2*b*tan(a/2 + b*x/2)**2 + b) - 2*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**
4 + 2*b*tan(a/2 + b*x/2)**2 + b))/c, Eq(m, -1)), (c**m*m*sin(a + b*x)*sin(a
 + b*x)**m*cos(a + b*x)**2/(b*m**2 + 4*b*m + 3*b) + 2*c**m*sin(a + b*x)**3*
sin(a + b*x)**m/(b*m**2 + 4*b*m + 3*b) + 3*c**m*sin(a + b*x)*sin(a + b*x)**
m*cos(a + b*x)**2/(b*m**2 + 4*b*m + 3*b), True))
```

Giac [B] time = 1.13056, size = 159, normalized size = 3.18

$$-\frac{\left(c\sin\left(bx+a\right)\right)^{m}c^{3}m\sin\left(bx+a\right)^{3}+\left(c\sin\left(bx+a\right)\right)^{m}c^{3}\sin\left(bx+a\right)^{3}-\left(c\sin\left(bx+a\right)\right)^{m}c^{3}m\sin\left(bx+a\right)-3\left(c\sin\left(b\right.\right.}{\left(c^{2}m^{2}+4c^{2}m+3c^{2}\right)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] -((c*sin(b*x + a))^m*c^3*m*sin(b*x + a)^3 + (c*sin(b*x + a))^m*c^3*sin(b*x
+ a)^3 - (c*sin(b*x + a))^m*c^3*m*sin(b*x + a) - 3*(c*sin(b*x + a))^m*c^3*s
in(b*x + a))/((c^2*m^2 + 4*c^2*m + 3*c^2)*b*c)
```

3.342 $\int \cos(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=24

$$\frac{(c\sin(a+bx))^{m+1}}{bc(m+1)}$$

```
[Out] (c*Sin[a + b*x])^(1 + m)/(b*c*(1 + m))
```

Rubi [A] time = 0.0248876, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 30}

$$\frac{(c\sin(a+bx))^{m+1}}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*(c*Sin[a + b*x])^m,x]

[Out] (c*Sin[a + b*x])^(1 + m)/(b*c*(1 + m))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)(c\sin(a+bx))^m\,dx &= \frac{\text{Subst}\left(\int x^m\,dx, x, c\sin(a+bx)\right)}{bc}\\ &= \frac{(c\sin(a+bx))^{1+m}}{bc(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0106087, size = 25, normalized size = 1.04

$$\frac{\sin(a+bx)(c\sin(a+bx))^m}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*(c*Sin[a + b*x])^m,x]

[Out] (Sin[a + b*x]*(c*Sin[a + b*x])^m)/(b*(1 + m))
```

Maple [A] time = 0., size = 25, normalized size = 1.

$$\frac{(c\sin(bx+a))^{1+m}}{bc(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*(c*sin(b*x+a))^m,x)
```

```
[Out] (c*sin(b*x+a))^(1+m)/b/c/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.45774, size = 58, normalized size = 2.42

$$\frac{(c\sin(bx+a))^m\sin(bx+a)}{bm+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] (c*sin(b*x + a))^m*sin(b*x + a)/(b*m + b)
```

Sympy [A] time = 1.67533, size = 58, normalized size = 2.42

$$\begin{cases} \frac{x\cos(a)}{c\sin(a)} & \text{for } b=0 \wedge m=-1 \\ x(c\sin(a))^m\cos(a) & \text{for } b=0 \\ \frac{\log(\sin(a+bx))}{bc} & \text{for } m=-1 \\ \frac{c^m\sin(a+bx)\sin^m(a+bx)}{bm+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(c*sin(b*x+a))**m,x)
```

```
[Out] Piecewise((x*cos(a)/(c*sin(a)), Eq(b, 0) & Eq(m, -1)), (x*(c*sin(a))**m*cos
(a), Eq(b, 0)), (log(sin(a + b*x))/(b*c), Eq(m, -1)), (c**m*sin(a + b*x)*si
n(a + b*x)**m/(b*m + b), True))
```

<u>Giac</u> [A] time = 1.13567, size = 32, normalized size = 1.33

$$\frac{(c\sin(bx+a))^{m+1}}{bc(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] (c*sin(b*x + a))^(m + 1)/(b*c*(m + 1))
```

3.343 $\int \sec(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=48

$$\frac{(c\sin(a+bx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x]
)^(1 + m))/(b*c*(1 + m))
```

Rubi [A] time = 0.0411817, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 364}

$$\frac{(c\sin(a+bx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x]
)^(1 + m))/(b*c*(1 + m))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \sec(a+bx)(c\sin(a+bx))^m\,dx &= \frac{\text{Subst}\left(\int \frac{x^m}{1-\frac{x^2}{c^2}}\,dx,x,c\sin(a+bx)\right)}{bc}\\ &= \frac{{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0215752, size = 51, normalized size = 1.06

$$\frac{\sin(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+1}{2}+1;\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, Sin[a + b*x]^2]*Sin[a + b*x
]*(c*Sin[a + b*x])^m)/(b*(1 + m))
```

Maple [F] time = 0.26, size = 0, normalized size = 0.

$$\int \sec(bx+a)\,(c\sin(bx+a))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*(c*sin(b*x+a))^m,x)
```

```
[Out] int(sec(b*x+a)*(c*sin(b*x+a))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^{m}\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((c\sin(bx+a))^{m}\sec(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^m*sec(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(a+bx))^{m}\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(c*sin(b*x+a))**m,x)
```

```
[Out] Integral((c*sin(a + b*x))**m*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^m \sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a), x)
```

3.344 $\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=48

$$\frac{(c\sin(a+bx))^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[Out] (Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x]
)^(1 + m))/(b*c*(1 + m))
```

Rubi [A] time = 0.046838, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2564, 364}

$$\frac{(c\sin(a+bx))^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x]
)^(1 + m))/(b*c*(1 + m))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx &= \frac{\text{Subst}\left(\int \frac{x^m}{\left(1-\frac{x^2}{c^2}\right)^2}\,dx,x,c\sin(a+bx)\right)}{bc}\\ &= \frac{{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0240761, size = 51, normalized size = 1.06

$$\frac{\sin(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+1}{2}+1;\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, Sin[a + b*x]^2]*Sin[a + b*x
]*(c*Sin[a + b*x])^m)/(b*(1 + m))
```

Maple [F] time = 0.259, size = 0, normalized size = 0.

$$\int (\sec (bx+a))^{3} (c \sin (bx+a))^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*(c*sin(b*x+a))^m,x)
```

```
[Out] int(sec(b*x+a)^3*(c*sin(b*x+a))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c \sin (bx+a))^{m} \sec (bx+a)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((c \sin (bx+a))^{m} \sec (bx+a)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^m*sec(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*(c*sin(b*x+a))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^m \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^3, x)
```

3.345 $\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=68

$$\frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[Cos[a + b*x]^2])
```

Rubi [A] time = 0.0409665, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2577}

$$\frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[Cos[a + b*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \frac{\cos(a+bx)\,{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.0533877, size = 63, normalized size = 0.93

$$\frac{\sqrt{\cos^2(a+bx)}\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, Sin[a +
 b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Maple [F] time = 0.885, size = 0, normalized size = 0.

$$\int (\cos (bx+a))^4 (c \sin (bx+a))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

```
[Out] int(cos(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c \sin (bx+a))^m \cos (bx+a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m*cos(b*x + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((c \sin (bx+a))^m \cos (bx+a)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^m*cos(b*x + a)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4*(c*sin(b*x+a))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c \sin (bx+a))^m \cos (bx+a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m*cos(b*x + a)^4, x)
```

3.346 $\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=68

$$\frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[Cos[a + b*x]^2])
```

Rubi [A] time = 0.0406509, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2577}

$$\frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[Cos[a + b*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx=\frac{\cos(a+bx)\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.047282, size = 63, normalized size = 0.93

$$\frac{\sqrt{\cos^2(a+bx)}\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, Sin[a +
 b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Maple [F] time = 0.783, size = 0, normalized size = 0.

$$\int \left(\cos\left(bx+a\right)\right)^{2}\left(c\sin\left(bx+a\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

```
[Out] int(cos(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx+a\right)\right)^{m}\cos\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m*cos(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(c\sin\left(bx+a\right)\right)^{m}\cos\left(bx+a\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^m*cos(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*(c*sin(b*x+a))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx+a\right)\right)^{m}\cos\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m*cos(b*x + a)^2, x)
```

3.347 $\int (c\sin(a+bx))^m\,dx$

Optimal. Leaf size=68

$$\frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[Cos[a + b*x]^2])
```

Rubi [A] time = 0.0146984, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2643}

$$\frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^m,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (c\sin(a+bx))^m\,dx = \frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.0389735, size = 63, normalized size = 0.93

$$\frac{\sqrt{\cos^2(a+bx)}\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Maple [F] time = 0.457, size = 0, normalized size = 0.

$$\int (c \sin (bx + a))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^m,x)
```

```
[Out] int((c*sin(b*x+a))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c \sin (bx + a))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((c \sin (bx + a))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c \sin (a + bx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**m,x)
```

```
[Out] Integral((c*sin(a + b*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c \sin (bx + a))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m, x)
```

3.348 $\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{\cos^2(a+bx)}\sec(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*Sec[a + b*x]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rubi [A] time = 0.0401584, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2577}

$$\frac{\sqrt{\cos^2(a+bx)}\sec(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*Sec[a + b*x]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx=\frac{\sqrt{\cos^2(a+bx)}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)\sec(a+bx)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

Mathematica [A] time = 0.0467693, size = 63, normalized size = 0.93

$$\frac{\sqrt{\cos^2(a+bx)}\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Maple [F] time = 0.349, size = 0, normalized size = 0.

$$\int \left(\sec \left(bx+a\right)\right)^{2} \left(c \sin \left(bx+a\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

```
[Out] int(sec(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin \left(bx+a\right)\right)^{m} \sec \left(bx+a\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(c \sin \left(bx+a\right)\right)^{m} \sec \left(bx+a\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^m*sec(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*(c*sin(b*x+a))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin \left(bx+a\right)\right)^{m} \sec \left(bx+a\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^2, x)
```

3.349 $\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{\cos^2(a+bx)}\sec(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*Sec[a + b*x]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rubi [A] time = 0.0399385, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2577}

$$\frac{\sqrt{\cos^2(a+bx)}\sec(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*Sec[a + b*x]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\,{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)\sec(a+bx)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

Mathematica [A] time = 0.0436321, size = 63, normalized size = 0.93

$$\frac{\sqrt{\cos^2(a+bx)}\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Maple [F] time = 0.3, size = 0, normalized size = 0.

$$\int (\sec (bx + a))^4 (c \sin (bx + a))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

```
[Out] int(sec(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c \sin (bx + a))^m \sec (bx + a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((c \sin (bx + a))^m \sec (bx + a)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^m*sec(b*x + a)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*(c*sin(b*x+a))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c \sin (bx + a))^m \sec (bx + a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^4, x)
```

3.350 $\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=75

$$\frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt[4]{\cos^2(a+bx)}}$$

```
[Out] (d*Sqrt[d*Cos[a + b*x]]*Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, Sin[a
 + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*(Cos[a + b*x]^2)^(1/4))
```

Rubi [A] time = 0.0546918, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2577}

$$\frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt[4]{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (d*Sqrt[d*Cos[a + b*x]]*Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, Sin[a
 + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*(Cos[a + b*x]^2)^(1/4))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx=\frac{d\sqrt{d\cos(a+bx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt[4]{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.139867, size = 78, normalized size = 1.04

$$\frac{d^2\cos^2(a+bx)^{3/4}\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(-\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (d^2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, S
in[a + b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m)*Sqrt[d*Cos[a + b
*x]])
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int (d\cos (bx+a))^{\frac{3}{2}} (c\sin (bx+a))^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

```
[Out] int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos (bx+a))^{\frac{3}{2}} (c\sin (bx+a))^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\cos (bx+a)}\,(c\sin (bx+a))^{m}\, d\cos (bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m*d*cos(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*(c*sin(b*x+a))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos (bx+a))^{\frac{3}{2}} (c\sin (bx+a))^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="giac")

[Out] integrate((d*cos(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

3.351 $\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=75

$$\frac{d\sqrt[4]{\cos^2(a+bx)}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{d\cos(a+bx)}}$$

```
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.0475906, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2577}

$$\frac{d\sqrt[4]{\cos^2(a+bx)}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^m,x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[d*Cos[a + b*x]])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx=\frac{d\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{d\cos(a+bx)}}$$

Mathematica [A] time = 0.06853, size = 75, normalized size = 1.

$$\frac{\sqrt[4]{\cos^2(a+bx)}\tan(a+bx)\sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,{}_2F_1\left(\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + m)
/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int \sqrt{d\cos(bx+a)}\,(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

```
[Out] int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cos(bx+a)}\,(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\cos(bx+a)}\,(c\sin(bx+a))^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(a+bx))^m\sqrt{d\cos(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**m,x)
```

```
[Out] Integral((c*sin(a + b*x))**m*sqrt(d*cos(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cos(bx+a)}\,(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m, x)
```

3.352 $\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=75

$$\frac{d\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)(d\cos(a+bx))^{3/2}}$$

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.0490397, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2577}

$$\frac{d\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^m/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*(d*Cos[a + b*x])^(3/2))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \frac{d\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)(d\cos(a+bx))^{3/2}}$$

Mathematica [A] time = 0.0712439, size = 75, normalized size = 1.

$$\frac{\cos^2(a+bx)^{3/4}\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^m/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m)*Sqrt[d*Cos[a + b*x]])
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int (c\sin (bx+a))^{m} \frac{1}{\sqrt{d\cos (bx+a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin (bx+a))^{m}}{\sqrt{d\cos (bx+a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m/sqrt(d*cos(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\cos (bx+a)}\,(c\sin (bx+a))^{m}}{d\cos (bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m/(d*cos(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin (a+bx))^{m}}{\sqrt{d\cos (a+bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**m/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral((c*sin(a + b*x))**m/sqrt(d*cos(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin (bx+a))^{m}}{\sqrt{d\cos (bx+a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m/sqrt(d*cos(b*x + a)), x)
```

3.353 $\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt[4]{\cos^2(a+bx)}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{5}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)\sqrt{d\cos(a+bx)}}$$

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m)*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.0578259, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2577}

$$\frac{\sqrt[4]{\cos^2(a+bx)}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{5}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m)*Sqrt[d*Cos[a + b*x]])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{5}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bcd(1+m)\sqrt{d\cos(a+bx)}}$$

Mathematica [A] time = 0.0928429, size = 78, normalized size = 1.01

$$\frac{\sqrt[4]{\cos^2(a+bx)}\tan(a+bx)\sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,{}_2F_1\left(\frac{5}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bd^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, (1 + m)
/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*d^2*(1 +
 m))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int (c \sin (bx + a))^m (d \cos (bx + a))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sin (bx + a))^m}{(d \cos (bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d \cos (bx + a)}\,(c \sin (bx + a))^m}{d^2 \cos (bx + a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m/(d^2*cos(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sin (a + bx))^m}{(d \cos (a + bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**m/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Integral((c*sin(a + b*x))**m/(d*cos(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sin (bx + a))^m}{(d \cos (bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(3/2), x)
```

3.354 $\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{7}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)(d\cos(a+bx))^{3/2}}$$

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m)*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.0549027, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2577}

$$\frac{\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{7}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m)*(d*Cos[a + b*x])^(3/2))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{7}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bcd(1+m)(d\cos(a+bx))^{3/2}}$$

Mathematica [A] time = 0.0837122, size = 78, normalized size = 1.01

$$\frac{\cos^2(a+bx)^{3/4}\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(\frac{7}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bd^2(m+1)\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*d^2*(1 + m)*Sqrt[d*Cos[a + b*
x]])
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^m (d\cos(bx+a))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^m}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\cos(bx+a)}\,(c\sin(bx+a))^m}{d^3\cos(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m/(d^3*cos(b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**m/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^m}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(5/2), x)
```

3.355 $\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx$

Optimal. Leaf size=76

$$\frac{2(d\cos(a+bx))^{n+3}}{bd^3(n+3)}-\frac{(d\cos(a+bx))^{n+5}}{bd^5(n+5)}-\frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

```
[Out] -((d*Cos[a + b*x])^(1 + n)/(b*d*(1 + n))) + (2*(d*Cos[a + b*x])^(3 + n))/(b
*d^3*(3 + n)) - (d*Cos[a + b*x])^(5 + n)/(b*d^5*(5 + n))
```

Rubi [A] time = 0.0655888, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 270}

$$\frac{2(d\cos(a+bx))^{n+3}}{bd^3(n+3)}-\frac{(d\cos(a+bx))^{n+5}}{bd^5(n+5)}-\frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^5,x]
```

```
[Out] -((d*Cos[a + b*x])^(1 + n)/(b*d*(1 + n))) + (2*(d*Cos[a + b*x])^(3 + n))/(b
*d^3*(3 + n)) - (d*Cos[a + b*x])^(5 + n)/(b*d^5*(5 + n))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^n\sin^5(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^n\left(1-\frac{x^2}{d^2}\right)^2 dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{\text{Subst}\left(\int \left(x^n-\frac{2x^{2+n}}{d^2}+\frac{x^{4+n}}{d^4}\right) dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{(d\cos(a+bx))^{1+n}}{bd(1+n)}+\frac{2(d\cos(a+bx))^{3+n}}{bd^3(3+n)}-\frac{(d\cos(a+bx))^{5+n}}{bd^5(5+n)}\end{aligned}$$

Mathematica [A] time = 0.299578, size = 83, normalized size = 1.09

$$-\frac{\cos(a+bx)\left(-4\left(n^2+8n+7\right)\cos(2(a+bx))+\left(n^2+4n+3\right)\cos(4(a+bx))+3n^2+28n+89\right)(d\cos(a+bx))^n}{8b(n+1)(n+3)(n+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^5,x]
```

```
[Out] -(Cos[a + b*x]*(d*Cos[a + b*x])^n*(89 + 28*n + 3*n^2 - 4*(7 + 8*n + n^2)*Co
s[2*(a + b*x)] + (3 + 4*n + n^2)*Cos[4*(a + b*x)]))/(8*b*(1 + n)*(3 + n)*(5
 + n))
```

Maple [F] time = 0.893, size = 0, normalized size = 0.

$$\int (d\cos (bx+a))^{n} (\sin (bx+a))^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*sin(b*x+a)^5,x)
```

```
[Out] int((d*cos(b*x+a))^n*sin(b*x+a)^5,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.7898, size = 209, normalized size = 2.75

$$-\frac{\left((n^{2}+4\,n+3)\cos (bx+a)^{5}-2\,(n^{2}+6\,n+5)\cos (bx+a)^{3}+(n^{2}+8\,n+15)\cos (bx+a)\right)(d\cos (bx+a))^{n}}{bn^{3}+9\,bn^{2}+23\,bn+15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -((n^2 + 4*n + 3)*cos(b*x + a)^5 - 2*(n^2 + 6*n + 5)*cos(b*x + a)^3 + (n^2
+ 8*n + 15)*cos(b*x + a))*(d*cos(b*x + a))^n/(b*n^3 + 9*b*n^2 + 23*b*n + 15
*b)
```

Sympy [A] time = 86.7078, size = 2462, normalized size = 32.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((x*(d*cos(a))**n*sin(a)**5, Eq(b, 0)), ((-log(cos(a + b*x))/b + s
in(a + b*x)**4/(4*b*cos(a + b*x)**4) - sin(a + b*x)**2/(2*b*cos(a + b*x)**2
```

```
))/d**5, Eq(n, -5)), ((2*log(tan(a/2 + b*x/2) - 1)*tan(a/2 + b*x/2)**8/(b*t
an(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) - 4*log(tan(a/2 + b*x/2)
- 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 +
 b) + 2*log(tan(a/2 + b*x/2) - 1)/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*
x/2)**4 + b) + 2*log(tan(a/2 + b*x/2) + 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 +
 b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) - 4*log(tan(a/2 + b*x/2) + 1)*tan
(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) + 2*
log(tan(a/2 + b*x/2) + 1)/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4
+ b) - 2*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/
2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) + 4*log(tan(a/2 + b*x/2)**2 + 1)*tan(a
/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) - 2*lo
g(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4
 + b) + 4*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)
**4 + b) + 4*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x
/2)**4 + b))/d**3, Eq(n, -3)), ((-log(tan(a/2 + b*x/2) - 1)*tan(a/2 + b*x/2
)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2
)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2) - 1)*tan(a/2 +
 b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 +
 b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 6*log(tan(a/2 + b*x/2) - 1)*tan
(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan
(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2) -
1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6
*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2
) - 1)/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x
/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) + 1)*tan(a/2 +
 b*x/2)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 +
 b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2) + 1)*tan
(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan
(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 6*log(tan(a/2 + b*x/2) +
1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6
*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*x
/2) + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)*
*6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 +
 b*x/2) + 1)/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2
 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2)**2 + 1)*
tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*
tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + 4*log(tan(a/2 + b*x/2)
**2 + 1)*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)*
*6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + 6*log(tan(a/2
 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2
+ b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + 4*lo
g(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b
*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 +
b) + log(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*
x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 2*tan(a/
2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/
2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 8*tan(a/2 + b*x/2)**4/(b*tan
(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*
tan(a/2 + b*x/2)**2 + b) - 2*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4
*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2
+ b))/d, Eq(n, -1)), (-d**n*n**2*sin(a + b*x)**4*cos(a + b*x)*cos(a + b*x)*
*n/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b) - 8*d**n*n*sin(a + b*x)**4*cos(a + b
*x)*cos(a + b*x)**n/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b) - 4*d**n*n*sin(a +
b*x)**2*cos(a + b*x)**3*cos(a + b*x)**n/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b)
 - 15*d**n*sin(a + b*x)**4*cos(a + b*x)*cos(a + b*x)**n/(b*n**3 + 9*b*n**2
+ 23*b*n + 15*b) - 20*d**n*sin(a + b*x)**2*cos(a + b*x)**3*cos(a + b*x)**n/
(b*n**3 + 9*b*n**2 + 23*b*n + 15*b) - 8*d**n*cos(a + b*x)**5*cos(a + b*x)**
n/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b), True))
```

Giac [B] time = 1.1605, size = 336, normalized size = 4.42

$$-\frac{(d\cos(bx+a))^n d^5n^2\cos(bx+a)^5 + 4\,(d\cos(bx+a))^n d^5n\cos(bx+a)^5 - 2\,(d\cos(bx+a))^n d^5n^2\cos(bx+a)^3 + 3\,(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -((d*cos(b*x + a))^n*d^5*n^2*cos(b*x + a)^5 + 4*(d*cos(b*x + a))^n*d^5*n*co
s(b*x + a)^5 - 2*(d*cos(b*x + a))^n*d^5*n^2*cos(b*x + a)^3 + 3*(d*cos(b*x +
 a))^n*d^5*cos(b*x + a)^5 - 12*(d*cos(b*x + a))^n*d^5*n*cos(b*x + a)^3 + (d
*cos(b*x + a))^n*d^5*n^2*cos(b*x + a) - 10*(d*cos(b*x + a))^n*d^5*cos(b*x +
 a)^3 + 8*(d*cos(b*x + a))^n*d^5*n*cos(b*x + a) + 15*(d*cos(b*x + a))^n*d^5
*cos(b*x + a))/((d^4*n^3 + 9*d^4*n^2 + 23*d^4*n + 15*d^4)*b*d)
```

3.356 $\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx$

Optimal. Leaf size=50

$$\frac{(d\cos(a+bx))^{n+3}}{bd^3(n+3)}-\frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

```
[Out] -((d*Cos[a + b*x])^(1 + n)/(b*d*(1 + n))) + (d*Cos[a + b*x])^(3 + n)/(b*d^3
*(3 + n))
```

Rubi [A] time = 0.0500521, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 14}

$$\frac{(d\cos(a+bx))^{n+3}}{bd^3(n+3)}-\frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^3,x]
```

```
[Out] -((d*Cos[a + b*x])^(1 + n)/(b*d*(1 + n))) + (d*Cos[a + b*x])^(3 + n)/(b*d^3
*(3 + n))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int x^n\left(1-\frac{x^2}{d^2}\right)dx,x,d\cos(a+bx)\right)}{bd}\\ &= -\frac{\operatorname{Subst}\left(\int \left(x^n-\frac{x^{2+n}}{d^2}\right)dx,x,d\cos(a+bx)\right)}{bd}\\ &= -\frac{(d\cos(a+bx))^{1+n}}{bd(1+n)}+\frac{(d\cos(a+bx))^{3+n}}{bd^3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.117085, size = 50, normalized size = 1.

$$\frac{\cos(a+bx)((n+1)\cos(2(a+bx))-n-5)(d\cos(a+bx))^n}{2b(n+1)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^3,x]
```

```
[Out] (Cos[a + b*x]*(d*Cos[a + b*x])^n*(-5 - n + (1 + n)*Cos[2*(a + b*x)]))/(2*b*
(1 + n)*(3 + n))
```

Maple [C] time = 1.467, size = 1076, normalized size = 21.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*sin(b*x+a)^3,x)
```

```
[Out] -1/8*(n+9)/b/(1+n)/(3+n)*(exp(2*I*(b*x+a))+1)^n*(1/2)^n*d^n*exp(I*(b*x+a))^
(-n)*exp(-1/2*I*(Pi*n*csgn(I*cos(b*x+a))^3-Pi*n*csgn(I*exp(-I*(b*x+a)))*csg
n(I*cos(b*x+a))^2-Pi*n*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))^2-Pi
*n*csgn(I*cos(b*x+a))*csgn(I*d*cos(b*x+a))^2+Pi*n*csgn(I*d)*csgn(I*cos(b*x+
a))*csgn(I*d*cos(b*x+a))+Pi*n*csgn(I*exp(-I*(b*x+a)))*csgn(I*(exp(2*I*(b*x+
a))+1))*csgn(I*cos(b*x+a))+Pi*n*csgn(I*d*cos(b*x+a))^3-Pi*n*csgn(I*d)*csgn(
I*d*cos(b*x+a))^2+2*b*x+2*a))-1/8*(n+9)/b/(1+n)/(3+n)*(exp(2*I*(b*x+a))+1)^
n*(1/2)^n*d^n*exp(I*(b*x+a))^(-n)*exp(-1/2*I*(Pi*n*csgn(I*cos(b*x+a))^3-Pi*
n*csgn(I*exp(-I*(b*x+a)))*csgn(I*cos(b*x+a))^2-Pi*n*csgn(I*(exp(2*I*(b*x+a)
)+1))*csgn(I*cos(b*x+a))^2-Pi*n*csgn(I*cos(b*x+a))*csgn(I*d*cos(b*x+a))^2+P
i*n*csgn(I*d)*csgn(I*cos(b*x+a))*csgn(I*d*cos(b*x+a))+Pi*n*csgn(I*exp(-I*(b
*x+a)))*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))+Pi*n*csgn(I*d*cos(b
*x+a))^3-Pi*n*csgn(I*d)*csgn(I*d*cos(b*x+a))^2-2*b*x-2*a))+1/8/(b*n+3*b)*(e
xp(2*I*(b*x+a))+1)^n*(1/2)^n*d^n*exp(I*(b*x+a))^(-n)*exp(-1/2*I*(Pi*n*csgn(
I*cos(b*x+a))^3-Pi*n*csgn(I*exp(-I*(b*x+a)))*csgn(I*cos(b*x+a))^2-Pi*n*csgn
(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))^2-Pi*n*csgn(I*cos(b*x+a))*csgn(
I*d*cos(b*x+a))^2+Pi*n*csgn(I*d)*csgn(I*cos(b*x+a))*csgn(I*d*cos(b*x+a))+Pi
*n*csgn(I*exp(-I*(b*x+a)))*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))+
Pi*n*csgn(I*d*cos(b*x+a))^3-Pi*n*csgn(I*d)*csgn(I*d*cos(b*x+a))^2+6*b*x+6*a
))+1/8/(b*n+3*b)*(exp(2*I*(b*x+a))+1)^n*(1/2)^n*d^n*exp(I*(b*x+a))^(-n)*exp
(-1/2*I*(Pi*n*csgn(I*cos(b*x+a))^3-Pi*n*csgn(I*exp(-I*(b*x+a)))*csgn(I*cos(
b*x+a))^2-Pi*n*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))^2-Pi*n*csgn(
I*cos(b*x+a))*csgn(I*d*cos(b*x+a))^2+Pi*n*csgn(I*d)*csgn(I*cos(b*x+a))*csgn
(I*d*cos(b*x+a))+Pi*n*csgn(I*exp(-I*(b*x+a)))*csgn(I*(exp(2*I*(b*x+a))+1))*
csgn(I*cos(b*x+a))+Pi*n*csgn(I*d*cos(b*x+a))^3-Pi*n*csgn(I*d)*csgn(I*d*cos(
b*x+a))^2-6*b*x-6*a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.26167, size = 122, normalized size = 2.44

$$\frac{\left((n+1)\cos\left(bx+a\right)^3-(n+3)\cos\left(bx+a\right)\right)\left(d\cos\left(bx+a\right)\right)^n}{bn^2+4\,bn+3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((n + 1)*cos(b*x + a)^3 - (n + 3)*cos(b*x + a))*(d*cos(b*x + a))^n/(b*n^2 +
 4*b*n + 3*b)
```

Sympy [A] time = 14.6605, size = 694, normalized size = 13.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((x*(d*cos(a))**n*sin(a)**3, Eq(b, 0)), ((log(cos(a + b*x))/b + si
n(a + b*x)**2/(2*b*cos(a + b*x)**2))/d**3, Eq(n, -3)), ((-log(tan(a/2 + b*x
/2) - 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)*
*2 + b) - 2*log(tan(a/2 + b*x/2) - 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/
2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) - 1)/(b*tan(a/2
 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) + 1)*tan
(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - 2*
log(tan(a/2 + b*x/2) + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4 + 2*b*
tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) + 1)/(b*tan(a/2 + b*x/2)**4
 + 2*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*
x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) + 2*log(tan(a
/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/
2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**4 +
2*b*tan(a/2 + b*x/2)**2 + b) - 2*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4
 + 2*b*tan(a/2 + b*x/2)**2 + b))/d, Eq(n, -1)), (-d**n*n*sin(a + b*x)**2*co
s(a + b*x)*cos(a + b*x)**n/(b*n**2 + 4*b*n + 3*b) - 3*d**n*sin(a + b*x)**2*
cos(a + b*x)*cos(a + b*x)**n/(b*n**2 + 4*b*n + 3*b) - 2*d**n*cos(a + b*x)**
3*cos(a + b*x)**n/(b*n**2 + 4*b*n + 3*b), True))
```

Giac [B] time = 1.16013, size = 158, normalized size = 3.16

$$\frac{\left(d\cos\left(bx+a\right)\right)^{n}d^{3}n\cos\left(bx+a\right)^{3}+\left(d\cos\left(bx+a\right)\right)^{n}d^{3}\cos\left(bx+a\right)^{3}-\left(d\cos\left(bx+a\right)\right)^{n}d^{3}n\cos\left(bx+a\right)-3\left(d\cos\left(b\right.\right.}{\left(d^{2}n^{2}+4\,d^{2}n+3\,d^{2}\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] ((d*cos(b*x + a))^n*d^3*n*cos(b*x + a)^3 + (d*cos(b*x + a))^n*d^3*cos(b*x +
 a)^3 - (d*cos(b*x + a))^n*d^3*n*cos(b*x + a) - 3*(d*cos(b*x + a))^n*d^3*co
s(b*x + a))/((d^2*n^2 + 4*d^2*n + 3*d^2)*b*d)
```

3.357 $\int (d\cos(a+bx))^n \sin(a+bx)\,dx$

Optimal. Leaf size=25

$$-\frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

```
[Out] -((d*Cos[a + b*x])^(1 + n)/(b*d*(1 + n)))
```

Rubi [A] time = 0.022271, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 30}

$$-\frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Sin[a + b*x],x]
```

```
[Out] -((d*Cos[a + b*x])^(1 + n)/(b*d*(1 + n)))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^n \sin(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^n\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{(d\cos(a+bx))^{1+n}}{bd(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0100037, size = 26, normalized size = 1.04

$$-\frac{\cos(a+bx)(d\cos(a+bx))^n}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x],x]
```

```
[Out] -((Cos[a + b*x]*(d*Cos[a + b*x])^n)/(b*(1 + n)))
```

Maple [A] time = 0.003, size = 26, normalized size = 1.

$$-\frac{(d\cos(bx+a))^{1+n}}{bd(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*sin(b*x+a),x)
```

```
[Out] -(d*cos(b*x+a))^(1+n)/b/d/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2406, size = 59, normalized size = 2.36

$$-\frac{(d\cos(bx+a))^n\cos(bx+a)}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -(d*cos(b*x + a))^n*cos(b*x + a)/(b*n + b)
```

Sympy [A] time = 1.80926, size = 61, normalized size = 2.44

$$\begin{cases} \frac{x\sin(a)}{d\cos(a)} & \text{for } b = 0 \wedge n = -1 \\ x(d\cos(a))^n\sin(a) & \text{for } b = 0 \\ -\frac{\log(\cos(a+bx))}{bd} & \text{for } n = -1 \\ -\frac{d^n\cos(a+bx)\cos^n(a+bx)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*sin(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a)/(d*cos(a)), Eq(b, 0) & Eq(n, -1)), (x*(d*cos(a))**n*sin
(a), Eq(b, 0)), (-log(cos(a + b*x))/(b*d), Eq(n, -1)), (-d**n*cos(a + b*x)*
cos(a + b*x)**n/(b*n + b), True))
```

Giac [A] time = 1.15563, size = 34, normalized size = 1.36

$$-\frac{(d\cos(bx+a))^{n+1}}{bd(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -(d*cos(b*x + a))^(n + 1)/(b*d*(n + 1))
```

3.358 $\int (d\cos(a+bx))^n \csc(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{(d\cos(a+bx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2])/(b*d*(1 + n)))
```

Rubi [A] time = 0.0404455, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 364}

$$-\frac{(d\cos(a+bx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Csc[a + b*x],x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2])/(b*d*(1 + n)))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (d\cos(a+bx))^n \csc(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^n}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\
&= -\frac{(d\cos(a+bx))^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)}{bd(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.0330812, size = 52, normalized size = 1.06

$$-\frac{\cos(a+bx)(d\cos(a+bx))^n\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+1}{2}+1;\cos^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x],x]
```

```
[Out] -((Cos[a + b*x]*(d*Cos[a + b*x])^n*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 +
 n)/2, Cos[a + b*x]^2])/(b*(1 + n)))
```

Maple [F] time = 0.385, size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n \csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*csc(b*x+a),x)
```

```
[Out] int((d*cos(b*x+a))^n*csc(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n \csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((d\cos(bx+a))^n \csc(bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*cos(b*x + a))^n*csc(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(a+bx))^n \csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*csc(b*x+a),x)
```

```
[Out] Integral((d*cos(a + b*x))**n*csc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n \csc(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a), x)
```

3.359 $\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{(d\cos(a+bx))^{n+1}\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2])/(b*d*(1 + n)))
```

Rubi [A] time = 0.046728, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 364}

$$-\frac{(d\cos(a+bx))^{n+1}\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^3,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2])/(b*d*(1 + n)))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^n\csc^3(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^n}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{(d\cos(a+bx))^{1+n}\,{}_2F_1\left(2,\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)}{bd(1+n)}\end{aligned}$$

Mathematica [A] time = 0.216714, size = 86, normalized size = 1.76

$$\frac{d\csc^2(a+bx)\left(-\cot^2(a+bx)\right)^{\frac{1-n}{2}}(d\cos(a+bx))^{n-1}\,{}_2F_1\left(\frac{1-n}{2},\frac{3-n}{2};\frac{5-n}{2};\csc^2(a+bx)\right)}{b(n-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^3,x]
```

```
[Out] (d*(d*Cos[a + b*x])^(-1 + n)*(-Cot[a + b*x]^2)^((1 - n)/2)*Csc[a + b*x]^2*H
ypergeometric2F1[(1 - n)/2, (3 - n)/2, (5 - n)/2, Csc[a + b*x]^2])/(b*(-3 +
 n))
```

Maple [F] time = 0.391, size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n (\csc(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*csc(b*x+a)^3,x)
```

```
[Out] int((d*cos(b*x+a))^n*csc(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n \csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((d\cos(bx+a))^n \csc(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*cos(b*x + a))^n*csc(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n \csc(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^3, x)
```

3.360 $\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{(d\cos(a+bx))^{n+1}\,{}_2F_1\left(3,\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2])/(b*d*(1 + n)))
```

Rubi [A] time = 0.0463348, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 364}

$$-\frac{(d\cos(a+bx))^{n+1}\,{}_2F_1\left(3,\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^5,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2])/(b*d*(1 + n)))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^n}{\left(1-\frac{x^2}{d^2}\right)^3}\,dx,x,d\cos(a+bx)\right)}{bd}\\ &= -\frac{(d\cos(a+bx))^{1+n}\,{}_2F_1\left(3,\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)}{bd(1+n)}\end{aligned}$$

Mathematica [A] time = 0.193424, size = 86, normalized size = 1.76

$$\frac{d\csc^4(a+bx)\left(-\cot^2(a+bx)\right)^{\frac{1-n}{2}}(d\cos(a+bx))^{n-1}\,{}_2F_1\left(\frac{1-n}{2},\frac{5-n}{2};\frac{7-n}{2};\csc^2(a+bx)\right)}{b(n-5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^5,x]
```

```
[Out] (d*(d*Cos[a + b*x])^(-1 + n)*(-Cot[a + b*x]^2)^((1 - n)/2)*Csc[a + b*x]^4*H
ypergeometric2F1[(1 - n)/2, (5 - n)/2, (7 - n)/2, Csc[a + b*x]^2])/(b*(-5 +
 n))
```

Maple [F] time = 0.219, size = 0, normalized size = 0.

$$\int \left(d\cos\left(bx+a\right)\right)^{n}\left(\csc\left(bx+a\right)\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*csc(b*x+a)^5,x)
```

```
[Out] int((d*cos(b*x+a))^n*csc(b*x+a)^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(bx+a\right)\right)^{n}\csc\left(bx+a\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\cos\left(bx+a\right)\right)^{n}\csc\left(bx+a\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] integral((d*cos(b*x + a))^n*csc(b*x + a)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*csc(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n \csc(bx+a)^5\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^5,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^5, x)
```

3.361 $\int (d\cos(a+bx))^n \sin^4(a+bx)\,dx$

Optimal. Leaf size=69

$$-\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, C
os[a + b*x]^2]*Sin[a + b*x])/(b*d*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rubi [A] time = 0.0391226, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2576}

$$-\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^4,x]
```


```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, C
os[a + b*x]^2]*Sin[a + b*x])/(b*d*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```


Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(a+bx))^n \sin^4(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sin(a+bx)}{bd(1+n)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.112452, size = 68, normalized size = 0.99

$$-\frac{\sin(2(a+bx))(d\cos(a+bx))^n\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{2b(n+1)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^4,x]
```

```
[Out] -((d*Cos[a + b*x])^n*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Cos[a +
b*x]^2]*Sin[2*(a + b*x)])/(2*b*(1 + n)*Sqrt[Sin[a + b*x]^2])
```

Maple [F] time = 0.655, size = 0, normalized size = 0.

$$\int \left(d\cos\left(bx+a\right)\right)^{n}\left(\sin\left(bx+a\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*sin(b*x+a)^4,x)
```

```
[Out] int((d*cos(b*x+a))^n*sin(b*x+a)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(bx+a\right)\right)^{n}\sin\left(bx+a\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n*sin(b*x + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}+1\right)\left(d\cos\left(bx+a\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*(d*cos(b*x + a))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(bx+a\right)\right)^{n}\sin\left(bx+a\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n*sin(b*x + a)^4, x)
```

3.362 $\int (d\cos(a+bx))^n \sin^2(a+bx)\,dx$

Optimal. Leaf size=69

$$-\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-1/2, (1 + n)/2, (3 + n)/2, C
os[a + b*x]^2]*Sin[a + b*x])/(b*d*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rubi [A] time = 0.0396596, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2576}

$$-\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^2,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-1/2, (1 + n)/2, (3 + n)/2, C
os[a + b*x]^2]*Sin[a + b*x])/(b*d*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(a+bx))^n \sin^2(a+bx)\,dx=-\frac{(d\cos(a+bx))^{1+n}\,{}_2F_1\left(-\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sin(a+bx)}{bd(1+n)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0834605, size = 68, normalized size = 0.99

$$-\frac{\sin(2(a+bx))(d\cos(a+bx))^n\,{}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{2b(n+1)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^2,x]
```

```
[Out] -((d*Cos[a + b*x])^n*Hypergeometric2F1[-1/2, (1 + n)/2, (3 + n)/2, Cos[a +
b*x]^2]*Sin[2*(a + b*x)])/(2*b*(1 + n)*Sqrt[Sin[a + b*x]^2])
```

Maple [F] time = 0.992, size = 0, normalized size = 0.

$$\int (d\cos (bx+a))^n (\sin (bx+a))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*sin(b*x+a)^2,x)
```

```
[Out] int((d*cos(b*x+a))^n*sin(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos (bx+a))^n \sin (bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n*sin(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos (bx+a)^2-1\right)(d\cos (bx+a))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*(d*cos(b*x + a))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos (a+bx))^n \sin^2 (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*sin(b*x+a)**2,x)
```

```
[Out] Integral((d*cos(a + b*x))**n*sin(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos (bx+a))^n \sin (bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n*sin(b*x + a)^2, x)
```

3.363 $\int (d\cos(a+bx))^n\,dx$

Optimal. Leaf size=69

$$-\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Co
s[a + b*x]^2]*Sin[a + b*x])/(b*d*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rubi [A] time = 0.0154633, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2643}

$$-\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Co
s[a + b*x]^2]*Sin[a + b*x])/(b*d*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (d\cos(a+bx))^n\,dx=-\frac{(d\cos(a+bx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sin(a+bx)}{bd(1+n)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0426758, size = 64, normalized size = 0.93

$$-\frac{\sqrt{\sin^2(a+bx)}\cot(a+bx)(d\cos(a+bx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n,x]
```

```
[Out] -(((d*Cos[a + b*x])^n*Cot[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n
)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(1 + n)))
```

Maple [F] time = 0.254, size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n,x)
```

```
[Out] int((d*cos(b*x+a))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((d\cos(bx+a))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*cos(b*x + a))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(a+bx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n,x)
```

```
[Out] Integral((d*cos(a + b*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n,x, algorithm="giac")

[Out] integrate((d*cos(b*x + a))^n, x)
```

3.364 $\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{\sin^2(a+bx)}\csc(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Csc[a + b*x]*Hypergeometric2F1[3/2, (1 + n)/2,
(3 + n)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*d*(1 + n)))
```

Rubi [A] time = 0.0397905, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2576}

$$-\frac{\sqrt{\sin^2(a+bx)}\csc(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^2,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Csc[a + b*x]*Hypergeometric2F1[3/2, (1 + n)/2,
(3 + n)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*d*(1 + n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(a+bx))^n\csc^2(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\csc(a+bx)\,{}_2F_1\left(\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{bd(1+n)}$$

Mathematica [A] time = 0.176787, size = 80, normalized size = 1.16

$$\frac{d\csc(a+bx)\left(-\cot^2(a+bx)\right)^{\frac{1-n}{2}}(d\cos(a+bx))^{n-1}\,{}_2F_1\left(\frac{1-n}{2},1-\frac{n}{2};2-\frac{n}{2};\csc^2(a+bx)\right)}{b(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^2,x]
```

```
[Out] (d*(d*Cos[a + b*x])^(-1 + n)*(-Cot[a + b*x]^2)^((1 - n)/2)*Csc[a + b*x]*Hyp
ergeometric2F1[(1 - n)/2, 1 - n/2, 2 - n/2, Csc[a + b*x]^2])/(b*(-2 + n))
```

Maple [F] time = 0.359, size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n (\csc(bx+a))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*csc(b*x+a)^2,x)
```

```
[Out] int((d*cos(b*x+a))^n*csc(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n \csc(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((d\cos(bx+a))^n \csc(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*cos(b*x + a))^n*csc(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*csc(b*x+a)**2,x)
```

```
[Out] Integral((d*cos(a + b*x))**n*csc(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n \csc(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^2, x)
```

3.365 $\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{\sin^2(a+bx)}\csc(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Csc[a + b*x]*Hypergeometric2F1[5/2, (1 + n)/2,
(3 + n)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*d*(1 + n)))
```

Rubi [A] time = 0.0380494, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2576}

$$-\frac{\sqrt{\sin^2(a+bx)}\csc(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^4,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Csc[a + b*x]*Hypergeometric2F1[5/2, (1 + n)/2,
(3 + n)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*d*(1 + n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(a+bx))^n\csc^4(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\csc(a+bx)\,{}_2F_1\left(\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{bd(1+n)}$$

Mathematica [A] time = 0.212804, size = 82, normalized size = 1.19

$$\frac{d\csc^3(a+bx)\left(-\cot^2(a+bx)\right)^{\frac{1-n}{2}}(d\cos(a+bx))^{n-1}\,{}_2F_1\left(\frac{1-n}{2},2-\frac{n}{2};3-\frac{n}{2};\csc^2(a+bx)\right)}{b(n-4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^4,x]
```

```
[Out] (d*(d*Cos[a + b*x])^(-1 + n)*(-Cot[a + b*x]^2)^((1 - n)/2)*Csc[a + b*x]^3*H
ypergeometric2F1[(1 - n)/2, 2 - n/2, 3 - n/2, Csc[a + b*x]^2])/(b*(-4 + n))
```

Maple [F] time = 0.2, size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n (\csc(bx+a))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*csc(b*x+a)^4,x)
```

```
[Out] int((d*cos(b*x+a))^n*csc(b*x+a)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n \csc(bx+a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((d\cos(bx+a))^n \csc(bx+a)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] integral((d*cos(b*x + a))^n*csc(b*x + a)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*csc(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n \csc(bx+a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^4, x)
```

3.366 $\int (d\cos(a+bx))^n (c\sin(a+bx))^{5/2}\,dx$

Optimal. Leaf size=76

$$-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{3}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sin^2(a+bx)^{3/4}}$$

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-3/4, (1 + n)/2, (3 + n)/2,
 Cos[a + b*x]^2]*(c*Sin[a + b*x])^(3/2))/(b*d*(1 + n)*(Sin[a + b*x]^2)^(3/4
)))
```

Rubi [A] time = 0.0534167, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2576}

$$-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{3}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sin^2(a+bx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-3/4, (1 + n)/2, (3 + n)/2,
 Cos[a + b*x]^2]*(c*Sin[a + b*x])^(3/2))/(b*d*(1 + n)*(Sin[a + b*x]^2)^(3/4
)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{5/2}\,dx = -\frac{c(d\cos(a+bx))^{1+n}\,{}_2F_1\left(-\frac{3}{4},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)(c\sin(a+bx))^{3/2}}{bd(1+n)\sin^2(a+bx)^{3/4}}$$

Mathematica [B] time = 0.415062, size = 158, normalized size = 2.08

$$\frac{\cot(a+bx)(c\sin(a+bx))^{5/2}(d\cos(a+bx))^n\left((n+1)\cos^2(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{n+3}{2};\frac{n+5}{2};\cos^2(a+bx)\right)-(n+3)\,{}_2F_1\left(-\frac{3}{4},\frac{n+1}{2};\right.\right.}{2b(n+1)(n+3)\sin^2(a+bx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] ((d*Cos[a + b*x])^n*Cot[a + b*x]*(-((3 + n)*Hypergeometric2F1[-3/4, (1 + n)
/2, (3 + n)/2, Cos[a + b*x]^2]) - (3 + n)*Hypergeometric2F1[1/4, (1 + n)/2,
```

```
 (3 + n)/2, Cos[a + b*x]^2] + (1 + n)*Cos[a + b*x]^2*Hypergeometric2F1[1/4,
 (3 + n)/2, (5 + n)/2, Cos[a + b*x]^2])*(c*Sin[a + b*x])^(5/2))/(2*b*(1 + n
)*(3 + n)*(Sin[a + b*x]^2)^(3/4))
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{n} (c\sin(bx+a))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x)
```

```
[Out] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^{\frac{5}{2}} (d\cos(bx+a))^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)*(d*cos(b*x + a))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(c^{2}\cos(bx+a)^{2}-c^{2}\right)\sqrt{c\sin(bx+a)}\,(d\cos(bx+a))^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^
n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*(c*sin(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^{\frac{5}{2}} (d\cos(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)*(d*cos(b*x + a))^n, x)
```

3.367 $\int (d\cos(a+bx))^n (c\sin(a+bx))^{3/2}\,dx$

Optimal. Leaf size=76

$$-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{1}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt[4]{\sin^2(a+bx)}}$$

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-1/4, (1 + n)/2, (3 + n)/2,
 Cos[a + b*x]^2]*Sqrt[c*Sin[a + b*x]])/(b*d*(1 + n)*(Sin[a + b*x]^2)^(1/4))
)
```

Rubi [A] time = 0.0533769, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2576}

$$-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{1}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt[4]{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-1/4, (1 + n)/2, (3 + n)/2,
 Cos[a + b*x]^2]*Sqrt[c*Sin[a + b*x]])/(b*d*(1 + n)*(Sin[a + b*x]^2)^(1/4))
)
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{3/2}\,dx = -\frac{c(d\cos(a+bx))^{1+n}\,{}_2F_1\left(-\frac{1}{4},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sqrt{c\sin(a+bx)}}{bd(1+n)\sqrt[4]{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.165952, size = 76, normalized size = 1.

$$-\frac{\cot(a+bx)(c\sin(a+bx))^{3/2}(d\cos(a+bx))^n\,{}_2F_1\left(-\frac{1}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{b(n+1)\sqrt[4]{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -(((d*Cos[a + b*x])^n*Cot[a + b*x]*Hypergeometric2F1[-1/4, (1 + n)/2, (3 +
n)/2, Cos[a + b*x]^2]*(c*Sin[a + b*x])^(3/2))/(b*(1 + n)*(Sin[a + b*x]^2)^(
1/4)))
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n (c\sin(bx+a))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

```
[Out] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^{\frac{3}{2}} (d\cos(bx+a))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)*(d*cos(b*x + a))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c\sin(bx+a)}\,(d\cos(bx+a))^n\, c\sin(bx+a),\, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n*c*sin(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*(c*sin(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^{\frac{3}{2}} (d\cos(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)*(d*cos(b*x + a))^n, x)
```

3.368 $\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx$

Optimal. Leaf size=76

$$-\frac{c\sqrt[4]{\sin^2(a+bx)}(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{1}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{c\sin(a+bx)}}$$

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1/4, (1 + n)/2, (3 + n)/2,
Cos[a + b*x]^2]*(Sin[a + b*x]^2)^(1/4))/(b*d*(1 + n)*Sqrt[c*Sin[a + b*x]]))
```

Rubi [A] time = 0.0454629, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2576}

$$-\frac{c\sqrt[4]{\sin^2(a+bx)}(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{1}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1/4, (1 + n)/2, (3 + n)/2,
Cos[a + b*x]^2]*(Sin[a + b*x]^2)^(1/4))/(b*d*(1 + n)*Sqrt[c*Sin[a + b*x]]))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx=-\frac{c(d\cos(a+bx))^{1+n}\,{}_2F_1\left(\frac{1}{4},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sqrt[4]{\sin^2(a+bx)}}{bd(1+n)\sqrt{c\sin(a+bx)}}$$

Mathematica [A] time = 0.106635, size = 82, normalized size = 1.08

$$-\frac{\sin(a+bx)\cos(a+bx)\sqrt{c\sin(a+bx)}(d\cos(a+bx))^n\,{}_2F_1\left(\frac{1}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{b(n+1)\sin^2(a+bx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((Cos[a + b*x]*(d*Cos[a + b*x])^n*Hypergeometric2F1[1/4, (1 + n)/2, (3 + n
)/2, Cos[a + b*x]^2]*Sin[a + b*x]*Sqrt[c*Sin[a + b*x]])/(b*(1 + n)*(Sin[a +
 b*x]^2)^(3/4)))
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n \sqrt{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
```

```
[Out] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\sin(bx+a)}\,(d\cos(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c\sin(bx+a)}\,(d\cos(bx+a))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\sin(a+bx)}\,(d\cos(a+bx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(c*sin(a + b*x))*(d*cos(a + b*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\sin(bx+a)}\,(d\cos(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n, x)
```

3.369 $\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=76

$$-\frac{c\sin^2(a+bx)^{3/4}(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{3}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)(c\sin(a+bx))^{3/2}}$$

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[3/4, (1 + n)/2, (3 + n)/2,
Cos[a + b*x]^2]*(Sin[a + b*x]^2)^(3/4))/(b*d*(1 + n)*(c*Sin[a + b*x])^(3/2)
))
```

Rubi [A] time = 0.0484558, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2576}

$$-\frac{c\sin^2(a+bx)^{3/4}(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{3}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)(c\sin(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[3/4, (1 + n)/2, (3 + n)/2,
Cos[a + b*x]^2]*(Sin[a + b*x]^2)^(3/4))/(b*d*(1 + n)*(c*Sin[a + b*x])^(3/2)
))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = -\frac{c(d\cos(a+bx))^{1+n}\,{}_2F_1\left(\frac{3}{4},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sin^2(a+bx)^{3/4}}{bd(1+n)(c\sin(a+bx))^{3/2}}$$

Mathematica [A] time = 0.118294, size = 82, normalized size = 1.08

$$-\frac{\sin(a+bx)\cos(a+bx)(d\cos(a+bx))^n\,{}_2F_1\left(\frac{3}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{b(n+1)\sqrt[4]{\sin^2(a+bx)}\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((Cos[a + b*x]*(d*Cos[a + b*x])^n*Hypergeometric2F1[3/4, (1 + n)/2, (3 + n
)/2, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(1 + n)*Sqrt[c*Sin[a + b*x]]*(Sin[a +
```

```
b*x]^2)^(1/4)))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n \frac{1}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\cos(bx+a))^n}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c\sin(bx+a)}\,(d\cos(bx+a))^n}{c\sin(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n/(c*sin(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral((d*cos(a + b*x))**n/sqrt(c*sin(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\cos(bx+a))^n}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

3.370 $\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{\sqrt[4]{\sin^2(a+bx)}(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{5}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bcd(n+1)\sqrt{c\sin(a+bx)}}$$

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[5/4, (1 + n)/2, (3 + n)/2, Co
s[a + b*x]^2]*(Sin[a + b*x]^2)^(1/4))/(b*c*d*(1 + n)*Sqrt[c*Sin[a + b*x]]))
```

Rubi [A] time = 0.0562769, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2576}

$$-\frac{\sqrt[4]{\sin^2(a+bx)}(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{5}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bcd(n+1)\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[5/4, (1 + n)/2, (3 + n)/2, Co
s[a + b*x]^2]*(Sin[a + b*x]^2)^(1/4))/(b*c*d*(1 + n)*Sqrt[c*Sin[a + b*x]]))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx=-\frac{(d\cos(a+bx))^{1+n}\,{}_2F_1\left(\frac{5}{4},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sqrt[4]{\sin^2(a+bx)}}{bcd(1+n)\sqrt{c\sin(a+bx)}}$$

Mathematica [A] time = 0.136956, size = 79, normalized size = 1.01

$$-\frac{\sqrt[4]{\sin^2(a+bx)}\cot(a+bx)\sqrt{c\sin(a+bx)}(d\cos(a+bx))^n\,{}_2F_1\left(\frac{5}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bc^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -(((d*Cos[a + b*x])^n*Cot[a + b*x]*Hypergeometric2F1[5/4, (1 + n)/2, (3 + n
)/2, Cos[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*(Sin[a + b*x]^2)^(1/4))/(b*c^2*(1
 + n)))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^n (c\sin(bx+a))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
```

```
[Out] int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\cos(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{c\sin(bx+a)}\,(d\cos(bx+a))^n}{c^2\cos(bx+a)^2-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n/(c^2*cos(b*x + a)^2 - c^2
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n/(c*sin(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*cos(a + b*x))**n/(c*sin(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\cos(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

3.371 $\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx$

Optimal. Leaf size=85

$$\frac{2b^7}{13f(b\sec(e+fx))^{13/2}}-\frac{2b^5}{3f(b\sec(e+fx))^{9/2}}+\frac{6b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] (2*b^7)/(13*f*(b*Sec[e + f*x])^(13/2)) - (2*b^5)/(3*f*(b*Sec[e + f*x])^(9/2
)) + (6*b^3)/(5*f*(b*Sec[e + f*x])^(5/2)) - (2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.0592543, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$\frac{2b^7}{13f(b\sec(e+fx))^{13/2}}-\frac{2b^5}{3f(b\sec(e+fx))^{9/2}}+\frac{6b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^7,x]
```

```
[Out] (2*b^7)/(13*f*(b*Sec[e + f*x])^(13/2)) - (2*b^5)/(3*f*(b*Sec[e + f*x])^(9/2
)) + (6*b^3)/(5*f*(b*Sec[e + f*x])^(5/2)) - (2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx&=\frac{b^7\operatorname{Subst}\left(\int\frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{15/2}}\,dx,x,b\sec(e+fx)\right)}{f}\\&=\frac{b^7\operatorname{Subst}\left(\int\left(-\frac{1}{x^{15/2}}+\frac{3}{b^2x^{11/2}}-\frac{3}{b^4x^{7/2}}+\frac{1}{b^6x^{3/2}}\right)dx,x,b\sec(e+fx)\right)}{f}\\&=\frac{2b^7}{13f(b\sec(e+fx))^{13/2}}-\frac{2b^5}{3f(b\sec(e+fx))^{9/2}}+\frac{6b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.340371, size = 58, normalized size = 0.68

$$\frac{(-8939\cos(e+fx)+887\cos(3(e+fx))-155\cos(5(e+fx))+15\cos(7(e+fx)))\sqrt{b\sec(e+fx)}}{6240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^7,x]
```

```
[Out] ((-8939*Cos[e + f*x] + 887*Cos[3*(e + f*x)] - 155*Cos[5*(e + f*x)] + 15*Cos
[7*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(6240*f)
```

Maple [B] time = 0.283, size = 517, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^7*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/390/f*(-1+cos(f*x+e))^2*(60*cos(f*x+e)^7-260*cos(f*x+e)^5+468*cos(f*x+e)^
3+195*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-195*cos(f*x+e)*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)+195*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-195*ln(-(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-78
0*cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(1/2)/sin(f*x+e)^4
```

Maxima [A] time = 1.05537, size = 85, normalized size = 1.

$$\frac{2\left(15\,b^{6}-\frac{65\,b^{6}}{\cos\left(fx+e\right)^{2}}+\frac{117\,b^{6}}{\cos\left(fx+e\right)^{4}}-\frac{195\,b^{6}}{\cos\left(fx+e\right)^{6}}\right)b}{195\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/195*(15*b^6 - 65*b^6/cos(f*x + e)^2 + 117*b^6/cos(f*x + e)^4 - 195*b^6/co
s(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(13/2))
```

Fricas [A] time = 2.24026, size = 149, normalized size = 1.75

$$\frac{2\left(15\,\cos\left(fx+e\right)^{7}-65\,\cos\left(fx+e\right)^{5}+117\,\cos\left(fx+e\right)^{3}-195\,\cos\left(fx+e\right)\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{195\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/195*(15*cos(f*x + e)^7 - 65*cos(f*x + e)^5 + 117*cos(f*x + e)^3 - 195*cos
(f*x + e))*sqrt(b/cos(f*x + e))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**7*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17161, size = 146, normalized size = 1.72

$$\frac{2\left(15\sqrt{b\cos\left(fx+e\right)}b^{6}\cos\left(fx+e\right)^{6}-65\sqrt{b\cos\left(fx+e\right)}b^{6}\cos\left(fx+e\right)^{4}+117\sqrt{b\cos\left(fx+e\right)}b^{6}\cos\left(fx+e\right)^{2}-1\right.}{195\,b^{6}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/195*(15*sqrt(b*cos(f*x + e))*b^6*cos(f*x + e)^6 - 65*sqrt(b*cos(f*x + e))
*b^6*cos(f*x + e)^4 + 117*sqrt(b*cos(f*x + e))*b^6*cos(f*x + e)^2 - 195*sqr
t(b*cos(f*x + e))*b^6)*sgn(cos(f*x + e))/(b^6*f)
```

3.372 $\int \sqrt{b \sec(e+fx)} \sin^5(e+fx) \, dx$

Optimal. Leaf size=63

$$-\frac{2b^5}{9f(b\sec(e+fx))^{9/2}}+\frac{4b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] (-2*b^5)/(9*f*(b*Sec[e + f*x])^(9/2)) + (4*b^3)/(5*f*(b*Sec[e + f*x])^(5/2)
) - (2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.0497374, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$-\frac{2b^5}{9f(b\sec(e+fx))^{9/2}}+\frac{4b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^5,x]
```

```
[Out] (-2*b^5)/(9*f*(b*Sec[e + f*x])^(9/2)) + (4*b^3)/(5*f*(b*Sec[e + f*x])^(5/2)
) - (2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx &= \frac{b^5 \operatorname{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{11/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\
&= \frac{b^5 \operatorname{Subst}\left(\int \left(\frac{1}{x^{11/2}}-\frac{2}{b^2x^{7/2}}+\frac{1}{b^4x^{3/2}}\right)dx, x, b\sec(e+fx)\right)}{f}\\
&= -\frac{2b^5}{9f(b\sec(e+fx))^{9/2}}+\frac{4b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.205191, size = 48, normalized size = 0.76

$$-\frac{(554\cos(e+fx)-47\cos(3(e+fx))+5\cos(5(e+fx)))\sqrt{b\sec(e+fx)}}{360f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^5,x]
```

```
[Out] -((554*Cos[e + f*x] - 47*Cos[3*(e + f*x)] + 5*Cos[5*(e + f*x)])*Sqrt[b*Sec[
e + f*x]])/(360*f)
```

Maple [B] time = 0.137, size = 507, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/90/f*(-1+cos(f*x+e))^2*(20*cos(f*x+e)^5-72*cos(f*x+e)^3+45*cos(f*x+e)*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-1)/sin(f*x+e)^2)-45*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*
ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*c
os(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+45*ln(-(2
*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-45*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+180*cos(f*x+e))*(cos(f*x
+e)+1)^2*(b/cos(f*x+e))^(1/2)/sin(f*x+e)^4
```

Maxima [A] time = 1.03814, size = 68, normalized size = 1.08

$$-\frac{2\left(5\,b^4-\frac{18\,b^4}{\cos\left(fx+e\right)^2}+\frac{45\,b^4}{\cos\left(fx+e\right)^4}\right)b}{45\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/45*(5*b^4 - 18*b^4/cos(f*x + e)^2 + 45*b^4/cos(f*x + e)^4)*b/(f*(b/cos(f
*x + e))^(9/2))
```

Fricas [A] time = 2.21484, size = 117, normalized size = 1.86

$$-\frac{2\left(5\,\cos\left(fx+e\right)^5-18\,\cos\left(fx+e\right)^3+45\,\cos\left(fx+e\right)\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{45\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(5*cos(f*x + e)^5 - 18*cos(f*x + e)^3 + 45*cos(f*x + e))*sqrt(b/cos(f
*x + e))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14488, size = 112, normalized size = 1.78

$$-\frac{2\left(5\sqrt{b\cos\left(fx+e\right)}b^4\cos\left(fx+e\right)^4-18\sqrt{b\cos\left(fx+e\right)}b^4\cos\left(fx+e\right)^2+45\sqrt{b\cos\left(fx+e\right)}b^4\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{45\,b^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -2/45*(5*sqrt(b*cos(f*x + e))*b^4*cos(f*x + e)^4 - 18*sqrt(b*cos(f*x + e))*
b^4*cos(f*x + e)^2 + 45*sqrt(b*cos(f*x + e))*b^4)*sgn(cos(f*x + e))/(b^4*f)
```

3.373 $\int \sqrt{b \sec(e+fx)} \sin^3(e+fx)\,dx$

Optimal. Leaf size=41

$$\frac{2b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] (2*b^3)/(5*f*(b*Sec[e + f*x])^(5/2)) - (2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.0428639, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 14}

$$\frac{2b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^3,x]
```

```
[Out] (2*b^3)/(5*f*(b*Sec[e + f*x])^(5/2)) - (2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx&=\frac{b^3\,\text{Subst}\left(\int\frac{-1+\frac{x^2}{b^2}}{x^{7/2}}\,dx,x,b\sec(e+fx)\right)}{f}\\&=\frac{b^3\,\text{Subst}\left(\int\left(-\frac{1}{x^{7/2}}+\frac{1}{b^2x^{3/2}}\right)dx,x,b\sec(e+fx)\right)}{f}\\&=\frac{2b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.1598, size = 36, normalized size = 0.88

$$\frac{(\cos(3(e+fx))-17\cos(e+fx))\sqrt{b\sec(e+fx)}}{10f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^3,x]
```

```
[Out] ((-17*Cos[e + f*x] + Cos[3*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(10*f)
```

Maple [B] time = 0.148, size = 497, normalized size = 12.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/10/f*(-1+cos(f*x+e))^2*(4*cos(f*x+e)^3+5*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x
+e)^2)-5*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(2*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+5*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)
-5*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-20*cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e))
^(1/2)/sin(f*x+e)^4
```

Maxima [A] time = 1.00714, size = 47, normalized size = 1.15

$$\frac{2\left(b^2 - \frac{5\,b^2}{\cos\left(fx+e\right)^2}\right)b}{5\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b^2 - 5*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(5/2))
```

Fricas [A] time = 2.16809, size = 84, normalized size = 2.05

$$\frac{2\left(\cos\left(fx+e\right)^3 - 5\,\cos\left(fx+e\right)\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(cos(f*x + e)^3 - 5*cos(f*x + e))*sqrt(b/cos(f*x + e))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1112, size = 77, normalized size = 1.88

$$\frac{2\left(\sqrt{b\cos\left(fx+e\right)}b^2\cos\left(fx+e\right)^2-5\sqrt{b\cos\left(fx+e\right)}b^2\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{5\,b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(sqrt(b*cos(f*x + e))*b^2*cos(f*x + e)^2 - 5*sqrt(b*cos(f*x + e))*b^2)*
sgn(cos(f*x + e))/(b^2*f)
```

3.374 $\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx$

Optimal. Leaf size=18

$$-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] (-2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.0296487, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x],x]
```

```
[Out] (-2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx &= \frac{b\,\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= -\frac{2b}{f\sqrt{b\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0336027, size = 18, normalized size = 1.

$$-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x],x]
```

```
[Out] (-2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Maple [A] time = 0.019, size = 17, normalized size = 0.9

$$-2\frac{b}{f\sqrt{b\sec\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2*b/f/(b*sec(f*x+e))^(1/2)
```

Maxima [A] time = 1.01509, size = 31, normalized size = 1.72

$$-\frac{2\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(b/cos(f*x + e))*cos(f*x + e)/f
```

Fricas [A] time = 2.11954, size = 54, normalized size = 3.

$$-\frac{2\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b/cos(f*x + e))*cos(f*x + e)/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(e+fx\right)}\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))*sin(e + f*x), x)
```

Giac [A] time = 1.15806, size = 32, normalized size = 1.78

$$-\frac{2\sqrt{b\cos\left(fx+e\right)}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*cos(f*x + e))*sgn(cos(f*x + e))/f
```

3.375 $\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}$$

```
[Out] (Sqrt[b]*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/f - (Sqrt[b]*ArcTanh[Sqrt[b*
Sec[e + f*x]]/Sqrt[b]])/f
```

Rubi [A] time = 0.0458734, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2622, 329, 298, 203, 206}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[b]*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/f - (Sqrt[b]*ArcTanh[Sqrt[b*
Sec[e + f*x]]/Sqrt[b]])/f
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{bf} \\
&= \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{bf} \\
&= -\frac{b\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f} \\
&= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}
\end{aligned}
$$

Mathematica [A] time = 0.34326, size = 73, normalized size = 1.26

$$
\frac{\sqrt{b\sec(e+fx)}\left(\log\left(1-\sqrt{\sec(e+fx)}\right)-\log\left(\sqrt{\sec(e+fx)}+1\right)+2\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)\right)}{2f\sqrt{\sec(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] ((2*ArcTan[Sqrt[Sec[e + f*x]]] + Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqrt
[Sec[e + f*x]]])*Sqrt[b*Sec[e + f*x]])/(2*f*Sqrt[Sec[e + f*x]])
```

Maple [B] time = 0.122, size = 169, normalized size = 2.9

$$
\frac{\cos(fx+e)\left(-1+\cos(fx+e)\right)}{2f\left(\sin(fx+e)\right)^2}\sqrt{\frac{b}{\cos(fx+e)}}\left(\ln\left(-2\frac{1}{\left(\sin(fx+e)\right)^2}\left(2\left(\cos(fx+e)\right)^2\sqrt{-\frac{\cos(fx+e)}{\left(\cos(fx+e)+1\right)^2}}-(\text{co}\right.\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(b/cos(f*x+e))^(1/2)*cos(f*x+e)*(ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-1)/sin(f*x+e)^2)-arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)))*(-1+cos(f*x+e))/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.65422, size = 660, normalized size = 11.38

$$\left[\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos(fx+e)+1\right)}{2b}\right)+\sqrt{-b}\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}-6b\cos(fx+e)+b}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)}{4f}, -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1
)/b) + sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*s
qrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*co
s(f*x + e) + 1)))/f, -1/4*(2*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f
*x + e) - 1)/sqrt(b)) - sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 +
 cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*
x + e)^2 - 2*cos(f*x + e) + 1)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(e+fx\right)}\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))*csc(e + f*x), x)
```

Giac [A] time = 1.15376, size = 86, normalized size = 1.48

$$\frac{b^2\left(\frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b}-\frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}}\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] b^2*(arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b) - arctan(sqrt(b*cos
(f*x + e))/sqrt(b))/b^(3/2))*sgn(cos(f*x + e))/f
```

3.376 $\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal. Leaf size=93

$$-\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf}+\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}-\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}$$

```
[Out] (3*Sqrt[b]*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (3*Sqrt[b]*ArcTanh
[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (Cot[e + f*x]^2*(b*Sec[e + f*x])^(3
/2))/(2*b*f)
```

Rubi [A] time = 0.0758129, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2622, 288, 329, 298, 203, 206}

$$-\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf}+\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}-\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[b]*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (3*Sqrt[b]*ArcTanh
[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (Cot[e + f*x]^2*(b*Sec[e + f*x])^(3
/2))/(2*b*f)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^{5/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{4bf}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{2bf}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4f} + \frac{(3b}{}$$

$$= \frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{\cot^2(e+fx)(b\sec(e+fx}{2bf}$$

Mathematica [A] time = 0.612232, size = 95, normalized size = 1.02

$$-\frac{\sqrt{b\sec(e+fx)}\left(-3\log\left(1-\sqrt{\sec(e+fx)}\right)+3\log\left(\sqrt{\sec(e+fx)}+1\right)+\frac{4\csc^2(e+fx)}{\sqrt{\sec(e+fx)}}-6\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)\right)}{8f\sqrt{\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -((-6*ArcTan[Sqrt[Sec[e + f*x]]] - 3*Log[1 - Sqrt[Sec[e + f*x]]] + 3*Log[1
+ Sqrt[Sec[e + f*x]]] + (4*Csc[e + f*x]^2)/Sqrt[Sec[e + f*x]])*Sqrt[b*Sec[e
 + f*x]])/(8*f*Sqrt[Sec[e + f*x]])
```

Maple [B] time = 0.148, size = 603, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/8/f*(-1+cos(f*x+e))*(8*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
+16*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-3*cos(f*x+e)^2*arctan(1
/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+4*cos(f*x+e)^2*ln(-2*(2*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+8*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(3/2)+4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-4*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)+3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
))+ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-4*ln(-2*
(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2))*cos(f*x+e)*(b/
cos(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/sin(f*x+e)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66575, size = 941, normalized size = 10.12

$$\left[\frac{6\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos(fx+e)+1\right)}{2b}\right)+3\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2-\cos\cdots}{\cos(fx+e\cdots}\right.}{16\left(f\cos\left(fx+e\right)^2-f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(6*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x +
 e))*(cos(f*x + e) + 1)/b) + 3*(cos(f*x + e)^2 - 1)*sqrt(-b)*log((b*cos(f*x
 + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) -
 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*sqrt(b/co
s(f*x + e))*cos(f*x + e))/(f*cos(f*x + e)^2 - f), -1/16*(6*(cos(f*x + e)^2
- 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(b)) -
3*(cos(f*x + e)^2 - 1)*sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 +
cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x
 + e)^2 - 2*cos(f*x + e) + 1)) - 8*sqrt(b/cos(f*x + e))*cos(f*x + e))/(f*co
s(f*x + e)^2 - f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(e+fx\right)}\csc^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))*csc(e + f*x)**3, x)
```

Giac [A] time = 1.12906, size = 139, normalized size = 1.49

$$\frac{b^4\left(\frac{2\sqrt{b\cos(fx+e)}}{\left(b^2\cos(fx+e)^2-b^2\right)b^2}+\frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3}-\frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}}\right)\operatorname{sgn}(\cos(fx+e))}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b^4*(2*sqrt(b*cos(f*x + e))/((b^2*cos(f*x + e)^2 - b^2)*b^2) + 3*arctan
(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^3) - 3*arctan(sqrt(b*cos(f*x +
e))/sqrt(b))/b^(7/2))*sgn(cos(f*x + e))/f
```

3.377 $\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal. Leaf size=123

$$-\frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3f}-\frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf}+\frac{21\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f}-\frac{21\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f}$$

```
[Out] (21*Sqrt[b]*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) - (21*Sqrt[b]*ArcT
anh[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) - (7*Cot[e + f*x]^2*(b*Sec[e + f*
x])^(3/2))/(16*b*f) - (Cot[e + f*x]^4*(b*Sec[e + f*x])^(7/2))/(4*b^3*f)
```

Rubi [A] time = 0.0857987, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2622, 288, 329, 298, 203, 206}

$$-\frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3f}-\frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf}+\frac{21\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f}-\frac{21\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (21*Sqrt[b]*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) - (21*Sqrt[b]*ArcT
anh[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) - (7*Cot[e + f*x]^2*(b*Sec[e + f*
x])^(3/2))/(16*b*f) - (Cot[e + f*x]^4*(b*Sec[e + f*x])^(7/2))/(4*b^3*f)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^{9/2}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\sec(e+fx)\right)}{b^5 f}$$

$$= -\frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3 f} + \frac{7\,\text{Subst}\left(\int \frac{x^{5/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{8b^3 f}$$

$$= -\frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3 f} + \frac{21\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3 f} + \frac{21\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3 f} - \frac{(21b)\,\text{Subst}}{}$$

$$= \frac{21\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{21\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{7\cot^2(e+fx)(b\sec(e+}{16bf}$$

Mathematica [A] time = 0.902167, size = 107, normalized size = 0.87

$$\frac{b\left(-16\csc^4(e+fx) - 28\csc^2(e+fx) + 21\sqrt{\sec(e+fx)}\left(\log\left(1-\sqrt{\sec(e+fx)}\right) - \log\left(\sqrt{\sec(e+fx)}+1\right)\right) + 42\sqrt{\sec}\right.}{64f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (b*(-28*Csc[e + f*x]^2 - 16*Csc[e + f*x]^4 + 42*ArcTan[Sqrt[Sec[e + f*x]]]*
Sqrt[Sec[e + f*x]] + 21*(Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqrt[Sec[e +
 f*x]]])*Sqrt[Sec[e + f*x]]))/(64*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] time = 0.158, size = 1089, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/64/f*(72*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+56*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+32*cos(f*x+e)^3*ln(-2*(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-21*cos(f*x+e)^3*arctan(1/2/(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-11*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-104*cos(f*x+e)*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(3/2)+44*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-32*
cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)
^2)+21*cos(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+11*cos
(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-8
8*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-88*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-32*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-1)/sin(f*x+e)^2)+21*cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2))+11*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)+44*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+32*ln(-2*(2*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-21*arctan(1/2/(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2))-11*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/
sin(f*x+e)^2))*cos(f*x+e)*(b/cos(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)/sin(f*x+e)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.6856, size = 1173, normalized size = 9.54

$$\left[\frac{42\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}+1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos\left(fx+e\right)}}\left(\cos\left(fx+e\right)+1\right)}{2b}\right)+21\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}+\right.}{128\left(f\cos\left(fx+e\right)^{4}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/128*(42*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(1/2*sqrt
(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) + 21*(cos(f*x + e)^4 - 2*co
s(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(
f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x +
e)^2 + 2*cos(f*x + e) + 1)) + 8*(7*cos(f*x + e)^3 - 11*cos(f*x + e))*sqrt(b
/cos(f*x + e)))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f), -1/128*(42*(co
s(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e)
)*(cos(f*x + e) - 1)/sqrt(b)) - 21*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*
sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*s
qrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e
) + 1)) - 8*(7*cos(f*x + e)^3 - 11*cos(f*x + e))*sqrt(b/cos(f*x + e)))/(f*c
os(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15552, size = 181, normalized size = 1.47

$$\frac{b^6\left(\frac{21\arctan\left(\frac{\sqrt{b\cos\left(fx+e\right)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^5}-\frac{21\arctan\left(\frac{\sqrt{b\cos\left(fx+e\right)}}{\sqrt{b}}\right)}{b^{\frac{11}{2}}}+\frac{2\left(7\sqrt{b\cos\left(fx+e\right)}b^2\cos\left(fx+e\right)^2-11\sqrt{b\cos\left(fx+e\right)}b^2\right)}{\left(b^2\cos\left(fx+e\right)^2-b^2\right)^2b^4}\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*b^6*(21*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^5) - 21*arct
an(sqrt(b*cos(f*x + e))/sqrt(b))/b^(11/2) + 2*(7*sqrt(b*cos(f*x + e))*b^2*c
os(f*x + e)^2 - 11*sqrt(b*cos(f*x + e))*b^2)/((b^2*cos(f*x + e)^2 - b^2)^2*
b^4))*sgn(cos(f*x + e))/f
```

3.378 $\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx$

Optimal. Leaf size=123

$$-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}-\frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}}+\frac{80\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{77f}$$

```
[Out] (80*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(77*
f) - (40*b*Sin[e + f*x])/(77*f*Sqrt[b*Sec[e + f*x]]) - (20*b*Sin[e + f*x]^3
)/(77*f*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e + f*x]^5)/(11*f*Sqrt[b*Sec[e + f
*x]])
```

Rubi [A] time = 0.143108, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2627, 3771, 2641}

$$-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}-\frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}}+\frac{80\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{77f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^6,x]
```


```
[Out] (80*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(77*
f) - (40*b*Sin[e + f*x])/(77*f*Sqrt[b*Sec[e + f*x]]) - (20*b*Sin[e + f*x]^3
)/(77*f*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e + f*x]^5)/(11*f*Sqrt[b*Sec[e + f
*x]])
```


Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx = -\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}} + \frac{10}{11}\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx$$
$$= -\frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}} - \frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}} + \frac{60}{77}\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,d$$
$$= -\frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}} - \frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}} - \frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}} + \frac{40}{77}\int \sqrt{b\sec(}$$
$$= -\frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}} - \frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}} - \frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}} + \frac{1}{77}\left(40\sqrt{\cos(e}\right.$$
$$= \frac{80\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{77f} - \frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}} - \frac{20b\sin}{77f\sqrt{b\,s}}$$

Mathematica [A] time = 0.150176, size = 73, normalized size = 0.59

$$\frac{\sqrt{b\sec(e+fx)}\left(-435\sin(2(e+fx))+68\sin(4(e+fx))-7\sin(6(e+fx))+1280\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{1232f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^6,x]
```

```
[Out] (Sqrt[b*Sec[e + f*x]]*(1280*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] -
435*Sin[2*(e + f*x)] + 68*Sin[4*(e + f*x)] - 7*Sin[6*(e + f*x)]))/(1232*f)
```

Maple [C] time = 0.253, size = 165, normalized size = 1.3

$$-\frac{\left(-2+2\cos\left(fx+e\right)\right)\left(\cos\left(fx+e\right)+1\right)^2}{77f\left(\sin\left(fx+e\right)\right)^3}\left(7\left(\cos\left(fx+e\right)\right)^6-7\left(\cos\left(fx+e\right)\right)^5+40i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\mathrm{c}}{\cos}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/77/f*(-1+cos(f*x+e))*(7*cos(f*x+e)^6-7*cos(f*x+e)^5+40*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*sin(f*x+e)-24*cos(f*x+e)^4+24*cos(f*x+e)^3+37*cos(f*x+e)^2-37*co
s(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(1/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos\left(fx+e\right)^{6}-3\cos\left(fx+e\right)^{4}+3\cos\left(fx+e\right)^{2}-1\right)\sqrt{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(b
*sec(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^6, x)
```

3.379 $\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx$

Optimal. Leaf size=95

$$-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}-\frac{4b\sin(e+fx)}{7f\sqrt{b\sec(e+fx)}}+\frac{8\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{7f}$$

```
[Out] (8*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(7*f)
 - (4*b*Sin[e + f*x])/(7*f*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e + f*x]^3)/(7*
f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.0984107, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2627, 3771, 2641}

$$-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}-\frac{4b\sin(e+fx)}{7f\sqrt{b\sec(e+fx)}}+\frac{8\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^4,x]
```

```
[Out] (8*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(7*f)
 - (4*b*Sin[e + f*x])/(7*f*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e + f*x]^3)/(7*
f*Sqrt[b*Sec[e + f*x]])
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx = -\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}+\frac{6}{7}\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx$$
$$= -\frac{4b\sin(e+fx)}{7f\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}+\frac{4}{7}\int \sqrt{b\sec(e+fx)}\,dx$$
$$= -\frac{4b\sin(e+fx)}{7f\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}+\frac{1}{7}\left(4\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{}{\sqrt{\mathrm{co}}}$$
$$= \frac{8\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{7f}-\frac{4b\sin(e+fx)}{7f\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+f}{7f\sqrt{b\sec(e+}}$$

Mathematica [A] time = 0.105073, size = 61, normalized size = 0.64

$$\frac{\sqrt{b\sec(e+fx)}\left(-10\sin(2(e+fx))+\sin(4(e+fx))+32\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{28f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^4,x]
```

```
[Out] (Sqrt[b*Sec[e + f*x]]*(32*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] - 10
*Sin[2*(e + f*x)] + Sin[4*(e + f*x)]))/(28*f)
```

Maple [C] time = 0.158, size = 143, normalized size = 1.5

$$\frac{\left(-2+2\cos\left(fx+e\right)\right)\left(\cos\left(fx+e\right)+1\right)^2}{7f\left(\sin\left(fx+e\right)\right)^3}\left(-4i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right.}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/7/f*(-1+cos(f*x+e))*(-4*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+cos(f*x+e)
^4-cos(f*x+e)^3-3*cos(f*x+e)^2+3*cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e)
)^(1/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}+1\right)\sqrt{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b*sec(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^4, x)
```

3.380 $\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx$

Optimal. Leaf size=67

$$\frac{4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

```
[Out] (4*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*f)
 - (2*b*Sin[e + f*x])/(3*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.0582376, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2627, 3771, 2641}

$$\frac{4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^2,x]
```

```
[Out] (4*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*f)
 - (2*b*Sin[e + f*x])/(3*f*Sqrt[b*Sec[e + f*x]])
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx&=-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{2}{3}\int \sqrt{b\sec(e+fx)}\,dx\\&=-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{1}{3}\left(2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx\\&=\frac{4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0939842, size = 51, normalized size = 0.76

$$-\frac{\sqrt{b\sec(e+fx)}\left(\sin(2(e+fx))-4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^2,x]
```

```
[Out] -(Sqrt[b*Sec[e + f*x]]*(-4*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + S
in[2*(e + f*x)]))/(3*f)
```

Maple [C] time = 0.143, size = 123, normalized size = 1.8

$$-\frac{\left(-2+2\cos\left(fx+e\right)\right)\left(\cos\left(fx+e\right)+1\right)^{2}}{3f\left(\sin\left(fx+e\right)\right)^{3}}\left(2i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx-\right.\right.}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/3/f*(-1+cos(f*x+e))*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+cos(f*x+e)
^2-cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(1/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos\left(fx+e\right)^{2}-1\right)\sqrt{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(b*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(e+fx\right)}\sin^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))*sin(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^2, x)
```

3.381 $\int \sqrt{b \sec(e+fx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/f
```

Rubi [A] time = 0.0207547, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2641}

$$\frac{2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/f
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sec(e+fx)}\,dx &= \left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx\\ &= \frac{2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{f}\end{aligned}$$

Mathematica [A] time = 0.0339519, size = 38, normalized size = 1.

$$\frac{2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/f
```

Maple [C] time = 0.119, size = 98, normalized size = 2.6

$$\frac{-2i\left(-1+\cos\left(fx+e\right)\right)\left(\cos\left(fx+e\right)+1\right)^{2}}{f\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{b}{\cos\left(fx+e\right)}}\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\mathrm{EllipticF}\left(\frac{i\left(-1+\right.}{\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2*I/f*(b/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(
f*x+e)+1)^2/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)), x)
```

3.382 $\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{f}-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] -((b*Csc[e + f*x])/(f*Sqrt[b*Sec[e + f*x]])) + (Sqrt[Cos[e + f*x]]*Elliptic
F[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/f
```

Rubi [A] time = 0.0572937, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2625, 3771, 2641}

$$\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{f}-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -((b*Csc[e + f*x])/(f*Sqrt[b*Sec[e + f*x]])) + (Sqrt[Cos[e + f*x]]*Elliptic
F[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/f
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx &= -\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}+\frac{1}{2}\int \sqrt{b\sec(e+fx)}\,dx\\ &= -\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}+\frac{1}{2}\left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx\\ &= -\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}+\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{f}\end{aligned}$$

Mathematica [A] time = 0.106407, size = 47, normalized size = 0.76

$$\frac{\sqrt{b\sec(e+fx)}\left(\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)-\cot(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] ((-Cot[e + f*x] + Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])*Sqrt[b*Sec[
e + f*x]])/f
```

Maple [C] time = 0.139, size = 184, normalized size = 3.

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(\cos\left(fx+e\right)+1\right)^{2}}{f\left(\sin\left(fx+e\right)\right)^{5}}\left(i\cos\left(fx+e\right)\text{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(-1+cos(f*x+e))^2*(I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+I*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*
x+e))^(1/2)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(e+fx\right)}\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))*csc(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^2, x)
```

3.383 $\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal. Leaf size=95

$$-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}-\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}}+\frac{5\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{6f}$$

```
[Out] (-5*b*Csc[e + f*x])/(6*f*Sqrt[b*Sec[e + f*x]]) - (b*Csc[e + f*x]^3)/(3*f*Sq
rt[b*Sec[e + f*x]]) + (5*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[
b*Sec[e + f*x]])/(6*f)
```

Rubi [A] time = 0.0956976, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2625, 3771, 2641}

$$-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}-\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}}+\frac{5\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-5*b*Csc[e + f*x])/(6*f*Sqrt[b*Sec[e + f*x]]) - (b*Csc[e + f*x]^3)/(3*f*Sq
rt[b*Sec[e + f*x]]) + (5*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[
b*Sec[e + f*x]])/(6*f)
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx = -\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{5}{6}\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$= -\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}} - \frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{5}{12}\int \sqrt{b\sec(e+fx)}\,dx$$
$$= -\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}} - \frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{1}{12}\left(5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{}{\sqrt{c}$$
$$= -\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}} - \frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{5\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+}}{6f}$$

Mathematica [A] time = 0.219647, size = 63, normalized size = 0.66

$$\frac{\sqrt{b\sec(e+fx)}\left(5\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)-\cot(e+fx)\left(2\csc^2(e+fx)+5\right)\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] ((-(Cot[e + f*x]*(5 + 2*Csc[e + f*x]^2)) + 5*Sqrt[Cos[e + f*x]]*EllipticF[(
e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]])/(6*f)
```

Maple [C] time = 0.165, size = 335, normalized size = 3.5

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^2\left(\cos\left(fx+e\right)+1\right)^2}{6f\left(\sin\left(fx+e\right)\right)^7}\left(5i\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right.}{\cos\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/6/f*(-1+cos(f*x+e))^2*(5*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*sin(f*
x+e)+5*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)-5*I*cos(f*x+e)*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-5*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e
)-5*cos(f*x+e)^3+7*cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(1/2)/sin(f*
x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^4, x)
```

3.384 $\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal. Leaf size=123

$$-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}-\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}}-\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}}+\frac{3\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{4f}$$

```
[Out] (-3*b*Csc[e + f*x])/(4*f*Sqrt[b*Sec[e + f*x]]) - (3*b*Csc[e + f*x]^3)/(10*f
*Sqrt[b*Sec[e + f*x]]) - (b*Csc[e + f*x]^5)/(5*f*Sqrt[b*Sec[e + f*x]]) + (3
*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(4*f)
```

Rubi [A] time = 0.137841, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2625, 3771, 2641}

$$-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}-\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}}-\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}}+\frac{3\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*Sqrt[b*Sec[e + f*x]],x]
```


```
[Out] (-3*b*Csc[e + f*x])/(4*f*Sqrt[b*Sec[e + f*x]]) - (3*b*Csc[e + f*x]^3)/(10*f
*Sqrt[b*Sec[e + f*x]]) - (b*Csc[e + f*x]^5)/(5*f*Sqrt[b*Sec[e + f*x]]) + (3
*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(4*f)
```


Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx = -\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}} + \frac{9}{10}\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$= -\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}} - \frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}} + \frac{3}{4}\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$= -\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}} - \frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}} - \frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}} + \frac{3}{8}\int \sqrt{b\sec(e+$$
$$= -\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}} - \frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}} - \frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}} + \frac{1}{8}\left(3\sqrt{\cos(e+fx\right.$$
$$= -\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}} - \frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}} - \frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}} + \frac{3\sqrt{\cos(e+fx)}F}{}$$

Mathematica [A] time = 0.441914, size = 73, normalized size = 0.59

$$\frac{\sqrt{b\sec(e+fx)}\left(15\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right) - \cot(e+fx)\left(4\csc^4(e+fx)+6\csc^2(e+fx)+15\right)\right)}{20f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] ((-(Cot[e + f*x]*(15 + 6*Csc[e + f*x]^2 + 4*Csc[e + f*x]^4)) + 15*Sqrt[Cos[
e + f*x]]*EllipticF[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]])/(20*f)
```

Maple [C] time = 0.181, size = 485, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/20/f*(-1+cos(f*x+e))^2*(15*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^5*sin(f*x+e)*(1/(cos(f*x+e)+1))^
(1/2)+15*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*cos(f*x+e)^4*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)-30*I*cos(f*x+e
)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-30*I*EllipticF(I*(-1+cos(f*x+e))/s
in(f*x+e),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*(1/(
cos(f*x+e)+1))^(1/2)+15*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-1
5*cos(f*x+e)^5+15*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+36*cos(f*x+e)^3-25
*cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(1/2)/sin(f*x+e)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^{6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^6, x)
```

3.385 $\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx$

Optimal. Leaf size=83

$$\frac{2b^7}{11f(b\sec(e+fx))^{11/2}}-\frac{6b^5}{7f(b\sec(e+fx))^{7/2}}+\frac{2b^3}{f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] (2*b^7)/(11*f*(b*Sec[e + f*x])^(11/2)) - (6*b^5)/(7*f*(b*Sec[e + f*x])^(7/2
)) + (2*b^3)/(f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rubi [A] time = 0.0627426, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$\frac{2b^7}{11f(b\sec(e+fx))^{11/2}}-\frac{6b^5}{7f(b\sec(e+fx))^{7/2}}+\frac{2b^3}{f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^7,x]
```

```
[Out] (2*b^7)/(11*f*(b*Sec[e + f*x])^(11/2)) - (6*b^5)/(7*f*(b*Sec[e + f*x])^(7/2
)) + (2*b^3)/(f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx&=\frac{b^7\operatorname{Subst}\left(\int\frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{13/2}}\,dx,x,b\sec(e+fx)\right)}{f}\\&=\frac{b^7\operatorname{Subst}\left(\int\left(-\frac{1}{x^{13/2}}+\frac{3}{b^2x^{9/2}}-\frac{3}{b^4x^{5/2}}+\frac{1}{b^6\sqrt{x}}\right)dx,x,b\sec(e+fx)\right)}{f}\\&=\frac{2b^7}{11f(b\sec(e+fx))^{11/2}}-\frac{6b^5}{7f(b\sec(e+fx))^{7/2}}+\frac{2b^3}{f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec}}{f}\end{aligned}$$

Mathematica [A] time = 0.142235, size = 52, normalized size = 0.63

$$\frac{b(809\cos(2(e+fx))-90\cos(4(e+fx))+7\cos(6(e+fx))+3370)\sqrt{b\sec(e+fx)}}{1232f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^7,x]
```

```
[Out] (b*(3370 + 809*Cos[2*(e + f*x)] - 90*Cos[4*(e + f*x)] + 7*Cos[6*(e + f*x)])
*Sqrt[b*Sec[e + f*x]])/(1232*f)
```

Maple [B] time = 0.237, size = 969, normalized size = 11.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x)
```

```
[Out] 1/154/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(28*cos(f*x+e)^7-77*cos(f*x+e)^3
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)+77*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-231*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+231*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)-132*cos(f*x+e)^5-231*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+231*cos(
f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-77*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+77*ln(-2*(2
*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(3/2)+308*cos(f*x+e)^3+308*cos(f*x+e))*(b/cos(f*x+e))^(3/2)/s
in(f*x+e)^4
```

Maxima [A] time = 0.995003, size = 97, normalized size = 1.17

$$\frac{2\,b\left(\frac{7\,b^{6}}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{11}{2}}}-\frac{33\,b^{4}}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}}+\frac{77\,b^{2}}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}+77\,\sqrt{\frac{b}{\cos\left(fx+e\right)}}\right)}{77\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x, algorithm="maxima")
```

```
[Out] 2/77*b*(7*b^6/(b/cos(f*x + e))^(11/2) - 33*b^4/(b/cos(f*x + e))^(7/2) + 77*
b^2/(b/cos(f*x + e))^(3/2) + 77*sqrt(b/cos(f*x + e)))/f
```

Fricas [A] time = 2.23216, size = 136, normalized size = 1.64

$$\frac{2\left(7\,b\cos\left(fx+e\right)^{6}-33\,b\cos\left(fx+e\right)^{4}+77\,b\cos\left(fx+e\right)^{2}+77\,b\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{77\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x, algorithm="fricas")
```

```
[Out] 2/77*(7*b*cos(f*x + e)^6 - 33*b*cos(f*x + e)^4 + 77*b*cos(f*x + e)^2 + 77*b
)*sqrt(b/cos(f*x + e))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14018, size = 143, normalized size = 1.72

$$\frac{2\left(7\,\sqrt{b\cos\left(fx+e\right)}b^{5}\cos\left(fx+e\right)^{5}-33\,\sqrt{b\cos\left(fx+e\right)}b^{5}\cos\left(fx+e\right)^{3}+77\,\sqrt{b\cos\left(fx+e\right)}b^{5}\cos\left(fx+e\right)+\frac{7}{\sqrt{b\,\mathrm{c}}}\right.}{77\,b^{4}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x, algorithm="giac")
```

```
[Out] 2/77*(7*sqrt(b*cos(f*x + e))*b^5*cos(f*x + e)^5 - 33*sqrt(b*cos(f*x + e))*b
^5*cos(f*x + e)^3 + 77*sqrt(b*cos(f*x + e))*b^5*cos(f*x + e) + 77*b^6/sqrt(
b*cos(f*x + e)))*sgn(cos(f*x + e))/(b^4*f)
```

3.386 $\int (b \sec(e+fx))^{3/2} \sin^5(e+fx)\,dx$

Optimal. Leaf size=63

$$-\frac{2b^5}{7f(b\sec(e+fx))^{7/2}}+\frac{4b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] (-2*b^5)/(7*f*(b*Sec[e + f*x])^(7/2)) + (4*b^3)/(3*f*(b*Sec[e + f*x])^(3/2)
) + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rubi [A] time = 0.0560433, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$-\frac{2b^5}{7f(b\sec(e+fx))^{7/2}}+\frac{4b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] (-2*b^5)/(7*f*(b*Sec[e + f*x])^(7/2)) + (4*b^3)/(3*f*(b*Sec[e + f*x])^(3/2)
) + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx &= \frac{b^5\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{9/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= \frac{b^5\,\text{Subst}\left(\int \left(\frac{1}{x^{9/2}}-\frac{2}{b^2x^{5/2}}+\frac{1}{b^4\sqrt{x}}\right)dx, x, b\sec(e+fx)\right)}{f}\\ &= -\frac{2b^5}{7f(b\sec(e+fx))^{7/2}}+\frac{4b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}\end{aligned}$$

Mathematica [A] time = 0.0914577, size = 42, normalized size = 0.67

$$\frac{b(44\cos(2(e+fx))-3\cos(4(e+fx))+215)\sqrt{b\sec(e+fx)}}{84f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] (b*(215 + 44*Cos[2*(e + f*x)] - 3*Cos[4*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(
84*f)
```

Maple [B] time = 0.172, size = 959, normalized size = 15.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x)
```

```
[Out] -1/42/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(-21*cos(f*x+e)^3*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/
sin(f*x+e)^2)+21*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*c
os(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-63*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-1)/sin(f*x+e)^2)+63*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+12*cos(f
*x+e)^5-63*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+63*cos(f*x+e)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)
/sin(f*x+e)^2)-21*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+21*ln(-(2*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-
56*cos(f*x+e)^3-84*cos(f*x+e))*(b/cos(f*x+e))^(3/2)/sin(f*x+e)^4
```

Maxima [A] time = 1.03334, size = 74, normalized size = 1.17

$$-\frac{2\,b\left(\frac{3\,b^4}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}}-\frac{14\,b^2}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}-21\,\sqrt{\frac{b}{\cos\left(fx+e\right)}}\right)}{21\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -2/21*b*(3*b^4/(b/cos(f*x + e))^(7/2) - 14*b^2/(b/cos(f*x + e))^(3/2) - 21*
sqrt(b/cos(f*x + e)))/f
```

Fricas [A] time = 2.24457, size = 108, normalized size = 1.71

$$-\frac{2\left(3\,b\cos\left(fx+e\right)^{4}-14\,b\cos\left(fx+e\right)^{2}-21\,b\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{21\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -2/21*(3*b*cos(f*x + e)^4 - 14*b*cos(f*x + e)^2 - 21*b)*sqrt(b/cos(f*x + e)
)/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12813, size = 109, normalized size = 1.73

$$-\frac{2\left(3\sqrt{b\cos\left(fx+e\right)}b^{3}\cos\left(fx+e\right)^{3}-14\sqrt{b\cos\left(fx+e\right)}b^{3}\cos\left(fx+e\right)-\frac{21\,b^{4}}{\sqrt{b\cos\left(fx+e\right)}}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{21\,b^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] -2/21*(3*sqrt(b*cos(f*x + e))*b^3*cos(f*x + e)^3 - 14*sqrt(b*cos(f*x + e))*
b^3*cos(f*x + e) - 21*b^4/sqrt(b*cos(f*x + e)))*sgn(cos(f*x + e))/(b^2*f)
```

3.387 $\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx$

Optimal. Leaf size=41

$$\frac{2b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] (2*b^3)/(3*f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rubi [A] time = 0.0501463, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 14}

$$\frac{2b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^3,x]
```

```
[Out] (2*b^3)/(3*f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx &= \frac{b^3\,\text{Subst}\left(\int\frac{-1+\frac{x^2}{b^2}}{x^{5/2}}\,dx,x,b\sec(e+fx)\right)}{f}\\ &= \frac{b^3\,\text{Subst}\left(\int\left(-\frac{1}{x^{5/2}}+\frac{1}{b^2\sqrt{x}}\right)dx,x,b\sec(e+fx)\right)}{f}\\ &= \frac{2b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}\end{aligned}$$

Mathematica [A] time = 0.0630676, size = 30, normalized size = 0.73

$$\frac{b(\cos(2(e+fx))+7)\sqrt{b\sec(e+fx)}}{3f}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^3,x]

[Out] (b*(7 + Cos[2*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(3*f)

Maple [B] time = 0.139, size = 949, normalized size = 23.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x)

[Out] 1/6/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(3*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+9*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-9*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+9*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-9*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+4*cos(f*x+e)^3+12*cos(f*x+e))*(b/cos(f*x+e))^(3/2)/sin(f*x+e)^4

Maxima [A] time = 0.992971, size = 50, normalized size = 1.22

$$\frac{2\,b\left(\frac{b^2}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}+3\,\sqrt{\frac{b}{\cos\left(fx+e\right)}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="maxima")

[Out] 2/3*b*(b^2/(b/cos(f*x + e))^(3/2) + 3*sqrt(b/cos(f*x + e)))/f

---

Fricas [A] time = 2.20181, size = 72, normalized size = 1.76

$$\frac{2\left(b\cos\left(fx+e\right)^{2}+3\,b\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 2/3*(b*cos(f*x + e)^2 + 3*b)*sqrt(b/cos(f*x + e))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18802, size = 68, normalized size = 1.66

$$\frac{2\left(\sqrt{b\cos\left(fx+e\right)}b\cos\left(fx+e\right)+\frac{3\,b^{2}}{\sqrt{b\cos\left(fx+e\right)}}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(b*cos(f*x + e))*b*cos(f*x + e) + 3*b^2/sqrt(b*cos(f*x + e)))*sgn(
cos(f*x + e))/f
```

3.388 $\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx$

Optimal. Leaf size=18

$$\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rubi [A] time = 0.0343327, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x],x]
```

```
[Out] (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx &= \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= \frac{2b\sqrt{b\sec(e+fx)}}{f}\end{aligned}$$

Mathematica [A] time = 0.0271977, size = 18, normalized size = 1.

$$\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x],x]
```

```
[Out] (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Maple [A] time = 0.013, size = 17, normalized size = 0.9

$$2\,\frac{b\sqrt{b\sec\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e),x)
```

```
[Out] 2*b*(b*sec(f*x+e))^(1/2)/f
```

Maxima [A] time = 0.995149, size = 31, normalized size = 1.72

$$\frac{2\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] 2*(b/cos(f*x + e))^(3/2)*cos(f*x + e)/f
```

Fricas [A] time = 2.19677, size = 38, normalized size = 2.11

$$\frac{2\,b\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] 2*b*sqrt(b/cos(f*x + e))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32588, size = 36, normalized size = 2.

$$\frac{2\,b^2 \mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{\sqrt{b\cos\left(fx+e\right)}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="giac")
```

```
[Out] 2*b^2*sgn(cos(f*x + e))/(sqrt(b*cos(f*x + e))*f)
```

3.389 $\int \csc(e + fx)(b\sec(e + fx))^{3/2}\,dx$

Optimal. Leaf size=77

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] -((b^(3/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/f) - (b^(3/2)*ArcTanh[Sqrt
[b*Sec[e + f*x]]/Sqrt[b]])/f + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rubi [A] time = 0.0533208, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2622, 321, 329, 212, 206, 203}

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((b^(3/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/f) - (b^(3/2)*ArcTanh[Sqrt
[b*Sec[e + f*x]]/Sqrt[b]])/f + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^{3/2}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{bf}$$

$$= \frac{2b\sqrt{b\sec(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{2b\sqrt{b\sec(e+fx)}}{f} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f}$$

$$= \frac{2b\sqrt{b\sec(e+fx)}}{f} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{b+x^2}\,dx,\right.}{f}$$

$$= -\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} + \frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Mathematica [A] time = 0.914073, size = 85, normalized size = 1.1

$$\frac{(b\sec(e+fx))^{3/2}\left(4\sqrt{\sec(e+fx)}+\log\left(1-\sqrt{\sec(e+fx)}\right)-\log\left(\sqrt{\sec(e+fx)}+1\right)-2\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)\right)}{2f\sec^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] ((-2*ArcTan[Sqrt[Sec[e + f*x]]] + Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqr
t[Sec[e + f*x]]] + 4*Sqrt[Sec[e + f*x]])*(b*Sec[e + f*x])^(3/2))/(2*f*Sec[e
 + f*x]^(3/2))
```

Maple [B] time = 0.148, size = 235, normalized size = 3.1

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^3\left(\cos\left(fx+e\right)\right)^2}{2\,f\left(\sin\left(fx+e\right)\right)^6}\left(4\,\cos\left(fx+e\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^2}}+\cos\left(fx+e\right)\ln\left(-2\,\frac{1}{\left(\sin\left(fx+e\right)\right)^2}\left(2\,(\mathrm{c}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(-1+cos(f*x+e))^3*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+
cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2
)+cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+4*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2))*(b/cos(f*x+e))^(3/2)*cos(f*x+e)^2/sin(f*x+e)^6/(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.05017, size = 737, normalized size = 9.57

$$\left[\frac{2\sqrt{-b}b\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)+\sqrt{-b}b\log\left(\frac{b\cos(fx+e)^2+4\left(\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}-6b\cos(fx+e)+b}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\cdot}{4f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b)*b*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) +
 1)/b) + sqrt(-b)*b*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(f*x + e
))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 +
2*cos(f*x + e) + 1)) + 8*b*sqrt(b/cos(f*x + e)))/f, 1/4*(2*b^(3/2)*arctan(1
/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(b)) + b^(3/2)*log((b*cos(f*
x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) +
 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) + 8*b*sqrt(b/
cos(f*x + e)))/f]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36877, size = 107, normalized size = 1.39

$$\frac{b^4\left(\frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}}+\frac{2}{\sqrt{b\cos(fx+e)}b^2}\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] b^4*(arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^2) + arctan(sqrt(b*c
os(f*x + e))/sqrt(b))/b^(5/2) + 2/(sqrt(b*cos(f*x + e))*b^2))*sgn(cos(f*x +
 e))/f
```

3.390 $\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=113

$$-\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}-\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}+\frac{5b\sqrt{b\sec(e+fx)}}{2f}-\frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf}$$

```
[Out] (-5*b^(3/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (5*b^(3/2)*ArcTan
h[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) + (5*b*Sqrt[b*Sec[e + f*x]])/(2*f) -
 (Cot[e + f*x]^2*(b*Sec[e + f*x])^(5/2))/(2*b*f)
```

Rubi [A] time = 0.0839355, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2622, 288, 321, 329, 212, 206, 203}

$$-\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}-\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}+\frac{5b\sqrt{b\sec(e+fx)}}{2f}-\frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-5*b^(3/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (5*b^(3/2)*ArcTan
h[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) + (5*b*Sqrt[b*Sec[e + f*x]])/(2*f) -
 (Cot[e + f*x]^2*(b*Sec[e + f*x])^(5/2))/(2*b*f)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^{7/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf} + \frac{5\,\text{Subst}\left(\int \frac{x^{3/2}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{4bf}$$

$$= \frac{5b\sqrt{b\sec(e+fx)}}{2f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx,\right.}{4f}$$

$$= \frac{5b\sqrt{b\sec(e+fx)}}{2f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{\right.}{2f}$$

$$= \frac{5b\sqrt{b\sec(e+fx)}}{2f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{\right.}{4f}$$

$$= -\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} + \frac{5b\sqrt{b\sec(e+fx)}}{2f} - \text{cot}$$

Mathematica [A] time = 2.27884, size = 97, normalized size = 0.86

$$-\frac{(b\sec(e+fx))^{3/2}\left(-5\log\left(1-\sqrt{\sec(e+fx)}\right)+5\log\left(\sqrt{\sec(e+fx)}+1\right)+4\left(\csc^2(e+fx)-5\right)\sqrt{\sec(e+fx)}+10\tan^{-}\right.}{8f\sec^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((10*ArcTan[Sqrt[Sec[e + f*x]]] - 5*Log[1 - Sqrt[Sec[e + f*x]]] + 5*Log[1
+ Sqrt[Sec[e + f*x]]] + 4*(-5 + Csc[e + f*x]^2)*Sqrt[Sec[e + f*x]])*(b*Sec[
```

```
e + f*x])^(3/2))/(8*f*Sec[e + f*x]^(3/2))
```

Maple [B] time = 0.129, size = 644, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/8/f*(cos(f*x+e)+1)*(-1+cos(f*x+e))^3*(4*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(3/2)+8*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+4*cos(
f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-16*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-5*cos(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2))-cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/s
in(f*x+e)^2)-4*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-1)/sin(f*x+e)^2)+5*cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2))+cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-c
os(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+
e)^2)+4*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(
f*x+e)^2)+16*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(b/cos(f*x+e))^(3/2)*cos
(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/sin(f*x+e)^8
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.89368, size = 991, normalized size = 8.77

$$\left[\frac{10\left(b\cos\left(fx+e\right)^2-b\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos\left(fx+e\right)}}\left(\cos\left(fx+e\right)+1\right)}{2b}\right)+5\left(b\cos\left(fx+e\right)^2-b\right)\sqrt{-b}\log\left(\frac{b\cos\left(fx+e\right)^2+4\left(\cos\left(f\right.\right.}{}\right.}{16\left(f\cos\left(fx+e\right)^2-f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(10*(b*cos(f*x + e)^2 - b)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*
x + e))*(cos(f*x + e) + 1)/b) + 5*(b*cos(f*x + e)^2 - b)*sqrt(-b)*log((b*co
s(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x +
```

```
e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(5*b
*cos(f*x + e)^2 - 4*b)*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)^2 - f), 1/16*(
10*(b*cos(f*x + e)^2 - b)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x
+ e) - 1)/sqrt(b)) + 5*(b*cos(f*x + e)^2 - b)*sqrt(b)*log((b*cos(f*x + e)^2
 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos
(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) + 8*(5*b*cos(f*x + e)
^2 - 4*b)*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)^2 - f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40269, size = 181, normalized size = 1.6

$$\frac{b^6\left(\frac{5\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4}+\frac{5\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{9}{2}}}+\frac{2\left(5b^2\cos(fx+e)^2-4b^2\right)}{\left(\sqrt{b\cos(fx+e)}b^2\cos(fx+e)^2-\sqrt{b\cos(fx+e)}b^2\right)b^4}\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*b^6*(5*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^4) + 5*arctan(
sqrt(b*cos(f*x + e))/sqrt(b))/b^(9/2) + 2*(5*b^2*cos(f*x + e)^2 - 4*b^2)/((
sqrt(b*cos(f*x + e))*b^2*cos(f*x + e)^2 - sqrt(b*cos(f*x + e))*b^2)*b^4))*s
gn(cos(f*x + e))/f
```

3.391 $\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx$

Optimal. Leaf size=128

$$\frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}+\frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}}-\frac{16b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{3f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2b\sin^5(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] (-16*b^2*EllipticE[(e + f*x)/2, 2])/(3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e +
f*x]]) + (8*b^3*Sin[e + f*x])/(3*f*(b*Sec[e + f*x])^(3/2)) + (20*b^3*Sin[e
+ f*x]^3)/(9*f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*Sec[e + f*x]]*Sin[e +
f*x]^5)/f
```

Rubi [A] time = 0.148598, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2624, 2627, 3771, 2639}

$$\frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}+\frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}}-\frac{16b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{3f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2b\sin^5(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^6,x]
```

```
[Out] (-16*b^2*EllipticE[(e + f*x)/2, 2])/(3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e +
f*x]]) + (8*b^3*Sin[e + f*x])/(3*f*(b*Sec[e + f*x])^(3/2)) + (20*b^3*Sin[e
+ f*x]^3)/(9*f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*Sec[e + f*x]]*Sin[e +
f*x]^5)/f
```

Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx = \frac{2b\sqrt{b\sec(e+fx)}\sin^5(e+fx)}{f} - \left(10b^2\right)\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$
$$= \frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\sin^5(e+fx)}{f} - \frac{1}{3}\left(20b^2\right)\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx}}$$
$$= \frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\sin^5(e+fx)}{f} - \frac{1}{3}($$
$$= \frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\sin^5(e+fx)}{f} - \frac{(}{3\surd}$$
$$= -\frac{16b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{3f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} +$$

Mathematica [A] time = 0.150728, size = 70, normalized size = 0.55

$$-\frac{b\sqrt{b\sec(e+fx)}\left(-158\sin(e+fx)-13\sin(3(e+fx))+\sin(5(e+fx))+384\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{72f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^6,x]
```

```
[Out] -(b*Sqrt[b*Sec[e + f*x]]*(384*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
- 158*Sin[e + f*x] - 13*Sin[3*(e + f*x)] + Sin[5*(e + f*x)]))/(72*f)
```

Maple [C] time = 0.233, size = 330, normalized size = 2.6

$$\frac{2\cos(fx+e)}{9f\sin(fx+e)}\left(\left(\cos(fx+e)\right)^6 - 24i\cos(fx+e)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\left(\cos(fx+e)+1\right)^{-1}}\sqrt{\frac{\cos(}{\cos(f}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x)
```

```
[Out] 2/9/f*(cos(f*x+e)^6-24*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+24
*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-24*I*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)+24*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-5*cos(f*x+e)^4+
19*cos(f*x+e)^2-24*cos(f*x+e)+9)*cos(f*x+e)*(b/cos(f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec(fx+e)\right)^{\frac{3}{2}}\sin(fx+e)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^{6}-3\,b\cos\left(fx+e\right)^{4}+3\,b\cos\left(fx+e\right)^{2}-b\right)\sqrt{b\sec\left(fx+e\right)}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^6 - 3*b*cos(f*x + e)^4 + 3*b*cos(f*x + e)^2 - b)*
sqrt(b*sec(f*x + e))*sec(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^6, x)
```

3.392 $\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx$

Optimal. Leaf size=98

$$\frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}-\frac{24b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2b\sin^3(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] (-24*b^2*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e +
f*x]]) + (12*b^3*Sin[e + f*x])/(5*f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*S
ec[e + f*x]]*Sin[e + f*x]^3)/f
```

Rubi [A] time = 0.107605, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2624, 2627, 3771, 2639}

$$\frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}-\frac{24b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2b\sin^3(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (-24*b^2*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e +
f*x]]) + (12*b^3*Sin[e + f*x])/(5*f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*S
ec[e + f*x]]*Sin[e + f*x]^3)/f
```

Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx = \frac{2b\sqrt{b\sec(e+fx)}\sin^3(e+fx)}{f} - \left(6b^2\right)\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$
$$= \frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\sin^3(e+fx)}{f} - \frac{1}{5}\left(12b^2\right)\int \frac{1}{\sqrt{b\sec(e+}$$
$$= \frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\sin^3(e+fx)}{f} - \frac{\left(12b^2\right)\int\sqrt{\cos(e+f}}{5\sqrt{\cos(e+fx)}\sqrt{b\sec(}}$$
$$= -\frac{24b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\sin}{f}$$

Mathematica [A] time = 0.114691, size = 60, normalized size = 0.61

$$\frac{b\sqrt{b\sec(e+fx)}\left(21\sin(e+fx)+\sin(3(e+fx))-48\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{10f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (b*Sqrt[b*Sec[e + f*x]]*(-48*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2] +
 21*Sin[e + f*x] + Sin[3*(e + f*x)]))/(10*f)
```

Maple [C] time = 0.179, size = 320, normalized size = 3.3

$$-\frac{2\cos\left(fx+e\right)}{5f\sin\left(fx+e\right)}\left(-12i\text{EllipticE}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos(}{\cos(f}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x)
```

```
[Out] -2/5/f*(-12*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+
e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+12*I*cos(f*x+
e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-12*I*sin(f*x+e)*EllipticE(I*(-1+cos
(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)+12*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)^4-8*cos(f*x+e)^
2+12*cos(f*x+e)-5)*cos(f*x+e)*(b/cos(f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4 - 2\,b\cos\left(fx+e\right)^2 + b\right)\sqrt{b\sec\left(fx+e\right)}\sec\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + b)*sqrt(b*sec(f*x + e))*s
ec(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^4, x)
```

3.393 $\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx$

Optimal. Leaf size=66

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] (-4*b^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]]) + (2*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/f
```

Rubi [A] time = 0.0657789, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2624, 3771, 2639}

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
```



```
[Out] (-4*b^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]]) + (2*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/f
```


Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx &= \frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}-\left(2b^2\right)\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\
&= \frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}-\frac{\left(2b^2\right)\int \sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= -\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.0789565, size = 48, normalized size = 0.73

$$\frac{2b\sqrt{b\sec(e+fx)}\left(\sin(e+fx)-2\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (2*b*Sqrt[b*Sec[e + f*x]]*(-2*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
+ Sin[e + f*x]))/f
```

Maple [C] time = 0.142, size = 312, normalized size = 4.7

$$-2\frac{\cos\left(fx+e\right)}{f\sin\left(fx+e\right)}\left(2i\cos\left(fx+e\right)\text{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\sin\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x)
```

```
[Out] -2/f*(2*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-2*I*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*EllipticE(I*(-1+co
s(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)
-2*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-cos(f*x+e)^2+2*cos(f*x+e)-1)*cos(
f*x+e)*(b/cos(f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^{2}-b\right)\sqrt{b\sec\left(fx+e\right)}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - b)*sqrt(b*sec(f*x + e))*sec(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx + e\right)\right)^{\frac{3}{2}} \sin \left(fx + e\right)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^2, x)
```

3.394 $\int (b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=66

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] (-2*b^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]]) + (2*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/f
```

Rubi [A] time = 0.0366553, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3771, 2639}

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*b^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]]) + (2*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/f
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^{3/2}\,dx &= \frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}-b^2\int\frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\ &= \frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}-\frac{b^2\int\sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\ &= -\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.0534289, size = 48, normalized size = 0.73

$$\frac{2b\sqrt{b\sec(e+fx)}\left(\sin(e+fx)-\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[b*Sec[e + f*x]]*(-(Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])
+ Sin[e + f*x]))/f
```

Maple [C] time = 0.174, size = 322, normalized size = 4.9

$$2\frac{\left(\cos\left(fx+e\right)+1\right)^{2}\left(-1+\cos\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)}{f\left(\sin\left(fx+e\right)\right)^{5}}\left(i\mathrm{EllipticE}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\cos\left(fx+e\right)\sin\left(fx+e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(I*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)-I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+I*Elliptic
E(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)-I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-cos(f*x
+e)+1)*cos(f*x+e)*(b/cos(f*x+e))^(3/2)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec\left(fx+e\right)}b\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*b*sec(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((b*sec(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2), x)
```

3.395 $\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=90

$$-\frac{3b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}+\frac{3b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] (-3*b^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]]) - (b*Csc[e + f*x]*Sqrt[b*Sec[e + f*x]])/f + (3*b*Sqrt[b*Sec[e + f*x]]*S
in[e + f*x])/f
```

Rubi [A] time = 0.0823653, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2625, 3768, 3771, 2639}

$$-\frac{3b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}+\frac{3b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-3*b^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]]) - (b*Csc[e + f*x]*Sqrt[b*Sec[e + f*x]])/f + (3*b*Sqrt[b*Sec[e + f*x]]*S
in[e + f*x])/f
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f} + \frac{3}{2}\int (b\sec(e+fx))^{3/2}\,dx$$
$$= -\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f} + \frac{3b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f} - \frac{1}{2}\left(3b^2\right)\int \frac{}{\sqrt{b\sec}}$$
$$= -\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f} + \frac{3b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f} - \frac{\left(3b^2\right)\int\sqrt{\cos(}}{2\sqrt{\cos(e+fx)}\sqrt{b}}$$
$$= -\frac{3b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f} + \frac{3b\sqrt{b\sec(e+fx}}{f}$$

Mathematica [A] time = 0.147256, size = 57, normalized size = 0.63

$$\frac{b\sqrt{b\sec(e+fx)}\left(3\sin(e+fx)-\csc(e+fx)-3\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (b*Sqrt[b*Sec[e + f*x]]*(-Csc[e + f*x] - 3*Sqrt[Cos[e + f*x]]*EllipticE[(e
+ f*x)/2, 2] + 3*Sin[e + f*x]))/f
```

Maple [C] time = 0.146, size = 322, normalized size = 3.6

$$-\frac{\left(\cos\left(fx+e\right)+1\right)^2\left(-1+\cos\left(fx+e\right)\right)^2\cos\left(fx+e\right)}{f\left(\sin\left(fx+e\right)\right)^5}\left(3i\cos\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(3*I*cos(f*x+e)*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*
sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)+3*cos(f*x+e)-2)*cos(f*x+e)*(b/cos(f*x+e))^(3/2)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)}b\csc\left(fx+e\right)^{2}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*b*csc(f*x + e)^2*sec(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^2, x)
```

3.396 $\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=124

$$-\frac{7b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}-\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f}+\frac{7b\sin(e+fx)\sqrt{b\sec}}{2f}$$

```
[Out] (-7*b^2*EllipticE[(e + f*x)/2, 2])/(2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f
*x]]) - (7*b*Csc[e + f*x]*Sqrt[b*Sec[e + f*x]])/(6*f) - (b*Csc[e + f*x]^3*S
qrt[b*Sec[e + f*x]])/(3*f) + (7*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/(2*f)
```

Rubi [A] time = 0.128983, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2625, 3768, 3771, 2639}

$$-\frac{7b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}-\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f}+\frac{7b\sin(e+fx)\sqrt{b\sec}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-7*b^2*EllipticE[(e + f*x)/2, 2])/(2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f
*x]]) - (7*b*Csc[e + f*x]*Sqrt[b*Sec[e + f*x]])/(6*f) - (b*Csc[e + f*x]^3*S
qrt[b*Sec[e + f*x]])/(3*f) + (7*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/(2*f)
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f} + \frac{7}{6}\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx$$
$$= -\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f} - \frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f} + \frac{7}{4}\int (b\sec(e$$
$$= -\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f} - \frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f} + \frac{7b\sqrt{b\sec(e+}}{}$$
$$= -\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f} - \frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f} + \frac{7b\sqrt{b\sec(e+}}{}$$
$$= -\frac{7b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f} - \frac{b\csc^3(e+}{}$$

Mathematica [A] time = 0.206314, size = 77, normalized size = 0.62

$$-\frac{b\sin(e+fx)\sqrt{b\sec(e+fx)}\left(2\csc^4(e+fx)+7\csc^2(e+fx)+21\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)-21\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(b*(-21 + 7*Csc[e + f*x]^2 + 2*Csc[e + f*x]^4 + 21*Sqrt[Cos[e + f*x]]*Csc[
e + f*x]*EllipticE[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/(6*f
)
```

Maple [C] time = 0.158, size = 622, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/6/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(21*I*cos(f*x+e)^3*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*sin(f*x+e)-21*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e
)+21*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-21*I*cos(f*x+e)^2*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-21*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*sin(f*x+e)+21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*si
n(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-21*I*El
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)+21*cos(f*x+e)^3-14*cos(f*x+e)^2-21*cos(f*x+e)+12)*cos(f*x+e)*(b/cos(f*x+e
))^(3/2)/sin(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)}b\csc\left(fx+e\right)^{4}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*b*csc(f*x + e)^4*sec(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^4, x)
```

3.397 $\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx$

Optimal. Leaf size=85

$$\frac{2b^7}{9f(b\sec(e+fx))^{9/2}}-\frac{6b^5}{5f(b\sec(e+fx))^{5/2}}+\frac{6b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] (2*b^7)/(9*f*(b*Sec[e + f*x])^(9/2)) - (6*b^5)/(5*f*(b*Sec[e + f*x])^(5/2))
 + (6*b^3)/(f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rubi [A] time = 0.0623046, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$\frac{2b^7}{9f(b\sec(e+fx))^{9/2}}-\frac{6b^5}{5f(b\sec(e+fx))^{5/2}}+\frac{6b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^7,x]
```

```
[Out] (2*b^7)/(9*f*(b*Sec[e + f*x])^(9/2)) - (6*b^5)/(5*f*(b*Sec[e + f*x])^(5/2))
 + (6*b^3)/(f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx &= \frac{b^7\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{11/2}}\,dx,x,b\sec(e+fx)\right)}{f}\\ &= \frac{b^7\,\text{Subst}\left(\int \left(-\frac{1}{x^{11/2}}+\frac{3}{b^2x^{7/2}}-\frac{3}{b^4x^{3/2}}+\frac{\sqrt{x}}{b^6}\right)dx,x,b\sec(e+fx)\right)}{f}\\ &= \frac{2b^7}{9f(b\sec(e+fx))^{9/2}}-\frac{6b^5}{5f(b\sec(e+fx))^{5/2}}+\frac{6b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}\end{aligned}$$

Mathematica [A] time = 0.425678, size = 52, normalized size = 0.61

$$\frac{b(1803\cos(2(e+fx))-78\cos(4(e+fx))+5\cos(6(e+fx))+2366)(b\sec(e+fx))^{3/2}}{720f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^7,x]
```

```
[Out] (b*(2366 + 1803*Cos[2*(e + f*x)] - 78*Cos[4*(e + f*x)] + 5*Cos[6*(e + f*x)]
)*(b*Sec[e + f*x])^(3/2))/(720*f)
```

Maple [B] time = 0.152, size = 532, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x)
```

```
[Out] 1/90/f*(-1+cos(f*x+e))^2*(20*cos(f*x+e)^6-108*cos(f*x+e)^4-135*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)+135*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-135
*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+135*cos(f*x+e)*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-c
os(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+
e)^2)+540*cos(f*x+e)^2+60)*cos(f*x+e)*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(5/2)
/sin(f*x+e)^4
```

Maxima [A] time = 1.05393, size = 89, normalized size = 1.05

$$\frac{2\left(15\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}+\frac{5\,b^6-\frac{27\,b^6}{\cos\left(fx+e\right)^2}+\frac{135\,b^6}{\cos\left(fx+e\right)^4}}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{9}{2}}}\right)b}{45\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x, algorithm="maxima")
```

```
[Out] 2/45*(15*(b/cos(f*x + e))^(3/2) + (5*b^6 - 27*b^6/cos(f*x + e)^2 + 135*b^6/
cos(f*x + e)^4)/(b/cos(f*x + e))^(9/2))*b/f
```

Fricas [A] time = 2.33944, size = 169, normalized size = 1.99

$$\frac{2\left(5\,b^2\cos\left(fx+e\right)^6-27\,b^2\cos\left(fx+e\right)^4+135\,b^2\cos\left(fx+e\right)^2+15\,b^2\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{45\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^2*cos(f*x + e)^6 - 27*b^2*cos(f*x + e)^4 + 135*b^2*cos(f*x + e)^2
 + 15*b^2)*sqrt(b/cos(f*x + e))/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20923, size = 146, normalized size = 1.72

$$\frac{2\left(5\sqrt{b\cos\left(fx+e\right)}b^4\cos\left(fx+e\right)^4 - 27\sqrt{b\cos\left(fx+e\right)}b^4\cos\left(fx+e\right)^2 + 135\sqrt{b\cos\left(fx+e\right)}b^4 + \frac{15\,b^5}{\sqrt{b\cos\left(fx+e\right)}\cos\left(f\right.}}{45\,b^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x, algorithm="giac")
```

```
[Out] 2/45*(5*sqrt(b*cos(f*x + e))*b^4*cos(f*x + e)^4 - 27*sqrt(b*cos(f*x + e))*b
^4*cos(f*x + e)^2 + 135*sqrt(b*cos(f*x + e))*b^4 + 15*b^5/(sqrt(b*cos(f*x +
 e))*cos(f*x + e)))*sgn(cos(f*x + e))/(b^2*f)
```

3.398 $\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx$

Optimal. Leaf size=63

$$-\frac{2b^5}{5f(b\sec(e+fx))^{5/2}}+\frac{4b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] (-2*b^5)/(5*f*(b*Sec[e + f*x])^(5/2)) + (4*b^3)/(f*Sqrt[b*Sec[e + f*x]]) +
(2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rubi [A] time = 0.0557179, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$-\frac{2b^5}{5f(b\sec(e+fx))^{5/2}}+\frac{4b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^5,x]
```

```
[Out] (-2*b^5)/(5*f*(b*Sec[e + f*x])^(5/2)) + (4*b^3)/(f*Sqrt[b*Sec[e + f*x]]) +
(2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx &= \frac{b^5\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{7/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\
&= \frac{b^5\,\text{Subst}\left(\int \left(\frac{1}{x^{7/2}}-\frac{2}{b^2x^{3/2}}+\frac{\sqrt{x}}{b^4}\right)dx, x, b\sec(e+fx)\right)}{f}\\
&= -\frac{2b^5}{5f(b\sec(e+fx))^{5/2}}+\frac{4b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}
\end{aligned}$$

Mathematica [A] time = 0.340456, size = 42, normalized size = 0.67

$$\frac{b(108\cos(2(e+fx))-3\cos(4(e+fx))+151)(b\sec(e+fx))^{3/2}}{60f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^5,x]
```

```
[Out] (b*(151 + 108*Cos[2*(e + f*x)] - 3*Cos[4*(e + f*x)])*(b*Sec[e + f*x])^(3/2)
)/(60*f)
```

Maple [B] time = 0.148, size = 522, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x)
```

```
[Out] -1/15/f*(-1+cos(f*x+e))^2*(6*cos(f*x+e)^4-15*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)+15*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(
f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-15*cos(f*x+e)*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)+15*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*
(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-60*cos(f*x+e)^2
-10)*cos(f*x+e)*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(5/2)/sin(f*x+e)^4
```

Maxima [A] time = 1.04202, size = 70, normalized size = 1.11

$$\frac{2\left(5\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}} - \frac{3\left(b^4 - \frac{10\,b^4}{\cos\left(fx+e\right)^2}\right)}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}}\right)b}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] 2/15*(5*(b/cos(f*x + e))^(3/2) - 3*(b^4 - 10*b^4/cos(f*x + e)^2)/(b/cos(f*x
 + e))^(5/2))*b/f
```

Fricas [A] time = 2.23493, size = 135, normalized size = 2.14

$$-\frac{2\left(3\,b^2\cos\left(fx+e\right)^4 - 30\,b^2\cos\left(fx+e\right)^2 - 5\,b^2\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{15\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -2/15*(3*b^2*cos(f*x + e)^4 - 30*b^2*cos(f*x + e)^2 - 5*b^2)*sqrt(b/cos(f*x
 + e))/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17027, size = 108, normalized size = 1.71

$$-\frac{2\left(3\sqrt{b\cos\left(fx+e\right)}b^2\cos\left(fx+e\right)^2-30\sqrt{b\cos\left(fx+e\right)}b^2-\frac{5\,b^3}{\sqrt{b\cos\left(fx+e\right)}\cos\left(fx+e\right)}\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] -2/15*(3*sqrt(b*cos(f*x + e))*b^2*cos(f*x + e)^2 - 30*sqrt(b*cos(f*x + e))*
b^2 - 5*b^3/(sqrt(b*cos(f*x + e))*cos(f*x + e)))*sgn(cos(f*x + e))/f
```

3.399 $\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx$

Optimal. Leaf size=41

$$\frac{2b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] (2*b^3)/(f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rubi [A] time = 0.0492443, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 14}

$$\frac{2b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^3,x]
```

```
[Out] (2*b^3)/(f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx &= \frac{b^3\,\text{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{3/2}}\,dx,x,b\sec(e+fx)\right)}{f}\\ &= \frac{b^3\,\text{Subst}\left(\int \left(-\frac{1}{x^{3/2}}+\frac{\sqrt{x}}{b^2}\right)dx,x,b\sec(e+fx)\right)}{f}\\ &= \frac{2b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}\end{aligned}$$

Mathematica [A] time = 0.197709, size = 32, normalized size = 0.78

$$\frac{b(3\cos(2(e+fx))+5)(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^3,x]
```

```
[Out] (b*(5 + 3*Cos[2*(e + f*x)])*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Maple [B] time = 0.151, size = 357, normalized size = 8.7

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)\cos\left(fx+e\right)}{6f\left(\sin\left(fx+e\right)\right)^{2}}\left(12\left(\cos\left(fx+e\right)\right)^{3}\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}+3\left(\cos\left(fx+e\right)\right)^{2}\ln\left(-2\frac{1}{\left(\sin\left(fx+e\right)\right)^{2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x)
```

```
[Out] -1/6/f*(-1+cos(f*x+e))*(12*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)+3*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*
x+e)^2)-3*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)+12*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+
e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2))*cos(f*x+e)*(b/cos(f*x+e))^(5/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/s
in(f*x+e)^2
```

Maple [A] is not this; **Maxima [A]** time = 1.03316, size = 49, normalized size = 1.2

$$\frac{2\left(\frac{3b^{2}}{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}+\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\right)b}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 2/3*(3*b^2/sqrt(b/cos(f*x + e)) + (b/cos(f*x + e))^(3/2))*b/f
```

Fricas [A] time = 2.08816, size = 97, normalized size = 2.37

$$\frac{2\left(3b^{2}\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{3f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*cos(f*x + e)^2 + b^2)*sqrt(b/cos(f*x + e))/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15711, size = 72, normalized size = 1.76

$$\frac{2\left(3\sqrt{b\cos\left(fx+e\right)}b+\frac{b^{2}}{\sqrt{b\cos\left(fx+e\right)}\cos\left(fx+e\right)}\right)b\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(b*cos(f*x + e))*b + b^2/(sqrt(b*cos(f*x + e))*cos(f*x + e)))*b*
sgn(cos(f*x + e))/f
```

3.400 $\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx$

Optimal. Leaf size=20

$$\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rubi [A] time = 0.0357261, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x],x]
```

```
[Out] (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx &= \frac{b\,\text{Subst}\left(\int \sqrt{x}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= \frac{2b(b\sec(e+fx))^{3/2}}{3f}\end{aligned}$$

Mathematica [A] time = 0.0422307, size = 20, normalized size = 1.

$$\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x],x]
```

```
[Out] (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Maple [A] time = 0.013, size = 17, normalized size = 0.9

$$\frac{2b}{3f}\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e),x)
```

```
[Out] 2/3*b*(b*sec(f*x+e))^(3/2)/f
```

Maxima [A] time = 1.04531, size = 31, normalized size = 1.55

$$\frac{2\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\cos\left(fx+e\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] 2/3*(b/cos(f*x + e))^(5/2)*cos(f*x + e)/f
```

Fricas [A] time = 2.16279, size = 63, normalized size = 3.15

$$\frac{2b^2\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{3f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] 2/3*b^2*sqrt(b/cos(f*x + e))/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15035, size = 49, normalized size = 2.45

$$\frac{2 b^3 \operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{3 \sqrt{b\cos\left(fx+e\right)} f \cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e),x, algorithm="giac")
```

```
[Out] 2/3*b^3*sgn(cos(f*x + e))/(sqrt(b*cos(f*x + e))*f*cos(f*x + e))
```

3.401 $\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=78

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] (b^(5/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/f - (b^(5/2)*ArcTanh[Sqrt[b*
Sec[e + f*x]]/Sqrt[b]])/f + (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rubi [A] time = 0.0543744, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2622, 321, 329, 298, 203, 206}

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b^(5/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/f - (b^(5/2)*ArcTanh[Sqrt[b*
Sec[e + f*x]]/Sqrt[b]])/f + (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^{5/2}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{bf}$$

$$= \frac{2b(b\sec(e+fx))^{3/2}}{3f} + \frac{b\,\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{2b(b\sec(e+fx))^{3/2}}{3f} + \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f}$$

$$= \frac{2b(b\sec(e+fx))^{3/2}}{3f} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{b+x^2}\,d\right.}{}$$

$$= \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} + \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Mathematica [A] time = 0.191296, size = 87, normalized size = 1.12

$$\frac{(b\sec(e+fx))^{5/2}\left(4\sec^{\frac{3}{2}}(e+fx)+3\log\left(1-\sqrt{\sec(e+fx)}\right)-3\log\left(\sqrt{\sec(e+fx)}+1\right)+6\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)\right)}{6f\sec^{\frac{5}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((b*Sec[e + f*x])^(5/2)*(6*ArcTan[Sqrt[Sec[e + f*x]]] + 3*Log[1 - Sqrt[Sec[
e + f*x]]] - 3*Log[1 + Sqrt[Sec[e + f*x]]] + 4*Sec[e + f*x]^(3/2)))/(6*f*Se
c[e + f*x]^(5/2))
```

Maple [B] time = 0.119, size = 237, normalized size = 3.

$$-\frac{(-1+\cos(fx+e))\cos(fx+e)}{6f(\sin(fx+e))^2}\left(3(\cos(fx+e))^2\arctan\left(1/2\frac{1}{\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)-3(\cos(fx+e))^2\ln\left(-2\frac{1}{(\sin(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/6/f*(-1+cos(f*x+e))*(3*cos(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2))-3*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-1)/sin(f*x+e)^2)+4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*cos(f*x+e)*(b/cos(f*x+e))^(5/2)/(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)/sin(f*x+e)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.69001, size = 868, normalized size = 11.13

$$\left[\frac{6\sqrt{-b}b^2\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)\cos(fx+e)+3\sqrt{-b}b^2\cos(fx+e)\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2-\cos(fx+e)\right.}{\cos(fx+e)^2+2\,\mathrm{co}}\right.}{12f\cos(fx+e)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(-b)*b^2*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e
) + 1)/b)*cos(f*x + e) + 3*sqrt(-b)*b^2*cos(f*x + e)*log((b*cos(f*x + e)^2
- 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos
(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*b^2*sqrt(b/cos(f*
x + e)))/(f*cos(f*x + e)), -1/12*(6*b^(5/2)*arctan(1/2*sqrt(b/cos(f*x + e))
*(cos(f*x + e) - 1)/sqrt(b))*cos(f*x + e) - 3*b^(5/2)*cos(f*x + e)*log((b*c
os(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x +
e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) - 8*b^2*
sqrt(b/cos(f*x + e)))/(f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1702, size = 123, normalized size = 1.58

$$\frac{b^6\left(\frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3}-\frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}}+\frac{2}{\sqrt{b\cos(fx+e)}b^3\cos(fx+e)}\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*b^6*(3*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^3) - 3*arctan(
sqrt(b*cos(f*x + e))/sqrt(b))/b^(7/2) + 2/(sqrt(b*cos(f*x + e))*b^3*cos(f*x
 + e)))*sgn(cos(f*x + e))/f
```

3.402 $\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=113

$$\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}-\frac{7b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}+\frac{7b(b\sec(e+fx))^{3/2}}{6f}-\frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf}$$

```
[Out] (7*b^(5/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (7*b^(5/2)*ArcTanh
[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) + (7*b*(b*Sec[e + f*x])^(3/2))/(6*f)
- (Cot[e + f*x]^2*(b*Sec[e + f*x])^(7/2))/(2*b*f)
```

Rubi [A] time = 0.0836008, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2622, 288, 321, 329, 298, 203, 206}

$$\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}-\frac{7b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}+\frac{7b(b\sec(e+fx))^{3/2}}{6f}-\frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (7*b^(5/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (7*b^(5/2)*ArcTanh
[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) + (7*b*(b*Sec[e + f*x])^(3/2))/(6*f)
- (Cot[e + f*x]^2*(b*Sec[e + f*x])^(7/2))/(2*b*f)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^{9/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf} + \frac{7\,\text{Subst}\left(\int \frac{x^{5/2}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{4bf}$$

$$= \frac{7b(b\sec(e+fx))^{3/2}}{6f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf} + \frac{(7b)\,\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x,\right.}{4f}$$

$$= \frac{7b(b\sec(e+fx))^{3/2}}{6f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf} + \frac{(7b)\,\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x,\right.}{2f}$$

$$= \frac{7b(b\sec(e+fx))^{3/2}}{6f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf} - \frac{\left(7b^3\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x,\right.}{4f}$$

$$= \frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{7b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} + \frac{7b(b\sec(e+fx))^{3/2}}{6f} - \text{co}$$

Mathematica [A] time = 1.95521, size = 109, normalized size = 0.96

$$\frac{b^3\left(-12\csc^2(e+fx)+16\sec^2(e+fx)+21\sqrt{\sec(e+fx)}\left(\log\left(1-\sqrt{\sec(e+fx)}\right)-\log\left(\sqrt{\sec(e+fx)}+1\right)\right)+42\sqrt{\sec(e}\right.}{24f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b^3*(-12*Csc[e + f*x]^2 + 42*ArcTan[Sqrt[Sec[e + f*x]]]*Sqrt[Sec[e + f*x]]
 + 21*(Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqrt[Sec[e + f*x]]])*Sqrt[Sec[
```

```
e + f*x]] + 16*Sec[e + f*x]^2))/(24*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] time = 0.16, size = 699, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/24/f*(-1+cos(f*x+e))*(24*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)+48*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-3*ln(-(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4-21*arctan(1/2/(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*cos(f*x+e)^4+24*cos(f*x+e)^4*ln(-2*(2*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+24*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-4*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-28*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*cos(f*x+
e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+21*cos
(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-24*cos(f*x+e)^2*
ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*c
os(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+16*cos(f*
x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+16*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2))*cos(f*x+e)*(b/cos(f*x+e))^(5/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)/sin(f*x+e)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.70897, size = 1133, normalized size = 10.03

$$\left[\frac{42\left(b^2\cos\left(fx+e\right)^3 - b^2\cos\left(fx+e\right)\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos\left(fx+e\right)+1\right)}{2\,b}\right) + 21\left(b^2\cos\left(fx+e\right)^3 - b^2\cos\left(fx+e\right.\right.}{48\left(f\cos\left(fx+e\right)^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(42*(b^2*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-b)*arctan(1/2*sqrt(
-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) + 21*(b^2*cos(f*x + e)^3 - b
^2*cos(f*x + e))*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f
*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e
)^2 + 2*cos(f*x + e) + 1)) + 8*(7*b^2*cos(f*x + e)^2 - 4*b^2)*sqrt(b/cos(f*
x + e)))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), -1/48*(42*(b^2*cos(f*x + e)^3
 - b^2*cos(f*x + e))*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e)
- 1)/sqrt(b)) - 21*(b^2*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(b)*log((b*c
os(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x +
e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) - 8*(7*b
^2*cos(f*x + e)^2 - 4*b^2)*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)^3 - f*cos(
f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15476, size = 173, normalized size = 1.53

$$\frac{b^8\left(\frac{6\sqrt{b\cos\left(fx+e\right)}}{\left(b^2\cos\left(fx+e\right)^2-b^2\right)b^4}+\frac{21\arctan\left(\frac{\sqrt{b\cos\left(fx+e\right)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^5}-\frac{21\arctan\left(\frac{\sqrt{b\cos\left(fx+e\right)}}{\sqrt{b}}\right)}{b^{\frac{11}{2}}}+\frac{8}{\sqrt{b\cos\left(fx+e\right)}b^5\cos\left(fx+e\right)}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*b^8*(6*sqrt(b*cos(f*x + e))/((b^2*cos(f*x + e)^2 - b^2)*b^4) + 21*arct
an(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^5) - 21*arctan(sqrt(b*cos(f*x
 + e))/sqrt(b))/b^(11/2) + 8/(sqrt(b*cos(f*x + e))*b^5*cos(f*x + e)))*sgn(c
os(f*x + e))/f
```

3.403 $\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=143

$$-\frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3f}+\frac{77b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f}-\frac{77b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f}+\frac{77b(b\sec(e+fx))^{3/2}}{48f}-$$

```
[Out] (77*b^(5/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) - (77*b^(5/2)*ArcT
anh[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) + (77*b*(b*Sec[e + f*x])^(3/2))/(
48*f) - (11*Cot[e + f*x]^2*(b*Sec[e + f*x])^(7/2))/(16*b*f) - (Cot[e + f*x]
^4*(b*Sec[e + f*x])^(11/2))/(4*b^3*f)
```

Rubi [A] time = 0.097547, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2622, 288, 321, 329, 298, 203, 206}

$$-\frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3f}+\frac{77b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f}-\frac{77b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f}+\frac{77b(b\sec(e+fx))^{3/2}}{48f}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(b*Sec[e + f*x])^(5/2),x]
```


```
[Out] (77*b^(5/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) - (77*b^(5/2)*ArcT
anh[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) + (77*b*(b*Sec[e + f*x])^(3/2))/(
48*f) - (11*Cot[e + f*x]^2*(b*Sec[e + f*x])^(7/2))/(16*b*f) - (Cot[e + f*x]
^4*(b*Sec[e + f*x])^(11/2))/(4*b^3*f)
```


Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^{13/2}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\sec(e+fx)\right)}{b^5 f}$$

$$= -\frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3 f} + \frac{11\operatorname{Subst}\left(\int \frac{x^{9/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{8b^3 f}$$

$$= -\frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3 f} + \frac{77\operatorname{Subst}\left(\right.}{}$$

$$= \frac{77b(b\sec(e+fx))^{3/2}}{48f} - \frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+}{4b^3 f}$$

$$= \frac{77b(b\sec(e+fx))^{3/2}}{48f} - \frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+}{4b^3 f}$$

$$= \frac{77b(b\sec(e+fx))^{3/2}}{48f} - \frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+}{4b^3 f}$$

$$= \frac{77b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{77b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} + \frac{77b(b\sec(e+fx))^{3/2}}{48f} -$$

Mathematica [A] time = 1.19865, size = 119, normalized size = 0.83

$$\frac{b^3\left(-48\csc^4(e+fx) - 180\csc^2(e+fx) + 128\sec^2(e+fx) + 231\sqrt{\sec(e+fx)}\left(\log\left(1-\sqrt{\sec(e+fx)}\right) - \log\left(\sqrt{\sec(e+\right.\right.\right.}{192f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b^3*(-180*Csc[e + f*x]^2 - 48*Csc[e + f*x]^4 + 462*ArcTan[Sqrt[Sec[e + f*x
]]]*Sqrt[Sec[e + f*x]] + 231*(Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqrt[Se
c[e + f*x]]])*Sqrt[Sec[e + f*x]] + 128*Sec[e + f*x]^2))/(192*f*Sqrt[b*Sec[e
 + f*x]])
```

Maple [B] time = 0.126, size = 1161, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/192/f*(408*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+360*cos(f*x
+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-57*ln(-(2*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^5-231*arctan(1/2/(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2))*cos(f*x+e)^5+288*cos(f*x+e)^5*ln(-2*(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-504*cos(f*x+e)^3*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(3/2)+100*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+
e)^4+57*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos
(f*x+e)^4+231*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*cos(f*x+e)^4
-288*cos(f*x+e)^4*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)-456*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-456*cos(f*x+
e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+57*cos(f*x+e)^3*ln(-(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+231*cos(f*x+e)^3*arctan(1/2/
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-288*cos(f*x+e)^3*ln(-2*(2*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+484*cos(f*x+e)^2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-57*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-1)/sin(f*x+e)^2)-231*cos(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2))+288*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)-128*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*cos(
f*x+e)*(b/cos(f*x+e))^(5/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/sin(f*x+e)
^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.0086, size = 1395, normalized size = 9.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(462*(b^2*cos(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*cos(f*x + e))*
sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) + 2
31*(b^2*cos(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*cos(f*x + e))*sqrt(-b)*
log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/c
os(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))
 + 8*(77*b^2*cos(f*x + e)^4 - 121*b^2*cos(f*x + e)^2 + 32*b^2)*sqrt(b/cos(f
*x + e)))/(f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e)), -1/384*
(462*(b^2*cos(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*cos(f*x + e))*sqrt(b)
*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(b)) - 231*(b^2*cos
(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*cos(f*x + e))*sqrt(b)*log((b*cos(f
*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e))
+ 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) - 8*(77*b^2*
cos(f*x + e)^4 - 121*b^2*cos(f*x + e)^2 + 32*b^2)*sqrt(b/cos(f*x + e)))/(f*
cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15855, size = 215, normalized size = 1.5

$$\frac{b^{10}\left(\frac{231\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^7}-\frac{231\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{15}{2}}}+\frac{6\left(15\sqrt{b\cos(fx+e)}b^2\cos(fx+e)^2-19\sqrt{b\cos(fx+e)}b^2\right)}{\left(b^2\cos(fx+e)^2-b^2\right)^2b^6}+\frac{64}{\sqrt{b\cos(fx+e)}b^7\cos(fx+e)}\right)\mathrm{sg}}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/96*b^10*(231*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^7) - 231*a
rctan(sqrt(b*cos(f*x + e))/sqrt(b))/b^(15/2) + 6*(15*sqrt(b*cos(f*x + e))*b
^2*cos(f*x + e)^2 - 19*sqrt(b*cos(f*x + e))*b^2)/((b^2*cos(f*x + e)^2 - b^2
)^2*b^6) + 64/(sqrt(b*cos(f*x + e))*b^7*cos(f*x + e)))*sgn(cos(f*x + e))/f
```

3.404 $\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx$

Optimal. Leaf size=130

$$\frac{20b^3\sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}}+\frac{40b^3\sin(e+fx)}{21f\sqrt{b\sec(e+fx)}}-\frac{80b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{21f}+\frac{2b\sin^5(e+fx)(b\sec}{3f}$$

```
[Out] (-80*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])
/(21*f) + (40*b^3*Sin[e + f*x])/(21*f*Sqrt[b*Sec[e + f*x]]) + (20*b^3*Sin[e
 + f*x]^3)/(21*f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f*x])^(3/2)*Sin[e
+ f*x]^5)/(3*f)
```

Rubi [A] time = 0.146661, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2624, 2627, 3771, 2641}

$$\frac{20b^3\sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}}+\frac{40b^3\sin(e+fx)}{21f\sqrt{b\sec(e+fx)}}-\frac{80b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{21f}+\frac{2b\sin^5(e+fx)(b\sec}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^6,x]
```

```
[Out] (-80*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])
/(21*f) + (40*b^3*Sin[e + f*x])/(21*f*Sqrt[b*Sec[e + f*x]]) + (20*b^3*Sin[e
 + f*x]^3)/(21*f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f*x])^(3/2)*Sin[e
+ f*x]^5)/(3*f)
```

Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx = \frac{2b(b\sec(e+fx))^{3/2}\sin^5(e+fx)}{3f} - \frac{1}{3}\left(10b^2\right)\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx$$
$$= \frac{20b^3\sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^5(e+fx)}{3f} - \frac{1}{7}\left(20b^2\right)\int \sqrt{b\sec(e+f}$$
$$= \frac{40b^3\sin(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{20b^3\sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^5(e+fx)}{3f} - \frac{1}{21}$$
$$= \frac{40b^3\sin(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{20b^3\sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^5(e+fx)}{3f} - \frac{1}{21}$$
$$= -\frac{80b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{21f} + \frac{40b^3\sin(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{20b^3}{21f\sqrt{}}$$

Mathematica [A] time = 0.205723, size = 74, normalized size = 0.57

$$-\frac{b^2\sqrt{b\sec(e+fx)}\left(-58\sin(2(e+fx))+3\sin(4(e+fx))-56\tan(e+fx)+320\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{84f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^6,x]
```

```
[Out] -(b^2*Sqrt[b*Sec[e + f*x]]*(320*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2
] - 58*Sin[2*(e + f*x)] + 3*Sin[4*(e + f*x)] - 56*Tan[e + f*x]))/(84*f)
```

Maple [C] time = 0.177, size = 168, normalized size = 1.3

$$\frac{\left(-2+2\cos\left(fx+e\right)\right)\cos\left(fx+e\right)\left(\cos\left(fx+e\right)+1\right)^2}{21f\left(\sin\left(fx+e\right)\right)^3}\left(40i\cos\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x)
```

```
[Out] 2/21/f*(-1+cos(f*x+e))*(40*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e
)-3*cos(f*x+e)^5+3*cos(f*x+e)^4+16*cos(f*x+e)^3-16*cos(f*x+e)^2+7*cos(f*x+e
)-7)*cos(f*x+e)*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(5/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2\cos\left(fx+e\right)^6-3\,b^2\cos\left(fx+e\right)^4+3\,b^2\cos\left(fx+e\right)^2-b^2\right)\sqrt{b\sec\left(fx+e\right)}\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^6 - 3*b^2*cos(f*x + e)^4 + 3*b^2*cos(f*x + e)^2
 - b^2)*sqrt(b*sec(f*x + e))*sec(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^6, x)
```

3.405 $\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx$

Optimal. Leaf size=100

$$\frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}-\frac{8b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b\sin^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] (-8*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/
(3*f) + (4*b^3*Sin[e + f*x])/(3*f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f
*x])^(3/2)*Sin[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.103034, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2624, 2627, 3771, 2641}

$$\frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}-\frac{8b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b\sin^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^4,x]
```

```
[Out] (-8*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/
(3*f) + (4*b^3*Sin[e + f*x])/(3*f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f
*x])^(3/2)*Sin[e + f*x]^3)/(3*f)
```

Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = \frac{2b(b\sec(e+fx))^{3/2}\sin^3(e+fx)}{3f} - \left(2b^2\right)\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx$$
$$= \frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^3(e+fx)}{3f} - \frac{1}{3}\left(4b^2\right)\int \sqrt{b\sec(e+f}$$
$$= \frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^3(e+fx)}{3f} - \frac{1}{3}\left(4b^2\sqrt{\cos(e+fx)}\sqrt{b}\right.$$
$$= -\frac{8b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{2b(b\,}{}$$

Mathematica [A] time = 0.134093, size = 64, normalized size = 0.64

$$-\frac{b^2\sqrt{b\sec(e+fx)}\left(-\sin(2(e+fx))-2\tan(e+fx)+8\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^4,x]
```

```
[Out] -(b^2*Sqrt[b*Sec[e + f*x]]*(8*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]
- Sin[2*(e + f*x)] - 2*Tan[e + f*x]))/(3*f)
```

Maple [C] time = 0.14, size = 144, normalized size = 1.4

$$\frac{\left(-2+2\cos\left(fx+e\right)\right)\cos\left(fx+e\right)\left(\cos\left(fx+e\right)+1\right)^2}{3f\left(\sin\left(fx+e\right)\right)^3}\left(4i\cos\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x)
```

```
[Out] 2/3/f*(-1+cos(f*x+e))*(4*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+
cos(f*x+e)^3-cos(f*x+e)^2+cos(f*x+e)-1)*cos(f*x+e)*(cos(f*x+e)+1)^2*(b/cos(
f*x+e))^(5/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos\left(fx+e\right)^4-2\,b^2\cos\left(fx+e\right)^2+b^2\right)\sqrt{b\sec\left(fx+e\right)}\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(f*x + e)^4 - 2*b^2*cos(f*x + e)^2 + b^2)*sqrt(b*sec(f*x +
 e))*sec(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^4, x)
```

3.406 $\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx$

Optimal. Leaf size=70

$$\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{4b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}$$

```
[Out] (-4*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/
(3*f) + (2*b*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)
```

Rubi [A] time = 0.0646299, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2624, 3771, 2641}

$$\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{4b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^2,x]
```

```
[Out] (-4*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/
(3*f) + (2*b*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)
```

Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx &= \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f}-\frac{1}{3}\left(2b^2\right)\int\sqrt{b\sec(e+fx)}\,dx\\ &= \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f}-\frac{1}{3}\left(2b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{}{\sqrt{\cos}}\\ &= -\frac{4b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b(b\sec(e+fx))^{3/2}\sin(e}{3f}\end{aligned}$$

Mathematica [A] time = 0.124455, size = 52, normalized size = 0.74

$$\frac{2b^2\sqrt{b\sec(e+fx)}\left(\tan(e+fx)-2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[b*Sec[e + f*x]]*(-2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2
] + Tan[e + f*x]))/(3*f)
```

Maple [C] time = 0.129, size = 126, normalized size = 1.8

$$\frac{\left(-2+2\cos\left(fx+e\right)\right)\cos\left(fx+e\right)\left(\cos\left(fx+e\right)+1\right)^2}{3f\left(\sin\left(fx+e\right)\right)^3}\left(2i\cos\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x)
```

```
[Out] 2/3/f*(-1+cos(f*x+e))*(2*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+
cos(f*x+e)-1)*cos(f*x+e)*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(5/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b^2\cos\left(fx+e\right)^2-b^2\right)\sqrt{b\sec\left(fx+e\right)}\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - b^2)*sqrt(b*sec(f*x + e))*sec(f*x + e)^2, x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^2, x)
```

3.407 $\int (b\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=70

$$\frac{2b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] (2*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(
3*f) + (2*b*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)
```

Rubi [A] time = 0.034645, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3771, 2641}

$$\frac{2b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(
3*f) + (2*b*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^{5/2}\,dx &= \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f}+\frac{1}{3}b^2\int \sqrt{b\sec(e+fx)}\,dx\\ &= \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f}+\frac{1}{3}\left(b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx\\ &= \frac{2b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.0753007, size = 51, normalized size = 0.73

$$\frac{2b^2\sqrt{b\sec(e+fx)}\left(\tan(e+fx)+\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[b*Sec[e + f*x]]*(Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] +
 Tan[e + f*x]))/(3*f)
```

Maple [C] time = 0.123, size = 128, normalized size = 1.8

$$-\frac{(-2+2\cos(fx+e))\cos(fx+e)(\cos(fx+e)+1)^2}{3f(\sin(fx+e))^3}\left(i\cos(fx+e)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{(\cos(f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(5/2),x)
```

```
[Out] -2/3/f*(-1+cos(f*x+e))*(I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-c
os(f*x+e)+1)*cos(f*x+e)*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(5/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(b\sec(fx+e))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec(fx+e)}b^2\sec(fx+e)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*b^2*sec(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2), x)
```

3.408 $\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=98

$$-\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{5b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] (-5*b^3*Csc[e + f*x])/(3*f*Sqrt[b*Sec[e + f*x]]) + (5*b^2*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*f) + (2*b*Csc[e + f*x]
*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rubi [A] time = 0.102253, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2626, 2625, 3771, 2641}

$$-\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{5b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-5*b^3*Csc[e + f*x])/(3*f*Sqrt[b*Sec[e + f*x]]) + (5*b^2*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*f) + (2*b*Csc[e + f*x]
*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f} + \frac{1}{3}\left(5b^2\right)\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$= -\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f} + \frac{1}{6}\left(5b^2\right)\int \sqrt{b\sec(e+fx)}$$
$$= -\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f} + \frac{1}{6}\left(5b^2\sqrt{\cos(e+fx)}\sqrt{b\sec}\right.$$
$$= -\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{5b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{2b\csc(e}{}$$

Mathematica [A] time = 0.181185, size = 67, normalized size = 0.68

$$\frac{b\sin(e+fx)(b\sec(e+fx))^{3/2}\left(-3\cot^2(e+fx)+5\cos^{\frac{3}{2}}(e+fx)\csc(e+fx)F\left(\frac{1}{2}(e+fx)\middle|2\right)+2\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b*(2 - 3*Cot[e + f*x]^2 + 5*Cos[e + f*x]^(3/2)*Csc[e + f*x]*EllipticF[(e +
 f*x)/2, 2])*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)
```

Maple [C] time = 0.145, size = 202, normalized size = 2.1

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^2\cos\left(fx+e\right)\left(\cos\left(fx+e\right)+1\right)^2}{3\,f\left(\sin\left(fx+e\right)\right)^5}\left(5\,i\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{}{\mathrm{co}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/3/f*(-1+cos(f*x+e))^2*(5*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x
+e)+5*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-5*cos(f*x+e)^2+2)*c
os(f*x+e)*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(5/2)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)}b^2\csc\left(fx+e\right)^2\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*b^2*csc(f*x + e)^2*sec(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^2, x)
```

3.409 $\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=123

$$-\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}}+\frac{5b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{2f}-\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}+\frac{b\csc(e+fx}{}$$

```
[Out] (-5*b^3*Csc[e + f*x])/(2*f*Sqrt[b*Sec[e + f*x]]) + (5*b^2*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(2*f) + (b*Csc[e + f*x]*(
b*Sec[e + f*x])^(3/2))/f - (b*Csc[e + f*x]^3*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rubi [A] time = 0.150323, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2625, 2626, 3771, 2641}

$$-\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}}+\frac{5b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{2f}-\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}+\frac{b\csc(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(b*Sec[e + f*x])^(5/2),x]
```


```
[Out] (-5*b^3*Csc[e + f*x])/(2*f*Sqrt[b*Sec[e + f*x]]) + (5*b^2*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(2*f) + (b*Csc[e + f*x]*(
b*Sec[e + f*x])^(3/2))/f - (b*Csc[e + f*x]^3*(b*Sec[e + f*x])^(3/2))/(3*f)
```


Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx = -\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f} + \frac{3}{2}\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx$$

$$= \frac{b\csc(e+fx)(b\sec(e+fx))^{3/2}}{f} - \frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f} + \frac{1}{2}\left(5b^2\right)\int \mathrm{c}$$

$$= -\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}} + \frac{b\csc(e+fx)(b\sec(e+fx))^{3/2}}{f} - \frac{b\csc^3(e+fx)(b\sec(e+$$

$$= -\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}} + \frac{b\csc(e+fx)(b\sec(e+fx))^{3/2}}{f} - \frac{b\csc^3(e+fx)(b\sec(e+$$

$$= -\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}} + \frac{5b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{2f} + \frac{b\csc}{}$$

Mathematica [A] time = 0.405542, size = 79, normalized size = 0.64

$$\frac{b\sin(e+fx)(b\sec(e+fx))^{3/2}\left(\cot^2(e+fx)\left(-\left(2\csc^2(e+fx)+11\right)\right)+15\cos^{\frac{3}{2}}(e+fx)\csc(e+fx)F\left(\frac{1}{2}(e+fx)\middle|2\right)+}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b*(4 - Cot[e + f*x]^2*(11 + 2*Csc[e + f*x]^2) + 15*Cos[e + f*x]^(3/2)*Csc[
e + f*x]*EllipticF[(e + f*x)/2, 2])*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(6
*f)
```

Maple [C] time = 0.175, size = 352, normalized size = 2.9

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^2\cos\left(fx+e\right)\left(\cos\left(fx+e\right)+1\right)^2}{6\,f\left(\sin\left(fx+e\right)\right)^7}\left(15\,i\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\left(\mathrm{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/6/f*(-1+cos(f*x+e))^2*(15*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4*sin(f*x+e)*(1/(cos(f*x+e)+1))^
(1/2)+15*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellip
ticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3*sin(f*x+e)-15*I*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e
))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)-15*I*cos(f*x+e)*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*sin(f*x+e)-15*cos(f*x+e)^4+21*cos(f*x+e)^2-4)*cos(f*x+e)*(cos(f*x+e
)+1)^2*(b/cos(f*x+e))^(5/2)/sin(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec\left(fx+e\right)}b^2\csc\left(fx+e\right)^4\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*b^2*csc(f*x + e)^4*sec(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^4, x)
```

3.410 $\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=87

$$\frac{2b^7}{15f(b\sec(e+fx))^{15/2}}-\frac{6b^5}{11f(b\sec(e+fx))^{11/2}}+\frac{6b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[Out] (2*b^7)/(15*f*(b*Sec[e + f*x])^(15/2)) - (6*b^5)/(11*f*(b*Sec[e + f*x])^(11
/2)) + (6*b^3)/(7*f*(b*Sec[e + f*x])^(7/2)) - (2*b)/(3*f*(b*Sec[e + f*x])^(
3/2))
```

Rubi [A] time = 0.0570967, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$\frac{2b^7}{15f(b\sec(e+fx))^{15/2}}-\frac{6b^5}{11f(b\sec(e+fx))^{11/2}}+\frac{6b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^7/Sqrt[b*Sec[e + f*x]],x]
```


```
[Out] (2*b^7)/(15*f*(b*Sec[e + f*x])^(15/2)) - (6*b^5)/(11*f*(b*Sec[e + f*x])^(11
/2)) + (6*b^3)/(7*f*(b*Sec[e + f*x])^(7/2)) - (2*b)/(3*f*(b*Sec[e + f*x])^(
3/2))
```


Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx&=\frac{b^7\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{17/2}}\,dx,x,b\sec(e+fx)\right)}{f}\\&=\frac{b^7\,\text{Subst}\left(\int \left(-\frac{1}{x^{17/2}}+\frac{3}{b^2x^{13/2}}-\frac{3}{b^4x^{9/2}}+\frac{1}{b^6x^{5/2}}\right)dx,x,b\sec(e+fx)\right)}{f}\\&=\frac{2b^7}{15f(b\sec(e+fx))^{15/2}}-\frac{6b^5}{11f(b\sec(e+fx))^{11/2}}+\frac{6b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.218208, size = 52, normalized size = 0.6

$$\frac{b(4035\cos(2(e+fx))-798\cos(4(e+fx))+77\cos(6(e+fx))-7410)}{18480f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^7/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (b*(-7410 + 4035*Cos[2*(e + f*x)] - 798*Cos[4*(e + f*x)] + 77*Cos[6*(e + f*
x)]))/(18480*f*(b*Sec[e + f*x])^(3/2))
```

Maple [A] time = 0.155, size = 56, normalized size = 0.6

$$\frac{\left(154\left(\cos\left(fx+e\right)\right)^{6}-630\left(\cos\left(fx+e\right)\right)^{4}+990\left(\cos\left(fx+e\right)\right)^{2}-770\right)\cos\left(fx+e\right)}{1155f}\frac{1}{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/1155/f*(77*cos(f*x+e)^6-315*cos(f*x+e)^4+495*cos(f*x+e)^2-385)*cos(f*x+e)
/(b/cos(f*x+e))^(1/2)
```

Maxima [A] time = 1.00202, size = 85, normalized size = 0.98

$$\frac{2\left(77\,b^{6}-\frac{315\,b^{6}}{\cos\left(fx+e\right)^{2}}+\frac{495\,b^{6}}{\cos\left(fx+e\right)^{4}}-\frac{385\,b^{6}}{\cos\left(fx+e\right)^{6}}\right)b}{1155\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(77*b^6 - 315*b^6/cos(f*x + e)^2 + 495*b^6/cos(f*x + e)^4 - 385*b^6/
cos(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(15/2))
```

Fricas [A] time = 2.71872, size = 159, normalized size = 1.83

$$\frac{2\left(77\cos\left(fx+e\right)^{8}-315\cos\left(fx+e\right)^{6}+495\cos\left(fx+e\right)^{4}-385\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{1155\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*cos(f*x + e)^8 - 315*cos(f*x + e)^6 + 495*cos(f*x + e)^4 - 385*c
os(f*x + e)^2)*sqrt(b/cos(f*x + e))/(b*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**7/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^7}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^7/sqrt(b*sec(f*x + e)), x)
```

3.411 $\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=65

$$-\frac{2b^5}{11f(b\sec(e+fx))^{11/2}}+\frac{4b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[Out] (-2*b^5)/(11*f*(b*Sec[e + f*x])^(11/2)) + (4*b^3)/(7*f*(b*Sec[e + f*x])^(7/
2)) - (2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.0496222, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$-\frac{2b^5}{11f(b\sec(e+fx))^{11/2}}+\frac{4b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-2*b^5)/(11*f*(b*Sec[e + f*x])^(11/2)) + (4*b^3)/(7*f*(b*Sec[e + f*x])^(7/
2)) - (2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx &= \frac{b^5\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{13/2}}\,dx,x,b\sec(e+fx)\right)}{f}\\ &= \frac{b^5\,\text{Subst}\left(\int \left(\frac{1}{x^{13/2}}-\frac{2}{b^2x^{9/2}}+\frac{1}{b^4x^{5/2}}\right)dx,x,b\sec(e+fx)\right)}{f}\\ &= -\frac{2b^5}{11f(b\sec(e+fx))^{11/2}}+\frac{4b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.162789, size = 42, normalized size = 0.65

$$\frac{b(180\cos(2(e+fx))-21\cos(4(e+fx))-415)}{924f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (b*(-415 + 180*Cos[2*(e + f*x)] - 21*Cos[4*(e + f*x)]))/(924*f*(b*Sec[e + f
*x])^(3/2))
```

Maple [A] time = 0.128, size = 46, normalized size = 0.7

$$-\frac{\left(42\left(\cos\left(fx+e\right)\right)^{4}-132\left(\cos\left(fx+e\right)\right)^{2}+154\right)\cos\left(fx+e\right)}{231f}\frac{1}{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/231/f*(21*cos(f*x+e)^4-66*cos(f*x+e)^2+77)*cos(f*x+e)/(b/cos(f*x+e))^(1/
2)
```

Maxima [A] time = 1.03342, size = 68, normalized size = 1.05

$$-\frac{2\left(21\,b^{4}-\frac{66\,b^{4}}{\cos\left(fx+e\right)^{2}}+\frac{77\,b^{4}}{\cos\left(fx+e\right)^{4}}\right)b}{231\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/231*(21*b^4 - 66*b^4/cos(f*x + e)^2 + 77*b^4/cos(f*x + e)^4)*b/(f*(b/cos
(f*x + e))^(11/2))
```

Fricas [A] time = 2.58573, size = 128, normalized size = 1.97

$$-\frac{2\left(21\cos\left(fx+e\right)^{6}-66\cos\left(fx+e\right)^{4}+77\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{231\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/231*(21*cos(f*x + e)^6 - 66*cos(f*x + e)^4 + 77*cos(f*x + e)^2)*sqrt(b/c
os(f*x + e))/(b*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)^5}{\sqrt{b\sec(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^5/sqrt(b*sec(f*x + e)), x)
```

3.412 $\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=43

$$\frac{2b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[Out] (2*b^3)/(7*f*(b*Sec[e + f*x])^(7/2)) - (2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.0448939, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 14}

$$\frac{2b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (2*b^3)/(7*f*(b*Sec[e + f*x])^(7/2)) - (2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx &= \frac{b^3\,\text{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{9/2}}\,dx,x,b\sec(e+fx)\right)}{f}\\
&= \frac{b^3\,\text{Subst}\left(\int \left(-\frac{1}{x^{9/2}}+\frac{1}{b^2x^{5/2}}\right)dx,x,b\sec(e+fx)\right)}{f}\\
&= \frac{2b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.104797, size = 32, normalized size = 0.74

$$\frac{b(3\cos(2(e+fx))-11)}{21f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (b*(-11 + 3*Cos[2*(e + f*x)]))/(21*f*(b*Sec[e + f*x])^(3/2))
```

Maple [A] time = 0.115, size = 36, normalized size = 0.8

$$\frac{\left(6\left(\cos\left(fx+e\right)\right)^{2}-14\right)\cos\left(fx+e\right)}{21f}\frac{1}{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/21/f*(3*cos(f*x+e)^2-7)*cos(f*x+e)/(b/cos(f*x+e))^(1/2)
```

Maxima [A] time = 1.0086, size = 50, normalized size = 1.16

$$\frac{2\left(3\,b^{2}-\frac{7\,b^{2}}{\cos\left(fx+e\right)^{2}}\right)b}{21\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*(3*b^2 - 7*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(7/2))
```

Fricas [A] time = 2.41556, size = 96, normalized size = 2.23

$$\frac{2\left(3\,\cos\left(fx+e\right)^{4}-7\,\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{21\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*cos(f*x + e)^4 - 7*cos(f*x + e)^2)*sqrt(b/cos(f*x + e))/(b*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48948, size = 68, normalized size = 1.58

$$\frac{2\left(3\,b^2-\frac{7\,b^2}{\cos\left(fx+e\right)^2}\right)\cos\left(fx+e\right)^3}{21\,b^2f\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*(3*b^2 - 7*b^2/cos(f*x + e)^2)*cos(f*x + e)^3/(b^2*f*sqrt(b/cos(f*x +
e)))
```

3.413 $\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=20

$$-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[Out] (-2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.0307073, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= -\frac{2b}{3f(b\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0450973, size = 20, normalized size = 1.

$$-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Maple [A] time = 0.019, size = 17, normalized size = 0.9

$$-\frac{2b}{3f}\left(b\sec\left(fx+e\right)\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/3*b/f/(b*sec(f*x+e))^(3/2)
```

Maxima [A] time = 1.0032, size = 31, normalized size = 1.55

$$-\frac{2\cos\left(fx+e\right)}{3f\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*cos(f*x + e)/(f*sqrt(b/cos(f*x + e)))
```

Fricas [A] time = 2.35535, size = 65, normalized size = 3.25

$$-\frac{2\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}}{3bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b/cos(f*x + e))*cos(f*x + e)^2/(b*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)/sqrt(b*sec(e + f*x)), x)
```

Giac [B] time = 1.43903, size = 51, normalized size = 2.55

$$-\frac{2\sqrt{b\cos\left(fx+e\right)}|f|\cos\left(fx+e\right)\operatorname{sgn}\left(f\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{3\,bf^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*cos(f*x + e))*abs(f)*cos(f*x + e)*sgn(f)*sgn(cos(f*x + e))/(b*f
^2)
```

3.414 $\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=59

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}$$

```
[Out] -(ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(Sqrt[b]*f)) - ArcTanh[Sqrt[b*Sec[e
+ f*x]]/Sqrt[b]]/(Sqrt[b]*f)
```

Rubi [A] time = 0.0422869, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2622, 329, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -(ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(Sqrt[b]*f)) - ArcTanh[Sqrt[b*Sec[e
+ f*x]]/Sqrt[b]]/(Sqrt[b]*f)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{bf}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{bf}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f} - \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}$$

Mathematica [A] time = 0.105999, size = 73, normalized size = 1.24

$$-\frac{\sqrt{\sec(e+fx)}\left(-\log\left(1-\sqrt{\sec(e+fx)}\right)+\log\left(\sqrt{\sec(e+fx)}+1\right)+2\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)\right)}{2f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[b*Sec[e + f*x]],x]

[Out] -((2*ArcTan[Sqrt[Sec[e + f*x]]] - Log[1 - Sqrt[Sec[e + f*x]]] + Log[1 + Sqr
t[Sec[e + f*x]]])*Sqrt[Sec[e + f*x]])/(2*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] time = 0.117, size = 161, normalized size = 2.7

$$-\frac{-1+\cos\left(fx+e\right)}{2f\left(\sin\left(fx+e\right)\right)^2}\left(\ln\left(-\frac{1}{\left(\sin\left(fx+e\right)\right)^2}\left(2\left(\cos\left(fx+e\right)\right)^2\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^2}}-\left(\cos\left(fx+e\right)\right)^2+2\cos\left(fx+e\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x)

[Out] -1/2/f*(ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+arc
tan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)))*(-1+cos(f*x+e))/sin(f*x+e)^2
/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b/cos(f*x+e))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.9606, size = 670, normalized size = 11.36

$$\left[\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)-\sqrt{-b}\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}-6b\cos(fx+e)+b}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)}{4bf}, \frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1
)/b) - sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*s
qrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*co
s(f*x + e) + 1)))/(b*f), 1/4*(2*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(co
s(f*x + e) - 1)/sqrt(b)) + sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^
2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos
(f*x + e)^2 - 2*cos(f*x + e) + 1)))/(b*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*sec(f*x + e)), x)
```

3.415 $\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=93

$$-\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf}-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f}$$

```
[Out] -ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(4*Sqrt[b]*f) - ArcTanh[Sqrt[b*Sec[e
+ f*x]]/Sqrt[b]]/(4*Sqrt[b]*f) - (Cot[e + f*x]^2*Sqrt[b*Sec[e + f*x]])/(2*b
*f)
```

Rubi [A] time = 0.0668614, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2622, 288, 329, 212, 206, 203}

$$-\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf}-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(4*Sqrt[b]*f) - ArcTanh[Sqrt[b*Sec[e
+ f*x]]/Sqrt[b]]/(4*Sqrt[b]*f) - (Cot[e + f*x]^2*Sqrt[b*Sec[e + f*x]])/(2*b
*f)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
```

```
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^{3/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}\\
&= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{4bf}\\
&= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{2bf}\\
&= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf} - \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4f} - \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x,\right.}{4f}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf}
\end{aligned}
$$

Mathematica [A] time = 1.16505, size = 93, normalized size = 1.

$$
\frac{\sqrt{\sec(e+fx)}\left(\log\left(1-\sqrt{\sec(e+fx)}\right)-\log\left(\sqrt{\sec(e+fx)}+1\right)-\frac{4\csc^2(e+fx)}{\sec^{\frac{3}{2}}(e+fx)}-2\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)\right)}{8f\sqrt{b\sec(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] ((-2*ArcTan[Sqrt[Sec[e + f*x]]] + Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqr
t[Sec[e + f*x]]] - (4*Csc[e + f*x]^2)/Sec[e + f*x]^(3/2))*Sqrt[Sec[e + f*x]
])/(8*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] time = 0.141, size = 425, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/8/f*(-1+cos(f*x+e))*(8*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
+16*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-4*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2))-cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)+8*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+4*cos(f*x+e)*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2))+ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2))/sin(
f*x+e)^4/(b/cos(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.91135, size = 950, normalized size = 10.22

$$\left[\frac{2\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos(fx+e)+1\right)}{2b}\right)+8\sqrt{\frac{b}{\cos(fx+e)}}\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\log}{16\left(bf\cos\left(fx+e\right)^2-bf\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x +
 e))*(cos(f*x + e) + 1)/b) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 - (cos(f
*x + e)^2 - 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x
 + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^
2 + 2*cos(f*x + e) + 1)))/(b*f*cos(f*x + e)^2 - b*f), 1/16*(2*(cos(f*x + e)
^2 - 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(b))
 + 8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sqrt(b)*log
((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f
*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)))/(b
*f*cos(f*x + e)^2 - b*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/sqrt(b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(b*sec(f*x + e)), x)
```

3.416 $\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3f}-\frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf}-\frac{5\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f}$$

```
[Out] (-5*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*Sqrt[b]*f) - (5*ArcTanh[Sqrt[
b*Sec[e + f*x]]/Sqrt[b]])/(32*Sqrt[b]*f) - (5*Cot[e + f*x]^2*Sqrt[b*Sec[e +
 f*x]])/(16*b*f) - (Cot[e + f*x]^4*(b*Sec[e + f*x])^(5/2))/(4*b^3*f)
```

Rubi [A] time = 0.0785193, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2622, 288, 329, 212, 206, 203}

$$-\frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3f}-\frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf}-\frac{5\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-5*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*Sqrt[b]*f) - (5*ArcTanh[Sqrt[
b*Sec[e + f*x]]/Sqrt[b]])/(32*Sqrt[b]*f) - (5*Cot[e + f*x]^2*Sqrt[b*Sec[e +
 f*x]])/(16*b*f) - (Cot[e + f*x]^4*(b*Sec[e + f*x])^(5/2))/(4*b^3*f)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^{7/2}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\sec(e+fx)\right)}{b^5 f}$$

$$= -\frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3 f} + \frac{5\,\text{Subst}\left(\int \frac{x^{3/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{8b^3 f}$$

$$= -\frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3 f} + \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x\right.}{32bf}$$

$$= -\frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3 f} + \frac{5\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b}\right.}{16bf}$$

$$= -\frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3 f} - \frac{5\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\,\text{s}}\right.}{32f}$$

$$= -\frac{5\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f} - \frac{5\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f} - \frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf} - \cot^4(e+$$

Mathematica [A] time = 2.23631, size = 107, normalized size = 0.87

$$-\frac{\sqrt{\sec(e+fx)}\left(-5\log\left(1-\sqrt{\sec(e+fx)}\right)+5\log\left(\sqrt{\sec(e+fx)}+1\right)+4\left(4\csc^4(e+fx)+\csc^2(e+fx)-5\right)\sqrt{\sec(e\right.}{64f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -((10*ArcTan[Sqrt[Sec[e + f*x]]] - 5*Log[1 - Sqrt[Sec[e + f*x]]] + 5*Log[1
+ Sqrt[Sec[e + f*x]]] + 4*(-5 + Csc[e + f*x]^2 + 4*Csc[e + f*x]^4)*Sqrt[Sec
[e + f*x]])*Sqrt[Sec[e + f*x]])/(64*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] time = 0.146, size = 729, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/64/f*(40*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+24*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-20*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-5*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-1)/sin(f*x+e)^2)-5*cos(f*x+e)^3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2))-72*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+40*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+5*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+5*cos(f*x+e)^2*arctan(1/2/(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-56*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-20
*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+5*cos(f*x+e)*ln(-(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+5*cos(f*x+e)*arctan(1/2/
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-5*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)-5*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)))/sin(f*x+e)^4/(b/cos(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.99466, size = 1188, normalized size = 9.66

$$\left[\frac{10\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}+1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos\left(fx+e\right)}}\left(\cos\left(fx+e\right)+1\right)}{2\,b}\right)-5\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}+1\right.}{128\left(bf\cos\left(fx+e\right)^{4}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/128*(10*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(1/2*sqrt
(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) - 5*(cos(f*x + e)^4 - 2*cos
(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f
*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e
```

```
)^2 + 2*cos(f*x + e) + 1)) + 8*(5*cos(f*x + e)^4 - 9*cos(f*x + e)^2)*sqrt(b
/cos(f*x + e)))/(b*f*cos(f*x + e)^4 - 2*b*f*cos(f*x + e)^2 + b*f), 1/128*(1
0*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x
 + e))*(cos(f*x + e) - 1)/sqrt(b)) + 5*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 +
 1)*sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(
b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x
 + e) + 1)) + 8*(5*cos(f*x + e)^4 - 9*cos(f*x + e)^2)*sqrt(b/cos(f*x + e)))
/(b*f*cos(f*x + e)^4 - 2*b*f*cos(f*x + e)^2 + b*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^5/sqrt(b*sec(f*x + e)), x)
```

3.417 $\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}-\frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}}-\frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}}+\frac{16E\left(\frac{1}{2}(e+fx)\middle|2\right)}{39f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] (16*EllipticE[(e + f*x)/2, 2])/(39*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]
]) - (8*b*Sin[e + f*x])/(39*f*(b*Sec[e + f*x])^(3/2)) - (20*b*Sin[e + f*x]^
3)/(117*f*(b*Sec[e + f*x])^(3/2)) - (2*b*Sin[e + f*x]^5)/(13*f*(b*Sec[e + f
*x])^(3/2))
```

Rubi [A] time = 0.143405, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2627, 3771, 2639}

$$-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}-\frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}}-\frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}}+\frac{16E\left(\frac{1}{2}(e+fx)\middle|2\right)}{39f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]
```


```
[Out] (16*EllipticE[(e + f*x)/2, 2])/(39*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]
]) - (8*b*Sin[e + f*x])/(39*f*(b*Sec[e + f*x])^(3/2)) - (20*b*Sin[e + f*x]^
3)/(117*f*(b*Sec[e + f*x])^(3/2)) - (2*b*Sin[e + f*x]^5)/(13*f*(b*Sec[e + f
*x])^(3/2))
```


Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}} + \frac{10}{13}\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$
$$= -\frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}} - \frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}} + \frac{20}{39}\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$
$$= -\frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}} - \frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}} - \frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}} + \frac{8}{39}\int \frac{1}{\sqrt{b\sec(e+}}$$
$$= -\frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}} - \frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}} - \frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}} + \frac{8\int\sqrt{\cos(e+}}{39\sqrt{\cos(e+fx)}\sqrt{}}$$
$$= \frac{16E\left(\frac{1}{2}(e+fx)\middle|2\right)}{39f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}} - \frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}} - \frac{2b\sin}{13f(b\sec}$$

Mathematica [A] time = 0.480945, size = 73, normalized size = 0.59

$$\frac{-317\sin(2(e+fx)) + 76\sin(4(e+fx)) - 9\sin(6(e+fx)) + \frac{768E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{1872f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] ((768*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] - 317*Sin[2*(e + f*x)]
+ 76*Sin[4*(e + f*x)] - 9*Sin[6*(e + f*x)])/(1872*f*Sqrt[b*Sec[e + f*x]])
```

Maple [C] time = 0.185, size = 338, normalized size = 2.8

$$-\frac{2}{117\,f\sin\left(fx+e\right)b}\left(-9\left(\cos\left(fx+e\right)\right)^{8} + 24\,i\mathrm{EllipticE}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{\left(\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/117/f*(-9*cos(f*x+e)^8+24*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*co
s(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)-24*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+37*cos(f*x+e)^6+24
*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-24*I*EllipticF(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(
f*x+e)-59*cos(f*x+e)^4+55*cos(f*x+e)^2-24*cos(f*x+e))*(b/cos(f*x+e))^(1/2)/
sin(f*x+e)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{6}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^{6}-3\,\cos\left(fx+e\right)^{4}+3\,\cos\left(fx+e\right)^{2}-1\right)\sqrt{b\sec\left(fx+e\right)}}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(b
*sec(f*x + e))/(b*sec(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(fx+e\right)^{6}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)), x)
```

3.418 $\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}-\frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}}+\frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] (8*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]
) - (4*b*Sin[e + f*x])/(15*f*(b*Sec[e + f*x])^(3/2)) - (2*b*Sin[e + f*x]^3)
/(9*f*(b*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.0998004, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2627, 3771, 2639}

$$-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}-\frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}}+\frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (8*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]
) - (4*b*Sin[e + f*x])/(15*f*(b*Sec[e + f*x])^(3/2)) - (2*b*Sin[e + f*x]^3)
/(9*f*(b*Sec[e + f*x])^(3/2))
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}+\frac{2}{3}\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$
$$= -\frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}}-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}+\frac{4}{15}\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx$$
$$= -\frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}}-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}+\frac{4\int \sqrt{\cos(e+fx)}\,dx}{15\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$= \frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}}-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 0.33787, size = 63, normalized size = 0.66

$$\frac{-68\sin(2(e+fx))+10\sin(4(e+fx))+\frac{192E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{360f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] ((192*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] - 68*Sin[2*(e + f*x)] +
 10*Sin[4*(e + f*x)])/(360*f*Sqrt[b*Sec[e + f*x]])
```

Maple [C] time = 0.15, size = 328, normalized size = 3.5

$$-\frac{2}{45\,f\sin\left(fx+e\right)b}\left(5\left(\cos\left(fx+e\right)\right)^{6}+12\,i\mathrm{EllipticE}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{\left(\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/45/f*(5*cos(f*x+e)^6+12*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(
f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)-12*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+12*I*EllipticE(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)-12*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-16*cos(f*x+
e)^4+23*cos(f*x+e)^2-12*cos(f*x+e))*(b/cos(f*x+e))^(1/2)/sin(f*x+e)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{4}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}+1\right)\sqrt{b\sec\left(fx+e\right)}}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b*sec(f*x + e))/(b*se
c(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^{4}\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**4/sqrt(b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{4}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

3.419 $\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=67

$$\frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

```
[Out] (4*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
 - (2*b*Sin[e + f*x])/(5*f*(b*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.0592002, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2627, 3771, 2639}

$$\frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (4*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
 - (2*b*Sin[e + f*x])/(5*f*(b*Sec[e + f*x])^(3/2))
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx &= -\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{2}{5}\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\ &= -\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{2\int \sqrt{\cos(e+fx)}\,dx}{5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\ &= \frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.133746, size = 60, normalized size = 0.9

$$-\frac{\sqrt{b\sec(e+fx)}\left(\sin(e+fx)+\sin(3(e+fx))-8\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{10bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -(Sqrt[b*Sec[e + f*x]]*(-8*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2] + S
in[e + f*x] + Sin[3*(e + f*x)]))/(10*b*f)
```

Maple [C] time = 0.185, size = 316, normalized size = 4.7

$$\frac{2}{5f\sin\left(fx+e\right)b}\left(2i\cos\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\sin\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/5/f*(2*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-2*I*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*EllipticE(I*(-1+c
os(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e
)-2*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+cos(f*x+e)^4-3*cos(f*x+e)^2+2*co
s(f*x+e))*(b/cos(f*x+e))^(1/2)/sin(f*x+e)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{2}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(\cos\left(fx+e\right)^{2}-1\right)\sqrt{b\sec\left(fx+e\right)}}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(b*sec(f*x + e))/(b*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/sqrt(b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

3.420 $\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=38

$$\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] (2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.0200134, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2639}

$$\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx &= \frac{\int \sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\ &= \frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0366981, size = 38, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Maple [C] time = 0.127, size = 306, normalized size = 8.1

$$2\frac{1}{f\sin\left(fx+e\right)b}\left(i\cos\left(fx+e\right)\text{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\sin\left(fx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f*(I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-I*EllipticE(I*(-1+co
s(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)+I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-I*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)^2+cos(f*x+e))*(b/cos(f*x+e))^(1
/2)/sin(f*x+e)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))/(b*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e)), x)
```

3.421 $\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=63

$$-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] -((b*Csc[e + f*x])/(f*(b*Sec[e + f*x])^(3/2))) - EllipticE[(e + f*x)/2, 2]/
(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.0580531, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2625, 3771, 2639}

$$-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -((b*Csc[e + f*x])/(f*(b*Sec[e + f*x])^(3/2))) - EllipticE[(e + f*x)/2, 2]/
(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx &= -\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}-\frac{1}{2}\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}-\frac{\int \sqrt{\cos(e+fx)}\,dx}{2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= -\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.151907, size = 48, normalized size = 0.76

$$\frac{-\cot(e+fx)-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-Cot[e + f*x] - EllipticE[(e + f*x)/2, 2]/Sqrt[Cos[e + f*x]])/(f*Sqrt[b*Se
c[e + f*x]])
```

Maple [C] time = 0.144, size = 316, normalized size = 5.

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(\cos\left(fx+e\right)+1\right)^{2}}{bf\left(\sin\left(fx+e\right)\right)^{5}}\left(i\cos\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(-1+cos(f*x+e))^2*(I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-I
*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+I*EllipticF(I*(-1+cos(f*x+
e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)-I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e))*(cos(f*x+e)
+1)^2*(b/cos(f*x+e))^(1/2)/b/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{2}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^{2}}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^2/(b*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

3.422 $\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}-\frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] -(b*Csc[e + f*x])/(2*f*(b*Sec[e + f*x])^(3/2)) - (b*Csc[e + f*x]^3)/(3*f*(b
*Sec[e + f*x])^(3/2)) - EllipticE[(e + f*x)/2, 2]/(2*f*Sqrt[Cos[e + f*x]]*S
qrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.101027, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2625, 3771, 2639}

$$-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}-\frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -(b*Csc[e + f*x])/(2*f*(b*Sec[e + f*x])^(3/2)) - (b*Csc[e + f*x]^3)/(3*f*(b
*Sec[e + f*x])^(3/2)) - EllipticE[(e + f*x)/2, 2]/(2*f*Sqrt[Cos[e + f*x]]*S
qrt[b*Sec[e + f*x]])
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{1}{2}\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$
$$= -\frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}} - \frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}} - \frac{1}{4}\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx$$
$$= -\frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}} - \frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}} - \frac{\int \sqrt{\cos(e+fx)}\,dx}{4\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$= -\frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}} - \frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}} - \frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Mathematica [A] time = 0.208904, size = 74, normalized size = 0.78

$$-\frac{\tan(e+fx)\left(2\csc^4(e+fx)+\csc^2(e+fx)+3\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)-3\right)}{6f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -((-3 + Csc[e + f*x]^2 + 2*Csc[e + f*x]^4 + 3*Sqrt[Cos[e + f*x]]*Csc[e + f*
x]*EllipticE[(e + f*x)/2, 2])*Tan[e + f*x])/(6*f*Sqrt[b*Sec[e + f*x]])
```

Maple [C] time = 0.177, size = 618, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/6/f*(-1+cos(f*x+e))^2*(3*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)-3*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*I*EllipticF(I*(
-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+
e)^2*sin(f*x+e)-3*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+3*I*Ell
ipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
sin(f*x+e)+3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*cos(f*x+e)^3-2*cos(f*
x+e)^2-3*cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(1/2)/b/sin(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^{4}}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^4/(b*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc^{4}\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/sqrt(b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc\left(fx+e\right)^{4}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

3.423 $\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}-\frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}}-\frac{7E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] (-7*b*Csc[e + f*x])/(20*f*(b*Sec[e + f*x])^(3/2)) - (7*b*Csc[e + f*x]^3)/(3
0*f*(b*Sec[e + f*x])^(3/2)) - (b*Csc[e + f*x]^5)/(5*f*(b*Sec[e + f*x])^(3/2
)) - (7*EllipticE[(e + f*x)/2, 2])/(20*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e +
f*x]])
```

Rubi [A] time = 0.143419, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2625, 3771, 2639}

$$-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}-\frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}}-\frac{7E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-7*b*Csc[e + f*x])/(20*f*(b*Sec[e + f*x])^(3/2)) - (7*b*Csc[e + f*x]^3)/(3
0*f*(b*Sec[e + f*x])^(3/2)) - (b*Csc[e + f*x]^5)/(5*f*(b*Sec[e + f*x])^(3/2
)) - (7*EllipticE[(e + f*x)/2, 2])/(20*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e +
f*x]])
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}dx = -\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{7}{10}\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$
$$= -\frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}} - \frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{7}{20}\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$
$$= -\frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}} - \frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}} - \frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}} - \frac{7}{40}\int \frac{1}{\sqrt{b\sec(e+fx}}$$
$$= -\frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}} - \frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}} - \frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}} - \frac{7\int\sqrt{\cos(e+f}}{40\sqrt{\cos(e+fx)}\sqrt{b\,s}}$$
$$= -\frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}} - \frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}} - \frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}} - \frac{7E\left(\frac{1}{2}(e+fx\right.}{20f\sqrt{\cos(e+fx)}\sqrt{b}}$$

Mathematica [A] time = 0.141438, size = 86, normalized size = 0.7

$$-\frac{\tan(e+fx)\left(12\csc^6(e+fx)+2\csc^4(e+fx)+7\csc^2(e+fx)+21\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)-21\right)}{60f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -((-21 + 7*Csc[e + f*x]^2 + 2*Csc[e + f*x]^4 + 12*Csc[e + f*x]^6 + 21*Sqrt[
Cos[e + f*x]]*Csc[e + f*x]*EllipticE[(e + f*x)/2, 2])*Tan[e + f*x])/(60*f*S
qrt[b*Sec[e + f*x]])
```

Maple [C] time = 0.187, size = 918, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/60/f*(-1+cos(f*x+e))^2*(-21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)+21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^5*sin(f*x+e)+21*I*cos(f*x+e
)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+21*I*EllipticF(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f
*x+e)^4*sin(f*x+e)-21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^5*sin(f*x+e)+4
2*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*sin(f*x+e)-21*I*EllipticE(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)-42*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)
+21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-21*I*EllipticE(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c
os(f*x+e)^4*sin(f*x+e)+21*cos(f*x+e)^5+42*I*EllipticE(I*(-1+cos(f*x+e))/sin
```

```
(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f
*x+e)^2*sin(f*x+e)-42*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*sin(f*x+e)-1
4*cos(f*x+e)^4-42*cos(f*x+e)^3+26*cos(f*x+e)^2+21*cos(f*x+e))*(cos(f*x+e)+1
)^2*(b/cos(f*x+e))^(1/2)/b/sin(f*x+e)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)^6}{\sqrt{b\sec(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^6/sqrt(b*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(fx+e)}\csc(fx+e)^6}{b\sec(fx+e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^6/(b*sec(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)^6}{\sqrt{b\sec(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/sqrt(b*sec(f*x + e)), x)
```

3.424 $\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{2b^7}{17f(b\sec(e+fx))^{17/2}}-\frac{6b^5}{13f(b\sec(e+fx))^{13/2}}+\frac{2b^3}{3f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[Out] (2*b^7)/(17*f*(b*Sec[e + f*x])^(17/2)) - (6*b^5)/(13*f*(b*Sec[e + f*x])^(13
/2)) + (2*b^3)/(3*f*(b*Sec[e + f*x])^(9/2)) - (2*b)/(5*f*(b*Sec[e + f*x])^(
5/2))
```

Rubi [A] time = 0.0632463, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$\frac{2b^7}{17f(b\sec(e+fx))^{17/2}}-\frac{6b^5}{13f(b\sec(e+fx))^{13/2}}+\frac{2b^3}{3f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^7/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*b^7)/(17*f*(b*Sec[e + f*x])^(17/2)) - (6*b^5)/(13*f*(b*Sec[e + f*x])^(13
/2)) + (2*b^3)/(3*f*(b*Sec[e + f*x])^(9/2)) - (2*b)/(5*f*(b*Sec[e + f*x])^(
5/2))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx &= \frac{b^7\operatorname{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{19/2}}\,dx,x,b\sec(e+fx)\right)}{f}\\
&= \frac{b^7\operatorname{Subst}\left(\int \left(-\frac{1}{x^{19/2}}+\frac{3}{b^2x^{15/2}}-\frac{3}{b^4x^{11/2}}+\frac{1}{b^6x^{7/2}}\right)dx,x,b\sec(e+fx)\right)}{f}\\
&= \frac{2b^7}{17f(b\sec(e+fx))^{17/2}}-\frac{6b^5}{13f(b\sec(e+fx))^{13/2}}+\frac{2b^3}{3f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.43607, size = 52, normalized size = 0.6

$$\frac{b(8365\cos(2(e+fx))-1890\cos(4(e+fx))+195\cos(6(e+fx))-10766)}{53040f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^7/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (b*(-10766 + 8365*Cos[2*(e + f*x)] - 1890*Cos[4*(e + f*x)] + 195*Cos[6*(e +
 f*x)]))/(53040*f*(b*Sec[e + f*x])^(5/2))
```

Maple [A] time = 0.146, size = 56, normalized size = 0.6

$$\frac{\left(390\left(\cos\left(fx+e\right)\right)^{6}-1530\left(\cos\left(fx+e\right)\right)^{4}+2210\left(\cos\left(fx+e\right)\right)^{2}-1326\right)\cos\left(fx+e\right)}{3315f}\left(\frac{b}{\cos\left(fx+e\right)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/3315/f*(195*cos(f*x+e)^6-765*cos(f*x+e)^4+1105*cos(f*x+e)^2-663)*cos(f*x+
e)/(b/cos(f*x+e))^(3/2)
```

Maxima [A] time = 1.00176, size = 85, normalized size = 0.98

$$\frac{2\left(195\,b^{6}-\frac{765\,b^{6}}{\cos\left(fx+e\right)^{2}}+\frac{1105\,b^{6}}{\cos\left(fx+e\right)^{4}}-\frac{663\,b^{6}}{\cos\left(fx+e\right)^{6}}\right)b}{3315\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3315*(195*b^6 - 765*b^6/cos(f*x + e)^2 + 1105*b^6/cos(f*x + e)^4 - 663*b^
6/cos(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(17/2))
```

Fricas [A] time = 2.70848, size = 165, normalized size = 1.9

$$\frac{2\left(195\,\cos\left(fx+e\right)^{9}-765\,\cos\left(fx+e\right)^{7}+1105\,\cos\left(fx+e\right)^{5}-663\,\cos\left(fx+e\right)^{3}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{3315\,b^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*cos(f*x + e)^9 - 765*cos(f*x + e)^7 + 1105*cos(f*x + e)^5 - 663
*cos(f*x + e)^3)*sqrt(b/cos(f*x + e))/(b^2*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**7/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^7}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^7/(b*sec(f*x + e))^(3/2), x)
```

3.425 $\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2b^5}{13f(b\sec(e+fx))^{13/2}}+\frac{4b^3}{9f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[Out] (-2*b^5)/(13*f*(b*Sec[e + f*x])^(13/2)) + (4*b^3)/(9*f*(b*Sec[e + f*x])^(9/
2)) - (2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Rubi [A] time = 0.0581976, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$-\frac{2b^5}{13f(b\sec(e+fx))^{13/2}}+\frac{4b^3}{9f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*b^5)/(13*f*(b*Sec[e + f*x])^(13/2)) + (4*b^3)/(9*f*(b*Sec[e + f*x])^(9/
2)) - (2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx &= \frac{b^5\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{15/2}}\,dx,x,b\sec(e+fx)\right)}{f}\\ &= \frac{b^5\,\text{Subst}\left(\int \left(\frac{1}{x^{15/2}}-\frac{2}{b^2x^{11/2}}+\frac{1}{b^4x^{7/2}}\right)dx,x,b\sec(e+fx)\right)}{f}\\ &= -\frac{2b^5}{13f(b\sec(e+fx))^{13/2}}+\frac{4b^3}{9f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.252138, size = 42, normalized size = 0.65

$$\frac{b(340\cos(2(e+fx))-45\cos(4(e+fx))-551)}{2340f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (b*(-551 + 340*Cos[2*(e + f*x)] - 45*Cos[4*(e + f*x)]))/(2340*f*(b*Sec[e +
f*x])^(5/2))
```

Maple [A] time = 0.114, size = 46, normalized size = 0.7

$$-\frac{\left(90 \left(\cos \left(fx+e\right)\right)^{4}-260 \left(\cos \left(fx+e\right)\right)^{2}+234\right) \cos \left(fx+e\right)}{585 f} \left(\frac{b}{\cos \left(fx+e\right)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -2/585/f*(45*cos(f*x+e)^4-130*cos(f*x+e)^2+117)*cos(f*x+e)/(b/cos(f*x+e))^(
3/2)
```

Maxima [A] time = 1.00042, size = 68, normalized size = 1.05

$$-\frac{2 \left(45 \, b^{4} - \frac{130 \, b^{4}}{\cos\left(fx+e\right)^{2}} + \frac{117 \, b^{4}}{\cos\left(fx+e\right)^{4}}\right) b}{585 \, f \left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/585*(45*b^4 - 130*b^4/cos(f*x + e)^2 + 117*b^4/cos(f*x + e)^4)*b/(f*(b/c
os(f*x + e))^(13/2))
```

Fricas [A] time = 2.64023, size = 134, normalized size = 2.06

$$-\frac{2 \left(45 \, \cos\left(fx+e\right)^{7} - 130 \, \cos\left(fx+e\right)^{5} + 117 \, \cos\left(fx+e\right)^{3}\right) \sqrt{\frac{b}{\cos\left(fx+e\right)}}}{585 \, b^{2} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/585*(45*cos(f*x + e)^7 - 130*cos(f*x + e)^5 + 117*cos(f*x + e)^3)*sqrt(b
/cos(f*x + e))/(b^2*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^5/(b*sec(f*x + e))^(3/2), x)
```

3.426 $\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2b^3}{9f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[Out] (2*b^3)/(9*f*(b*Sec[e + f*x])^(9/2)) - (2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Rubi [A] time = 0.0502984, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 14}

$$\frac{2b^3}{9f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*b^3)/(9*f*(b*Sec[e + f*x])^(9/2)) - (2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx &= \frac{b^3\,\text{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{11/2}}\,dx,x,b\sec(e+fx)\right)}{f}\\ &= \frac{b^3\,\text{Subst}\left(\int \left(-\frac{1}{x^{11/2}}+\frac{1}{b^2x^{7/2}}\right)dx,x,b\sec(e+fx)\right)}{f}\\ &= \frac{2b^3}{9f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.171556, size = 32, normalized size = 0.74

$$\frac{b(5\cos(2(e+fx))-13)}{45f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (b*(-13 + 5*Cos[2*(e + f*x)]))/(45*f*(b*Sec[e + f*x])^(5/2))
```

Maple [A] time = 0.102, size = 36, normalized size = 0.8

$$\frac{\left(10\left(\cos\left(fx+e\right)\right)^{2}-18\right)\cos\left(fx+e\right)}{45f}\left(\frac{b}{\cos\left(fx+e\right)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/45/f*(5*cos(f*x+e)^2-9)*cos(f*x+e)/(b/cos(f*x+e))^(3/2)
```

Maxima [A] time = 1.001, size = 50, normalized size = 1.16

$$\frac{2\left(5b^{2}-\frac{9b^{2}}{\cos\left(fx+e\right)^{2}}\right)b}{45f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45*(5*b^2 - 9*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(9/2))
```

Fricas [A] time = 2.67593, size = 99, normalized size = 2.3

$$\frac{2\left(5\cos\left(fx+e\right)^{5}-9\cos\left(fx+e\right)^{3}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{45b^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*cos(f*x + e)^5 - 9*cos(f*x + e)^3)*sqrt(b/cos(f*x + e))/(b^2*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.47836, size = 68, normalized size = 1.58

$$\frac{2\left(5\,b^2-\frac{9\,b^2}{\cos\left(fx+e\right)^2}\right)\cos\left(fx+e\right)^4}{45\,b^3f\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 2/45*(5*b^2 - 9*b^2/cos(f*x + e)^2)*cos(f*x + e)^4/(b^3*f*sqrt(b/cos(f*x +
e)))
```

3.427 $\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[Out] (-2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Rubi [A] time = 0.0361541, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{1}{x^{7/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= -\frac{2b}{5f(b\sec(e+fx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0536722, size = 20, normalized size = 1.

$$-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Maple [A] time = 0.012, size = 17, normalized size = 0.9

$$-\frac{2b}{5f}\left(b\sec\left(fx+e\right)\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -2/5*b/f/(b*sec(f*x+e))^(5/2)
```

Maxima [A] time = 0.984656, size = 31, normalized size = 1.55

$$-\frac{2\cos\left(fx+e\right)}{5f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*cos(f*x + e)/(f*(b/cos(f*x + e))^(3/2))
```

Fricas [A] time = 2.48782, size = 68, normalized size = 3.4

$$-\frac{2\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{3}}{5b^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(b/cos(f*x + e))*cos(f*x + e)^3/(b^2*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)/(b*sec(e + f*x))**(3/2), x)
```

Giac [A] time = 1.46603, size = 41, normalized size = 2.05

$$-\frac{2\,\cos\left(fx+e\right)^{2}}{5\,bf\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -2/5*cos(f*x + e)^2/(b*f*sqrt(b/cos(f*x + e)))
```

3.428 $\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f}+\frac{2}{bf\sqrt{b\sec(e+fx)}}$$

```
[Out] ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(b^(3/2)*f) - ArcTanh[Sqrt[b*Sec[e + f
*x]]/Sqrt[b]]/(b^(3/2)*f) + 2/(b*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.0543373, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2622, 325, 329, 298, 203, 206}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f}+\frac{2}{bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(b^(3/2)*f) - ArcTanh[Sqrt[b*Sec[e + f
*x]]/Sqrt[b]]/(b^(3/2)*f) + 2/(b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^{3/2}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{bf}$$

$$= \frac{2}{bf\sqrt{b\sec(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= \frac{2}{bf\sqrt{b\sec(e+fx)}} + \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{b^3 f}$$

$$= \frac{2}{bf\sqrt{b\sec(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{bf} + \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+f}\right.}{bf}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f} + \frac{2}{bf\sqrt{b\sec(e+fx)}}$$

Mathematica [A] time = 1.96262, size = 89, normalized size = 1.14

$$\frac{\sqrt{\sec(e+fx)}\left(\log\left(1-\sqrt{\sec(e+fx)}\right)-\log\left(\sqrt{\sec(e+fx)}+1\right)\right)+2\sqrt{\sec(e+fx)}\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)+4}{2bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4 + 2*ArcTan[Sqrt[Sec[e + f*x]]]*Sqrt[Sec[e + f*x]] + (Log[1 - Sqrt[Sec[e
+ f*x]]] - Log[1 + Sqrt[Sec[e + f*x]]])*Sqrt[Sec[e + f*x]])/(2*b*f*Sqrt[b*S
ec[e + f*x]])
```

Maple [B] time = 0.108, size = 221, normalized size = 2.8

$$-\frac{-1+\cos(fx+e)}{2f\cos(fx+e)(\sin(fx+e))^2}\left(4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\ln\left(-\frac{1}{(\sin(fx+e))^2}\left(2(\cos(fx+e))^2\sqrt{-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(-1+cos(f*x+e))*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-l
n(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(
f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+arctan(1/2/(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
)/cos(f*x+e)/sin(f*x+e)^2/(b/cos(f*x+e))^(3/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.07057, size = 813, normalized size = 10.42

$$\left[\frac{2\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)+8\sqrt{\frac{b}{\cos(fx+e)}}\cos\left(fx+e\right)-\sqrt{-b}\log\left(-\frac{b\cos(fx+e)^{2}+4\left(\cos(fx+e)^{2}-\cos(fx+e)\right)\sqrt{-b}\cdots}{\cos(fx+e)^{2}+2\cos(fx\cdots}\right)}{4b^{2}f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b)*arctan(2*sqrt(-b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos
(f*x + e) + b)) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e) - sqrt(-b)*log(-(b*co
s(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x +
e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)))/(b^2*f)
, 1/4*(2*sqrt(b)*arctan(2*sqrt(b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(
f*x + e) - b)) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e) + sqrt(b)*log(-(b*cos(
f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e))
 + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)))/(b^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)/(b*sec(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/(b*sec(f*x + e))^(3/2), x)
```

3.429 $\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3f}-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f}$$

```
[Out] -ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(4*b^(3/2)*f) + ArcTanh[Sqrt[b*Sec[e
+ f*x]]/Sqrt[b]]/(4*b^(3/2)*f) - (Cot[e + f*x]^2*(b*Sec[e + f*x])^(3/2))/(2
*b^3*f)
```

Rubi [A] time = 0.0738407, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2622, 290, 329, 298, 203, 206}

$$-\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3f}-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(4*b^(3/2)*f) + ArcTanh[Sqrt[b*Sec[e
+ f*x]]/Sqrt[b]]/(4*b^(3/2)*f) - (Cot[e + f*x]^2*(b*Sec[e + f*x])^(3/2))/(2
*b^3*f)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\sqrt{x}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}\\
&= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3 f} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{4b^3 f}\\
&= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3 f} - \frac{\operatorname{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{2b^3 f}\\
&= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3 f} + \frac{\operatorname{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4bf} - \frac{\operatorname{Subst}\left(\int \frac{1}{b+x^2}\,dx, x,\right.}{4bf}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2} f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2} f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3 f}
\end{aligned}
$$

Mathematica [A] time = 0.466018, size = 98, normalized size = 1.05

$$
\frac{-4\csc^2(e+fx)+\sqrt{\sec(e+fx)}\left(\log\left(\sqrt{\sec(e+fx)}+1\right)-\log\left(1-\sqrt{\sec(e+fx)}\right)\right)-2\sqrt{\sec(e+fx)}\tan^{-1}\left(\sqrt{\sec(e+fx)}\right.}{8bf\sqrt{b\sec(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-4*Csc[e + f*x]^2 - 2*ArcTan[Sqrt[Sec[e + f*x]]]*Sqrt[Sec[e + f*x]] + (-Lo
g[1 - Sqrt[Sec[e + f*x]]] + Log[1 + Sqrt[Sec[e + f*x]]])*Sqrt[Sec[e + f*x]]
)/(8*b*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] time = 0.125, size = 426, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/8/f*(-1+cos(f*x+e))*(8*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
+16*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+cos(f*x+e)^2*arctan(1/2
/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+8*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)+4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-4*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+ln(-(2*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2))/cos(f*x+e)/sin(f*
x+e)^4/(b/cos(f*x+e))^(3/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.0209, size = 957, normalized size = 10.29

$$\left[-\frac{2\left(\cos\left(fx+e\right)^{2}-1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos(fx+e)+1\right)}{2b}\right)+\left(\cos\left(fx+e\right)^{2}-1\right)\sqrt{-b}\log\left(\frac{b\cos(fx+e)^{2}-4\left(\cos(fx+e)^{2}-\cdots}{\cos(f\cdots}\right)}{16\left(b^{2}f\cos\left(fx+e\right)^{2}-b^{2}f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x
+ e))*(cos(f*x + e) + 1)/b) + (cos(f*x + e)^2 - 1)*sqrt(-b)*log((b*cos(f*x
+ e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) -
6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 8*sqrt(b/cos
(f*x + e))*cos(f*x + e))/(b^2*f*cos(f*x + e)^2 - b^2*f), 1/16*(2*(cos(f*x +
 e)^2 - 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(
b)) + (cos(f*x + e)^2 - 1)*sqrt(b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^
2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos
(f*x + e)^2 - 2*cos(f*x + e) + 1)) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e))/(
b^2*f*cos(f*x + e)^2 - b^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^{3}\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(b*sec(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/(b*sec(f*x + e))^(3/2), x)
```

3.430 $\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3f}-\frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3f}-\frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f}$$

```
[Out] (-3*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*b^(3/2)*f) + (3*ArcTanh[Sqrt[
b*Sec[e + f*x]]/Sqrt[b]])/(32*b^(3/2)*f) - (3*Cot[e + f*x]^2*(b*Sec[e + f*x
])^(3/2))/(16*b^3*f) - (Cot[e + f*x]^4*(b*Sec[e + f*x])^(3/2))/(4*b^3*f)
```

Rubi [A] time = 0.0858205, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2622, 288, 290, 329, 298, 203, 206}

$$-\frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3f}-\frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3f}-\frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
```



```
[Out] (-3*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*b^(3/2)*f) + (3*ArcTanh[Sqrt[
b*Sec[e + f*x]]/Sqrt[b]])/(32*b^(3/2)*f) - (3*Cot[e + f*x]^2*(b*Sec[e + f*x
])^(3/2))/(16*b^3*f) - (Cot[e + f*x]^4*(b*Sec[e + f*x])^(3/2))/(4*b^3*f)
```


Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^{5/2}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\sec(e+fx)\right)}{b^5 f}$$

$$= -\frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3 f} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{x}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{8b^3 f}$$

$$= -\frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3 f} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3 f} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\right.}{32b^3 f}$$

$$= -\frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3 f} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3 f} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{\right.}}{16b^3 f}$$

$$= -\frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3 f} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3 f} + \frac{3\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b}\right.}{32bf}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f} + \frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f} - \frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3 f} - \frac{\cot^4(e+\right.}{}$$

Mathematica [A] time = 0.654316, size = 109, normalized size = 0.89

$$\frac{-16\csc^4(e+fx)+4\csc^2(e+fx)+3\sqrt{\sec(e+fx)}\left(\log\left(\sqrt{\sec(e+fx)}+1\right)-\log\left(1-\sqrt{\sec(e+fx)}\right)\right)-6\sqrt{\sec(e+fx)}\,\text{t}}{64bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*Csc[e + f*x]^2 - 16*Csc[e + f*x]^4 - 6*ArcTan[Sqrt[Sec[e + f*x]]]*Sqrt[S
ec[e + f*x]] + 3*(-Log[1 - Sqrt[Sec[e + f*x]]] + Log[1 + Sqrt[Sec[e + f*x]]
])*Sqrt[Sec[e + f*x]])/(64*b*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] time = 0.133, size = 729, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/64/f*(8*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-8*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-3*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*cos(f*x+e)^3*arctan(1/2/(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2))-40*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)+12*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*cos(f*x+e)^2*
ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos
(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*cos(f*x+e
)^2*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-24*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(3/2)-24*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*cos(
f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*co
s(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+12*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/
sin(f*x+e)^2)+3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)))/cos(f*x+e
)/sin(f*x+e)^4/(b/cos(f*x+e))^(3/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.05884, size = 1192, normalized size = 9.69

$$\left[-\frac{6\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}+1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos\left(fx+e\right)}}\left(\cos\left(fx+e\right)+1\right)}{2\,b}\right)+3\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}\cdots\right.}{128\left(b^{2}f\cos\left(fx+e\right)^{\cdots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(1/2*sqrt
(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) + 3*(cos(f*x + e)^4 - 2*cos
(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f
*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e
)^2 + 2*cos(f*x + e) + 1)) + 8*(cos(f*x + e)^3 + 3*cos(f*x + e))*sqrt(b/cos
(f*x + e)))/(b^2*f*cos(f*x + e)^4 - 2*b^2*f*cos(f*x + e)^2 + b^2*f), 1/128*
(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*
x + e))*(cos(f*x + e) - 1)/sqrt(b)) + 3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2
+ 1)*sqrt(b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt
(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*
x + e) + 1)) - 8*(cos(f*x + e)^3 + 3*cos(f*x + e))*sqrt(b/cos(f*x + e)))/(b
^2*f*cos(f*x + e)^4 - 2*b^2*f*cos(f*x + e)^2 + b^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{5}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^5/(b*sec(f*x + e))^(3/2), x)
```

3.431 $\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{8\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{77b^2f}-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}+\frac{8\sin(e+fx)}{77bf\sqrt{b\sec(e+fx)}}-\frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}}$$

```
[Out] (8*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(77*b
^2*f) - (12*b*Sin[e + f*x])/(77*f*(b*Sec[e + f*x])^(5/2)) + (8*Sin[e + f*x]
)/(77*b*f*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e + f*x]^3)/(11*f*(b*Sec[e + f*x
])^(5/2))
```

Rubi [A] time = 0.133504, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2627, 3769, 3771, 2641}

$$\frac{8\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{77b^2f}-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}+\frac{8\sin(e+fx)}{77bf\sqrt{b\sec(e+fx)}}-\frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (8*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(77*b
^2*f) - (12*b*Sin[e + f*x])/(77*f*(b*Sec[e + f*x])^(5/2)) + (8*Sin[e + f*x]
)/(77*b*f*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e + f*x]^3)/(11*f*(b*Sec[e + f*x
])^(5/2))
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx &= -\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}+\frac{6}{11}\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx\\
&= -\frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}}-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}+\frac{12}{77}\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx\\
&= -\frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}}+\frac{8\sin(e+fx)}{77bf\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}+\frac{4\int \sqrt{b\sec(e+fx)}\,dx}{77b^2}\\
&= -\frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}}+\frac{8\sin(e+fx)}{77bf\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}+\frac{\left(4\sqrt{\cos(e+fx)}\sqrt{b\sec\ldots}\right.}{}\\
&= \frac{8\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{77b^2f}-\frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}}+\frac{8\sin(e+fx)}{77bf\sqrt{b\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.163047, size = 81, normalized size = 0.64

$$\frac{\sec^2(e+fx)\left(-5\sin(2(e+fx))-24\sin(4(e+fx))+7\sin(6(e+fx))+128\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{1232f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^2*(128*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] - 5*Sin[2
*(e + f*x)] - 24*Sin[4*(e + f*x)] + 7*Sin[6*(e + f*x)]))/(1232*f*(b*Sec[e +
 f*x])^(3/2))
```

Maple [C] time = 0.167, size = 173, normalized size = 1.4

$$-\frac{2\left(\cos(fx+e)+1\right)^2\left(-1+\cos(fx+e)\right)}{77f\left(\cos(fx+e)\right)^2\left(\sin(fx+e)\right)^3}\left(-7\left(\cos(fx+e)\right)^6+4\,i\mathrm{EllipticF}\left(\frac{i\left(-1+\cos(fx+e)\right)}{\sin(fx+e)},i\right)\sqrt{\left(\cos(fx+e)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -2/77/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))*(-7*cos(f*x+e)^6+4*I*EllipticF(I*(
-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*sin(f*x+e)+7*cos(f*x+e)^5+13*cos(f*x+e)^4-13*cos(f*x+e)^3-4*co
s(f*x+e)^2+4*cos(f*x+e))/cos(f*x+e)^2/sin(f*x+e)^3/(b/cos(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}+1\right)\sqrt{b\sec\left(fx+e\right)}}{b^{2}\sec\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b*sec(f*x + e))/(b^2*
sec(f*x + e)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^{4}\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(b*sec(f*x+e))**(3/2),x)

[Out] Integral(sin(e + f*x)**4/(b*sec(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)
```

3.432 $\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{21b^2f}+\frac{4\sin(e+fx)}{21bf\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}$$

```
[Out] (4*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(21*b
^2*f) - (2*b*Sin[e + f*x])/(7*f*(b*Sec[e + f*x])^(5/2)) + (4*Sin[e + f*x])/
(21*b*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.0821301, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2627, 3769, 3771, 2641}

$$\frac{4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{21b^2f}+\frac{4\sin(e+fx)}{21bf\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(21*b
^2*f) - (2*b*Sin[e + f*x])/(7*f*(b*Sec[e + f*x])^(5/2)) + (4*Sin[e + f*x])/
(21*b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}} + \frac{2}{7}\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx$$

$$= -\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}} + \frac{4\sin(e+fx)}{21bf\sqrt{b\sec(e+fx)}} + \frac{2\int\sqrt{b\sec(e+fx)}\,dx}{21b^2}$$

$$= -\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}} + \frac{4\sin(e+fx)}{21bf\sqrt{b\sec(e+fx)}} + \frac{\left(2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx}}}{21b^2}$$

$$= \frac{4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{21b^2f} - \frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}} + \frac{4\sin(e+fx)}{21bf\sqrt{b\sec(e+fx}}$$

Mathematica [A] time = 0.129931, size = 71, normalized size = 0.72

$$\frac{\sec^2(e+fx)\left(2\sin(2(e+fx)) - 3\sin(4(e+fx)) + 16\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{84f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^2*(16*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + 2*Sin[2*
(e + f*x)] - 3*Sin[4*(e + f*x)]))/(84*f*(b*Sec[e + f*x])^(3/2))
```

Maple [C] time = 0.133, size = 153, normalized size = 1.6

$$-\frac{2\left(\cos\left(fx+e\right)+1\right)^2\left(-1+\cos\left(fx+e\right)\right)}{21f\left(\cos\left(fx+e\right)\right)^2\left(\sin\left(fx+e\right)\right)^3}\left(2i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+\right.\right.}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -2/21/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin
(f*x+e)+3*cos(f*x+e)^4-3*cos(f*x+e)^3-2*cos(f*x+e)^2+2*cos(f*x+e))/cos(f*x+
e)^2/sin(f*x+e)^3/(b/cos(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^{2}-1\right)\sqrt{b\sec\left(fx+e\right)}}{b^{2}\sec\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(b*sec(f*x + e))/(b^2*sec(f*x + e)^2), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin^{2}\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/(b*sec(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(fx+e\right)^{2}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

3.433 $\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3b^2f}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

```
[Out] (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*b^
2*f) + (2*Sin[e + f*x])/(3*b*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.035942, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3769, 3771, 2641}

$$\frac{2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3b^2f}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(-3/2),x]
```

```
[Out] (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*b^
2*f) + (2*Sin[e + f*x])/(3*b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx &= \frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}+\frac{\int \sqrt{b\sec(e+fx)}\,dx}{3b^2}\\
&= \frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}+\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{3b^2}\\
&= \frac{2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3b^2f}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.0598092, size = 59, normalized size = 0.82

$$\frac{\sec^2(e+fx)\left(\sin(2(e+fx))+2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{3f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(-3/2),x]
```

```
[Out] (Sec[e + f*x]^2*(2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + Sin[2*(e
+ f*x)]))/(3*f*(b*Sec[e + f*x])^(3/2))
```

Maple [C] time = 0.115, size = 131, normalized size = 1.8

$$-\frac{2\left(\cos\left(fx+e\right)+1\right)^2\left(-1+\cos\left(fx+e\right)\right)}{3f\left(\cos\left(fx+e\right)\right)^2\left(\sin\left(fx+e\right)\right)^3}\left(i\text{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -2/3/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))*(I*EllipticF(I*(-1+cos(f*x+e))/sin(
f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*
x+e)-cos(f*x+e)^2+cos(f*x+e))/(b/cos(f*x+e))^(3/2)/cos(f*x+e)^2/sin(f*x+e)^
3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}}{b^2\sec\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))/(b^2*sec(f*x + e)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((b*sec(e + f*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(-3/2), x)
```

3.434 $\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{b^2f}-\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}}$$

```
[Out] -(Csc[e + f*x]/(b*f*Sqrt[b*Sec[e + f*x]])) - (Sqrt[Cos[e + f*x]]*EllipticF[
(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(b^2*f)
```

Rubi [A] time = 0.0655459, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2623, 3771, 2641}

$$-\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{b^2f}-\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(Csc[e + f*x]/(b*f*Sqrt[b*Sec[e + f*x]])) - (Sqrt[Cos[e + f*x]]*EllipticF[
(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(b^2*f)
```

Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}} - \frac{\int \sqrt{b\sec(e+fx)}\,dx}{2b^2}$$
$$= -\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}} - \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{2b^2}$$
$$= -\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}} - \frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{b^2f}$$

Mathematica [A] time = 0.102077, size = 58, normalized size = 0.85

$$\frac{-F\left(\frac{1}{2}(e+fx)\middle|2\right)-\sqrt{\cos(e+fx)}\csc(e+fx)}{f\cos^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-(Sqrt[Cos[e + f*x]]*Csc[e + f*x]) - EllipticF[(e + f*x)/2, 2])/(f*Cos[e +
 f*x]^(3/2)*(b*Sec[e + f*x])^(3/2))
```

Maple [C] time = 0.124, size = 191, normalized size = 2.8

$$-\frac{\left(\cos\left(fx+e\right)+1\right)^2\left(-1+\cos\left(fx+e\right)\right)^2}{f\left(\cos\left(fx+e\right)\right)^2\left(\sin\left(fx+e\right)\right)^5}\left(i\cos\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(I*cos(f*x+e)*EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*sin(f*x+e)+I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e))/cos(f*x+
e)^2/sin(f*x+e)^5/(b/cos(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^{2}}{b^{2}\sec\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^2/(b^2*sec(f*x + e)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc^{2}\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(b*sec(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc\left(fx+e\right)^{2}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

3.435 $\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{6b^2f}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}+\frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}}$$

```
[Out] Csc[e + f*x]/(6*b*f*Sqrt[b*Sec[e + f*x]]) - Csc[e + f*x]^3/(3*b*f*Sqrt[b*Se
c[e + f*x]]) - (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e +
 f*x]])/(6*b^2*f)
```

Rubi [A] time = 0.104377, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2623, 2625, 3771, 2641}

$$-\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{6b^2f}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}+\frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]
```



```
[Out] Csc[e + f*x]/(6*b*f*Sqrt[b*Sec[e + f*x]]) - Csc[e + f*x]^3/(3*b*f*Sqrt[b*Se
c[e + f*x]]) - (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e +
 f*x]])/(6*b^2*f)
```


Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}} - \frac{\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx}{6b^2}$$
$$= \frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}} - \frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}} - \frac{\int \sqrt{b\sec(e+fx)}\,dx}{12b^2}$$
$$= \frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}} - \frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}} - \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{12b^2}$$
$$= \frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}} - \frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}} - \frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{6b^2f}$$

Mathematica [A] time = 0.221751, size = 62, normalized size = 0.61

$$\frac{-2\csc^3(e+fx)+\csc(e+fx)-\frac{F\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{6bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Csc[e + f*x] - 2*Csc[e + f*x]^3 - EllipticF[(e + f*x)/2, 2]/Sqrt[Cos[e + f
*x]])/(6*b*f*Sqrt[b*Sec[e + f*x]])
```

Maple [C] time = 0.142, size = 343, normalized size = 3.4

$$\frac{\left(\cos\left(fx+e\right)+1\right)^2\left(-1+\cos\left(fx+e\right)\right)^2}{6f\left(\cos\left(fx+e\right)\right)^2\left(\sin\left(fx+e\right)\right)^7}\left(i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\text{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/6/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f
*x+e)^3*sin(f*x+e)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)-I*co
s(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-I*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*s
in(f*x+e)-cos(f*x+e)^3-cos(f*x+e))/cos(f*x+e)^2/sin(f*x+e)^7/(b/cos(f*x+e))
^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^{4}}{b^{2}\sec\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^4/(b^2*sec(f*x + e)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc^{4}\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(b*sec(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)
```

3.436 $\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=132

$$-\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{12b^2f}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}+\frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}}+\frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}}$$

```
[Out] Csc[e + f*x]/(12*b*f*Sqrt[b*Sec[e + f*x]]) + Csc[e + f*x]^3/(30*b*f*Sqrt[b*
Sec[e + f*x]]) - Csc[e + f*x]^5/(5*b*f*Sqrt[b*Sec[e + f*x]]) - (Sqrt[Cos[e
+ f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(12*b^2*f)
```

Rubi [A] time = 0.14399, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2623, 2625, 3771, 2641}

$$-\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{12b^2f}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}+\frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}}+\frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] Csc[e + f*x]/(12*b*f*Sqrt[b*Sec[e + f*x]]) + Csc[e + f*x]^3/(30*b*f*Sqrt[b*
Sec[e + f*x]]) - Csc[e + f*x]^5/(5*b*f*Sqrt[b*Sec[e + f*x]]) - (Sqrt[Cos[e
+ f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(12*b^2*f)
```

Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}} - \frac{\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx}{10b^2}$$
$$= \frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}} - \frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}} - \frac{\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx}{12b^2}$$
$$= \frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}} + \frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}} - \frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}} - \frac{\int \sqrt{b\sec(e+fx)}\,dx}{24b^2}$$
$$= \frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}} + \frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}} - \frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}} - \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e\right.}{24}$$
$$= \frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}} + \frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}} - \frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}} - \frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+\right.}{12b}$$

Mathematica [A] time = 0.311429, size = 74, normalized size = 0.56

$$\frac{-12\csc^5(e+fx)+2\csc^3(e+fx)+5\csc(e+fx)-\frac{5F\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{60bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (5*Csc[e + f*x] + 2*Csc[e + f*x]^3 - 12*Csc[e + f*x]^5 - (5*EllipticF[(e +
f*x)/2, 2])/Sqrt[Cos[e + f*x]])/(60*b*f*Sqrt[b*Sec[e + f*x]])
```

Maple [C] time = 0.178, size = 493, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/60/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(5*I*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*c
os(f*x+e)^5*sin(f*x+e)+5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*sin(f*x+e
)-10*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3*sin(f*x+e)-10*I*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/s
in(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)+5*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*sin(f*x+e)+5*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-5*cos(f*x+e)^5+12*co
s(f*x+e)^3+5*cos(f*x+e))/cos(f*x+e)^2/sin(f*x+e)^9/(b/cos(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^{6}}{b^{2}\sec\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^6/(b^2*sec(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(3/2), x)
```

3.437 $\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=87

$$\frac{2b^7}{19f(b\sec(e+fx))^{19/2}}-\frac{2b^5}{5f(b\sec(e+fx))^{15/2}}+\frac{6b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[Out] (2*b^7)/(19*f*(b*Sec[e + f*x])^(19/2)) - (2*b^5)/(5*f*(b*Sec[e + f*x])^(15/
2)) + (6*b^3)/(11*f*(b*Sec[e + f*x])^(11/2)) - (2*b)/(7*f*(b*Sec[e + f*x])^
(7/2))
```

Rubi [A] time = 0.0630548, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$\frac{2b^7}{19f(b\sec(e+fx))^{19/2}}-\frac{2b^5}{5f(b\sec(e+fx))^{15/2}}+\frac{6b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^7/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*b^7)/(19*f*(b*Sec[e + f*x])^(19/2)) - (2*b^5)/(5*f*(b*Sec[e + f*x])^(15/
2)) + (6*b^3)/(11*f*(b*Sec[e + f*x])^(11/2)) - (2*b)/(7*f*(b*Sec[e + f*x])^
(7/2))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx&=\frac{b^7\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{21/2}}\,dx,x,b\sec(e+fx)\right)}{f}\\&=\frac{b^7\,\text{Subst}\left(\int \left(-\frac{1}{x^{21/2}}+\frac{3}{b^2x^{17/2}}-\frac{3}{b^4x^{13/2}}+\frac{1}{b^6x^{9/2}}\right)dx,x,b\sec(e+fx)\right)}{f}\\&=\frac{2b^7}{19f(b\sec(e+fx))^{19/2}}-\frac{2b^5}{5f(b\sec(e+fx))^{15/2}}+\frac{6b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.361263, size = 62, normalized size = 0.71

$$\frac{\cos^4(e+fx)(14287\cos(2(e+fx))-3542\cos(4(e+fx))+385\cos(6(e+fx))-15226)\sqrt{b\sec(e+fx)}}{117040b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^7/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]^4*(-15226 + 14287*Cos[2*(e + f*x)] - 3542*Cos[4*(e + f*x)] +
385*Cos[6*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(117040*b^3*f)
```

Maple [A] time = 0.19, size = 56, normalized size = 0.6

$$\frac{\left(770\left(\cos\left(fx+e\right)\right)^{6}-2926\left(\cos\left(fx+e\right)\right)^{4}+3990\left(\cos\left(fx+e\right)\right)^{2}-2090\right)\cos\left(fx+e\right)}{7315f}\left(\frac{b}{\cos\left(fx+e\right)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 2/7315/f*(385*cos(f*x+e)^6-1463*cos(f*x+e)^4+1995*cos(f*x+e)^2-1045)*cos(f*
x+e)/(b/cos(f*x+e))^(5/2)
```

Maxima [A] time = 1.01907, size = 85, normalized size = 0.98

$$\frac{2\left(385\,b^{6}-\frac{1463\,b^{6}}{\cos\left(fx+e\right)^{2}}+\frac{1995\,b^{6}}{\cos\left(fx+e\right)^{4}}-\frac{1045\,b^{6}}{\cos\left(fx+e\right)^{6}}\right)b}{7315\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{19}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7315*(385*b^6 - 1463*b^6/cos(f*x + e)^2 + 1995*b^6/cos(f*x + e)^4 - 1045*
b^6/cos(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(19/2))
```

Fricas [A] time = 2.92419, size = 169, normalized size = 1.94

$$\frac{2\left(385\cos\left(fx+e\right)^{10}-1463\cos\left(fx+e\right)^{8}+1995\cos\left(fx+e\right)^{6}-1045\cos\left(fx+e\right)^{4}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{7315\,b^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*cos(f*x + e)^10 - 1463*cos(f*x + e)^8 + 1995*cos(f*x + e)^6 - 1
045*cos(f*x + e)^4)*sqrt(b/cos(f*x + e))/(b^3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**7/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^7}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^7/(b*sec(f*x + e))^(5/2), x)
```

3.438 $\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2b^5}{15f(b\sec(e+fx))^{15/2}}+\frac{4b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[Out] (-2*b^5)/(15*f*(b*Sec[e + f*x])^(15/2)) + (4*b^3)/(11*f*(b*Sec[e + f*x])^(1
1/2)) - (2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Rubi [A] time = 0.0585754, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$-\frac{2b^5}{15f(b\sec(e+fx))^{15/2}}+\frac{4b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*b^5)/(15*f*(b*Sec[e + f*x])^(15/2)) + (4*b^3)/(11*f*(b*Sec[e + f*x])^(1
1/2)) - (2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx &= \frac{b^5\operatorname{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{17/2}}\,dx,x,b\sec(e+fx)\right)}{f}\\ &= \frac{b^5\operatorname{Subst}\left(\int \left(\frac{1}{x^{17/2}}-\frac{2}{b^2x^{13/2}}+\frac{1}{b^4x^{9/2}}\right)dx,x,b\sec(e+fx)\right)}{f}\\ &= -\frac{2b^5}{15f(b\sec(e+fx))^{15/2}}+\frac{4b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.219008, size = 52, normalized size = 0.8

$$\frac{\cos^4(e+fx)(532\cos(2(e+fx))-77\cos(4(e+fx))-711)\sqrt{b\sec(e+fx)}}{4620b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]^4*(-711 + 532*Cos[2*(e + f*x)] - 77*Cos[4*(e + f*x)])*Sqrt[b*
Sec[e + f*x]])/(4620*b^3*f)
```

Maple [A] time = 0.165, size = 46, normalized size = 0.7

$$-\frac{\left(154\left(\cos\left(fx+e\right)\right)^{4}-420\left(\cos\left(fx+e\right)\right)^{2}+330\right)\cos\left(fx+e\right)}{1155\,f}\left(\frac{b}{\cos\left(fx+e\right)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -2/1155/f*(77*cos(f*x+e)^4-210*cos(f*x+e)^2+165)*cos(f*x+e)/(b/cos(f*x+e))^
(5/2)
```

Maxima [A] time = 1.00631, size = 68, normalized size = 1.05

$$-\frac{2\left(77\,b^{4}-\frac{210\,b^{4}}{\cos\left(fx+e\right)^{2}}+\frac{165\,b^{4}}{\cos\left(fx+e\right)^{4}}\right)b}{1155\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/1155*(77*b^4 - 210*b^4/cos(f*x + e)^2 + 165*b^4/cos(f*x + e)^4)*b/(f*(b/
cos(f*x + e))^(15/2))
```

Fricas [A] time = 2.67109, size = 135, normalized size = 2.08

$$-\frac{2\left(77\,\cos\left(fx+e\right)^{8}-210\,\cos\left(fx+e\right)^{6}+165\,\cos\left(fx+e\right)^{4}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{1155\,b^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(77*cos(f*x + e)^8 - 210*cos(f*x + e)^6 + 165*cos(f*x + e)^4)*sqrt(
b/cos(f*x + e))/(b^3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)^5}{(b\sec(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^5/(b*sec(f*x + e))^(5/2), x)
```

3.439 $\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[Out] (2*b^3)/(11*f*(b*Sec[e + f*x])^(11/2)) - (2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Rubi [A] time = 0.0526156, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 14}

$$\frac{2b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*b^3)/(11*f*(b*Sec[e + f*x])^(11/2)) - (2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx &= \frac{b^3\operatorname{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{13/2}}\,dx,x,b\sec(e+fx)\right)}{f}\\ &= \frac{b^3\operatorname{Subst}\left(\int \left(-\frac{1}{x^{13/2}}+\frac{1}{b^2x^{9/2}}\right)dx,x,b\sec(e+fx)\right)}{f}\\ &= \frac{2b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.148857, size = 42, normalized size = 0.98

$$\frac{\cos^4(e+fx)(7\cos(2(e+fx))-15)\sqrt{b\sec(e+fx)}}{77b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]^4*(-15 + 7*Cos[2*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(77*b^3*f)
```

Maple [A] time = 0.113, size = 36, normalized size = 0.8

$$\frac{\left(14\left(\cos\left(fx+e\right)\right)^{2}-22\right)\cos\left(fx+e\right)}{77f}\left(\frac{b}{\cos\left(fx+e\right)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 2/77/f*(7*cos(f*x+e)^2-11)*cos(f*x+e)/(b/cos(f*x+e))^(5/2)
```

Maxima [A] time = 1.01223, size = 50, normalized size = 1.16

$$\frac{2\left(7\,b^{2}-\frac{11\,b^{2}}{\cos\left(fx+e\right)^{2}}\right)b}{77\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/77*(7*b^2 - 11*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(11/2))
```

Fricas [A] time = 2.4716, size = 100, normalized size = 2.33

$$\frac{2\left(7\,\cos\left(fx+e\right)^{6}-11\,\cos\left(fx+e\right)^{4}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{77\,b^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/77*(7*cos(f*x + e)^6 - 11*cos(f*x + e)^4)*sqrt(b/cos(f*x + e))/(b^3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.56658, size = 68, normalized size = 1.58

$$\frac{2\left(7\,b^{2}-\frac{11\,b^{2}}{\cos\left(fx+e\right)^{2}}\right)\cos\left(fx+e\right)^{5}}{77\,b^{4}f\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 2/77*(7*b^2 - 11*b^2/cos(f*x + e)^2)*cos(f*x + e)^5/(b^4*f*sqrt(b/cos(f*x +
 e)))
```

3.440 $\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[Out] (-2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Rubi [A] time = 0.036703, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{1}{x^{9/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= -\frac{2b}{7f(b\sec(e+fx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.0737125, size = 20, normalized size = 1.

$$-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Maple [A] time = 0.011, size = 17, normalized size = 0.9

$$-\frac{2b}{7f}\left(b\sec\left(fx+e\right)\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -2/7*b/f/(b*sec(f*x+e))^(7/2)
```

Maxima [A] time = 1.00297, size = 31, normalized size = 1.55

$$-\frac{2\cos\left(fx+e\right)}{7f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/7*cos(f*x + e)/(f*(b/cos(f*x + e))^(5/2))
```

Fricas [A] time = 2.47557, size = 68, normalized size = 3.4

$$-\frac{2\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{4}}{7b^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/7*sqrt(b/cos(f*x + e))*cos(f*x + e)^4/(b^3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.46223, size = 41, normalized size = 2.05

$$-\frac{2\cos\left(fx+e\right)^{3}}{7\,b^{2}f\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -2/7*cos(f*x + e)^3/(b^2*f*sqrt(b/cos(f*x + e)))
```

3.441 $\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f}+\frac{2}{3bf(b\sec(e+fx))^{3/2}}$$

```
[Out] -(ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(b^(5/2)*f)) - ArcTanh[Sqrt[b*Sec[e
+ f*x]]/Sqrt[b]]/(b^(5/2)*f) + 2/(3*b*f*(b*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.055331, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2622, 325, 329, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f}+\frac{2}{3bf(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(b^(5/2)*f)) - ArcTanh[Sqrt[b*Sec[e
+ f*x]]/Sqrt[b]]/(b^(5/2)*f) + 2/(3*b*f*(b*Sec[e + f*x])^(3/2))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^{5/2}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{bf}$$

$$= \frac{2}{3bf(b\sec(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= \frac{2}{3bf(b\sec(e+fx))^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{b^3 f}$$

$$= \frac{2}{3bf(b\sec(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{b^2 f} - \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{b^2 f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f} + \frac{2}{3bf(b\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 0.205727, size = 90, normalized size = 1.11

$$\frac{\sqrt{\sec(e+fx)}\left(\frac{4}{\sec^{\frac{3}{2}}(e+fx)} + 3\log\left(1-\sqrt{\sec(e+fx)}\right) - 3\log\left(\sqrt{\sec(e+fx)}+1\right) - 6\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)\right)}{6b^2 f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((-6*ArcTan[Sqrt[Sec[e + f*x]]] + 3*Log[1 - Sqrt[Sec[e + f*x]]] - 3*Log[1 +
 Sqrt[Sec[e + f*x]]] + 4/Sec[e + f*x]^(3/2))*Sqrt[Sec[e + f*x]])/(6*b^2*f*S
qrt[b*Sec[e + f*x]])
```

Maple [B] time = 0.121, size = 377, normalized size = 4.7

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^2\left(\cos\left(fx+e\right)+1\right)^2}{6f\left(\cos\left(fx+e\right)\right)^3\left(\sin\left(fx+e\right)\right)^4}\left(-3\cos\left(fx+e\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^2}}\ln\left(-\frac{1}{\left(\sin\left(fx+e\right)\right)^2}\left(2\left(\cos\left(fx+e\right)\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/6/f*(-1+cos(f*x+e))^2*(-3*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
))*cos(f*x+e)+4*cos(f*x+e)^2-3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-3*arctan(1/2/(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(cos
(f*x+e)+1)^2/cos(f*x+e)^3/sin(f*x+e)^4/(b/cos(f*x+e))^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.73854, size = 826, normalized size = 10.2

$$\left[\frac{8\sqrt{\frac{b}{\cos(fx+e)}}\cos\left(fx+e\right)^{2}+6\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)-3\sqrt{-b}\log\left(-\frac{b\cos(fx+e)^{2}-4\left(\cos(fx+e)^{2}-\cos(fx+e)\right)\cdots}{\cos(fx+e)^{2}+2\cos\cdots}\right.}{12\,b^{3}f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 + 6*sqrt(-b)*arctan(2*sqrt(-b)
*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) + b)) - 3*sqrt(-b)*log(-
(b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f
*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)))/(b
^3*f), 1/12*(8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 - 6*sqrt(b)*arctan(2*sqr
t(b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) - b)) + 3*sqrt(b)*lo
g(-(b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos
(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)))/
(b^3*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)}{(b\sec(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/(b*sec(f*x + e))^(5/2), x)
```

3.442 $\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3f}+\frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f}$$

```
[Out] (3*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*b^(5/2)*f) + (3*ArcTanh[Sqrt[b*
Sec[e + f*x]]/Sqrt[b]])/(4*b^(5/2)*f) - (Cot[e + f*x]^2*Sqrt[b*Sec[e + f*x]
])/(2*b^3*f)
```

Rubi [A] time = 0.0728972, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2622, 290, 329, 212, 206, 203}

$$-\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3f}+\frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (3*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*b^(5/2)*f) + (3*ArcTanh[Sqrt[b*
Sec[e + f*x]]/Sqrt[b]])/(4*b^(5/2)*f) - (Cot[e + f*x]^2*Sqrt[b*Sec[e + f*x]
])/(2*b^3*f)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3 f} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{4b^3 f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3 f} - \frac{3\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{2b^3 f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3 f} + \frac{3\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4b^2 f} + \frac{3\,\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x\right.}{4b^2}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2} f} + \frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2} f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3 f}$$

Mathematica [A] time = 2.34729, size = 98, normalized size = 1.05

$$\frac{\sqrt{\sec(e+fx)}\left(-3\log\left(1-\sqrt{\sec(e+fx)}\right)+3\log\left(\sqrt{\sec(e+fx)}+1\right)-\frac{4\csc^2(e+fx)}{\sec^{\frac{3}{2}}(e+fx)}+6\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)\right)}{8b^2 f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((6*ArcTan[Sqrt[Sec[e + f*x]]] - 3*Log[1 - Sqrt[Sec[e + f*x]]] + 3*Log[1 +
Sqrt[Sec[e + f*x]]] - (4*Csc[e + f*x]^2)/Sec[e + f*x]^(3/2))*Sqrt[Sec[e + f
*x]])/(8*b^2*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] time = 0.127, size = 437, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/8/f*(-1+cos(f*x+e))*(8*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
+16*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-4*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*cos(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2))+8*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+4*cos(f*x+e)*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)-3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)))/cos(f*x+e)^2/sin(f*x+e)^4/(b/cos(f*x+e))^(5/2)/(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.75014, size = 969, normalized size = 10.42

$$\left[-\frac{6\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos(fx+e)+1\right)}{2b}\right)-8\sqrt{\frac{b}{\cos(fx+e)}}\cos\left(fx+e\right)^2+3\left(\cos\left(fx+e\right)^2-1\right)\sqrt{}}{16\left(b^3f\cos\left(fx+e\right)^2-b^3f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(6*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x
+ e))*(cos(f*x + e) + 1)/b) - 8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 + 3*(co
s(f*x + e)^2 - 1)*sqrt(-b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(
f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x +
e)^2 + 2*cos(f*x + e) + 1)))/(b^3*f*cos(f*x + e)^2 - b^3*f), -1/16*(6*(cos(
f*x + e)^2 - 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/
sqrt(b)) - 8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 - 3*(cos(f*x + e)^2 - 1)*s
qrt(b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sq
rt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e)
 + 1)))/(b^3*f*cos(f*x + e)^2 - b^3*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/(b*sec(f*x + e))^(5/2), x)
```

3.443 $\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3f}-\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3f}+\frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f}$$

```
[Out] (3*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*b^(5/2)*f) + (3*ArcTanh[Sqrt[b
*Sec[e + f*x]]/Sqrt[b]])/(32*b^(5/2)*f) - (Cot[e + f*x]^2*Sqrt[b*Sec[e + f*
x]])/(16*b^3*f) - (Cot[e + f*x]^4*Sqrt[b*Sec[e + f*x]])/(4*b^3*f)
```

Rubi [A] time = 0.0840227, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2622, 288, 290, 329, 212, 206, 203}

$$-\frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3f}-\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3f}+\frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```


```
[Out] (3*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*b^(5/2)*f) + (3*ArcTanh[Sqrt[b
*Sec[e + f*x]]/Sqrt[b]])/(32*b^(5/2)*f) - (Cot[e + f*x]^2*Sqrt[b*Sec[e + f*
x]])/(16*b^3*f) - (Cot[e + f*x]^4*Sqrt[b*Sec[e + f*x]])/(4*b^3*f)
```


Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^{3/2}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\sec(e+fx)\right)}{b^5 f}$$

$$= -\frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3 f} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{8b^3 f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3 f} - \frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3 f} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\,\text{se}\right.}{32b^3 f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3 f} - \frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3 f} - \frac{3\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(}\right.}{16b^3 f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3 f} - \frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3 f} + \frac{3\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e}\right.}{32b^2 f}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2} f} + \frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2} f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3 f} - \frac{\cot^4(e+fx)\sqrt{}}{4b}$$

Mathematica [A] time = 2.53302, size = 110, normalized size = 0.89

$$\frac{\sqrt{\sec(e+fx)}\left(-3\log\left(1-\sqrt{\sec(e+fx)}\right)+3\log\left(\sqrt{\sec(e+fx)}+1\right)+6\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)-\frac{2(3\cos(2(e+fx))+5)\csc^4(e+fx}{\sec^{\frac{3}{2}}(e+fx)}\right.}{64b^2 f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((6*ArcTan[Sqrt[Sec[e + f*x]]] - 3*Log[1 - Sqrt[Sec[e + f*x]]] + 3*Log[1 +
Sqrt[Sec[e + f*x]]] - (2*(5 + 3*Cos[2*(e + f*x)])*Csc[e + f*x]^4)/Sec[e + f
*x]^(3/2))*Sqrt[Sec[e + f*x]])/(64*b^2*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] time = 0.141, size = 737, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/64/f*(24*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+40*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-12*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-3*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-1)/sin(f*x+e)^2)-3*cos(f*x+e)^3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2))+8*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+24*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*cos(f*x+e)^2*arctan(1/2/(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2))-8*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-12*co
s(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*cos(f*x+e)*ln(-(2*cos(f*x+e
)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*cos(f*x+e)*arctan(1/2/(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)-3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
))/cos(f*x+e)^2/sin(f*x+e)^4/(b/cos(f*x+e))^(5/2)/(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.78732, size = 1199, normalized size = 9.75

$$\left[-\frac{6\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}+1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos\left(fx+e\right)}}\left(\cos\left(fx+e\right)+1\right)}{2b}\right)+3\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}\cdots}{128\left(b^{3}f\cos\left(fx+e\right)\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(1/2*sqrt
(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) + 3*(cos(f*x + e)^4 - 2*cos
(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(f
*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e
)^2 + 2*cos(f*x + e) + 1)) + 8*(3*cos(f*x + e)^4 + cos(f*x + e)^2)*sqrt(b/c
os(f*x + e)))/(b^3*f*cos(f*x + e)^4 - 2*b^3*f*cos(f*x + e)^2 + b^3*f), -1/1
28*(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arctan(1/2*sqrt(b/cos
(f*x + e))*(cos(f*x + e) - 1)/sqrt(b)) - 3*(cos(f*x + e)^4 - 2*cos(f*x + e)
^2 + 1)*sqrt(b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + cos(f*x + e))*s
qrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos
(f*x + e) + 1)) + 8*(3*cos(f*x + e)^4 + cos(f*x + e)^2)*sqrt(b/cos(f*x + e)
))/(b^3*f*cos(f*x + e)^4 - 2*b^3*f*cos(f*x + e)^2 + b^3*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{5}}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^5/(b*sec(f*x + e))^(5/2), x)
```

3.444 $\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{65b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}+\frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}}-\frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}}$$

```
[Out] (8*EllipticE[(e + f*x)/2, 2])/(65*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f
*x]]) - (4*b*Sin[e + f*x])/(39*f*(b*Sec[e + f*x])^(7/2)) + (8*Sin[e + f*x])
/(195*b*f*(b*Sec[e + f*x])^(3/2)) - (2*b*Sin[e + f*x]^3)/(13*f*(b*Sec[e + f
*x])^(7/2))
```

Rubi [A] time = 0.13199, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2627, 3769, 3771, 2639}

$$\frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{65b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}+\frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}}-\frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (8*EllipticE[(e + f*x)/2, 2])/(65*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f
*x]]) - (4*b*Sin[e + f*x])/(39*f*(b*Sec[e + f*x])^(7/2)) + (8*Sin[e + f*x])
/(195*b*f*(b*Sec[e + f*x])^(3/2)) - (2*b*Sin[e + f*x]^3)/(13*f*(b*Sec[e + f
*x])^(7/2))
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}} + \frac{6}{13}\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$$
$$= -\frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}} - \frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}} + \frac{4}{39}\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx$$
$$= -\frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}} + \frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}} - \frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}} + \frac{4\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx}{65b^2}$$
$$= -\frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}} + \frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}} - \frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}} + \frac{4\int \sqrt{\cos(e}{65b^2\sqrt{\cos(e+fx}$$
$$= \frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{65b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}} + \frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}} - \frac{2b}{13f(b}$$

Mathematica [A] time = 0.4978, size = 83, normalized size = 0.66

$$\frac{192E\left(\frac{1}{2}(e+fx)\middle|2\right) + (-6\sin(e+fx) - 55\sin(3(e+fx)) + 15\sin(5(e+fx)))\cos^{\frac{3}{2}}(e+fx)}{1560f\cos^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (192*EllipticE[(e + f*x)/2, 2] + Cos[e + f*x]^(3/2)*(-6*Sin[e + f*x] - 55*S
in[3*(e + f*x)] + 15*Sin[5*(e + f*x)]))/(1560*f*Cos[e + f*x]^(5/2)*(b*Sec[e
 + f*x])^(5/2))
```

Maple [C] time = 0.158, size = 343, normalized size = 2.7

$$\frac{2}{195f\left(\cos\left(fx+e\right)\right)^3\sin\left(fx+e\right)}\left(-15\left(\cos\left(fx+e\right)\right)^8 + 12i\cos\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}, i\right)\sqrt{\left(\cos\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 2/195/f*(-15*cos(f*x+e)^8+12*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f
*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x
+e)-12*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+40*cos(f*x+e)^6+12
*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-12*I*sin(f*x+e)*EllipticE(I*(-1+cos
(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)-29*cos(f*x+e)^4-8*cos(f*x+e)^2+12*cos(f*x+e))/cos(f*x+e)^3/sin(f*x+e
)/(b/cos(f*x+e))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{b\sec\left(fx+e\right)}}{b^3\sec\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b*sec(f*x + e))/(b^3*
sec(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)
```

3.445 $\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=98

$$\frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}}-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}$$

```
[Out] (4*EllipticE[(e + f*x)/2, 2])/(15*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f
*x]]) - (2*b*Sin[e + f*x])/(9*f*(b*Sec[e + f*x])^(7/2)) + (4*Sin[e + f*x])/
(45*b*f*(b*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.0818197, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2627, 3769, 3771, 2639}

$$\frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}}-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]
```


```
[Out] (4*EllipticE[(e + f*x)/2, 2])/(15*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f
*x]]) - (2*b*Sin[e + f*x])/(9*f*(b*Sec[e + f*x])^(7/2)) + (4*Sin[e + f*x])/
(45*b*f*(b*Sec[e + f*x])^(3/2))
```


Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}+\frac{2}{9}\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx$$
$$= -\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}+\frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}}+\frac{2\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx}{15b^2}$$
$$= -\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}+\frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}}+\frac{2\int \sqrt{\cos(e+fx)}\,dx}{15b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$= \frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}+\frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 0.392435, size = 66, normalized size = 0.67

$$\frac{-4\sin(2(e+fx))-10\sin(4(e+fx))+\frac{96E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{360b^2f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((96*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] - 4*Sin[2*(e + f*x)] - 1
0*Sin[4*(e + f*x)])/(360*b^2*f*Sqrt[b*Sec[e + f*x]])
```

Maple [C] time = 0.129, size = 333, normalized size = 3.4

$$\frac{2}{45\,f\left(\cos\left(fx+e\right)\right)^{3}\sin\left(fx+e\right)}\left(6\,i\cos\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos}{\cos(f}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 2/45/f*(6*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-6*I*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5*cos(f*x+e)^6+6*I*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)-6*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-7*cos(f*x+e)^4-4
*cos(f*x+e)^2+6*cos(f*x+e))/cos(f*x+e)^3/sin(f*x+e)/(b/cos(f*x+e))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{2}}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^2-1\right)\sqrt{b\sec\left(fx+e\right)}}{b^3\sec\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(b*sec(f*x + e))/(b^3*sec(f*x + e)^3), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

3.446 $\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

```
[Out] (6*EllipticE[(e + f*x)/2, 2])/(5*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*
x]]) + (2*Sin[e + f*x])/(5*b*f*(b*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.034779, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3769, 3771, 2639}

$$\frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(-5/2),x]
```

```
[Out] (6*EllipticE[(e + f*x)/2, 2])/(5*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*
x]]) + (2*Sin[e + f*x])/(5*b*f*(b*Sec[e + f*x])^(3/2))
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx &= \frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}+\frac{3\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx}{5b^2}\\ &= \frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}+\frac{3\int \sqrt{\cos(e+fx)}\,dx}{5b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\ &= \frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0849257, size = 60, normalized size = 0.83

$$\frac{\sqrt{b\sec(e+fx)}\left(\sin(e+fx)+\sin(3(e+fx))+12\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{10b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(-5/2),x]
```

```
[Out] (Sqrt[b*Sec[e + f*x]]*(12*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2] + Si
n[e + f*x] + Sin[3*(e + f*x)]))/(10*b^3*f)
```

Maple [C] time = 0.15, size = 323, normalized size = 4.5

$$\frac{2}{5f\left(\cos\left(fx+e\right)\right)^{3}\sin\left(fx+e\right)}\left(3i\cos\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx\right.}{\cos\left(fx+\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 2/5/f*(3*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-3*I*EllipticE(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e
)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)^4-2*cos(f*x+e)^2+3*co
s(f*x+e))/(b/cos(f*x+e))^(5/2)/cos(f*x+e)^3/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}}{b^3\sec\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))/(b^3*sec(f*x + e)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((b*sec(e + f*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(-5/2), x)
```

3.447 $\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}$$

```
[Out] -(Csc[e + f*x]/(b*f*(b*Sec[e + f*x])^(3/2))) - (3*EllipticE[(e + f*x)/2, 2]
)/(b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.0659776, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2623, 3771, 2639}

$$-\frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(Csc[e + f*x]/(b*f*(b*Sec[e + f*x])^(3/2))) - (3*EllipticE[(e + f*x)/2, 2]
)/(b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx &= -\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}-\frac{3\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx}{2b^2}\\ &= -\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}-\frac{3\int \sqrt{\cos(e+fx)}\,dx}{2b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\ &= -\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}-\frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.163899, size = 51, normalized size = 0.75

$$\frac{-\cot(e+fx)-\frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{b^2f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-Cot[e + f*x] - (3*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]])/(b^2*f*S
qrt[b*Sec[e + f*x]])
```

Maple [C] time = 0.137, size = 313, normalized size = 4.6

$$-\frac{1}{f\left(\cos\left(fx+e\right)\right)^3\sin\left(fx+e\right)}\left(3i\cos\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos(\ldots}{\cos(fx\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/f*(3*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-3*I*EllipticE(I*(
-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)
-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-2*cos(f*x+e)^2+3*cos(f*x+e))/cos(
f*x+e)^3/sin(f*x+e)/(b/cos(f*x+e))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^2}{b^3\sec\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^2/(b^3*sec(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

3.448 $\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=102

$$\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}+\frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}}$$

```
[Out] Csc[e + f*x]/(2*b*f*(b*Sec[e + f*x])^(3/2)) - Csc[e + f*x]^3/(3*b*f*(b*Sec[
e + f*x])^(3/2)) + EllipticE[(e + f*x)/2, 2]/(2*b^2*f*Sqrt[Cos[e + f*x]]*Sq
rt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.108161, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2623, 2625, 3771, 2639}

$$\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}+\frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] Csc[e + f*x]/(2*b*f*(b*Sec[e + f*x])^(3/2)) - Csc[e + f*x]^3/(3*b*f*(b*Sec[
e + f*x])^(3/2)) + EllipticE[(e + f*x)/2, 2]/(2*b^2*f*Sqrt[Cos[e + f*x]]*Sq
rt[b*Sec[e + f*x]])
```

Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}} - \frac{\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx}{2b^2}$$
$$= \frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}} - \frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}} + \frac{\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx}{4b^2}$$
$$= \frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}} - \frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}} + \frac{\int \sqrt{\cos(e+fx)}\,dx}{4b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$= \frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}} - \frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}} + \frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Mathematica [A] time = 0.25234, size = 79, normalized size = 0.77

$$\frac{\sin(e+fx)\sqrt{b\sec(e+fx)}\left(-2\csc^4(e+fx)+5\csc^2(e+fx)+3\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)-3\right)}{6b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((-3 + 5*Csc[e + f*x]^2 - 2*Csc[e + f*x]^4 + 3*Sqrt[Cos[e + f*x]]*Csc[e + f
*x]*EllipticE[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/(6*b^3*f)
```

Maple [C] time = 0.162, size = 623, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/6/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(3*I*cos(f*x+e)^3*sin(f*x+e)*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(
f*x+e))/sin(f*x+e),I)-3*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
+3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)-3*I*EllipticE(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)-3*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
sin(f*x+e)+3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x
+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*Elliptic
F(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)+3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*cos
(f*x+e)^3+2*cos(f*x+e)^2-3*cos(f*x+e))/cos(f*x+e)^3/sin(f*x+e)^7/(b/cos(f*x
+e))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)^4}{(b\sec(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(fx+e)}\csc(fx+e)^4}{b^3\sec(fx+e)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^4/(b^3*sec(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)^4}{(b\sec(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)
```

3.449 $\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}} dx$

Optimal. Leaf size=132

$$\frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}+\frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}}+\frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}}$$

```
[Out] (3*Csc[e + f*x])/(20*b*f*(b*Sec[e + f*x])^(3/2)) + Csc[e + f*x]^3/(10*b*f*(
b*Sec[e + f*x])^(3/2)) - Csc[e + f*x]^5/(5*b*f*(b*Sec[e + f*x])^(3/2)) + (3
*EllipticE[(e + f*x)/2, 2])/(20*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]])
```

Rubi [A] time = 0.148684, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2623, 2625, 3771, 2639}

$$\frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}+\frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}}+\frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(b*Sec[e + f*x])^(5/2),x]
```



```
[Out] (3*Csc[e + f*x])/(20*b*f*(b*Sec[e + f*x])^(3/2)) + Csc[e + f*x]^3/(10*b*f*(
b*Sec[e + f*x])^(3/2)) - Csc[e + f*x]^5/(5*b*f*(b*Sec[e + f*x])^(3/2)) + (3
*EllipticE[(e + f*x)/2, 2])/(20*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]])
```


Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}} - \frac{3\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx}{10b^2}$$

$$= \frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}} - \frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}} - \frac{3\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx}{20b^2}$$

$$= \frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}} + \frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}} - \frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{b\sec(e+fx)}}\,d}{40b^2}$$

$$= \frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}} + \frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}} - \frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}} + \frac{3\int \sqrt{\cos}}{40b^2\sqrt{\cos(e+f}}$$

$$= \frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}} + \frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}} - \frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}} + \frac{3E\left(\frac{1}{2}(e\right.}{20b^2f\sqrt{\cos(e+}}$$

Mathematica [A] time = 0.177929, size = 87, normalized size = 0.66

$$\frac{\sin(e+fx)\sqrt{b\sec(e+fx)}\left(-4\csc^6(e+fx)+6\csc^4(e+fx)+\csc^2(e+fx)+3\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e+fx)\right.\right.}{20b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((-3 + Csc[e + f*x]^2 + 6*Csc[e + f*x]^4 - 4*Csc[e + f*x]^6 + 3*Sqrt[Cos[e
+ f*x]]*Csc[e + f*x]*EllipticE[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]]*Sin[e
+ f*x])/(20*b^3*f)
```

Maple [C] time = 0.181, size = 923, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/20/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(6*I*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*c
os(f*x+e)^3*sin(f*x+e)-3*I*cos(f*x+e)^4*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)+3*I*cos(f*x+e)^4*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*I*cos(f*x+e)^5*si
n(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipti
cE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-6*I*cos(f*x+e)^2*sin(f*x+e)*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)-3*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-6*I*cos(f
*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+6*I*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
cos(f*x+e)^2*sin(f*x+e)-3*I*cos(f*x+e)^5*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2
```

```
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),
I)-3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-3*cos(f*x+e)^5+3*I*EllipticE(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)+3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(
f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)+2*cos(f*x+e)^4+6*cos(f*x+e)^3+2*cos(f*x+e)^2-3*cos(f*x+e))/cos(f*x+e)^3/s
in(f*x+e)^9/(b/cos(f*x+e))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^{6}}{b^{3}\sec\left(fx+e\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^6/(b^3*sec(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(5/2), x)
```

3.450 $\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx$

Optimal. Leaf size=449

$$-\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}}-\frac{21a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{32\sqrt{2}\sqrt{b}f}+\frac{21a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec}}{}$$

```
[Out] (-21*a^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqr
t[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(32*Sqrt[2]*
Sqrt[b]*f) + (21*a^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/
(Sqrt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/
(32*Sqrt[2]*Sqrt[b]*f) + (21*a^(9/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sq
rt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e +
f*x]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*Sqrt[b]*f) - (21*a^(9/2)*Sqrt[b*Cos
[e + f*x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[
e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*Sqrt[b]
*f) - (7*a^3*b*(a*Sin[e + f*x])^(3/2))/(16*f*Sqrt[b*Sec[e + f*x]]) - (a*b*(
a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.475627, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {2583, 2585, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}}-\frac{21a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{32\sqrt{2}\sqrt{b}f}+\frac{21a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(9/2),x]
```


```
[Out] (-21*a^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqr
t[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(32*Sqrt[2]*
Sqrt[b]*f) + (21*a^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/
(Sqrt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/
(32*Sqrt[2]*Sqrt[b]*f) + (21*a^(9/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sq
rt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e +
f*x]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*Sqrt[b]*f) - (21*a^(9/2)*Sqrt[b*Cos
[e + f*x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[
e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*Sqrt[b]
*f) - (7*a^3*b*(a*Sin[e + f*x])^(3/2))/(16*f*Sqrt[b*Sec[e + f*x]]) - (a*b*(
a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[b*Sec[e + f*x]])
```


Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
```

```
f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx = -\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}} + \frac{1}{8}\left(7a^2\right)\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx$$

$$= -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}} + \frac{1}{32}\left(21a^4\right)\int \sqrt{b\sec(e+fx)}\sqrt{a$$

$$= -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}} + \frac{1}{32}\left(21a^4\sqrt{b\cos(e+fx)}\sqrt{b\sec$$

$$= -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}} + \frac{\left(21a^5b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e}{}$$

$$= -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}} - \frac{\left(21a^5\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+}{}$$

$$= -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}} + \frac{\left(21a^5\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+}{}$$

$$= \frac{21a^{9/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+f}{64\sqrt{2}\sqrt{b}f}$$

$$= -\frac{21a^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{32\sqrt{2}\sqrt{b}f} + \frac{21a^{9/2}\tan^{}{}$$

Mathematica [C] time = 0.366912, size = 80, normalized size = 0.18

$$\frac{a^4\tan(e+fx)\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}\left(14\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)-7\cos(2(e+fx))+\cos(4(e+fx))-8\right)}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^4*(-8 - 7*Cos[2*(e + f*x)] + Cos[4*(e + f*x)] + 14*Hypergeometric2F1[3/4
, 1, 7/4, -Tan[e + f*x]^2])*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]]*Tan[e
 + f*x])/(32*f)
```

Maple [C] time = 0.162, size = 546, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(9/2)*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/64/f*2^(1/2)*(8*2^(1/2)*cos(f*x+e)^4-21*I*(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),1/2-1/2*I,1/2*2^(1/2))+21*I*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
```

```
+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/
2*I,1/2*2^(1/2))-8*2^(1/2)*cos(f*x+e)^3+21*(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),1/2-1/2*I,1/2*2^(1/2))+21*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I
,1/2*2^(1/2))-22*2^(1/2)*cos(f*x+e)^2+22*2^(1/2)*cos(f*x+e))*(a*sin(f*x+e))
^(9/2)*(b/cos(f*x+e))^(1/2)/sin(f*x+e)^3/(-1+cos(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(9/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(9/2)*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(9/2), x)
```

3.451 $\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=414

$$-\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{4\sqrt{2}\sqrt{b}f}+\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(}{\sqrt{a}\sqrt{b\cos(e}}\right.}{4\sqrt{2}\sqrt{b}f}$$

```
[Out] (-3*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt
[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(4*Sqrt[2]*Sq
rt[b]*f) + (3*a^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sq
rt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(4*
Sqrt[2]*Sqrt[b]*f) + (3*a^(5/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]
*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]
*Sqrt[b*Sec[e + f*x]])/(8*Sqrt[2]*Sqrt[b]*f) - (3*a^(5/2)*Sqrt[b*Cos[e + f*
x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x
]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(8*Sqrt[2]*Sqrt[b]*f) - (a
*b*(a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.348411, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {2583, 2585, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{4\sqrt{2}\sqrt{b}f}+\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(}{\sqrt{a}\sqrt{b\cos(e}}\right.}{4\sqrt{2}\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2),x]
```


```
[Out] (-3*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt
[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(4*Sqrt[2]*Sq
rt[b]*f) + (3*a^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sq
rt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(4*
Sqrt[2]*Sqrt[b]*f) + (3*a^(5/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]
*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]
*Sqrt[b*Sec[e + f*x]])/(8*Sqrt[2]*Sqrt[b]*f) - (3*a^(5/2)*Sqrt[b*Cos[e + f*
x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x
]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(8*Sqrt[2]*Sqrt[b]*f) - (a
*b*(a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[b*Sec[e + f*x]])
```


Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx = -\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}} + \frac{1}{4}\left(3a^2\right)\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx$$

$$= -\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}} + \frac{1}{4}\left(3a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}$$

$$= -\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}} + \frac{\left(3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{a^2+b^2x^4}d\right.}{2f}$$

$$= -\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}} - \frac{\left(3a^3\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{a-bx^2}{a^2+b^2x^4}dx\right.}{4f}$$

$$= -\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}} + \frac{\left(3a^3\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x}\right.}{8bf}$$

$$= \frac{3a^{5/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx}}{8\sqrt{2}\sqrt{b}f}$$

$$= -\frac{3a^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}\sqrt{b}f} + \frac{3a^{5/2}\tan^{-1}}{}$$

Mathematica [C] time = 0.251696, size = 65, normalized size = 0.16

$$-\frac{a(a\sin(e+fx))^{3/2}(b\sec(e+fx))^{3/2}\left(-2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)+\cos(2(e+fx))+1\right)}{4bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a*(1 + Cos[2*(e + f*x)] - 2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^
2])*(b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(3/2))/(4*b*f)
```

Maple [C] time = 0.107, size = 512, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(5/2)*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/8/f*2^(1/2)*(3*I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-
3*I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*((1-cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*((1-cos(f*x+e)+sin(f*x+e))/sin(f
```

```
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),
1/2+1/2*I,1/2*2^(1/2))+2*2^(1/2)*cos(f*x+e)^2-2*2^(1/2)*cos(f*x+e))*(a*sin(
f*x+e))^(5/2)*(b/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(5/2)*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.452 $\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx$

Optimal. Leaf size=376

$$-\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{b}f}+\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right.}{\sqrt{2}\sqrt{b}f}$$

```
[Out] -((Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[
b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2]*Sqrt[
b]*f)) + (Sqrt[a]*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a
]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2
]*Sqrt[b]*f) + (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]*Sqrt[b]
*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*
Sec[e + f*x]])/(2*Sqrt[2]*Sqrt[b]*f) - (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Log[Sq
rt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[
a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(2*Sqrt[2]*Sqrt[b]*f)
```

Rubi [A] time = 0.264938, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2585, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{b}f}+\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right.}{\sqrt{2}\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[
b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2]*Sqrt[
b]*f)) + (Sqrt[a]*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a
]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2
]*Sqrt[b]*f) + (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]*Sqrt[b]
*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*
Sec[e + f*x]])/(2*Sqrt[2]*Sqrt[b]*f) - (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Log[Sq
rt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[
a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(2*Sqrt[2]*Sqrt[b]*f)
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx = \left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\,dx$$

$$= \frac{\left(2ab\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\mathrm{Subst}\left(\int \frac{x^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{f}$$

$$= -\frac{\left(a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\mathrm{Subst}\left(\int \frac{a-bx^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{f} + \frac{\left(a\sqrt{b\cos\right.}{}$$

$$= \frac{\left(a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{2bf} + \frac{\left(a\sqrt{b\right.}{}$$

$$= \frac{\sqrt{a}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{2\sqrt{2}\sqrt{b}f} -$$

$$= -\frac{\sqrt{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}\sqrt{b}f} + \frac{\sqrt{a}\tan^{-1}\left(1+\right.}{}$$

Mathematica [C] time = 0.128543, size = 55, normalized size = 0.15

$$\frac{2\tan(e+fx)\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]*Sqr
t[a*Sin[e + f*x]]*Tan[e + f*x])/(3*f)
```

Maple [C] time = 0.105, size = 273, normalized size = 0.7

$$-\frac{\sqrt{2}\sin\left(fx+e\right)}{2f\left(-1+\cos\left(fx+e\right)\right)}\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b}{\cos\left(fx+e\right)}}\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx\right.}{\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*2^(1/2)*(a*sin(f*x+e))^(1/2)*(b/cos(f*x+e))^(1/2)*((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(I*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*sin(f*x+e)/(-1+cos(f
*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(e+fx\right)}\sqrt{b\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(1/2)*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sin(e + f*x))*sqrt(b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e)), x)
```

3.453 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=33

$$-\frac{2b}{af\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

[Out] (-2*b)/(a*f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])

Rubi [A] time = 0.0516478, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2578}

$$-\frac{2b}{af\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]

[Out] (-2*b)/(a*f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{2b}{af\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

Mathematica [A] time = 0.0740362, size = 37, normalized size = 1.12

$$-\frac{\sin(2(e+fx))\sqrt{b\sec(e+fx)}}{f(a\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]

[Out] -((Sqrt[b*Sec[e + f*x]]*Sin[2*(e + f*x)])/(f*(a*Sin[e + f*x])^(3/2)))

Maple [A] time = 0.123, size = 40, normalized size = 1.2

$$-2\,\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{f\left(a\sin\left(fx+e\right)\right)^{3/2}}\sqrt{\frac{b}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/f*sin(f*x+e)*cos(f*x+e)*(b/cos(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(3/2), x)
```

Fricas [A] time = 3.85517, size = 108, normalized size = 3.27

$$-\frac{2\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{a^{2}f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*cos(f*x + e)/(a^2*f*sin(f*x +
e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(3/2), x)
```

3.454 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{8b}{5a^3f\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b}{5af(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}}$$

```
[Out] (-2*b)/(5*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2)) - (8*b)/(5*a^3*f
*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.108842, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2584, 2578}

$$-\frac{8b}{5a^3f\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b}{5af(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*b)/(5*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2)) - (8*b)/(5*a^3*f
*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx &= -\frac{2b}{5af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}}+\frac{4\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx}{5a^2}\\ &= -\frac{2b}{5af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}}-\frac{8b}{5a^3f\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.197654, size = 52, normalized size = 0.73

$$\frac{2(2\cos(2(e+fx))-3)\cot(e+fx)\sqrt{b\sec(e+fx)}}{5a^2f(a\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*(-3 + 2*Cos[2*(e + f*x)])*Cot[e + f*x]*Sqrt[b*Sec[e + f*x]])/(5*a^2*f*(a
*Sin[e + f*x])^(3/2))
```

Maple [A] time = 0.122, size = 52, normalized size = 0.7

$$\frac{\left(8\left(\cos\left(fx+e\right)\right)^{2}-10\right)\cos\left(fx+e\right)\sin\left(fx+e\right)}{5f}\sqrt{\frac{b}{\cos\left(fx+e\right)}}\left(a\sin\left(fx+e\right)\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/5/f*(4*cos(f*x+e)^2-5)*cos(f*x+e)*(b/cos(f*x+e))^(1/2)*sin(f*x+e)/(a*sin(
f*x+e))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(7/2), x)
```

Fricas [A] time = 3.61584, size = 176, normalized size = 2.48

$$-\frac{2\left(4\cos\left(fx+e\right)^{3}-5\cos\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{5\left(a^{4}f\cos\left(fx+e\right)^{2}-a^{4}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(4*cos(f*x + e)^3 - 5*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*
x + e))/((a^4*f*cos(f*x + e)^2 - a^4*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec \left(fx+e\right)}}{\left(a \sin \left(fx+e\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(7/2), x)
```

3.455 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{64b}{45a^5f\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{16b}{45a^3f(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}}-\frac{2b}{9af(a\sin(e+fx))^{9/2}\sqrt{b\sec(e+fx)}}$$

```
[Out] (-2*b)/(9*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(9/2)) - (16*b)/(45*a^3
*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2)) - (64*b)/(45*a^5*f*Sqrt[b*S
ec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.165228, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2584, 2578}

$$-\frac{64b}{45a^5f\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{16b}{45a^3f(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}}-\frac{2b}{9af(a\sin(e+fx))^{9/2}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(11/2),x]
```

```
[Out] (-2*b)/(9*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(9/2)) - (16*b)/(45*a^3
*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2)) - (64*b)/(45*a^5*f*Sqrt[b*S
ec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx &= -\frac{2b}{9af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}}+\frac{8\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx}{9a^2}\\ &= -\frac{2b}{9af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}}-\frac{16b}{45a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}}+\frac{32\int \frac{\sqrt{b\sec}}{(a\sin(e+}}{45a}\\ &= -\frac{2b}{9af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}}-\frac{16b}{45a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}}-\frac{}{45a^5f\sqrt{b\se}}\end{aligned}$$

Mathematica [A] time = 0.214393, size = 65, normalized size = 0.61

$$\frac{2b(20\cos(2(e+fx))-4\cos(4(e+fx))-21)\csc^5(e+fx)\sqrt{a\sin(e+fx)}}{45a^6f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(11/2),x]
```

```
[Out] (2*b*(-21 + 20*Cos[2*(e + f*x)] - 4*Cos[4*(e + f*x)])*Csc[e + f*x]^5*Sqrt[a
*Sin[e + f*x]])/(45*a^6*f*Sqrt[b*Sec[e + f*x]])
```

Maple [A] time = 0.133, size = 62, normalized size = 0.6

$$-\frac{\left(64\left(\cos\left(fx+e\right)\right)^4-144\left(\cos\left(fx+e\right)\right)^2+90\right)\cos\left(fx+e\right)\sin\left(fx+e\right)}{45f}\sqrt{\frac{b}{\cos\left(fx+e\right)}}\left(a\sin\left(fx+e\right)\right)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x)
```

```
[Out] -2/45/f*(32*cos(f*x+e)^4-72*cos(f*x+e)^2+45)*cos(f*x+e)*(b/cos(f*x+e))^(1/2
)*sin(f*x+e)/(a*sin(f*x+e))^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(11/2), x)
```

Fricas [A] time = 4.38894, size = 240, normalized size = 2.26

$$-\frac{2\left(32\cos\left(fx+e\right)^5-72\cos\left(fx+e\right)^3+45\cos\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b}{\cos(fx+e)}}}{45\left(a^6f\cos\left(fx+e\right)^4-2a^6f\cos\left(fx+e\right)^2+a^6f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x, algorithm="fricas")
```

```
[Out] -2/45*(32*cos(f*x + e)^5 - 72*cos(f*x + e)^3 + 45*cos(f*x + e))*sqrt(a*sin(
f*x + e))*sqrt(b/cos(f*x + e))/((a^6*f*cos(f*x + e)^4 - 2*a^6*f*cos(f*x + e
```

```
)^2 + a^6*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(11/2), x)
```

3.456 $\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=128

$$-\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}}+\frac{5a^4\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{12f\sqrt{a\sin(e+fx)}}-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}$$

```
[Out] (-5*a^3*b*Sqrt[a*Sin[e + f*x]])/(6*f*Sqrt[b*Sec[e + f*x]]) - (a*b*(a*Sin[e
+ f*x])^(5/2))/(3*f*Sqrt[b*Sec[e + f*x]]) + (5*a^4*EllipticF[e - Pi/4 + f*x
, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(12*f*Sqrt[a*Sin[e + f*x]
])
```

Rubi [A] time = 0.214773, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2583, 2585, 2573, 2641}

$$-\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}}+\frac{5a^4\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{12f\sqrt{a\sin(e+fx)}}-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-5*a^3*b*Sqrt[a*Sin[e + f*x]])/(6*f*Sqrt[b*Sec[e + f*x]]) - (a*b*(a*Sin[e
+ f*x])^(5/2))/(3*f*Sqrt[b*Sec[e + f*x]]) + (5*a^4*EllipticF[e - Pi/4 + f*x
, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(12*f*Sqrt[a*Sin[e + f*x]
])
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = -\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}+\frac{1}{6}\left(5a^2\right)\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx$$
$$= -\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}+\frac{1}{12}\left(5a^4\right)\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$$
$$= -\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}+\frac{1}{12}\left(5a^4\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+\right.$$
$$= -\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}+\frac{\left(5a^4\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2f}\right.}{12\sqrt{a\sin(e+f}}$$
$$= -\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}+\frac{5a^4F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{b\sec(e+}}{12f\sqrt{a\sin(e+f}}$$

Mathematica [C] time = 0.871719, size = 90, normalized size = 0.7

$$\frac{a^3b\sqrt{a\sin(e+fx)}\left(5\left(-\tan^2(e+fx)\right)^{3/4}\csc^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)+2(\cos(2(e+fx))-6)\right)}{12f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*b*Sqrt[a*Sin[e + f*x]]*(2*(-6 + Cos[2*(e + f*x)]) + 5*Csc[e + f*x]^2*H
ypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(
12*f*Sqrt[b*Sec[e + f*x]])
```

Maple [A] time = 0.158, size = 212, normalized size = 1.7

$$-\frac{\sqrt{2}}{12\,f\left(\sin\left(fx+e\right)\right)^3\left(-1+\cos\left(fx+e\right)\right)}\left(5\,\sin\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(f\right.}{\sin\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(7/2)*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/12/f*2^(1/2)*(5*sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2
*2^(1/2)*cos(f*x+e)^4+2*2^(1/2)*cos(f*x+e)^3+7*2^(1/2)*cos(f*x+e)^2-7*2^(1/
2)*cos(f*x+e))*(a*sin(f*x+e))^(7/2)*(b/cos(f*x+e))^(1/2)/sin(f*x+e)^3/(-1+c
os(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^3\cos\left(fx+e\right)^2-a^3\right)\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^3*cos(f*x + e)^2 - a^3)*sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x +
e))*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(7/2)*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.457 $\int \sqrt{b \sec(e+fx)}(a \sin(e+fx))^{3/2} \, dx$

Optimal. Leaf size=91

$$\frac{a^2\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{2f\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] -((a*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]])) + (a^2*EllipticF[e -
 Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sqrt[a*Si
n[e + f*x]])
```

Rubi [A] time = 0.150605, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2583, 2585, 2573, 2641}

$$\frac{a^2\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{2f\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((a*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]])) + (a^2*EllipticF[e -
 Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sqrt[a*Si
n[e + f*x]])
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = -\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}} + \frac{1}{2}a^2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$$

$$= -\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}} + \frac{1}{2}\left(a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{b\cos(e+fx)}\cdots}$$

$$= -\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}} + \frac{\left(a^2\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}} + \frac{a^2F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{2f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] time = 1.2486, size = 66, normalized size = 0.73

$$\frac{(a\sin(e+fx))^{5/2}(b\sec(e+fx))^{3/2}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{1}{2};\sec^2(e+fx)\right)}{abf\left(-\tan^2(e+fx)\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Hypergeometric2F1[-1/2, -1/4, 1/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/2)*
(a*Sin[e + f*x])^(5/2))/(a*b*f*(-Tan[e + f*x]^2)^(5/4))
```

Maple [A] time = 0.125, size = 184, normalized size = 2.

$$-\frac{\sqrt{2}}{2f\left(-1+\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}\left(\sin\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+\cdots\right.}{\sin\left(fx+e\right)}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(3/2)*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*2^(1/2)*(sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2^(1
/2)*cos(f*x+e)^2-2^(1/2)*cos(f*x+e))*(a*sin(f*x+e))^(3/2)*(b/cos(f*x+e))^(1
/2)/(-1+cos(f*x+e))/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}a\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))*a*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(3/2)*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(3/2), x)
```

3.458 $\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

```
[Out] (EllipticF[e - Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/
(f*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.0990154, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2585, 2573, 2641}

$$\frac{\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (EllipticF[e - Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/
(f*Sqrt[a*Sin[e + f*x]])
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx &= \left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}\,dx\\ &= \frac{\left(\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{\sqrt{a\sin(e+fx)}}\\ &= \frac{F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{f\sqrt{a\sin(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.371877, size = 66, normalized size = 1.25

$$\frac{\left(-\tan^2(e+fx)\right)^{3/4}\cot(e+fx)\sqrt{b\sec(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)}{f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (Cot[e + f*x]*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*Sqrt[b*Sec[e
 + f*x]]*(-Tan[e + f*x]^2)^(3/4))/(f*Sqrt[a*Sin[e + f*x]])
```

Maple [B] time = 0.13, size = 153, normalized size = 2.9

$$-\frac{\sqrt{2}\left(\sin\left(fx+e\right)\right)^2}{f\left(-1+\cos\left(fx+e\right)\right)}\sqrt{\frac{b}{\cos\left(fx+e\right)}}\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(\right.}{\sin\left(fx\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*2^(1/2)*(b/cos(f*x+e))^(1/2)*sin(f*x+e)^2*((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2),1/2*2^(1/2))/(a*sin(f*x+e))^(1/2)/(-1+cos(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}}{a\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/(a*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(e+fx\right)}}{\sqrt{a\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))/sqrt(a*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

3.459 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{2\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{3a^2f\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

```
[Out] (-2*b)/(3*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) + (2*EllipticF[e
 - Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*f*Sqr
t[a*Sin[e + f*x]])
```

Rubi [A] time = 0.153644, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2584, 2585, 2573, 2641}

$$\frac{2\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{3a^2f\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*b)/(3*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) + (2*EllipticF[e
 - Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*f*Sqr
t[a*Sin[e + f*x]])
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = -\frac{2b}{3af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} + \frac{2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{3a^2}$$

$$= -\frac{2b}{3af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} + \frac{\left(2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx}{3a^2}$$

$$= -\frac{2b}{3af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} + \frac{\left(2\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3a^2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{2b}{3af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} + \frac{2F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{3a^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] time = 0.432101, size = 75, normalized size = 0.79

$$\frac{2\cot(e+fx)\sqrt{b\sec(e+fx)}\left(\left(-\tan^2(e+fx)\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)-1\right)}{3a^2f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*(-1 + Hypergeometric2F1[1/2, 3/4, 3/2,
 Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(3*a^2*f*Sqrt[a*Sin[e + f*x]])
```

Maple [B] time = 0.135, size = 279, normalized size = 2.9

$$\frac{\sqrt{2}\sin\left(fx+e\right)}{3f}\left(2\sin\left(fx+e\right)\cos\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/3/f*2^(1/2)*(2*sin(f*x+e)*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2
^(1/2))+2*sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2^(1/2)*co
s(f*x+e))*(b/cos(f*x+e))^(1/2)*sin(f*x+e)/(a*sin(f*x+e))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}}{\left(a^3\cos\left(fx+e\right)^2-a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/((a^3*cos(f*x + e)^2 -
a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

3.460 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{4b}{7a^3f(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}+\frac{4\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{7a^4f\sqrt{a\sin(e+fx)}}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

```
[Out] (-2*b)/(7*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2)) - (4*b)/(7*a^3*f
*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) + (4*EllipticF[e - Pi/4 + f*x
, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(7*a^4*f*Sqrt[a*Sin[e + f
*x]])
```

Rubi [A] time = 0.213507, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2584, 2585, 2573, 2641}

$$-\frac{4b}{7a^3f(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}+\frac{4\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{7a^4f\sqrt{a\sin(e+fx)}}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-2*b)/(7*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2)) - (4*b)/(7*a^3*f
*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) + (4*EllipticF[e - Pi/4 + f*x
, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(7*a^4*f*Sqrt[a*Sin[e + f
*x]])
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = -\frac{2b}{7af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} + \frac{6\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx}{7a^2}$$

$$= -\frac{2b}{7af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} - \frac{4b}{7a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} + \frac{4\int \frac{\sqrt{b\sec(e+fx}}{\sqrt{a\sin(e+fx}}}{7a^4}$$

$$= -\frac{2b}{7af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} - \frac{4b}{7a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} + \frac{\left(4\sqrt{b\cos(e+}\right.}{}$$

$$= -\frac{2b}{7af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} - \frac{4b}{7a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} + \frac{\left(4\sqrt{b\sec(e+}\right.}{}$$

$$= -\frac{2b}{7af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} - \frac{4b}{7a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} + \frac{4F\left(e-\frac{\pi}{4}+f\right.}{}$$

Mathematica [C] time = 1.01634, size = 111, normalized size = 0.85

$$-\frac{2\cos(2(e+fx))(b\sec(e+fx))^{3/2}\left(2\left(-\tan^2(e+fx)\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)+(\cos(2(e+fx))-2)\csc^2(e+fx)\right)}{7a^3bf\left(\sec^2(e+fx)-2\right)(a\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-2*Cos[2*(e + f*x)]*(b*Sec[e + f*x])^(3/2)*((-2 + Cos[2*(e + f*x)])*Csc[e
+ f*x]^2 + 2*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x
]^2)^(3/4)))/(7*a^3*b*f*(-2 + Sec[e + f*x]^2)*(a*Sin[e + f*x])^(3/2))
```

Maple [B] time = 0.154, size = 532, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/7/f*2^(1/2)*(4*sin(f*x+e)*cos(f*x+e)^3*((1-cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/
sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/
2*2^(1/2))+4*sin(f*x+e)*cos(f*x+e)^2*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(
1/2))-4*sin(f*x+e)*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-4
*sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((
1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2*2^(1/2)*cos(f*x+e
)^3+3*2^(1/2)*cos(f*x+e))*(b/cos(f*x+e))^(1/2)*sin(f*x+e)/(a*sin(f*x+e))^(9
```

```
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}}{\left(a^{5}\cos\left(fx+e\right)^{4}-2\,a^{5}\cos\left(fx+e\right)^{2}+a^{5}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/((a^5*cos(f*x + e)^4 - 2
*a^5*cos(f*x + e)^2 + a^5)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(9/2), x)
```

3.461 $\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=115

$$-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}-\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}+\frac{7\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{20f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] (-7*b*Sin[e + f*x]^(3/2))/(30*f*(b*Sec[e + f*x])^(3/2)) - (b*Sin[e + f*x]^(
7/2))/(5*f*(b*Sec[e + f*x])^(3/2)) + (7*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[S
in[e + f*x]])/(20*f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 0.164466, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2583, 2585, 2572, 2639}

$$-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}-\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}+\frac{7\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{20f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^(9/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-7*b*Sin[e + f*x]^(3/2))/(30*f*(b*Sec[e + f*x])^(3/2)) - (b*Sin[e + f*x]^(
7/2))/(5*f*(b*Sec[e + f*x])^(3/2)) + (7*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[S
in[e + f*x]])/(20*f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx &= -\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{7}{10}\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx \\
&= -\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{7}{20}\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx \\
&= -\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{7\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{20\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} \\
&= -\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{\left(7\sqrt{\sin(e+fx)}\right)\int \sqrt{\sin(2e+2fx)}\,dx}{20\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}} \\
&= -\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{7E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{20f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}
\end{aligned}$$

Mathematica [C] time = 0.536627, size = 86, normalized size = 0.75

$$-\frac{b\left(42\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\sec^2(e+fx)\right)-26\cos(2(e+fx))+3\cos(4(e+fx))+23\right)}{120f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^(9/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -(b*(23 - 26*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)] + 42*Hypergeometric2F1[-
1/2, 1/4, 1/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(1/4)))/(120*f*(b*Sec[e +
f*x])^(3/2)*Sqrt[Sin[e + f*x]])
```

Maple [B] time = 0.118, size = 532, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/120/f*2^(1/2)*(12*cos(f*x+e)^6*2^(1/2)-38*2^(1/2)*cos(f*x+e)^4+42*cos(f*
x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-21*cos(f*x+e)*(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+42*(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),1/2*2^(1/2))-21*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+47*2^(1/
2)*cos(f*x+e)^2-21*2^(1/2)*cos(f*x+e))/cos(f*x+e)/sin(f*x+e)^(1/2)/(b/cos(f
*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{\frac{9}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^(9/2)/sqrt(b*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}+1\right)\sqrt{b\sec\left(fx+e\right)}\sqrt{\sin\left(fx+e\right)}}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b*sec(f*x + e))*sqrt(
sin(f*x + e))/(b*sec(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**(9/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{\frac{9}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^(9/2)/sqrt(b*sec(f*x + e)), x)
```

3.462 $\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{2f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

```
[Out] -(b*Sin[e + f*x]^(3/2))/(3*f*(b*Sec[e + f*x])^(3/2)) + (EllipticE[e - Pi/4
+ f*x, 2]*Sqrt[Sin[e + f*x]])/(2*f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*
x]])
```

Rubi [A] time = 0.118849, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2583, 2585, 2572, 2639}

$$\frac{\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{2f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^(5/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -(b*Sin[e + f*x]^(3/2))/(3*f*(b*Sec[e + f*x])^(3/2)) + (EllipticE[e - Pi/4
+ f*x, 2]*Sqrt[Sin[e + f*x]])/(2*f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*
x]])
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{1}{2}\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx$$

$$= -\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$= -\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{\sqrt{\sin(e+fx)}\int \sqrt{\sin(2e+2fx)}\,dx}{2\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

$$= -\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{2f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

Mathematica [C] time = 0.292999, size = 74, normalized size = 0.87

$$\frac{b\left(-3\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\sec^2(e+fx)\right)+\cos(2(e+fx))-1\right)}{6f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^(5/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (b*(-1 + Cos[2*(e + f*x)] - 3*Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e + f*x
]^2]*(-Tan[e + f*x]^2)^(1/4)))/(6*f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x
]])
```

Maple [B] time = 0.136, size = 511, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/12/f*2^(1/2)*(2*2^(1/2)*cos(f*x+e)^4+3*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2),1/2*2^(1/2))-6*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2
))+3*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-6*((1-cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2),1/2*2^(1/2))-5*2^(1/2)*cos(f*x+e)^2+3*2^(1/2)*cos(f*x+e))/cos(f*x+e)
/sin(f*x+e)^(1/2)/(b/cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^(5/2)/sqrt(b*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^{2}-1\right)\sqrt{b\sec\left(fx+e\right)}\sqrt{\sin\left(fx+e\right)}}{b\sec\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))/(b*s
ec(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**(5/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^(5/2)/sqrt(b*sec(f*x + e)), x)
```

3.463 $\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] (EllipticE[e - Pi/4 + f*x, 2]*Sqrt[Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]]*S
qrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 0.0807287, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2585, 2572, 2639}

$$\frac{\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[e + f*x]]/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (EllipticE[e - Pi/4 + f*x, 2]*Sqrt[Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]]*S
qrt[Sin[2*e + 2*f*x]])
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx &= \frac{\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= \frac{\sqrt{\sin(e+fx)}\int \sqrt{\sin(2e+2fx)}\,dx}{\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}\\
&= \frac{E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}
\end{aligned}$$

Mathematica [C] time = 1.08359, size = 60, normalized size = 1.18

$$-\frac{b\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\sec^2(e+fx)\right)}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sin[e + f*x]]/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -((b*Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(1
/4))/(f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x]]))
```

Maple [B] time = 0.118, size = 497, normalized size = 9.8

$$-\frac{\sqrt{2}}{2f\cos\left(fx+e\right)}\left(2\cos\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(f\right.}{\sin\left(fx\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*2^(1/2)*(2*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-co
s(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*((1-cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2),1/2*2^(1/2))-((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*
EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2^(1/2)
*cos(f*x+e)^2-2^(1/2)*cos(f*x+e))/cos(f*x+e)/sin(f*x+e)^(1/2)/(b/cos(f*x+e)
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\sin\left(fx+e\right)}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(f*x + e))/sqrt(b*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{\sin\left(fx+e\right)}}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))/(b*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\sin\left(e+fx\right)}}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**(1/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(e + f*x))/sqrt(b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\sin\left(fx+e\right)}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(f*x + e))/sqrt(b*sec(f*x + e)), x)
```

3.464 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx$

Optimal. Leaf size=81

$$-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}-\frac{2\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] (-2*b)/(f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x]]) - (2*EllipticE[e - Pi/
4 + f*x, 2]*Sqrt[Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*
x]])
```

Rubi [A] time = 0.119976, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2584, 2585, 2572, 2639}

$$-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}-\frac{2\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(3/2)),x]
```

```
[Out] (-2*b)/(f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x]]) - (2*EllipticE[e - Pi/
4 + f*x, 2]*Sqrt[Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*
x]])
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx = -\frac{2b}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - 2\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx$$
$$= -\frac{2b}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{2\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$= -\frac{2b}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{\left(2\sqrt{\sin(e+fx)}\right)\int \sqrt{\sin(2e+2fx)}\,dx}{\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$
$$= -\frac{2b}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{2E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

Mathematica [C] time = 0.245375, size = 63, normalized size = 0.78

$$\frac{2b\left(\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\sec^2(e+fx)\right)-1\right)}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(3/2)),x]
```

```
[Out] (2*b*(-1 + Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e + f*x]^2]*(-Tan[e + f*x]
^2)^(1/4)))/(f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x]])
```

Maple [B] time = 0.109, size = 484, normalized size = 6.

$$\frac{\sqrt{2}}{f\cos\left(fx+e\right)}\left(2\cos\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*2^(1/2)*(2*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos(f
*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*((1-cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2),1/2*2^(1/2))-((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2^(1/2)*co
s(f*x+e))/cos(f*x+e)/sin(f*x+e)^(1/2)/(b/cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{\sin\left(fx+e\right)}}{\left(b\cos\left(fx+e\right)^{2}-b\right)\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))/((b*cos(f*x + e)^2 - b)*s
ec(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)**(3/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(3/2)), x)
```

3.465 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx$

Optimal. Leaf size=115

$$-\frac{2b}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{4b}{5f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}-\frac{4\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{5f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] (-2*b)/(5*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(5/2)) - (4*b)/(5*f*(b*Sec[
e + f*x])^(3/2)*Sqrt[Sin[e + f*x]]) - (4*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[
Sin[e + f*x]])/(5*f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 0.162471, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2584, 2585, 2572, 2639}

$$-\frac{2b}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{4b}{5f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}-\frac{4\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{5f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(7/2)),x]
```


```
[Out] (-2*b)/(5*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(5/2)) - (4*b)/(5*f*(b*Sec[
e + f*x])^(3/2)*Sqrt[Sin[e + f*x]]) - (4*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[
Sin[e + f*x]])/(5*f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])
```


Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}dx = -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)} + \frac{2}{5}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}dx$$

$$= -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)} - \frac{4b}{5f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{4}{5}\int$$

$$= -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)} - \frac{4b}{5f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{4\int\sqrt{}}{5\sqrt{b}}$$

$$= -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)} - \frac{4b}{5f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{(4\sqrt{\sin}}{5\sqrt{}}$$

$$= -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)} - \frac{4b}{5f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{4E(e}{5f\sqrt{b}}$$

Mathematica [C] time = 0.461951, size = 82, normalized size = 0.71

$$\frac{2b\left(2\sin^2(e+fx)\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\sec^2(e+fx)\right)+\cos(2(e+fx))-2\right)}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(7/2)),x]
```

```
[Out] (2*b*(-2 + Cos[2*(e + f*x)] + 2*Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e + f
*x]^2]*Sin[e + f*x]^2*(-Tan[e + f*x]^2)^(1/4)))/(5*f*(b*Sec[e + f*x])^(3/2)
*Sin[e + f*x]^(5/2))
```

Maple [B] time = 0.159, size = 1030, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)^(7/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -16/5/f*2^(1/2)*(-1+cos(f*x+e))^4*(4*cos(f*x+e)^3*((1-cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2),1/2*2^(1/2))-2*cos(f*x+e)^3*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2
))+4*cos(f*x+e)^2*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2*cos(f*x+e)^
2*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-4*cos(f*x+e)*((1-cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2),1/2*2^(1/2))+2*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
```

```
in(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2
*2^(1/2))-2*2^(1/2)*cos(f*x+e)^3-4*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/
2))+2*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(f*x+e)^2+2*2^
(1/2)*cos(f*x+e))/sin(f*x+e)^(5/2)/(b/cos(f*x+e))^(1/2)/(-1+cos(f*x+e)+sin(
f*x+e))/(1-cos(f*x+e)+sin(f*x+e))/(sin(f*x+e)^2+cos(f*x+e)^2-2*cos(f*x+e)+1
)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(7/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{\sin\left(fx+e\right)}}{\left(b\cos\left(fx+e\right)^{4}-2\,b\cos\left(fx+e\right)^{2}+b\right)\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(7/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))/((b*cos(f*x + e)^4 - 2*b*c
os(f*x + e)^2 + b)*sec(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)**(7/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(7/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(7/2)), x)
```

3.466 $\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=363

$$-\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}+\frac{\sqrt{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\log\left(\sqrt{b}\cot(e\right.}{8\sqrt{2}f\sqrt{b\cos}}$$

```
[Out] (Sqrt[b]*ArcTan[1 - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*
x]])])/(4*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*A
rcTan[1 + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*x]])])/(4*
Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*Log[Sqrt[b]
 + Sqrt[b]*Cot[e + f*x] - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e + f*x]]
])/(8*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) + (Sqrt[b]*Log[S
qrt[b] + Sqrt[b]*Cot[e + f*x] + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e +
 f*x]]])/(8*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (b*Sqrt[
Sin[e + f*x]])/(2*f*(b*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.267997, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2583, 2585, 2575, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}+\frac{\sqrt{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\log\left(\sqrt{b}\cot(e\right.}{8\sqrt{2}f\sqrt{b\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^(3/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[b]*ArcTan[1 - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*
x]])])/(4*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*A
rcTan[1 + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*x]])])/(4*
Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*Log[Sqrt[b]
 + Sqrt[b]*Cot[e + f*x] - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e + f*x]]
])/(8*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) + (Sqrt[b]*Log[S
qrt[b] + Sqrt[b]*Cot[e + f*x] + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e +
 f*x]]])/(8*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (b*Sqrt[
Sin[e + f*x]])/(2*f*(b*Sec[e + f*x])^(3/2))
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}} + \frac{1}{4}\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx$$

$$= -\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}} + \frac{\int \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\,dx}{4\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$= -\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}} - \frac{b\,\text{Subst}\left(\int \frac{x^2}{b^2+x^4}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$= -\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}} + \frac{b\,\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{4f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{b\,\text{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{4f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$= -\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-x^2}\right.}{8\sqrt{2}f\sqrt{b\cos(e+fx)}}$$

$$= -\frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$= \frac{\sqrt{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\,c\right.}{8\sqrt{2}f\sqrt{b\cos(}$$

Mathematica [A] time = 1.54861, size = 218, normalized size = 0.6

$$-\frac{\sqrt{\sin(e+fx)}\sqrt{b\sec(e+fx)}\left(2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{\tan^2(e+fx)}\right)-2\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt[4]{\tan^2(e+fx)}+1\right)+4\sqrt[4]{\tan^2(e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^(3/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -(Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*(Ta
n[e + f*x]^2)^(1/4)] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*(Tan[e + f*x]^2)^(1/4)]
 + Sqrt[2]*Log[1 - Sqrt[2]*(Tan[e + f*x]^2)^(1/4) + Sqrt[Tan[e + f*x]^2]] -
 Sqrt[2]*Log[1 + Sqrt[2]*(Tan[e + f*x]^2)^(1/4) + Sqrt[Tan[e + f*x]^2]] + 4
*(Tan[e + f*x]^2)^(1/4) + 4*Cos[2*(e + f*x)]*(Tan[e + f*x]^2)^(1/4)))/(16*b
*f*(Tan[e + f*x]^2)^(1/4))
```

Maple [C] time = 0.129, size = 648, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/8/f*2^(1/2)*(I*sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2
^(1/2))-I*sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
```

```
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))
-sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((
(1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-sin(f*x+
e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+2*sin(f*x+e)*((1-
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2*2^(1/2)*cos(f*x+e)^3+2*2^(1/2)*co
s(f*x+e)^2)*sin(f*x+e)^(1/2)/(-1+cos(f*x+e))/(b/cos(f*x+e))^(1/2)/cos(f*x+e
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^(3/2)/sqrt(b*sec(f*x + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**(3/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^(3/2)/sqrt(b*sec(f*x + e)), x)
```

3.467 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx$

Optimal. Leaf size=328

$$\frac{\sqrt{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\log\left(\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}+\right.}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] (Sqrt[b]*ArcTan[1 - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*
x]])])/(Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*Arc
Tan[1 + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*x]])])/(Sqrt
[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*Log[Sqrt[b] + S
qrt[b]*Cot[e + f*x] - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e + f*x]]])/(
2*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) + (Sqrt[b]*Log[Sqrt[
b] + Sqrt[b]*Cot[e + f*x] + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e + f*x
]]])/(2*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.192076, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2585, 2575, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\log\left(\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}+\right.}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]),x]
```

```
[Out] (Sqrt[b]*ArcTan[1 - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*
x]])])/(Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*Arc
Tan[1 + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*x]])])/(Sqrt
[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*Log[Sqrt[b] + S
qrt[b]*Cot[e + f*x] - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e + f*x]]])/(
2*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) + (Sqrt[b]*Log[Sqrt[
b] + Sqrt[b]*Cot[e + f*x] + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e + f*x
]]])/(2*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \frac{\int \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\,dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$= -\frac{(2b)\,\mathrm{Subst}\left(\int \frac{x^2}{b^2+x^4}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$= \frac{b\,\mathrm{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{b\,\mathrm{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$= -\frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-x^2}\,dx, x,\right.}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec}}$$
$$= -\frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)\right.}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b}}$$
$$= \frac{\sqrt{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\,\mathrm{l}}{}$$

Mathematica [A] time = 0.938711, size = 166, normalized size = 0.51

$$\frac{\sqrt{\sin(e+fx)}\sqrt{b\sec(e+fx)}\left(-2\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{\tan^2(e+fx)}\right)+2\tan^{-1}\left(\sqrt{2}\sqrt[4]{\tan^2(e+fx)}+1\right)-\log\left(\sqrt{\tan^2(e+\right.\right.}{2\sqrt{2}bf\sqrt[4]{\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]),x]
```

```
[Out] ((-2*ArcTan[1 - Sqrt[2]*(Tan[e + f*x]^2)^(1/4)] + 2*ArcTan[1 + Sqrt[2]*(Tan
[e + f*x]^2)^(1/4)] - Log[1 - Sqrt[2]*(Tan[e + f*x]^2)^(1/4) + Sqrt[Tan[e +
 f*x]^2]] + Log[1 + Sqrt[2]*(Tan[e + f*x]^2)^(1/4) + Sqrt[Tan[e + f*x]^2]])
*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(2*Sqrt[2]*b*f*(Tan[e + f*x]^2)^(
1/4))
```

Maple [C] time = 0.11, size = 304, normalized size = 0.9

$$-\frac{\sqrt{2}}{2f\cos(fx+e)\left(-1+\cos(fx+e)\right)}\sqrt{\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{\frac{-1+\mathrm{co}}{\sin(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*2^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(I*Ellipt
icPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*
EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/
2))+EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2
```

```
^(1/2))+EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1
/2*2^(1/2))-2*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^
(1/2)))*sin(f*x+e)^(3/2)/(b/cos(f*x+e))^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sqrt{\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(e+fx\right)}\sqrt{\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)**(1/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*sec(e + f*x))*sqrt(sin(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sqrt{\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))), x)
```

3.468 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx$

Optimal. Leaf size=30

$$-\frac{2b}{3f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

```
[Out] (-2*b)/(3*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Rubi [A] time = 0.0378593, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2578}

$$-\frac{2b}{3f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(5/2)),x]
```

```
[Out] (-2*b)/(3*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = -\frac{2b}{3f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

Mathematica [A] time = 0.102346, size = 30, normalized size = 1.

$$-\frac{2b}{3f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(5/2)),x]
```

```
[Out] (-2*b)/(3*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Maple [B] time = 0.096, size = 70, normalized size = 2.3

$$-\frac{8\cos(fx+e)\left(-1+\cos(fx+e)\right)^{2}}{3f\left(\left(\sin(fx+e)\right)^{2}+\left(\cos(fx+e)\right)^{2}-2\cos(fx+e)+1\right)^{2}}\left(\sin(fx+e)\right)^{-\frac{3}{2}}\frac{1}{\sqrt{\frac{b}{\cos(fx+e)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -8/3/f*cos(f*x+e)*(-1+cos(f*x+e))^2/sin(f*x+e)^(3/2)/(sin(f*x+e)^2+cos(f*x+
e)^2-2*cos(f*x+e)+1)^2/(b/cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(5/2)), x)
```

Fricas [B] time = 2.32199, size = 117, normalized size = 3.9

$$\frac{2\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}\sqrt{\sin\left(fx+e\right)}}{3\left(bf\cos\left(fx+e\right)^{2}-bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b/cos(f*x + e))*cos(f*x + e)^2*sqrt(sin(f*x + e))/(b*f*cos(f*x + e
)^2 - b*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)**(5/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(5/2)), x)
```

3.469 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx$

Optimal. Leaf size=61

$$-\frac{8b}{21f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{2b}{7f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

```
[Out] (-2*b)/(7*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(7/2)) - (8*b)/(21*f*(b*Sec
[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Rubi [A] time = 0.0780422, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2584, 2578}

$$-\frac{8b}{21f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{2b}{7f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(9/2)),x]
```

```
[Out] (-2*b)/(7*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(7/2)) - (8*b)/(21*f*(b*Sec
[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx &= -\frac{2b}{7f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)}+\frac{4}{7}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx\\ &= -\frac{2b}{7f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)}-\frac{8b}{21f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}\end{aligned}$$

Mathematica [A] time = 0.138539, size = 42, normalized size = 0.69

$$\frac{2b(2\cos(2(e+fx))-5)}{21f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(9/2)),x]
```

```
[Out] (2*b*(-5 + 2*Cos[2*(e + f*x)]))/(21*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(
7/2))
```

Maple [A] time = 0.107, size = 82, normalized size = 1.3

$$\frac{32\cos\left(fx+e\right)\left(4\left(\cos\left(fx+e\right)\right)^{2}-7\right)\left(-1+\cos\left(fx+e\right)\right)^{4}}{21f\left(\left(\sin\left(fx+e\right)\right)^{2}+\left(\cos\left(fx+e\right)\right)^{2}-2\cos\left(fx+e\right)+1\right)^{4}}\left(\sin\left(fx+e\right)\right)^{-\frac{7}{2}}\frac{1}{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 32/21/f*cos(f*x+e)*(4*cos(f*x+e)^2-7)*(-1+cos(f*x+e))^4/sin(f*x+e)^(7/2)/(s
in(f*x+e)^2+cos(f*x+e)^2-2*cos(f*x+e)+1)^4/(b/cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(9/2)), x)
```

Fricas [A] time = 2.47585, size = 181, normalized size = 2.97

$$\frac{2\left(4\cos\left(fx+e\right)^{4}-7\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}\sqrt{\sin\left(fx+e\right)}}{21\left(bf\cos\left(fx+e\right)^{4}-2\,bf\cos\left(fx+e\right)^{2}+bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(4*cos(f*x + e)^4 - 7*cos(f*x + e)^2)*sqrt(b/cos(f*x + e))*sqrt(sin(f*
x + e))/(b*f*cos(f*x + e)^4 - 2*b*f*cos(f*x + e)^2 + b*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)**(9/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sec (fx+e)} \sin (fx+e)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(9/2)), x)
```

3.470 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx$

Optimal. Leaf size=91

$$-\frac{64b}{231f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{16b}{77f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{2b}{11f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

```
[Out] (-2*b)/(11*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(11/2)) - (16*b)/(77*f*(b*
Sec[e + f*x])^(3/2)*Sin[e + f*x]^(7/2)) - (64*b)/(231*f*(b*Sec[e + f*x])^(3
/2)*Sin[e + f*x]^(3/2))
```

Rubi [A] time = 0.121811, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2584, 2578}

$$-\frac{64b}{231f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{16b}{77f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{2b}{11f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(13/2)),x]
```

```
[Out] (-2*b)/(11*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(11/2)) - (16*b)/(77*f*(b*
Sec[e + f*x])^(3/2)*Sin[e + f*x]^(7/2)) - (64*b)/(231*f*(b*Sec[e + f*x])^(3
/2)*Sin[e + f*x]^(3/2))
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx &= -\frac{2b}{11f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)}+\frac{8}{11}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx\\ &= -\frac{2b}{11f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)}-\frac{16b}{77f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)}+\frac{32}{77}\\ &= -\frac{2b}{11f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)}-\frac{16b}{77f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)}-\frac{}{23}\end{aligned}$$

Mathematica [A] time = 0.201415, size = 52, normalized size = 0.57

$$\frac{2b(28\cos(2(e+fx))-4\cos(4(e+fx))-45)}{231f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(13/2)),x]
```

```
[Out] (2*b*(-45 + 28*Cos[2*(e + f*x)] - 4*Cos[4*(e + f*x)]))/(231*f*(b*Sec[e + f*
x])^(3/2)*Sin[e + f*x]^(11/2))
```

Maple [A] time = 0.129, size = 92, normalized size = 1.

$$-\frac{128\cos\left(fx+e\right)\left(32\left(\cos\left(fx+e\right)\right)^{4}-88\left(\cos\left(fx+e\right)\right)^{2}+77\right)\left(-1+\cos\left(fx+e\right)\right)^{6}}{231f\left(\left(\sin\left(fx+e\right)\right)^{2}+\left(\cos\left(fx+e\right)\right)^{2}-2\cos\left(fx+e\right)+1\right)^{6}}\left(\sin\left(fx+e\right)\right)^{-\frac{11}{2}}\frac{1}{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)^(13/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -128/231/f*cos(f*x+e)*(32*cos(f*x+e)^4-88*cos(f*x+e)^2+77)*(-1+cos(f*x+e))^
6/sin(f*x+e)^(11/2)/(sin(f*x+e)^2+cos(f*x+e)^2-2*cos(f*x+e)+1)^6/(b/cos(f*x
+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(13/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(13/2)), x)
```

Fricas [A] time = 2.6834, size = 243, normalized size = 2.67

$$\frac{2\left(32\cos\left(fx+e\right)^{6}-88\cos\left(fx+e\right)^{4}+77\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}\sqrt{\sin\left(fx+e\right)}}{231\left(bf\cos\left(fx+e\right)^{6}-3bf\cos\left(fx+e\right)^{4}+3bf\cos\left(fx+e\right)^{2}-bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(13/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(32*cos(f*x + e)^6 - 88*cos(f*x + e)^4 + 77*cos(f*x + e)^2)*sqrt(b/co
s(f*x + e))*sqrt(sin(f*x + e))/(b*f*cos(f*x + e)^6 - 3*b*f*cos(f*x + e)^4 +
```

```
3*b*f*cos(f*x + e)^2 - b*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)**(13/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(13/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(13/2)), x)
```

3.471 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx$

Optimal. Leaf size=121

$$-\frac{256b}{1155f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{64b}{385f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{8b}{55f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{\cdots}{15f\cdots}$$

```
[Out] (-2*b)/(15*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(15/2)) - (8*b)/(55*f*(b*S
ec[e + f*x])^(3/2)*Sin[e + f*x]^(11/2)) - (64*b)/(385*f*(b*Sec[e + f*x])^(3
/2)*Sin[e + f*x]^(7/2)) - (256*b)/(1155*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*
x]^(3/2))
```

Rubi [A] time = 0.162258, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2584, 2578}

$$-\frac{256b}{1155f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{64b}{385f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{8b}{55f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{\cdots}{15f\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(17/2)),x]
```

```
[Out] (-2*b)/(15*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(15/2)) - (8*b)/(55*f*(b*S
ec[e + f*x])^(3/2)*Sin[e + f*x]^(11/2)) - (64*b)/(385*f*(b*Sec[e + f*x])^(3
/2)*Sin[e + f*x]^(7/2)) - (256*b)/(1155*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*
x]^(3/2))
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = -\frac{2b}{15f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)} + \frac{4}{5}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx$$
$$= -\frac{2b}{15f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)} - \frac{8b}{55f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)} + \frac{32}{55}\ldots$$
$$= -\frac{2b}{15f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)} - \frac{8b}{55f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)} - \frac{\ldots}{38\ldots}$$
$$= -\frac{2b}{15f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)} - \frac{8b}{55f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)} - \frac{\ldots}{38\ldots}$$

Mathematica [A] time = 0.254319, size = 62, normalized size = 0.51

$$\frac{2b(150\cos(2(e+fx)) - 36\cos(4(e+fx)) + 4\cos(6(e+fx)) - 195)}{1155f\sin^{\frac{15}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(17/2)),x]
```

```
[Out] (2*b*(-195 + 150*Cos[2*(e + f*x)] - 36*Cos[4*(e + f*x)] + 4*Cos[6*(e + f*x)
]))/(1155*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(15/2))
```

Maple [A] time = 0.15, size = 102, normalized size = 0.8

$$\frac{512\cos(fx+e)\left(128(\cos(fx+e))^6 - 480(\cos(fx+e))^4 + 660(\cos(fx+e))^2 - 385\right)(-1+\cos(fx+e))^8}{1155f\left((\sin(fx+e))^2 + (\cos(fx+e))^2 - 2\cos(fx+e) + 1\right)^8}(\sin\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)^(17/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 512/1155/f*cos(f*x+e)*(128*cos(f*x+e)^6-480*cos(f*x+e)^4+660*cos(f*x+e)^2-3
85)*(-1+cos(f*x+e))^8/sin(f*x+e)^(15/2)/(sin(f*x+e)^2+cos(f*x+e)^2-2*cos(f*
x+e)+1)^8/(b/cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(fx+e)}\sin(fx+e)^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(17/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(17/2)), x)
```

Fricas [A] time = 3.01009, size = 308, normalized size = 2.55

$$\frac{2\left(128\cos\left(fx+e\right)^{8}-480\cos\left(fx+e\right)^{6}+660\cos\left(fx+e\right)^{4}-385\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{\cos(fx+e)}}\sqrt{\sin\left(fx+e\right)}}{1155\left(bf\cos\left(fx+e\right)^{8}-4\,bf\cos\left(fx+e\right)^{6}+6\,bf\cos\left(fx+e\right)^{4}-4\,bf\cos\left(fx+e\right)^{2}+bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(17/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(128*cos(f*x + e)^8 - 480*cos(f*x + e)^6 + 660*cos(f*x + e)^4 - 385*
cos(f*x + e)^2)*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e))/(b*f*cos(f*x + e)^8
 - 4*b*f*cos(f*x + e)^6 + 6*b*f*cos(f*x + e)^4 - 4*b*f*cos(f*x + e)^2 + b*f
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)**(17/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(17/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(17/2)), x)
```

3.472 $\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=490

$$-\frac{7a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{128\sqrt{2}b^{5/2}f}+\frac{7a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a}\cdots}{\sqrt{a}\sqrt{b\,c\cdots}}\right.}{128\sqrt{2}b^{5/2}f}$$

```
[Out] (-7*a^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt
[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(128*Sqrt[2]*
b^(5/2)*f) + (7*a^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(
Sqrt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(
128*Sqrt[2]*b^(5/2)*f) + (7*a^(9/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqr
t[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f
*x]]*Sqrt[b*Sec[e + f*x]])/(256*Sqrt[2]*b^(5/2)*f) - (7*a^(9/2)*Sqrt[b*Cos[
e + f*x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e
 + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(256*Sqrt[2]*b^(5/2)
*f) - (7*a^3*(a*Sin[e + f*x])^(3/2))/(192*b*f*Sqrt[b*Sec[e + f*x]]) - (a*(a
*Sin[e + f*x])^(7/2))/(48*b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x])^(11/
2)/(6*a*b*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.553205, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2582, 2583, 2585, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{128\sqrt{2}b^{5/2}f}+\frac{7a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a}\cdots}{\sqrt{a}\sqrt{b\,c\cdots}}\right.}{128\sqrt{2}b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(9/2)/(b*Sec[e + f*x])^(3/2),x]
```


```
[Out] (-7*a^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt
[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(128*Sqrt[2]*
b^(5/2)*f) + (7*a^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(
Sqrt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(
128*Sqrt[2]*b^(5/2)*f) + (7*a^(9/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqr
t[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f
*x]]*Sqrt[b*Sec[e + f*x]])/(256*Sqrt[2]*b^(5/2)*f) - (7*a^(9/2)*Sqrt[b*Cos[
e + f*x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e
 + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(256*Sqrt[2]*b^(5/2)
*f) - (7*a^3*(a*Sin[e + f*x])^(3/2))/(192*b*f*Sqrt[b*Sec[e + f*x]]) - (a*(a
*Sin[e + f*x])^(7/2))/(48*b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x])^(11/
2)/(6*a*b*f*Sqrt[b*Sec[e + f*x]])
```


Rule 2582

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m - n)), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx}{12b^2}$$

$$= -\frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{\left(7a^2\right)\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx}{96b^2}$$

$$= -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{\left(7a^4\right)\int \sqrt{b\sec(e+}{1}$$

$$= -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{\left(7a^4\sqrt{b\cos(e+fx}{}$$

$$= -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{\left(7a^5\sqrt{b\cos(e+fx}{}$$

$$= -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} - \frac{\left(7a^5\sqrt{b\cos(e+fx}{}$$

$$= -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{\left(7a^5\sqrt{b\cos(e+fx}{}$$

$$= \frac{7a^{9/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a} - \frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}} + \sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{256\sqrt{2}b^{5/2}f} - \frac{7a^{9/2}\sqrt{}}{}$$

$$= -\frac{7a^{9/2}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{128\sqrt{2}b^{5/2}f} + \frac{7a^{9/2}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt{b}\sqrt{}}{\sqrt{a}\sqrt{b}}\right.}{}$$

Mathematica [C] time = 0.444454, size = 97, normalized size = 0.2

$$-\frac{a^5\tan^2(e+fx)\left(-14\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right) + 9\cos(2(e+fx)) + 3\cos(4(e+fx)) - 2\cos(6(e+fx)) + 4\right)}{384bf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(9/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(a^5*(4 + 9*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)] - 2*Cos[6*(e + f*x)] - 1
4*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2])*Tan[e + f*x]^2)/(384*b*f
*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Maple [C] time = 0.145, size = 572, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x)

[Out] 1/768/f*2^(1/2)*(64*cos(f*x+e)^6*2^(1/2)-64*cos(f*x+e)^5*2^(1/2)-120*2^(1/2)*cos(f*x+e)^4+21*I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-21*I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+120*2^(1/2)*cos(f*x+e)^3+21*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+21*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+42*2^(1/2)*cos(f*x+e)^2-42*2^(1/2)*cos(f*x+e))*(a*sin(f*x+e))^(9/2)/(-1+cos(f*x+e))/sin(f*x+e)^3/cos(f*x+e)^2/(b/cos(f*x+e))^(3/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sin\left(fx+e\right)\right)^{\frac{9}{2}}}{\left(b \sec\left(fx+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e))^(9/2)/(b*sec(f*x + e))^(3/2), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sin(f*x+e))**(9/2)/(b*sec(f*x+e))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(9/2)/(b*sec(f*x + e))^(3/2), x)
```

3.473 $\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=453

$$-\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{32\sqrt{2}b^{5/2}f}+\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(}}{\sqrt{a}\sqrt{b\cos(e}}\right.}{32\sqrt{2}b^{5/2}f}$$

```
[Out] (-3*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt
[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(32*Sqrt[2]*b
^(5/2)*f) + (3*a^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(S
qrt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(3
2*Sqrt[2]*b^(5/2)*f) + (3*a^(5/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[
2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x
]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*b^(5/2)*f) - (3*a^(5/2)*Sqrt[b*Cos[e +
 f*x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e +
f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*b^(5/2)*f)
- (a*(a*Sin[e + f*x])^(3/2))/(16*b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x
])^(7/2)/(4*a*b*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.447008, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2582, 2583, 2585, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{32\sqrt{2}b^{5/2}f}+\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(}}{\sqrt{a}\sqrt{b\cos(e}}\right.}{32\sqrt{2}b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(5/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-3*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt
[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(32*Sqrt[2]*b
^(5/2)*f) + (3*a^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(S
qrt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(3
2*Sqrt[2]*b^(5/2)*f) + (3*a^(5/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[
2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x
]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*b^(5/2)*f) - (3*a^(5/2)*Sqrt[b*Cos[e +
 f*x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e +
f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*b^(5/2)*f)
- (a*(a*Sin[e + f*x])^(3/2))/(16*b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x
])^(7/2)/(4*a*b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 2582

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m - n)), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
```

```
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}} + \frac{\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx}{8b^2}$$

$$= -\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}} + \frac{\left(3a^2\right)\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx}{32b^2}$$

$$= -\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}} + \frac{\left(3a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{\sqrt{a\sin(e+}}{\sqrt{b\cos(e+}}}{32b^2}$$

$$= -\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}} + \frac{\left(3a^3\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\right.}{16bf}$$

$$= -\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}} - \frac{\left(3a^3\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\right.}{32b^2f}$$

$$= -\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}} + \frac{\left(3a^3\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\right.}{64b^3f}$$

$$= \frac{3a^{5/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{64\sqrt{2}b^{5/2}f} - \frac{3a^{5/2}\sqrt{b\cos}}{}$$

$$= -\frac{3a^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{32\sqrt{2}b^{5/2}f} + \frac{3a^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin}}{\sqrt{a}\sqrt{b\cos(}}\right.}{}$$

Mathematica [C] time = 0.332361, size = 82, normalized size = 0.18

$$-\frac{a^3\tan^2(e+fx)\left(-2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)+\cos(2(e+fx))+\cos(4(e+fx))\right)}{32bf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(5/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(a^3*(Cos[2*(e + f*x)] + Cos[4*(e + f*x)] - 2*Hypergeometric2F1[3/4, 1, 7/
4, -Tan[e + f*x]^2])*Tan[e + f*x]^2)/(32*b*f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Si
n[e + f*x]])
```

Maple [C] time = 0.123, size = 546, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/64/f*2^(1/2)*(8*2^(1/2)*cos(f*x+e)^4+3*I*((1-cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
,1/2-1/2*I,1/2*2^(1/2))-3*I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2
^(1/2))-8*2^(1/2)*cos(f*x+e)^3-3*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,
1/2*2^(1/2))-3*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*2^(
1/2)*cos(f*x+e)^2+6*2^(1/2)*cos(f*x+e))*(a*sin(f*x+e))^(5/2)/(-1+cos(f*x+e)
)/sin(f*x+e)/cos(f*x+e)^2/(b/cos(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(5/2)/(b*sec(f*x + e))^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(5/2)/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(5/2)/(b*sec(f*x + e))^(3/2), x)
```

3.474 $\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=418

$$-\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{4\sqrt{2}b^{5/2}f}+\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(\cdots}{\sqrt{a}\sqrt{b\cos(e\cdots}\right.}{4\sqrt{2}b^{5/2}f}$$

```
[Out] -(Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b
*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(4*Sqrt[2]*b^(5
/2)*f) + (Sqrt[a]*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a
]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(4*Sqrt
[2]*b^(5/2)*f) + (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]*Sqrt[
b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[
b*Sec[e + f*x]])/(8*Sqrt[2]*b^(5/2)*f) - (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Log[
Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqr
t[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(8*Sqrt[2]*b^(5/2)*f) + (a*Sin[e +
 f*x])^(3/2)/(2*a*b*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] time = 0.356122, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {2582, 2585, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{4\sqrt{2}b^{5/2}f}+\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(\cdots}{\sqrt{a}\sqrt{b\cos(e\cdots}\right.}{4\sqrt{2}b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[e + f*x]]/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b
*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(4*Sqrt[2]*b^(5
/2)*f) + (Sqrt[a]*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a
]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(4*Sqrt
[2]*b^(5/2)*f) + (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]*Sqrt[
b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[
b*Sec[e + f*x]])/(8*Sqrt[2]*b^(5/2)*f) - (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Log[
Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqr
t[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(8*Sqrt[2]*b^(5/2)*f) + (a*Sin[e +
 f*x])^(3/2)/(2*a*b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 2582

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m - n)), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
```

```
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} + \frac{\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx}{4b^2}$$

$$= \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} + \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\,dx}{4b^2}$$

$$= \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} + \frac{\left(a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right.}{2bf}$$

$$= \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} - \frac{\left(a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{a-bx^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right.}{4b^2f}$$

$$= \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} + \frac{\left(a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx, x, \frac{\sqrt{a\sin(e}}{\sqrt{b\cos(e}}\right.}{8b^3f}$$

$$= \frac{\sqrt{a}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{8\sqrt{2}b^{5/2}f} - \frac{\sqrt{a}\sqrt{b\,\text{co}}}{}$$

$$= -\frac{\sqrt{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}b^{5/2}f} + \frac{\sqrt{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin}}{\sqrt{a}\sqrt{b\cos(e}}\right.}{}$$

Mathematica [C] time = 0.231035, size = 76, normalized size = 0.18

$$\frac{\sec^2(e+fx)\sqrt{a\sin(e+fx)}\left(2\tan(e+fx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)+3\sin(2(e+fx))\right)}{12f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^2*Sqrt[a*Sin[e + f*x]]*(3*Sin[2*(e + f*x)] + 2*Hypergeometric
2F1[3/4, 1, 7/4, -Tan[e + f*x]^2]*Tan[e + f*x]))/(12*f*(b*Sec[e + f*x])^(3/
2))
```

Maple [C] time = 0.144, size = 516, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/8/f*2^(1/2)*(-I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+I*
((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+((1-cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
```

```
+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*
I,1/2*2^(1/2))+2*2^(1/2)*cos(f*x+e)^2-2*2^(1/2)*cos(f*x+e))*(a*sin(f*x+e))^
(1/2)*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^2/(b/cos(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))/(b*sec(f*x + e))^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(1/2)/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))/(b*sec(f*x + e))^(3/2), x)
```

3.475 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=411

$$\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}a^{3/2}b^{5/2}f}-\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}a^{3/2}b^{5/2}f}$$

```
[Out] (ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b*Cos[e +
f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2]*a^(3/2)*b^(5/2)
*f) - (ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b*Co
s[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2]*a^(3/2)*b
^(5/2)*f) - (Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin
[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x
]])/(2*Sqrt[2]*a^(3/2)*b^(5/2)*f) + (Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] + (Sq
rt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e +
f*x]]*Sqrt[b*Sec[e + f*x]])/(2*Sqrt[2]*a^(3/2)*b^(5/2)*f) - 2/(a*b*f*Sqrt[b
*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.35963, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {2581, 2585, 2574, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}a^{3/2}b^{5/2}f}-\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}a^{3/2}b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(3/2)),x]
```


```
[Out] (ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b*Cos[e +
f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2]*a^(3/2)*b^(5/2)
*f) - (ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b*Co
s[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2]*a^(3/2)*b
^(5/2)*f) - (Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin
[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x
]])/(2*Sqrt[2]*a^(3/2)*b^(5/2)*f) + (Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] + (Sq
rt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e +
f*x]]*Sqrt[b*Sec[e + f*x]])/(2*Sqrt[2]*a^(3/2)*b^(5/2)*f) - 2/(a*b*f*Sqrt[b
*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```


Rule 2581

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m + 1)), x] - Dist[(n + 1)/(a^2*b^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n,
 -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
```

```
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}dx = -\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}} - \frac{\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}dx}{a^2b^2}$$

$$= -\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}} - \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{\sqrt{}}{\sqrt{}}}{a^2b^2}$$

$$= -\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}} - \frac{\left(2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Su}}{abf}$$

$$= -\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}} + \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Su}}{ab^2f}$$

$$= -\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}} - \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Su}}{2ab}$$

$$= -\frac{\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+}}{2\sqrt{2}a^{3/2}b^{5/2}f}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}a^{3/2}b^{5/2}f} - \frac{\tan^{-1}\left(1+\frac{\sqrt{}}{}\right.}{}$$

Mathematica [C] time = 0.20698, size = 66, normalized size = 0.16

$$-\frac{2\left(\tan^2(e+fx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)+3\right)}{3abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (-2*(3 + Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2]*Tan[e + f*x]^2))/(
3*a*b*f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Maple [C] time = 0.111, size = 957, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*2^(1/2)*(I*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*
2^(1/2))-I*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*El
lipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2)
)-cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(
```

```
((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-cos(f*x
+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+I*((1-cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*((1-cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),
1/2+1/2*I,1/2*2^(1/2))-((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2
))-((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*2^(1/2)*cos(f*x
+e))*sin(f*x+e)/cos(f*x+e)^2/(b/cos(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(3/2)), x)
```

3.476 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=35

$$-\frac{2b}{5af(a\sin(e+fx))^{5/2}(b\sec(e+fx))^{5/2}}$$

```
[Out] (-2*b)/(5*a*f*(b*Sec[e + f*x])^(5/2)*(a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.0575665, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2578}

$$-\frac{2b}{5af(a\sin(e+fx))^{5/2}(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(7/2)),x]
```

```
[Out] (-2*b)/(5*a*f*(b*Sec[e + f*x])^(5/2)*(a*Sin[e + f*x])^(5/2))
```

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = -\frac{2b}{5af(b\sec(e+fx))^{5/2}(a\sin(e+fx))^{5/2}}$$

Mathematica [A] time = 0.109555, size = 45, normalized size = 1.29

$$-\frac{2\cot^3(e+fx)\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}{5a^4b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(7/2)),x]
```

```
[Out] (-2*Cot[e + f*x]^3*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])/(5*a^4*b^2*f)
```

Maple [A] time = 0.085, size = 40, normalized size = 1.1

$$-\frac{2\sin(fx+e)\cos(fx+e)}{5f}\left(\frac{b}{\cos(fx+e)}\right)^{-\frac{3}{2}}(a\sin(fx+e))^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x)
```

```
[Out] -2/5/f*sin(f*x+e)*cos(f*x+e)/(b/cos(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(7/2)), x)
```

Fricas [B] time = 2.46116, size = 157, normalized size = 4.49

$$\frac{2\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{3}}{5\left(a^{4}b^{2}f\cos\left(fx+e\right)^{2}-a^{4}b^{2}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x, algorithm="fricas"
)
```

```
[Out] 2/5*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*cos(f*x + e)^3/((a^4*b^2*f*co
s(f*x + e)^2 - a^4*b^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(7/2)), x)
```

3.477 $\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=172

$$\frac{a^4\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{24b^2f\sqrt{a\sin(e+fx)}}-\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}}+\frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}}-\frac{a(a\sin(e+fx))^{5/}}{30bf\sqrt{b\sec(e+f}}$$

```
[Out] -(a^3*Sqrt[a*Sin[e + f*x]])/(12*b*f*Sqrt[b*Sec[e + f*x]]) - (a*(a*Sin[e + f
*x])^(5/2))/(30*b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x])^(9/2)/(5*a*b*f
*Sqrt[b*Sec[e + f*x]]) + (a^4*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f
*x]]*Sqrt[Sin[2*e + 2*f*x]])/(24*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.274728, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2582, 2583, 2585, 2573, 2641}

$$\frac{a^4\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{24b^2f\sqrt{a\sin(e+fx)}}-\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}}+\frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}}-\frac{a(a\sin(e+fx))^{5/}}{30bf\sqrt{b\sec(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(7/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(a^3*Sqrt[a*Sin[e + f*x]])/(12*b*f*Sqrt[b*Sec[e + f*x]]) - (a*(a*Sin[e + f
*x])^(5/2))/(30*b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x])^(9/2)/(5*a*b*f
*Sqrt[b*Sec[e + f*x]]) + (a^4*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f
*x]]*Sqrt[Sin[2*e + 2*f*x]])/(24*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2582

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m - n)), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
```

```
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx}{10b^2}$$

$$= -\frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{a^2\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx}{12b^2}$$

$$= -\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{a^4\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{24b^2}$$

$$= -\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{\left(a^4\sqrt{b\cos(e+fx)}\sqrt{b\,\mathrm{se}\right.}{}$$

$$= -\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{\left(a^4\sqrt{b\sec(e+fx)}\sqrt{\sin\right.}{24b^2\sqrt{a}}$$

$$= -\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{a^4F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{b}}{24b^2f\sqrt{a}}$$

Mathematica [C] time = 0.861685, size = 103, normalized size = 0.6

$$-\frac{a^5\left(-20\left(-\tan^2(e+fx)\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)+17\cos(2(e+fx))-16\cos(4(e+fx))+3\cos(6(e+fx))-4\right)}{480bf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(7/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(a^5*(-4 + 17*Cos[2*(e + f*x)] - 16*Cos[4*(e + f*x)] + 3*Cos[6*(e + f*x)]
- 20*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/
4)))/(480*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2))
```

Maple [A] time = 0.138, size = 246, normalized size = 1.4

$$-\frac{\sqrt{2}}{120f\left(-1+\cos(fx+e)\right)\left(\sin(fx+e)\right)^3\left(\cos(fx+e)\right)^2}\left(-12\left(\cos(fx+e)\right)^6\sqrt{2}+12\left(\cos(fx+e)\right)^5\sqrt{2}+5\sin(fx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/120/f*2^(1/2)*(-12*cos(f*x+e)^6*2^(1/2)+12*cos(f*x+e)^5*2^(1/2)+5*sin(f*
x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
```

```
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+22*2^(1/2)*cos(f*x+e)^4-22
*2^(1/2)*cos(f*x+e)^3-5*2^(1/2)*cos(f*x+e)^2+5*2^(1/2)*cos(f*x+e))*(a*sin(f
*x+e))^(7/2)/(-1+cos(f*x+e))/sin(f*x+e)^3/cos(f*x+e)^2/(b/cos(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e))^{\frac{7}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(7/2)/(b*sec(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^3\cos\left(fx+e\right)^2-a^3\right)\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}\sin\left(fx+e\right)}{b^2\sec\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^3*cos(f*x + e)^2 - a^3)*sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x +
e))*sin(f*x + e)/(b^2*sec(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(7/2)/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e))^{\frac{7}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(7/2)/(b*sec(f*x + e))^(3/2), x)
```

3.478 $\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{a^2\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{12b^2f\sqrt{a\sin(e+fx)}}+\frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}}-\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}}$$

```
[Out] -(a*Sqrt[a*Sin[e + f*x]])/(6*b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x])^(
5/2)/(3*a*b*f*Sqrt[b*Sec[e + f*x]]) + (a^2*EllipticF[e - Pi/4 + f*x, 2]*Sqr
t[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(12*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.215809, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2582, 2583, 2585, 2573, 2641}

$$\frac{a^2\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{12b^2f\sqrt{a\sin(e+fx)}}+\frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}}-\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(3/2)/(b*Sec[e + f*x])^(3/2),x]
```


```
[Out] -(a*Sqrt[a*Sin[e + f*x]])/(6*b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x])^(
5/2)/(3*a*b*f*Sqrt[b*Sec[e + f*x]]) + (a^2*EllipticF[e - Pi/4 + f*x, 2]*Sqr
t[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(12*b^2*f*Sqrt[a*Sin[e + f*x]])
```


Rule 2582

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m - n)), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
```

```
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}} + \frac{\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx}{6b^2}$$

$$= -\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}} + \frac{a^2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{12b^2}$$

$$= -\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}} + \frac{\left(a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{b\cos(e+fx)}\ldots}}{12b^2}$$

$$= -\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}} + \frac{\left(a^2\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2f\ldots}}}{12b^2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}} + \frac{a^2F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2\ldots}}{12b^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] time = 0.432342, size = 87, normalized size = 0.64

$$\frac{a\sqrt{a\sin(e+fx)}\left(\left(-\tan^2(e+fx)\right)^{3/4}\csc^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)-2\cos(2(e+fx))\right)}{12bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*Sin[e + f*x]]*(-2*Cos[2*(e + f*x)] + Csc[e + f*x]^2*Hypergeometri
c2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(12*b*f*Sqrt[
b*Sec[e + f*x]])
```

Maple [A] time = 0.123, size = 218, normalized size = 1.6

$$-\frac{\sqrt{2}}{12f\left(-1+\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2}\left(\sin\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+\ldots\right.}{\sin\left(\ldots\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/12/f*2^(1/2)*(sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*2
^(1/2)*cos(f*x+e)^4-2*2^(1/2)*cos(f*x+e)^3-2^(1/2)*cos(f*x+e)^2+2^(1/2)*cos
(f*x+e))*(a*sin(f*x+e))^(3/2)/(-1+cos(f*x+e))/sin(f*x+e)/(b/cos(f*x+e))^(3/
```

```
2)/cos(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)/(b*sec(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}a\sin\left(fx+e\right)}{b^2\sec\left(fx+e\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))*a*sin(f*x + e)/(b^2*sec(
f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(3/2)/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)/(b*sec(f*x + e))^(3/2), x)
```

3.479 $\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx$

Optimal. Leaf size=94

$$\frac{\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{2b^2f\sqrt{a\sin(e+fx)}}+\frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}$$

```
[Out] Sqrt[a*Sin[e + f*x]]/(a*b*f*Sqrt[b*Sec[e + f*x]]) + (EllipticF[e - Pi/4 + f
*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(2*b^2*f*Sqrt[a*Sin[e +
 f*x]])
```

Rubi [A] time = 0.152337, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2582, 2585, 2573, 2641}

$$\frac{\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{2b^2f\sqrt{a\sin(e+fx)}}+\frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*Sqrt[a*Sin[e + f*x]]),x]
```

```
[Out] Sqrt[a*Sin[e + f*x]]/(a*b*f*Sqrt[b*Sec[e + f*x]]) + (EllipticF[e - Pi/4 + f
*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(2*b^2*f*Sqrt[a*Sin[e +
 f*x]])
```

Rule 2582

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m - n)), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}dx = \frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}} + \frac{\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2b^2}$$

$$= \frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}} + \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+f}}}{2b^2}$$

$$= \frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}} + \frac{\left(\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2b^2\sqrt{a\sin(e+fx)}}$$

$$= \frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}} + \frac{F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{2b^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] time = 0.53812, size = 84, normalized size = 0.89

$$-\frac{\cot(e+fx)\sqrt{b\sec(e+fx)}\left(-\left(-\tan^2(e+fx)\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)+\cos(2(e+fx))-1\right)}{2b^2f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*Sqrt[a*Sin[e + f*x]]),x]
```

```
[Out] -(Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*(-1 + Cos[2*(e + f*x)] - Hypergeometric
2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(2*b^2*f*Sqrt[
a*Sin[e + f*x]])
```

Maple [A] time = 0.118, size = 190, normalized size = 2.

$$-\frac{\sqrt{2}\sin(fx+e)}{2f(-1+\cos(fx+e))(\cos(fx+e))^2}\left(\sin(fx+e)\sqrt{\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(f}{\sin(fx+e)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*2^(1/2)*(sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2^(1
/2)*cos(f*x+e)^2+2^(1/2)*cos(f*x+e))*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^
2/(b/cos(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(fx+e))^{\frac{3}{2}}\sqrt{a\sin(fx+e)}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*sqrt(a*sin(f*x + e))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}}{ab^2\sec\left(fx+e\right)^2\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/(a*b^2*sec(f*x + e)^2*si
n(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*sqrt(a*sin(f*x + e))), x)
```

3.480 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{3a^2b^2f\sqrt{a\sin(e+fx)}}-\frac{2}{3abf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

```
[Out] -2/(3*a*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) - (EllipticF[e - P
i/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*b^2*f*Sqr
t[a*Sin[e + f*x]])
```

Rubi [A] time = 0.156332, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2581, 2585, 2573, 2641}

$$-\frac{\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{3a^2b^2f\sqrt{a\sin(e+fx)}}-\frac{2}{3abf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -2/(3*a*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) - (EllipticF[e - P
i/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*b^2*f*Sqr
t[a*Sin[e + f*x]])
```

Rule 2581

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m + 1)), x] - Dist[(n + 1)/(a^2*b^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n,
 -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = -\frac{2}{3abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} - \frac{\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{3a^2b^2}$$

$$= -\frac{2}{3abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} - \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{}{}}{3a^2b^2}$$

$$= -\frac{2}{3abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} - \frac{\left(\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int}{3a^2b^2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{2}{3abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} - \frac{F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{b\sec(e+fx)}\sqrt{s}}{3a^2b^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] time = 0.495503, size = 78, normalized size = 0.78

$$-\frac{\cot(e+fx)\sqrt{b\sec(e+fx)}\left(\left(-\tan^2(e+fx)\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)+2\right)}{3a^2b^2f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -(Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*(2 + Hypergeometric2F1[1/2, 3/4, 3/2, S
ec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(3*a^2*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Maple [B] time = 0.102, size = 284, normalized size = 2.8

$$-\frac{\sqrt{2}\sin\left(fx+e\right)}{3f\left(\cos\left(fx+e\right)\right)^2}\left(\sin\left(fx+e\right)\cos\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-}{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/3/f*2^(1/2)*(sin(f*x+e)*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^
(1/2))+sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(f
*x+e))*sin(f*x+e)/cos(f*x+e)^2/(a*sin(f*x+e))^(5/2)/(b/cos(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}}{\left(a^3b^2\cos\left(fx+e\right)^2-a^3b^2\right)\sec\left(fx+e\right)^2\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/((a^3*b^2*cos(f*x + e)^
2 - a^3*b^2)*sec(f*x + e)^2*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(5/2)), x)
```

3.481 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{2\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{21a^4b^2f\sqrt{a\sin(e+fx)}}+\frac{2}{21a^3bf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}-\frac{2}{7abf(a\sin(e+fx))^{7/2}\sqrt{\cdots}}$$

```
[Out] -2/(7*a*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2)) + 2/(21*a^3*b*f*Sq
rt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) - (2*EllipticF[e - Pi/4 + f*x, 2
]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(21*a^4*b^2*f*Sqrt[a*Sin[e +
 f*x]])
```

Rubi [A] time = 0.21898, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2581, 2584, 2585, 2573, 2641}

$$-\frac{2\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{21a^4b^2f\sqrt{a\sin(e+fx)}}+\frac{2}{21a^3bf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}-\frac{2}{7abf(a\sin(e+fx))^{7/2}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(9/2)),x]
```


```
[Out] -2/(7*a*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2)) + 2/(21*a^3*b*f*Sq
rt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) - (2*EllipticF[e - Pi/4 + f*x, 2
]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(21*a^4*b^2*f*Sqrt[a*Sin[e +
 f*x]])
```


Rule 2581

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m + 1)), x] - Dist[(n + 1)/(a^2*b^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n,
 -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
```

```
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = -\frac{2}{7abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} - \frac{\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx}{7a^2b^2}$$

$$= -\frac{2}{7abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} + \frac{2}{21a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+}$$

$$= -\frac{2}{7abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} + \frac{2}{21a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+}$$

$$= -\frac{2}{7abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} + \frac{2}{21a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+}$$

$$= -\frac{2}{7abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} + \frac{2}{21a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+}$$

Mathematica [C] time = 0.867706, size = 119, normalized size = 0.87

$$\frac{\cos(2(e+fx))\csc^4(e+fx)\sqrt{a\sin(e+fx)}\left((\cos(2(e+fx))+5)\sec^2(e+fx)-2\left(-\tan^2(e+fx)\right)^{7/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2\right.\right.}{21a^5bf\left(\sec^2(e+fx)-2\right)\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(9/2)),x]
```

```
[Out] (Cos[2*(e + f*x)]*Csc[e + f*x]^4*Sqrt[a*Sin[e + f*x]]*((5 + Cos[2*(e + f*x)
])*Sec[e + f*x]^2 - 2*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Ta
n[e + f*x]^2)^(7/4)))/(21*a^5*b*f*Sqrt[b*Sec[e + f*x]]*(-2 + Sec[e + f*x]^2
))
```

Maple [B] time = 0.122, size = 540, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/21/f*2^(1/2)*(2*sin(f*x+e)*cos(f*x+e)^3*((1-cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/
sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/
2*2^(1/2))+2*sin(f*x+e)*cos(f*x+e)^2*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
```

```
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(
1/2))-2*sin(f*x+e)*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2
*sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((
1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2^(1/2)*cos(f*x+e)^
3-2*2^(1/2)*cos(f*x+e))*sin(f*x+e)/cos(f*x+e)^2/(b/cos(f*x+e))^(3/2)/(a*sin
(f*x+e))^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}}{\left(a^{5}b^{2}\cos\left(fx+e\right)^{4}-2\,a^{5}b^{2}\cos\left(fx+e\right)^{2}+a^{5}b^{2}\right)\sec\left(fx+e\right)^{2}\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/((a^5*b^2*cos(f*x + e)^4
 - 2*a^5*b^2*cos(f*x + e)^2 + a^5*b^2)*sec(f*x + e)^2*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(9/2)), x)
```

3.482 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{4\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{77a^6b^2f\sqrt{a\sin(e+fx)}}+\frac{4}{77a^5bf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}+\frac{2}{77a^3bf(a\sin(e+fx))^{7/2}}$$

```
[Out] -2/(11*a*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(11/2)) + 2/(77*a^3*b*f*
Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2)) + 4/(77*a^5*b*f*Sqrt[b*Sec[e +
 f*x]]*(a*Sin[e + f*x])^(3/2)) - (4*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[b*Sec
[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(77*a^6*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.282325, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2581, 2584, 2585, 2573, 2641}

$$-\frac{4\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{77a^6b^2f\sqrt{a\sin(e+fx)}}+\frac{4}{77a^5bf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}+\frac{2}{77a^3bf(a\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(13/2)),x]
```

```
[Out] -2/(11*a*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(11/2)) + 2/(77*a^3*b*f*
Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2)) + 4/(77*a^5*b*f*Sqrt[b*Sec[e +
 f*x]]*(a*Sin[e + f*x])^(3/2)) - (4*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[b*Sec
[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(77*a^6*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2581

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m + 1)), x] - Dist[(n + 1)/(a^2*b^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n,
 -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
```

```
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = -\frac{2}{11abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{11/2}} - \frac{\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx}{11a^2b^2}$$

$$= -\frac{2}{11abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{11/2}} + \frac{2}{77a^3bf\sqrt{b\sec(e+fx)}(a\sin(e$$

$$= -\frac{2}{11abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{11/2}} + \frac{2}{77a^3bf\sqrt{b\sec(e+fx)}(a\sin(e$$

$$= -\frac{2}{11abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{11/2}} + \frac{2}{77a^3bf\sqrt{b\sec(e+fx)}(a\sin(e$$

$$= -\frac{2}{11abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{11/2}} + \frac{2}{77a^3bf\sqrt{b\sec(e+fx)}(a\sin(e$$

$$= -\frac{2}{11abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{11/2}} + \frac{2}{77a^3bf\sqrt{b\sec(e+fx)}(a\sin(e$$

Mathematica [C] time = 1.20343, size = 131, normalized size = 0.75

$$\frac{2\cot(2(e+fx))\csc(2(e+fx))\sqrt{a\sin(e+fx)}\left(8\left(-\tan^2(e+fx)\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)+(6\cos(2(e+fx))-}{77a^7bf\left(\sec^2(e+fx)-2\right)\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(13/2)),x]
```

```
[Out] (2*Cot[2*(e + f*x)]*Csc[2*(e + f*x)]*Sqrt[a*Sin[e + f*x]]*((23 + 6*Cos[2*(e
 + f*x)] - Cos[4*(e + f*x)])*Csc[e + f*x]^4 + 8*Hypergeometric2F1[1/2, 3/4,
 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(77*a^7*b*f*Sqrt[b*Sec[e +
f*x]]*(-2 + Sec[e + f*x]^2))
```

Maple [B] time = 0.168, size = 793, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x)
```

```
[Out] -1/77/f*2^(1/2)*(4*sin(f*x+e)*cos(f*x+e)^5*((1-cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
```

```
/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1
/2*2^(1/2))+4*sin(f*x+e)*cos(f*x+e)^4*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^
(1/2))-8*sin(f*x+e)*cos(f*x+e)^3*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2)
)-8*sin(f*x+e)*cos(f*x+e)^2*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+4*s
in(f*x+e)*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2*cos(f*x+
e)^5*2^(1/2)+4*sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+5*2^(
1/2)*cos(f*x+e)^3+4*2^(1/2)*cos(f*x+e))*sin(f*x+e)/cos(f*x+e)^2/(b/cos(f*x+
e))^(3/2)/(a*sin(f*x+e))^(13/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sec \left(fx+e\right)\right)^{\frac{3}{2}} \left(a \sin \left(fx+e\right)\right)^{\frac{13}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(13/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b \sec \left(fx+e\right)}\sqrt{a \sin \left(fx+e\right)}}{\left(a^7b^2 \cos \left(fx+e\right)^6 - 3\,a^7b^2 \cos \left(fx+e\right)^4 + 3\,a^7b^2 \cos \left(fx+e\right)^2 - a^7b^2\right) \sec \left(fx+e\right)^2 \sin \left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/((a^7*b^2*cos(f*x + e)^
6 - 3*a^7*b^2*cos(f*x + e)^4 + 3*a^7*b^2*cos(f*x + e)^2 - a^7*b^2)*sec(f*x
+ e)^2*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(13/2)), x)
```

3.483 $\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=75

$$\frac{d\cos^2(a+bx)^{3/4}(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{7}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rubi [A] time = 0.108838, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2587, 2577}

$$\frac{d\cos^2(a+bx)^{3/4}(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{7}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[a + b*x])^(5/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = \left(d^2(d\cos(a+bx))^{3/2}(d\sec(a+bx))^{3/2}\right)\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx$$

$$= \frac{d\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{7}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(d\sec(a+bx))^{3/2}(c\sin(a+bx)}{bc(1+m)}$$

Mathematica [A] time = 8.716, size = 96, normalized size = 1.28

$$-\frac{2\cot(a+bx)(d\sec(a+bx))^{5/2}\left(-\tan^2(a+bx)\right)^{\frac{1-m}{2}}(c\sin(a+bx))^m\,{}_2F_1\left(\frac{1}{4}(5-2m),\frac{1-m}{2};\frac{1}{4}(9-2m);\sec^2(a+bx)\right)}{b(2m-5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[a + b*x])^(5/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (-2*Cot[a + b*x]*Hypergeometric2F1[(5 - 2*m)/4, (1 - m)/2, (9 - 2*m)/4, Sec
[a + b*x]^2]*(d*Sec[a + b*x])^(5/2)*(c*Sin[a + b*x])^m*(-Tan[a + b*x]^2)^((
1 - m)/2))/(b*(-5 + 2*m))
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int (d \sec (bx + a))^{\frac{5}{2}} (c \sin (bx + a))^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x)
```

```
[Out] int((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d \sec (bx + a))^{\frac{5}{2}} (c \sin (bx + a))^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(b*x + a))^(5/2)*(c*sin(b*x + a))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d \sec (bx + a)} (c \sin (bx + a))^{m} d^{2} \sec (bx + a)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m*d^2*sec(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(5/2)*(c*sin(b*x+a))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\sec(bx+a))^{\frac{5}{2}}(c\sin(bx+a))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((d*sec(b*x + a))^(5/2)*(c*sin(b*x + a))^m, x)
```

3.484 $\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=75

$$\frac{d\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{5}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rubi [A] time = 0.107566, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2587, 2577}

$$\frac{d\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{5}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx &= \left(d^2\sqrt{d\cos(a+bx)}\sqrt{d\sec(a+bx)}\right)\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx\\ &= \frac{d\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{5}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)\sqrt{d\sec(a+bx)}(c\sin(a+bx)}{bc(1+m)}\end{aligned}$$

Mathematica [A] time = 1.3574, size = 96, normalized size = 1.28

$$-\frac{2\cot(a+bx)(d\sec(a+bx))^{3/2}\left(-\tan^2(a+bx)\right)^{\frac{1-m}{2}}(c\sin(a+bx))^m\,{}_2F_1\left(\frac{1}{4}(3-2m),\frac{1-m}{2};\frac{1}{4}(7-2m);\sec^2(a+bx)\right)}{b(2m-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (-2*Cot[a + b*x]*Hypergeometric2F1[(3 - 2*m)/4, (1 - m)/2, (7 - 2*m)/4, Sec
[a + b*x]^2]*(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^m*(-Tan[a + b*x]^2)^((
1 - m)/2))/(b*(-3 + 2*m))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int (d\sec(bx+a))^{\frac{3}{2}} (c\sin(bx+a))^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

```
[Out] int((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\sec(bx+a))^{\frac{3}{2}} (c\sin(bx+a))^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\sec(bx+a)}\,(c\sin(bx+a))^m\, d\sec(bx+a)\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m*d*sec(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(3/2)*(c*sin(b*x+a))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\sec(bx+a))^{\frac{3}{2}}(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((d*sec(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

3.485 $\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=77

$$\frac{\cos^2(a+bx)^{3/4}(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)}$$

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m))
```

Rubi [A] time = 0.0952061, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2586, 2577}

$$\frac{\cos^2(a+bx)^{3/4}(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^m,x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m))
```

Rule 2586

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n + 1))
/b^2, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && LtQ[n, 1]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx &= \frac{\left((d\cos(a+bx))^{3/2}(d\sec(a+bx))^{3/2}\right)\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx}{d^2}\\ &= \frac{\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{1+m}}{bcd(1+m)}\end{aligned}$$

Mathematica [A] time = 1.37376, size = 106, normalized size = 1.38

$$-\frac{\sin(2(a+bx))\csc^2(a+bx)\sqrt{d\sec(a+bx)}\left(-\tan^2(a+bx)\right)^{\frac{1-m}{2}}(c\sin(a+bx))^m\,{}_2F_1\left(\frac{1}{4}(1-2m),\frac{1-m}{2};\frac{1}{4}(5-2m);\sec^2(a+\right.}{b(2m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^m,x]
```

```
[Out] -((Csc[a + b*x]^2*Hypergeometric2F1[(1 - 2*m)/4, (1 - m)/2, (5 - 2*m)/4, Se
c[a + b*x]^2]*Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^m*Sin[2*(a + b*x)]*(-Ta
n[a + b*x]^2)^((1 - m)/2))/(b*(-1 + 2*m)))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \sqrt{d \sec \left(bx+a\right)}\, \left(c \sin \left(bx+a\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

```
[Out] int((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \sec \left(bx+a\right)}\, \left(c \sin \left(bx+a\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d \sec \left(bx+a\right)}\, \left(c \sin \left(bx+a\right)\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin \left(a+bx\right)\right)^{m} \sqrt{d \sec \left(a+bx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(1/2)*(c*sin(b*x+a))**m,x)
```

```
[Out] Integral((c*sin(a + b*x))**m*sqrt(d*sec(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \sec (bx + a)}\, (c \sin (bx + a))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m, x)
```

3.486 $\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)}$$

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m))
```

Rubi [A] time = 0.0966463, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2586, 2577}

$$\frac{\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^m/Sqrt[d*Sec[a + b*x]],x]
```

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m))
```

Rule 2586

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n + 1))
/b^2, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && LtQ[n, 1]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx &= \frac{\left(\sqrt{d\cos(a+bx)}\sqrt{d\sec(a+bx)}\right)\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx}{d^2}\\ &= \frac{\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)\sqrt{d\sec(a+bx)}(c\sin(a+bx))^{1+m}}{bcd(1+m)}\end{aligned}$$

Mathematica [C] time = 1.7626, size = 289, normalized size = 3.75

$$\frac{8c(m+3)\sin^2\left(\frac{1}{2}(a+bx)\right)\cos^4\left(\frac{1}{2}(a+bx)\right)}{b(m+1)\sqrt{d\sec(a+bx)}\left((\cos(a+bx)-1)\left((2m+3)F_1\left(\frac{m+3}{2};-\frac{1}{2},m+\frac{5}{2};\frac{m+5}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*Sin[a + b*x])^m/Sqrt[d*Sec[a + b*x]],x]
```

```
[Out] (8*c*(3 + m)*AppellF1[(1 + m)/2, -1/2, 3/2 + m, (3 + m)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^4*Sin[(a + b*x)/2]^2*(c*Sin[a + b
*x])^(-1 + m))/(b*(1 + m)*(((3 + 2*m)*AppellF1[(3 + m)/2, -1/2, 5/2 + m, (5
 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + AppellF1[(3 + m)/2, 1/2
, 3/2 + m, (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos[a
 + b*x]) + (3 + m)*AppellF1[(1 + m)/2, -1/2, 3/2 + m, (3 + m)/2, Tan[(a + b
*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x]))*Sqrt[d*Sec[a + b*x]])
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int (c\sin (bx+a))^{m} \frac{1}{\sqrt{d\sec (bx+a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x)
```

```
[Out] int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin (bx+a))^{m}}{\sqrt{d\sec (bx+a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m/sqrt(d*sec(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\sec (bx+a)}\,(c\sin (bx+a))^{m}}{d\sec (bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m/(d*sec(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin (a+bx))^{m}}{\sqrt{d\sec (a+bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**m/(d*sec(b*x+a))**(1/2),x)
```

```
[Out] Integral((c*sin(a + b*x))**m/sqrt(d*sec(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin(bx+a))^m}{\sqrt{d\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m/sqrt(d*sec(b*x + a)), x)
```

3.487 $\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}}$$

```
[Out] (Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b
*x])^(1 + m))/(b*c*d*(1 + m)*(Cos[a + b*x]^2)^(1/4)*Sqrt[d*Sec[a + b*x]])
```

Rubi [A] time = 0.107111, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2586, 2577}

$$\frac{(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^m/(d*Sec[a + b*x])^(3/2),x]
```



```
[Out] (Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b
*x])^(1 + m))/(b*c*d*(1 + m)*(Cos[a + b*x]^2)^(1/4)*Sqrt[d*Sec[a + b*x]])
```


Rule 2586

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n + 1))
/b^2, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && LtQ[n, 1]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx &= \frac{\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx}{d^2\sqrt{d\cos(a+bx)}\sqrt{d\sec(a+bx)}}\\ &= \frac{{}_2F_1\left(-\frac{1}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bcd(1+m)\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}}\end{aligned}$$

Mathematica [A] time = 3.82065, size = 116, normalized size = 1.51

$$\frac{2c\cos(2(a+bx))\left(-\tan^2(a+bx)\right)^{\frac{1-m}{2}}(c\sin(a+bx))^{m-1}\,{}_2F_1\left(\frac{1}{4}(-2m-3),\frac{1-m}{2};\frac{1}{4}(1-2m);\sec^2(a+bx)\right)}{bd(2m+3)\left(\sec^2(a+bx)-2\right)\sqrt{d\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^m/(d*Sec[a + b*x])^(3/2),x]
```

```
[Out] (2*c*Cos[2*(a + b*x)]*Hypergeometric2F1[(-3 - 2*m)/4, (1 - m)/2, (1 - 2*m)/
4, Sec[a + b*x]^2]*(c*Sin[a + b*x])^(-1 + m)*(-Tan[a + b*x]^2)^((1 - m)/2))
/(b*d*(3 + 2*m)*Sqrt[d*Sec[a + b*x]]*(-2 + Sec[a + b*x]^2))
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int (c\sin (bx+a))^{m} (d\sec (bx+a))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x)
```

```
[Out] int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sin (bx+a))^{m}}{(d\sec (bx+a))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m/(d*sec(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\sec (bx+a)}\,(c\sin (bx+a))^{m}}{d^{2}\sec (bx+a)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m/(d^2*sec(b*x + a)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**m/(d*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sin (bx + a))^m}{(d \sec (bx + a))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m/(d*sec(b*x + a))^(3/2), x)
```

3.488 $\int \sec^n(e+fx)\sin^m(e+fx)\,dx$

Optimal. Leaf size=86

$$-\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

```
[Out] -((Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e
 + f*x]^(-1 + n)*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(f*(1
- n)))
```

Rubi [A] time = 0.0764943, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2587, 2576}

$$-\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*Sin[e + f*x]^m,x]
```

```
[Out] -((Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e
 + f*x]^(-1 + n)*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(f*(1
- n)))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\begin{aligned}\int \sec^n(e+fx)\sin^m(e+fx)\,dx &= \left(\cos^n(e+fx)\sec^n(e+fx)\right)\int \cos^{-n}(e+fx)\sin^m(e+fx)\,dx\\ &= -\frac{{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin^{-1+m}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}}{f(1-n)}\end{aligned}$$

Mathematica [C] time = 1.38711, size = 285, normalized size = 3.31

$$\frac{4(m+3)\sin\left(\frac{1}{2}(e+fx)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)\sin^{m}\cdots}{f(m+1)\left((m+3)(\cos(e+fx)+1)F_1\left(\frac{m+1}{2};n,m-n+1;\frac{m+3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4\sin^2\left(\frac{1}{2}(e+\cdots\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*Sin[e + f*x]^m,x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Sec[e + f*x]^n*Sin[(e + f*x)/2]*Si
n[e + f*x]^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) - 4*((1 + m
 - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5 + m)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F] time = 0.516, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{n} \left(\sin \left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*sin(f*x+e)^m,x)
```

```
[Out] int(sec(f*x+e)^n*sin(f*x+e)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec \left(fx+e\right)^{n} \sin \left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*sin(f*x+e)^m,x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n*sin(f*x + e)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sec \left(fx+e\right)^{n} \sin \left(fx+e\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*sin(f*x+e)^m,x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^n*sin(f*x + e)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^{m}\left(e+fx\right) \sec^{n}\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*sin(f*x+e)**m,x)
```

```
[Out] Integral(sin(e + f*x)**m*sec(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)^{n} \sin\left(fx+e\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*sin(f*x+e)^m,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n*sin(f*x + e)^m, x)
```

3.489 $\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx$

Optimal. Leaf size=89

$$-\frac{a\sin^2(e+fx)^{\frac{1-m}{2}}\sec^{n-1}(e+fx)(a\sin(e+fx))^{m-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

```
[Out] -((a*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec
[e + f*x]^(-1 + n)*(a*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/
(f*(1 - n)))
```

Rubi [A] time = 0.0876263, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2587, 2576}

$$-\frac{a\sin^2(e+fx)^{\frac{1-m}{2}}\sec^{n-1}(e+fx)(a\sin(e+fx))^{m-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec
[e + f*x]^(-1 + n)*(a*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/
(f*(1 - n)))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx = \left(\cos^n(e+fx)\sec^n(e+fx)\right)\int \cos^{-n}(e+fx)(a\sin(e+fx))^m\,dx$$
$$= -\frac{a\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)(a\sin(e+fx))^{-1+m}\sin^2(e+fx}{f(1-n)}$$

Mathematica [C] time = 0.16955, size = 287, normalized size = 3.22

$$\frac{4(m+3)\sin\left(\frac{1}{2}(e+fx)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)\sec^n(e+}{f(m+1)\left((m+3)(\cos(e+fx)+1)F_1\left(\frac{m+1}{2};n,m-n+1;\frac{m+3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4\sin^2\left(\frac{1}{2}(e+fx)\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Sec[e + f*x]^n*Sin[(e + f*x)/2]*(a
*Sin[e + f*x])^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3
+ m)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) - 4*((1
 + m - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5 + m)/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F] time = 0.49, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{n} \left(a\sin \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a*sin(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^n*(a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin \left(fx+e\right)\right)^{m} \sec \left(fx+e\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*sec(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin \left(fx+e\right)\right)^{m} \sec \left(fx+e\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*sec(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin \left(e+fx\right)\right)^{m} \sec^{n}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*sec(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*sec(f*x + e)^n, x)
```

3.490 $\int (b\sec(e+fx))^n \sin^m(e+fx)\,dx$

Optimal. Leaf size=89

$$-\frac{b\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

```
[Out] -((b*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*
Sec[e + f*x])^(-1 + n)*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/
(f*(1 - n)))
```

Rubi [A] time = 0.0891234, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2587, 2576}

$$-\frac{b\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^m,x]
```

```
[Out] -((b*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*
Sec[e + f*x])^(-1 + n)*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/
(f*(1 - n)))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (b\sec(e+fx))^n\sin^m(e+fx)\,dx = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}\sin^m(e+fx)\,dx$$
$$= -\frac{b\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin^{-1+m}(e+fx)\sin^2(e+}{f(1-n)}$$

Mathematica [C] time = 0.134162, size = 287, normalized size = 3.22

$$\frac{4(m+3)\sin\left(\frac{1}{2}(e+fx)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)\sin^m(}{f(m+1)\left((m+3)(\cos(e+fx)+1)F_1\left(\frac{m+1}{2};n,m-n+1;\frac{m+3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4\sin^2\left(\frac{1}{2}(e+\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^m,x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(b*Sec[e + f*x])^n*Sin[(e + f*x)/2
]*Sin[e + f*x]^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3
+ m)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) - 4*((1
 + m - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5 + m)/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F] time = 0.432, size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)\right)^{n} \left(\sin \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^m,x)
```

```
[Out] int((b*sec(f*x+e))^n*sin(f*x+e)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)\right)^{n} \sin \left(fx+e\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sec \left(fx+e\right)\right)^{n} \sin \left(fx+e\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^n*sin(f*x + e)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(e+fx\right)\right)^{n} \sin^{m}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**m,x)
```

```
[Out] Integral((b*sec(e + f*x))**n*sin(e + f*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(fx+e))^n \sin(fx+e)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^m, x)
```

3.491 $\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx$

Optimal. Leaf size=92

$$-\frac{ab\sin^2(e+fx)^{\frac{1-m}{2}}(a\sin(e+fx))^{m-1}(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

```
[Out] -((a*b*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(
b*Sec[e + f*x])^(-1 + n)*(a*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m
)/2))/(f*(1 - n)))
```

Rubi [A] time = 0.103437, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2587, 2576}

$$-\frac{ab\sin^2(e+fx)^{\frac{1-m}{2}}(a\sin(e+fx))^{m-1}(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a*b*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(
b*Sec[e + f*x])^(-1 + n)*(a*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m
)/2))/(f*(1 - n)))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}(a\sin(e+fx))^m\,dx$$

$$= -\frac{ab\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}(a\sin(e+fx))^{-1+m}\sin^2}{f(1-n)}$$

Mathematica [C] time = 0.127795, size = 289, normalized size = 3.14

$$\frac{4(m+3)\sin\left(\frac{1}{2}(e+fx)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)(a\sin(e+}{f(m+1)\left((m+3)(\cos(e+fx)+1)F_1\left(\frac{m+1}{2};n,m-n+1;\frac{m+3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4\sin^2\left(\frac{1}{2}(e+fx)\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(b*Sec[e + f*x])^n*Sin[(e + f*x)/2
]*(a*Sin[e + f*x])^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n,
 (3 + m)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) - 4
*((1 + m - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5 + m)/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F] time = 0.455, size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\left(a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

```
[Out] int((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\left(a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)\right)^{n}\left(a\sin\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(e+fx\right)\right)^{m}\left(b\sec\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n*(a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*(b*sec(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\left(a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

3.492 $\int (b\sec(e+fx))^n \sin^5(e+fx)\,dx$

Optimal. Leaf size=80

$$-\frac{b^5(b\sec(e+fx))^{n-5}}{f(5-n)}+\frac{2b^3(b\sec(e+fx))^{n-3}}{f(3-n)}-\frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

```
[Out] -((b^5*(b*Sec[e + f*x])^(-5 + n))/(f*(5 - n))) + (2*b^3*(b*Sec[e + f*x])^(-
3 + n))/(f*(3 - n)) - (b*(b*Sec[e + f*x])^(-1 + n))/(f*(1 - n))
```

Rubi [A] time = 0.0732299, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 270}

$$-\frac{b^5(b\sec(e+fx))^{n-5}}{f(5-n)}+\frac{2b^3(b\sec(e+fx))^{n-3}}{f(3-n)}-\frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^5,x]
```

```
[Out] -((b^5*(b*Sec[e + f*x])^(-5 + n))/(f*(5 - n))) + (2*b^3*(b*Sec[e + f*x])^(-
3 + n))/(f*(3 - n)) - (b*(b*Sec[e + f*x])^(-1 + n))/(f*(1 - n))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^n \sin^5(e+fx)\,dx &= \frac{b^5\,\text{Subst}\left(\int x^{-6+n}\left(-1+\frac{x^2}{b^2}\right)^2\,dx, x, b\sec(e+fx)\right)}{f}\\ &= \frac{b^5\,\text{Subst}\left(\int \left(x^{-6+n}-\frac{2x^{-4+n}}{b^2}+\frac{x^{-2+n}}{b^4}\right)dx, x, b\sec(e+fx)\right)}{f}\\ &= -\frac{b^5(b\sec(e+fx))^{-5+n}}{f(5-n)}+\frac{2b^3(b\sec(e+fx))^{-3+n}}{f(3-n)}-\frac{b(b\sec(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A] time = 0.367233, size = 80, normalized size = 1.

$$\frac{b\left(-4\left(n^2-8n+7\right)\cos(2(e+fx))+\left(n^2-4n+3\right)\cos(4(e+fx))+3n^2-28n+89\right)(b\sec(e+fx))^{n-1}}{8f(n-5)(n-3)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^5,x]
```

```
[Out] (b*(89 - 28*n + 3*n^2 - 4*(7 - 8*n + n^2)*Cos[2*(e + f*x)] + (3 - 4*n + n^2
)*Cos[4*(e + f*x)])*(b*Sec[e + f*x])^(-1 + n))/(8*f*(-5 + n)*(-3 + n)*(-1 +
 n))
```

Maple [F] time = 1.006, size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\left(\sin\left(fx+e\right)\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^5,x)
```

```
[Out] int((b*sec(f*x+e))^n*sin(f*x+e)^5,x)
```

Maxima [A] time = 1.03923, size = 115, normalized size = 1.44

$$\frac{\frac{b^{n}\cos\left(fx+e\right)^{-n}\cos\left(fx+e\right)^{5}}{n-5}-\frac{2\,b^{n}\cos\left(fx+e\right)^{-n}\cos\left(fx+e\right)^{3}}{n-3}+\frac{b^{n}\cos\left(fx+e\right)^{-n}\cos\left(fx+e\right)}{n-1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] (b^n*cos(f*x + e)^(-n)*cos(f*x + e)^5/(n - 5) - 2*b^n*cos(f*x + e)^(-n)*cos
(f*x + e)^3/(n - 3) + b^n*cos(f*x + e)^(-n)*cos(f*x + e)/(n - 1))/f
```

Fricas [A] time = 1.80017, size = 208, normalized size = 2.6

$$\frac{\left(\left(n^{2}-4\,n+3\right)\cos\left(fx+e\right)^{5}-2\left(n^{2}-6\,n+5\right)\cos\left(fx+e\right)^{3}+\left(n^{2}-8\,n+15\right)\cos\left(fx+e\right)\right)\left(\frac{b}{\cos\left(fx+e\right)}\right)^{n}}{fn^{3}-9\,fn^{2}+23\,fn-15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] ((n^2 - 4*n + 3)*cos(f*x + e)^5 - 2*(n^2 - 6*n + 5)*cos(f*x + e)^3 + (n^2 -
 8*n + 15)*cos(f*x + e))*(b/cos(f*x + e))^n/(f*n^3 - 9*f*n^2 + 23*f*n - 15*
f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^5, x)
```

3.493 $\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx$

Optimal. Leaf size=52

$$\frac{b^3(b\sec(e+fx))^{n-3}}{f(3-n)}-\frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

```
[Out] (b^3*(b*Sec[e + f*x])^(-3 + n))/(f*(3 - n)) - (b*(b*Sec[e + f*x])^(-1 + n))
/(f*(1 - n))
```

Rubi [A] time = 0.0526494, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 14}

$$\frac{b^3(b\sec(e+fx))^{n-3}}{f(3-n)}-\frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^3,x]
```

```
[Out] (b^3*(b*Sec[e + f*x])^(-3 + n))/(f*(3 - n)) - (b*(b*Sec[e + f*x])^(-1 + n))
/(f*(1 - n))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^n\sin^3(e+fx)\,dx &= \frac{b^3\,\text{Subst}\left(\int x^{-4+n}\left(-1+\frac{x^2}{b^2}\right)dx, x, b\sec(e+fx)\right)}{f}\\ &= \frac{b^3\,\text{Subst}\left(\int\left(-x^{-4+n}+\frac{x^{-2+n}}{b^2}\right)dx, x, b\sec(e+fx)\right)}{f}\\ &= \frac{b^3(b\sec(e+fx))^{-3+n}}{f(3-n)}-\frac{b(b\sec(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A] time = 0.145121, size = 47, normalized size = 0.9

$$-\frac{b((n-1)\cos(2(e+fx))-n+5)(b\sec(e+fx))^{n-1}}{2f(n-3)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^3,x]
```

```
[Out] -(b*(5 - n + (-1 + n)*Cos[2*(e + f*x)])*(b*Sec[e + f*x])^(-1 + n))/(2*f*(-3
 + n)*(-1 + n))
```

Maple [C] time = 1.523, size = 1732, normalized size = 33.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^3,x)
```

```
[Out] -1/8/(f*n-3*f)*exp(I*(f*x+e))^n*b^n*(exp(2*I*(f*x+e))+1)^(-n)*2^n*exp(-1/2*
I*(Pi*n*csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*(f
*x+e))/(exp(2*I*(f*x+e))+1))-Pi*n*csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I
*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-Pi*n*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*
(f*x+e))/(exp(2*I*(f*x+e))+1))^2+Pi*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e
))+1))^3-Pi*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f
*x+e))/(exp(2*I*(f*x+e))+1))^2+Pi*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))
+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b)+Pi*n*csgn(I*b*
exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3-Pi*n*csgn(I*b*exp(I*(f*x+e))/(exp(2*
I*(f*x+e))+1))^2*csgn(I*b)+6*f*x+6*e))-1/8/(f*n-3*f)*exp(I*(f*x+e))^n*b^n*(
exp(2*I*(f*x+e))+1)^(-n)*2^n*exp(-1/2*I*(Pi*n*csgn(I/(exp(2*I*(f*x+e))+1))*
csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))-Pi*n*csg
n(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-Pi*
n*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2+Pi*n
*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3-Pi*n*csgn(I*exp(I*(f*x+e))/(
exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2+Pi*n*c
sgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I
*(f*x+e))+1))*csgn(I*b)+Pi*n*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^
3-Pi*n*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*b)-6*f*x-6*e)
)+1/8*(n-9)/f/(-1+n)/(-3+n)*exp(I*(f*x+e))^n*b^n*(exp(2*I*(f*x+e))+1)^(-n)*
2^n*exp(-1/2*I*(Pi*n*csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*cs
gn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))-Pi*n*csgn(I/(exp(2*I*(f*x+e))+1))
*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-Pi*n*csgn(I*exp(I*(f*x+e)))*
csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2+Pi*n*csgn(I*exp(I*(f*x+e))/(e
xp(2*I*(f*x+e))+1))^3-Pi*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn
(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2+Pi*n*csgn(I*exp(I*(f*x+e))/(exp
(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b)+P
i*n*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3-Pi*n*csgn(I*b*exp(I*(f*
x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*b)+2*f*x+2*e))+1/8*(n-9)/f/(-1+n)/(-3+
n)*exp(I*(f*x+e))^n*b^n*(exp(2*I*(f*x+e))+1)^(-n)*2^n*exp(-1/2*I*(Pi*n*csgn
(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*(f*x+e))/(exp(
2*I*(f*x+e))+1))-Pi*n*csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e))/(e
xp(2*I*(f*x+e))+1))^2-Pi*n*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*(f*x+e))/(ex
p(2*I*(f*x+e))+1))^2+Pi*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3-Pi*
n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(
2*I*(f*x+e))+1))^2+Pi*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*
b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b)+Pi*n*csgn(I*b*exp(I*(f*x+e
))/(exp(2*I*(f*x+e))+1))^3-Pi*n*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1
))^2*csgn(I*b)-2*f*x-2*e))
```

Maxima [A] time = 1.03439, size = 80, normalized size = 1.54

$$-\frac{\frac{b^{n}\cos\left(fx+e\right)^{-n}\cos\left(fx+e\right)^{3}}{n-3}-\frac{b^{n}\cos\left(fx+e\right)^{-n}\cos\left(fx+e\right)}{n-1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] -(b^n*cos(f*x + e)^(-n)*cos(f*x + e)^3/(n - 3) - b^n*cos(f*x + e)^(-n)*cos(
f*x + e)/(n - 1))/f
```

Fricas [A] time = 1.76699, size = 123, normalized size = 2.37

$$-\frac{\left((n-1)\cos\left(fx+e\right)^{3}-(n-3)\cos\left(fx+e\right)\right)\left(\frac{b}{\cos\left(fx+e\right)}\right)^{n}}{fn^{2}-4\,fn+3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -((n - 1)*cos(f*x + e)^3 - (n - 3)*cos(f*x + e))*(b/cos(f*x + e))^n/(f*n^2
- 4*f*n + 3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^3, x)
```

3.494 $\int (b\sec(e+fx))^n \sin(e+fx)\,dx$

Optimal. Leaf size=25

$$-\frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

```
[Out] -((b*(b*Sec[e + f*x])^(-1 + n))/(f*(1 - n)))
```

Rubi [A] time = 0.0333282, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2622, 30}

$$-\frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x],x]
```

```
[Out] -((b*(b*Sec[e + f*x])^(-1 + n))/(f*(1 - n)))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^n \sin(e+fx)\,dx &= \frac{b\,\text{Subst}\left(\int x^{-2+n}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= -\frac{b(b\sec(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A] time = 0.0211982, size = 22, normalized size = 0.88

$$\frac{b(b\sec(e+fx))^{n-1}}{f(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x],x]
```

```
[Out] (b*(b*Sec[e + f*x])^(-1 + n))/(f*(-1 + n))
```

Maple [B] time = 0.029, size = 120, normalized size = 4.8

$$\left(\frac{1}{f\left(-1+n\right)}\mathrm{e}^{n\ln\left(b\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{2}\right)\left(1-\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{2}\right)^{-1}\right)}-\frac{1}{f\left(-1+n\right)}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{2}\mathrm{e}^{n\ln\left(b\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{2}\right)\left(1-\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{2}\right)^{-1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e),x)
```

```
[Out] (1/f/(-1+n)*exp(n*ln(b*(1+tan(1/2*f*x+1/2*e)^2)/(1-tan(1/2*f*x+1/2*e)^2)))-
1/f/(-1+n)*tan(1/2*f*x+1/2*e)^2*exp(n*ln(b*(1+tan(1/2*f*x+1/2*e)^2)/(1-tan(
1/2*f*x+1/2*e)^2))))/(1+tan(1/2*f*x+1/2*e)^2)
```

Maxima [A] time = 1.00783, size = 38, normalized size = 1.52

$$\frac{b^{n}\cos\left(fx+e\right)^{-n}\cos\left(fx+e\right)}{f(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] b^n*cos(f*x + e)^(-n)*cos(f*x + e)/(f*(n - 1))
```

Fricas [A] time = 1.64906, size = 58, normalized size = 2.32

$$\frac{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{n}\cos\left(fx+e\right)}{fn-f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] (b/cos(f*x + e))^n*cos(f*x + e)/(f*n - f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(e+fx\right)\right)^{n}\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e),x)
```

```
[Out] Integral((b*sec(e + f*x))**n*sin(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e), x)
```

3.495 $\int \csc(e+fx)(b\sec(e+fx))^n\,dx$

Optimal. Leaf size=49

$$-\frac{(b\sec(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\sec^2(e+fx)\right)}{bf(n+1)}$$

```
[Out] -((Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2]*(b*Sec[e + f*
x])^(1 + n))/(b*f*(1 + n)))
```

Rubi [A] time = 0.0382708, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2622, 364}

$$-\frac{(b\sec(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\sec^2(e+fx)\right)}{bf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(b*Sec[e + f*x])^n,x]
```

```
[Out] -((Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2]*(b*Sec[e + f*
x])^(1 + n))/(b*f*(1 + n)))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc(e+fx)(b\sec(e+fx))^n\,dx &= \frac{\text{Subst}\left(\int \frac{x^n}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{bf}\\
&= -\frac{{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\sec^2(e+fx)\right)(b\sec(e+fx))^{1+n}}{bf(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.33198, size = 92, normalized size = 1.88

$$\frac{(b\sec(e+fx))^n\left({}_2F_1(1,-n;1-n;\cos(e+fx))-2^n\sec^2\left(\frac{1}{2}(e+fx)\right)^{-n}\,{}_2F_1\left(-n,-n;1-n;\frac{1}{2}\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)\right)}{2fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(b*Sec[e + f*x])^n,x]
```

```
[Out] ((Hypergeometric2F1[1, -n, 1 - n, Cos[e + f*x]] - (2^n*Hypergeometric2F1[-n
, -n, 1 - n, (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/2])/(Sec[(e + f*x)/2]^2)^n)*
(b*Sec[e + f*x])^n)/(2*f*n)
```

Maple [F] time = 0.403, size = 0, normalized size = 0.

$$\int \csc \left(fx+e\right) \left(b\sec \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(b*sec(f*x+e))^n,x)
```

```
[Out] int(csc(f*x+e)*(b*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)\right)^{n} \csc \left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec \left(fx+e\right)\right)^{n} \csc \left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^n*csc(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(e+fx\right)\right)^{n} \csc \left(e+fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))**n,x)
```

```
[Out] Integral((b*sec(e + f*x))**n*csc(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx + e\right)\right)^{n} \csc \left(fx + e\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e), x)
```

3.496 $\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx$

Optimal. Leaf size=48

$$\frac{(b\sec(e+fx))^{n+3}\,{}_2F_1\left(2,\frac{n+3}{2};\frac{n+5}{2};\sec^2(e+fx)\right)}{b^3f(n+3)}$$

```
[Out] (Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x]
)^(3 + n))/(b^3*f*(3 + n))
```

Rubi [A] time = 0.0516906, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 364}

$$\frac{(b\sec(e+fx))^{n+3}\,{}_2F_1\left(2,\frac{n+3}{2};\frac{n+5}{2};\sec^2(e+fx)\right)}{b^3f(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(b*Sec[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x]
)^(3 + n))/(b^3*f*(3 + n))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx &= \frac{\text{Subst}\left(\int \frac{x^{2+n}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx,x,b\sec(e+fx)\right)}{b^3f}\\ &= \frac{{}_2F_1\left(2,\frac{3+n}{2};\frac{5+n}{2};\sec^2(e+fx)\right)(b\sec(e+fx))^{3+n}}{b^3f(3+n)}\end{aligned}$$

Mathematica [C] time = 16.8315, size = 2113, normalized size = 44.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3*(b*Sec[e + f*x])^n,x]
```

```
[Out] (Csc[e + f*x]^3*(b*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*((2^
(1 + n)*AppellF1[1 - n, -n, 1, 2 - n, (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/2,
Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Cos[e + f*x]*Sec[(e + f*x)/2]^2)/(-1 + n)
- (AppellF1[1, n, -n, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e +
f*x)/2]^2*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^n*(Sec[(e + f*x)/2]^2)^n)/(C
sc[(e + f*x)/2]^2)^n + AppellF1[1, n, -n, 2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^2))/(8*f*((
(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*(AppellF1[1, n, -n, 2, Cot[(e + f*x)/2]
^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]*(Csc[(e + f*x)/2]^2)^(1 - n)*(-(C
os[e + f*x]*Csc[(e + f*x)/2]^2))^n*(Sec[(e + f*x)/2]^2)^n - (n*AppellF1[1,
n, -n, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]*(-(Cos[
e + f*x]*Csc[(e + f*x)/2]^2))^n*(Sec[(e + f*x)/2]^2)^n)/(Csc[(e + f*x)/2]^2
)^n - (n*AppellF1[1, n, -n, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot
[(e + f*x)/2]^3*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^n*(Sec[(e + f*x)/2]^2)
^n)/(Csc[(e + f*x)/2]^2)^n - (Cot[(e + f*x)/2]^2*(-(Cos[e + f*x]*Csc[(e + f
*x)/2]^2))^n*(-(n*AppellF1[2, n, 1 - n, 3, Cot[(e + f*x)/2]^2, -Cot[(e + f*
x)/2]^2]*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2)/2 - (n*AppellF1[2, 1 + n, -n,
 3, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]*Csc[(e + f*x)
/2]^2)/2)*(Sec[(e + f*x)/2]^2)^n)/(Csc[(e + f*x)/2]^2)^n - (2^(1 + n)*Appel
lF1[1 - n, -n, 1, 2 - n, (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/2, Cos[e + f*x]*
Sec[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(-1 + n) - (n*AppellF1
[1, n, -n, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^2*(
-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^(-1 + n)*(Sec[(e + f*x)/2]^2)^n*(Cos[e
+ f*x]*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2 + Csc[(e + f*x)/2]^2*Sin[e + f*x
]))/(Csc[(e + f*x)/2]^2)^n + (2^(1 + n)*AppellF1[1 - n, -n, 1, 2 - n, (Cos[
e + f*x]*Sec[(e + f*x)/2]^2)/2, Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Cos[e + f*
x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(-1 + n) + AppellF1[1, n, -n, 2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec
[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2] + (Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*T
an[(e + f*x)/2]^2*((n*AppellF1[2, n, 1 - n, 3, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2 + (n*AppellF1[2, 1 + n,
 -n, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
+ f*x)/2])/2) + n*AppellF1[1, n, -n, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + n)*Tan[(e + f*x)/2]^2*(-(Sec[
(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x
)/2]) + (2^(1 + n)*Cos[e + f*x]*Sec[(e + f*x)/2]^2*(-(((1 - n)*n*AppellF1[2
 - n, 1 - n, 1, 3 - n, (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/2, Cos[e + f*x]*Se
c[(e + f*x)/2]^2]*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x])/2 + (Cos[e + f*x]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2))/(2 - n)) + ((1 - n)*AppellF1[2 - n, -
n, 2, 3 - n, (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/2, Cos[e + f*x]*Sec[(e + f*x
)/2]^2]*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2]))/(2 - n)))/(-1 + n)))/8 + (n*(Cos[(e + f*x)/2]^2*Sec[e
 + f*x])^(-1 + n)*((2^(1 + n)*AppellF1[1 - n, -n, 1, 2 - n, (Cos[e + f*x]*S
ec[(e + f*x)/2]^2)/2, Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Cos[e + f*x]*Sec[(e
+ f*x)/2]^2)/(-1 + n) - (AppellF1[1, n, -n, 2, Cot[(e + f*x)/2]^2, -Cot[(e
+ f*x)/2]^2]*Cot[(e + f*x)/2]^2*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^n*(Sec
[(e + f*x)/2]^2)^n)/(Csc[(e + f*x)/2]^2)^n + AppellF1[1, n, -n, 2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e
 + f*x)/2]^2)*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e +
 f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/8))
```

Maple [F] time = 0.397, size = 0, normalized size = 0.

$$\int \left(\csc \left(fx+e\right)\right)^{3} \left(b \sec \left(fx+e\right)\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(b*sec(f*x+e))^n,x)
```

```
[Out] int(csc(f*x+e)^3*(b*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)\right)^{n} \csc \left(fx+e\right)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sec \left(fx+e\right)\right)^{n} \csc \left(fx+e\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^n*csc(f*x + e)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(e+fx\right)\right)^{n} \csc^{3} \left(e+fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(b*sec(f*x+e))**n,x)
```

```
[Out] Integral((b*sec(e + f*x))**n*csc(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)\right)^{n} \csc \left(fx+e\right)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^3, x)
```

3.497 $\int (b\sec(e+fx))^n \sin^6(e+fx)\,dx$

Optimal. Leaf size=73

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(-\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -((b*Hypergeometric2F1[-5/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
 + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rubi [A] time = 0.0845627, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(-\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^6,x]
```

```
[Out] -((b*Hypergeometric2F1[-5/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
 + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^n\sin^6(e+fx)\,dx &= \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}\sin^6(e+fx)\,dx\\ &= -\frac{b\,{}_2F_1\left(-\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [C] time = 25.6185, size = 8327, normalized size = 114.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^6,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.859, size = 0, normalized size = 0.

$$\int \left(b \sec \left(f x+e\right)\right)^{n} \left(\sin \left(f x+e\right)\right)^{6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^6,x)
```

```
[Out] int((b*sec(f*x+e))^n*sin(f*x+e)^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(f x+e\right)\right)^{n} \sin \left(f x+e\right)^{6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos \left(f x+e\right)^{6}-3 \, \cos \left(f x+e\right)^{4}+3 \, \cos \left(f x+e\right)^{2}-1\right) \left(b \sec \left(f x+e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*(b*sec
(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^n \sin\left(fx+e\right)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^6, x)
```

3.498 $\int (b\sec(e+fx))^n \sin^4(e+fx)\,dx$

Optimal. Leaf size=73

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(-\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -((b*Hypergeometric2F1[-3/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
 + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rubi [A] time = 0.0818204, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(-\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^4,x]
```

```
[Out] -((b*Hypergeometric2F1[-3/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
 + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (b\sec(e+fx))^n\sin^4(e+fx)\,dx = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}\sin^4(e+fx)\,dx$$
$$= -\frac{b\,{}_2F_1\left(-\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [C] time = 24.7412, size = 6192, normalized size = 84.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^4,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.701, size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)\right)^{n} \left(\sin \left(fx+e\right)\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^4,x)
```

```
[Out] int((b*sec(f*x+e))^n*sin(f*x+e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)\right)^{n} \sin \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\cos \left(fx+e\right)^{4} - 2 \cos \left(fx+e\right)^{2} + 1\right) \left(b \sec \left(fx+e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**4,x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^4, x)
```

3.499 $\int (b\sec(e+fx))^n\sin^2(e+fx)\,dx$

Optimal. Leaf size=73

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -((b*Hypergeometric2F1[-1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
 + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rubi [A] time = 0.081193, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^2,x]
```

```
[Out] -((b*Hypergeometric2F1[-1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
 + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^n\sin^2(e+fx)\,dx &= \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}\sin^2(e+fx)\,dx\\ &= -\frac{b\,{}_2F_1\left(-\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [C] time = 18.7373, size = 4143, normalized size = 56.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^2,x]
```

```
[Out] (24*(Sec[(e + f*x)/2]^2)^(-3 + n)*(b*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*Se
c[e + f*x])^n*Sin[e + f*x]^2*Tan[(e + f*x)/2]*((AppellF1[1/2, n, 2 - n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF1[
1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-2 + n)*
AppellF1[3/2, n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*A
ppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2) - AppellF1[1/2, n, 3 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]/(3*AppellF1[1/2, n, 3 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + 2*((-3 + n)*AppellF1[3/2, n, 4 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))/(f*(12*(Sec[(e + f*x)/2]^2
)^(-2 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*((AppellF1[1/2, n, 2 - n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF1
[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-2 + n)
*AppellF1[3/2, n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*
AppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Tan[(e + f*x)/2]^2) - AppellF1[1/2, n, 3 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]/(3*AppellF1[1/2, n, 3 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + 2*((-3 + n)*AppellF1[3/2, n, 4 - n, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + 24*(-3 + n)*(Sec[(e + f*
x)/2]^2)^(-3 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2]^2*((
AppellF1[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2)/(3*AppellF1[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + 2*((-2 + n)*AppellF1[3/2, n, 3 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - AppellF1[1/2, n, 3 - n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, n, 3 - n, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-3 + n)*AppellF1[3/2, n, 4 -
 n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n,
3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
+ 24*(Sec[(e + f*x)/2]^2)^(-3 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[
(e + f*x)/2]*((AppellF1[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3*AppellF1[1/2, n, 2 - n, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-2 + n)*AppellF1[3/2, n,
 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2) + (Sec[(e + f*x)/2]^2*(-((2 - n)*AppellF1[3/2, n, 3 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (n*
AppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, n, 2 - n, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-2 + n)*AppellF1[3/2, n, 3 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2 -
 n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (-
((3 - n)*AppellF1[3/2, n, 4 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (n*AppellF1[3/2, 1 + n, 3 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2])/3)/(3*AppellF1[1/2, n, 3 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + 2*((-3 + n)*AppellF1[3/2, n, 4 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (AppellF1[1/2, n, 2 - n, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*(2*((-2 + n)*Ap
pellF1[3/2, n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*App
ellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-((2 - n)*AppellF1[3/2, n, 3 - n, 5/2
```

```
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2])/3 + (n*AppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*(
(-2 + n)*((-3*(3 - n)*AppellF1[5/2, n, 4 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*n*AppellF1[5/2
, 1 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/5) + n*((-3*(2 - n)*AppellF1[5/2, 1 + n, 3 - n, 7/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2])/5 + (3*(1 + n)*AppellF1[5/2, 2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1
/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-2 + n)*A
ppellF1[3/2, n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*Ap
pellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Ta
n[(e + f*x)/2]^2)^2 + (AppellF1[1/2, n, 3 - n, 3/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*(2*((-3 + n)*AppellF1[3/2, n, 4 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-((3
 - n)*AppellF1[3/2, n, 4 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (n*AppellF1[3/2, 1 + n, 3 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x
)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-3 + n)*((-3*(4 - n)*AppellF1[5/2, n, 5 -
 n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
 + f*x)/2])/5 + (3*n*AppellF1[5/2, 1 + n, 4 - n, 7/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + n*((-3*(3 - n
)*AppellF1[5/2, 1 + n, 4 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + n)*AppellF1[5/2, 2 + n, 3
 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[
(e + f*x)/2])/5))))/(3*AppellF1[1/2, n, 3 - n, 3/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + 2*((-3 + n)*AppellF1[3/2, n, 4 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2) + 24*n*(Sec[(e + f*x)
/2]^2)^(-3 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + n)*Tan[(e + f*x)/2]
*((AppellF1[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2)/(3*AppellF1[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + 2*((-2 + n)*AppellF1[3/2, n, 3 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - AppellF1[1/2, n, 3 - n,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, n, 3 - n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-3 + n)*AppellF1[3/2, n,
4 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 +
n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2
))*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*
Sec[e + f*x]*Tan[e + f*x])))
```

Maple [F] time = 0.917, size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\left(\sin\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^2,x)
```

```
[Out] int((b*sec(f*x+e))^n*sin(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^{2}-1\right)\left(b\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(e+fx\right)\right)^{n}\sin^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**2,x)
```

```
[Out] Integral((b*sec(e + f*x))**n*sin(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^2, x)
```

3.500 $\int (b\sec(e+fx))^n\,dx$

Optimal. Leaf size=73

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -((b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
+ f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rubi [A] time = 0.0325035, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3772, 2643}

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n,x]
```

```
[Out] -((b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
+ f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^n\,dx &= \left(\frac{\cos(e+fx)}{b}\right)^n (b\sec(e+fx))^n \int \left(\frac{\cos(e+fx)}{b}\right)^{-n} dx\\ &= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^n\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0471877, size = 61, normalized size = 0.84

$$\frac{\sqrt{-\tan^2(e+fx)}\cot(e+fx)(b\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\sec^2(e+fx)\right)}{fn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^n,x]
```

```
[Out] (Cot[e + f*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[e + f*x]^2]*(b*Sec
[e + f*x])^n*Sqrt[-Tan[e + f*x]^2])/(f*n)
```

Maple [F] time = 0.169, size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n,x)
```

```
[Out] int((b*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sec \left(fx+e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(e+fx\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n,x)
```

```
[Out] Integral((b*sec(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n, x)
```

3.501 $\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx$

Optimal. Leaf size=73

$$-\frac{b\sqrt{\sin^2(e+fx)}\csc(e+fx)(b\sec(e+fx))^{n-1}{}_2F_1\left(\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

```
[Out] -((b*Csc[e + f*x]*Hypergeometric2F1[3/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]
^2]*(b*Sec[e + f*x])^(-1 + n)*Sqrt[Sin[e + f*x]^2])/(f*(1 - n)))
```

Rubi [A] time = 0.0794675, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$-\frac{b\sqrt{\sin^2(e+fx)}\csc(e+fx)(b\sec(e+fx))^{n-1}{}_2F_1\left(\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(b*Sec[e + f*x])^n,x]
```

```
[Out] -((b*Csc[e + f*x]*Hypergeometric2F1[3/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]
^2]*(b*Sec[e + f*x])^(-1 + n)*Sqrt[Sin[e + f*x]^2])/(f*(1 - n)))
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx &= \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}\csc^2(e+fx)\,dx\\ &= -\frac{b\csc(e+fx)\,{}_2F_1\left(\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sqrt{\sin^2(e+fx)}}{f(1-n)}\end{aligned}$$

Mathematica [C] time = 14.7078, size = 2638, normalized size = 36.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2*(b*Sec[e + f*x])^n,x]
```

```
[Out] (Cot[(e + f*x)/2]*Csc[e + f*x]^2*(b*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*Sec
[e + f*x])^n*(-(AppellF1[-1/2, n, -n, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n) + (3*AppellF1[1/2, n, -n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Tan[(e +
f*x)/2]^2)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2])*Tan[(e + f*x)/2]^2)))/(2*f*(-(Csc[(e + f*x)/2]^2*(Cos[(e + f*x)/2]^2
*Sec[e + f*x])^n*(-(AppellF1[-1/2, n, -n, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n) + (3*AppellF1[1/2, n, -n,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Tan[(
e + f*x)/2]^2)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2])*Tan[(e + f*x)/2]^2)))/4 + (Cot[(e + f*x)/2]*(Cos[(e + f*x)/2]^2*
Sec[e + f*x])^n*(-((Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(-(n*AppellF1[1/2, n
, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*T
an[(e + f*x)/2]) - n*AppellF1[1/2, 1 + n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])) - n*AppellF1[-1/2, n
, -n, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e +
f*x)/2]^2)^(-1 + n)*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[
(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(1 + n)*Tan[(e + f*x)/2]
)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2
*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Ta
n[(e + f*x)/2]^2) + (3*n*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^3)/(3*AppellF1[1/2
, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2,
 n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1
+ n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)
 + (3*(Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^2*((n*AppellF1[3/2, n, 1 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2])/3 + (n*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, n, -n
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 -
 n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*A
ppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e +
 f*x)/2]^2)^n*Tan[(e + f*x)/2]^2*(2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*((
n*AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (n*AppellF1[3/2, 1 + n, -n, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3
) + 2*n*Tan[(e + f*x)/2]^2*((-3*(1 - n)*AppellF1[5/2, n, 2 - n, 7/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 +
 (6*n*AppellF1[5/2, 1 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + n)*AppellF1[5/2, 2 +
n, -n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2])/5)))/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2))/2 + (n*Cot[(e + f*x)/2]*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(-1 + n)*(-(AppellF1[-1/2, n, -n, 1/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n) + (3*Appel
lF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x
```

```
)/2]^2)^n*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))*(-(Cos[(e + f*x)/2]*Sec[e +
f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/2))
```

Maple [F] time = 0.295, size = 0, normalized size = 0.

$$\int \left(\csc \left(fx+e\right)\right)^{2} \left(b\sec \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(b*sec(f*x+e))^n,x)

[Out] int(csc(f*x+e)^2*(b*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)\right)^{n} \csc \left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec \left(fx+e\right)\right)^{n} \csc \left(fx+e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^n,x, algorithm="fricas")

[Out] integral((b*sec(f*x + e))^n*csc(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(e+fx\right)\right)^{n} \csc^{2}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(b*sec(f*x+e))**n,x)

[Out] Integral((b*sec(e + f*x))**n*csc(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^n \csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^2, x)
```

3.502 $\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx$

Optimal. Leaf size=73

$$-\frac{b\sqrt{\sin^2(e+fx)}\csc(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

```
[Out] -((b*Csc[e + f*x]*Hypergeometric2F1[5/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]
^2]*(b*Sec[e + f*x])^(-1 + n)*Sqrt[Sin[e + f*x]^2])/(f*(1 - n)))
```

Rubi [A] time = 0.0797302, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$-\frac{b\sqrt{\sin^2(e+fx)}\csc(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(b*Sec[e + f*x])^n,x]
```

```
[Out] -((b*Csc[e + f*x]*Hypergeometric2F1[5/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]
^2]*(b*Sec[e + f*x])^(-1 + n)*Sqrt[Sin[e + f*x]^2])/(f*(1 - n)))
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\begin{aligned}\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx &= \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}\csc^4(e+fx)\,dx\\ &= -\frac{b\csc(e+fx)\,{}_2F_1\left(\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sqrt{\sin^2(e+fx)}}{f(1-n)}\end{aligned}$$

Mathematica [C] time = 17.415, size = 3833, normalized size = 52.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^4*(b*Sec[e + f*x])^n,x]

[Out] (Cot[(e + f*x)/2]^3*Csc[e + f*x]^4*(b*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*S
ec[e + f*x])^n*(-(AppellF1[-3/2, n, -n, -1/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n) - 9*AppellF1[-1/2, n, -n,
1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2
]^2)^n*Tan[(e + f*x)/2]^2 + AppellF1[3/2, n, -n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^6
+ (27*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(S
ec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^4)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))/(24*f*(-(Cot[(e
+ f*x)/2]^2*Csc[(e + f*x)/2]^2*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*(-(Appel
lF1[-3/2, n, -n, -1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*
x]*Sec[(e + f*x)/2]^2)^n) - 9*AppellF1[-1/2, n, -n, 1/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]
^2 + AppellF1[3/2, n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Co
s[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^6 + (27*AppellF1[1/2, n,
-n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Ta
n[(e + f*x)/2]^4)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)))/16 + (Cot[(e + f*x)/2]^3*(Cos[(e + f*x)
/2]^2*Sec[e + f*x])^n*(-9*AppellF1[-1/2, n, -n, 1/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*T
an[(e + f*x)/2] + 3*AppellF1[3/2, n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e +
 f*x)/2]^5 - (Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(3*n*AppellF1[-1/2, n, 1 -
 n, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
 + f*x)/2] + 3*n*AppellF1[-1/2, 1 + n, -n, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) - 9*(Cos[e + f*x]*Sec[(e
 + f*x)/2]^2)^n*Tan[(e + f*x)/2]^2*(-(n*AppellF1[1/2, n, 1 - n, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) - n
*AppellF1[1/2, 1 + n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan
[(e + f*x)/2]^6*((3*n*AppellF1[5/2, n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*n*AppellF1[5/2
, 1 + n, -n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2])/5) - n*AppellF1[-3/2, n, -n, -1/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + n)*(-(Sec[(e
+ f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2
]) - 9*n*AppellF1[-1/2, n, -n, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + n)*Tan[(e + f*x)/2]^2*(-(Sec[(e +
 f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
) + n*AppellF1[3/2, n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(C
os[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + n)*Tan[(e + f*x)/2]^6*(-(Sec[(e + f*x
)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) +
(54*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec
[(e + f*x)/2]^2)^(1 + n)*Tan[(e + f*x)/2]^3)/(3*AppellF1[1/2, n, -n, 3/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (27*n*Appell
F1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)
/2]^2)^n*Tan[(e + f*x)/2]^5)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (27*(Sec[(e + f*x)/2]^2)^n*T
an[(e + f*x)/2]^4*((n*AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
```

```
[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (n*AppellF1[3/2,
1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2
*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (27*AppellF1[1/2, n, -n, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2
]^4*(2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*((n*AppellF1[3/2, n, 1 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2])/3 + (n*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*n*Tan[(e + f*x)/2]^2*((
-3*(1 - n)*AppellF1[5/2, n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (6*n*AppellF1[5/2, 1 + n, 1
- n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(
e + f*x)/2])/5 + (3*(1 + n)*AppellF1[5/2, 2 + n, -n, 7/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)))/(3*Appell
F1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF
1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3
/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)
/2]^2)^2))/24 + (n*Cot[(e + f*x)/2]^3*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1
 + n)*(-(AppellF1[-3/2, n, -n, -1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n) - 9*AppellF1[-1/2, n, -n, 1/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Ta
n[(e + f*x)/2]^2 + AppellF1[3/2, n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^6 + (27*App
ellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f
*x)/2]^2)^n*Tan[(e + f*x)/2]^4)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))*(-(Cos[(e + f*x)/2]*Sec[e
+ f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/2
4))
```

Maple [F] time = 0.208, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^{4}\left(b\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(b*sec(f*x+e))^n,x)
```

```
[Out] int(csc(f*x+e)^4*(b*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^n*csc(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(b*sec(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^4, x)
```

3.503 $\int (b\sec(a+bx))^n (c\sin(a+bx))^{3/2}\,dx$

Optimal. Leaf size=76

$$-\frac{c\sqrt{c\sin(a+bx)}(b\sec(a+bx))^{n-1}\,{}_2F_1\left(-\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{(1-n)\sqrt[4]{\sin^2(a+bx)}}$$

```
[Out] -((c*Hypergeometric2F1[-1/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a
 + b*x])^(-1 + n)*Sqrt[c*Sin[a + b*x]])/((1 - n)*(Sin[a + b*x]^2)^(1/4)))
```

Rubi [A] time = 0.109772, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2587, 2576}

$$-\frac{c\sqrt{c\sin(a+bx)}(b\sec(a+bx))^{n-1}\,{}_2F_1\left(-\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{(1-n)\sqrt[4]{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -((c*Hypergeometric2F1[-1/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a
 + b*x])^(-1 + n)*Sqrt[c*Sin[a + b*x]])/((1 - n)*(Sin[a + b*x]^2)^(1/4)))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (b\sec(a+bx))^n (c\sin(a+bx))^{3/2}\,dx = \left(b^2(b\cos(a+bx))^{-1+n}(b\sec(a+bx))^{-1+n}\right)\int (b\cos(a+bx))^{-n}(c\sin(a+bx))^{\cdots}$$

$$= -\frac{c\,{}_2F_1\left(-\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sqrt{c\sin(a+bx)}}{(1-n)\sqrt[4]{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.757648, size = 104, normalized size = 1.37

$$\frac{2(c\sin(a+bx))^{5/2}\cos^2(a+bx)^{\frac{n-1}{2}}(b\sec(a+bx))^{n-1}\left(5\sin^2(a+bx)\,{}_2F_1\left(\frac{9}{4},\frac{n+1}{2};\frac{13}{4};\sin^2(a+bx)\right)+9\,{}_2F_1\left(\frac{5}{4},\frac{n-1}{2};\frac{9}{4};\cdots\right.\right.}{45c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^((-1 + n)/2)*(b*Sec[a + b*x])^(-1 + n)*(c*Sin[a + b*x])
^(5/2)*(9*Hypergeometric2F1[5/4, (-1 + n)/2, 9/4, Sin[a + b*x]^2] + 5*Hyper
geometric2F1[9/4, (1 + n)/2, 13/4, Sin[a + b*x]^2]*Sin[a + b*x]^2))/(45*c)
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int \left(b \sec \left(bx+a\right)\right)^{n} \left(c \sin \left(bx+a\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

```
[Out] int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin \left(bx+a\right)\right)^{\frac{3}{2}} \left(b \sec \left(bx+a\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)*(b*sec(b*x + a))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c \sin \left(bx+a\right)} \left(b \sec \left(bx+a\right)\right)^{n} c \sin \left(bx+a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n*c*sin(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))**n*(c*sin(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sin(bx+a))^{\frac{3}{2}} (b\sec(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)*(b*sec(b*x + a))^n, x)
```

3.504 $\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx$

Optimal. Leaf size=76

$$-\frac{c\sqrt[4]{\sin^2(a+bx)}(b\sec(a+bx))^{n-1}\,{}_2F_1\left(\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{(1-n)\sqrt{c\sin(a+bx)}}$$

```
[Out] -((c*Hypergeometric2F1[1/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a
+ b*x])^(-1 + n)*(Sin[a + b*x]^2)^(1/4))/((1 - n)*Sqrt[c*Sin[a + b*x]]))
```

Rubi [A] time = 0.0942671, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2587, 2576}

$$-\frac{c\sqrt[4]{\sin^2(a+bx)}(b\sec(a+bx))^{n-1}\,{}_2F_1\left(\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{(1-n)\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((c*Hypergeometric2F1[1/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a
+ b*x])^(-1 + n)*(Sin[a + b*x]^2)^(1/4))/((1 - n)*Sqrt[c*Sin[a + b*x]]))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\begin{aligned}\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx &= \left(b^2(b\cos(a+bx))^{-1+n}(b\sec(a+bx))^{-1+n}\right)\int (b\cos(a+bx))^{-n}\sqrt{c\sin(a+bx)}\,dx\\ &= -\frac{c\,{}_2F_1\left(\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sqrt[4]{\sin^2(a+bx)}}{(1-n)\sqrt{c\sin(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.122712, size = 75, normalized size = 0.99

$$\frac{\sin(2(a+bx))\sqrt{c\sin(a+bx)}\cos^2(a+bx)^{\frac{n-1}{2}}(b\sec(a+bx))^n\,{}_2F_1\left(\frac{3}{4},\frac{n+1}{2};\frac{7}{4};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] ((Cos[a + b*x]^2)^((-1 + n)/2)*Hypergeometric2F1[3/4, (1 + n)/2, 7/4, Sin[a
 + b*x]^2]*(b*Sec[a + b*x])^n*Sqrt[c*Sin[a + b*x]]*Sin[2*(a + b*x)])/(3*b)
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int (b \sec (bx + a))^n \sqrt{c \sin (bx + a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
```

```
[Out] int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \sin (bx + a)} (b \sec (bx + a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c \sin (bx + a)} (b \sec (bx + a))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (a + bx))^n \sqrt{c \sin (a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))**n*(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral((b*sec(a + b*x))**n*sqrt(c*sin(a + b*x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \sin(bx + a)} \, (b \sec(bx + a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n, x)
```

3.505 $\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=76

$$-\frac{c\sin^2(a+bx)^{3/4}(b\sec(a+bx))^{n-1}\,{}_2F_1\left(\frac{3}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{(1-n)(c\sin(a+bx))^{3/2}}$$

```
[Out] -((c*Hypergeometric2F1[3/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a
+ b*x])^(-1 + n)*(Sin[a + b*x]^2)^(3/4))/((1 - n)*(c*Sin[a + b*x])^(3/2)))
```

Rubi [A] time = 0.0962568, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2587, 2576}

$$-\frac{c\sin^2(a+bx)^{3/4}(b\sec(a+bx))^{n-1}\,{}_2F_1\left(\frac{3}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{(1-n)(c\sin(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((c*Hypergeometric2F1[3/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a
+ b*x])^(-1 + n)*(Sin[a + b*x]^2)^(3/4))/((1 - n)*(c*Sin[a + b*x])^(3/2)))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx &= \left(b^2(b\cos(a+bx))^{-1+n}(b\sec(a+bx))^{-1+n}\right)\int \frac{(b\cos(a+bx))^{-n}}{\sqrt{c\sin(a+bx)}}\,dx\\ &= -\frac{c\,{}_2F_1\left(\frac{3}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sin^2(a+bx)^{3/4}}{(1-n)(c\sin(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.13394, size = 72, normalized size = 0.95

$$\frac{\sin(2(a+bx))\cos^2(a+bx)^{\frac{n-1}{2}}(b\sec(a+bx))^n\,{}_2F_1\left(\frac{1}{4},\frac{n+1}{2};\frac{5}{4};\sin^2(a+bx)\right)}{b\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] ((Cos[a + b*x]^2)^((-1 + n)/2)*Hypergeometric2F1[1/4, (1 + n)/2, 5/4, Sin[a
 + b*x]^2]*(b*Sec[a + b*x])^n*Sin[2*(a + b*x)])/(b*Sqrt[c*Sin[a + b*x]])
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int (b\sec (bx+a))^n \frac{1}{\sqrt{c\sin (bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec (bx+a))^n}{\sqrt{c\sin (bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c\sin (bx+a)}\,(b\sec (bx+a))^n}{c\sin (bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n/(c*sin(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec (a+bx))^n}{\sqrt{c\sin (a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))**n/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral((b*sec(a + b*x))**n/sqrt(c*sin(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(bx+a))^n}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

3.506 $\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{\sqrt[4]{\sin^2(a+bx)}(b\sec(a+bx))^{n-1}\,{}_2F_1\left(\frac{5}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{c(1-n)\sqrt{c\sin(a+bx)}}$$

```
[Out] -((Hypergeometric2F1[5/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a +
b*x])^(-1 + n)*(Sin[a + b*x]^2)^(1/4))/(c*(1 - n)*Sqrt[c*Sin[a + b*x]]))
```

Rubi [A] time = 0.116657, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2587, 2576}

$$-\frac{\sqrt[4]{\sin^2(a+bx)}(b\sec(a+bx))^{n-1}\,{}_2F_1\left(\frac{5}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{c(1-n)\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -((Hypergeometric2F1[5/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a +
b*x])^(-1 + n)*(Sin[a + b*x]^2)^(1/4))/(c*(1 - n)*Sqrt[c*Sin[a + b*x]]))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx &= \left(b^2(b\cos(a+bx))^{-1+n}(b\sec(a+bx))^{-1+n}\right)\int \frac{(b\cos(a+bx))^{-n}}{(c\sin(a+bx))^{3/2}}\,dx\\ &= -\frac{{}_2F_1\left(\frac{5}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sqrt[4]{\sin^2(a+bx)}}{c(1-n)\sqrt{c\sin(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.148636, size = 73, normalized size = 0.94

$$-\frac{\sin(2(a+bx))\cos^2(a+bx)^{\frac{n-1}{2}}(b\sec(a+bx))^n\,{}_2F_1\left(-\frac{1}{4},\frac{n+1}{2};\frac{3}{4};\sin^2(a+bx)\right)}{b(c\sin(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -(((Cos[a + b*x]^2)^((-1 + n)/2)*Hypergeometric2F1[-1/4, (1 + n)/2, 3/4, Si
n[a + b*x]^2]*(b*Sec[a + b*x])^n*Sin[2*(a + b*x)])/(b*(c*Sin[a + b*x])^(3/2
)))
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int \left(b \sec \left(bx+a\right)\right)^{n} \left(c \sin \left(bx+a\right)\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
```

```
[Out] int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \sec \left(bx+a\right)\right)^{n}}{\left(c \sin \left(bx+a\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{c \sin \left(bx+a\right)} \left(b \sec \left(bx+a\right)\right)^{n}}{c^{2} \cos \left(bx+a\right)^{2}-c^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n/(c^2*cos(b*x + a)^2 - c^2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))**n/(c*sin(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

3.507 $\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx$

Optimal. Leaf size=100

$$-\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{10\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21f}$$

```
[Out] (-2*d^3*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*d*Cos[e + f*x])/(2
1*f*Sqrt[d*Csc[e + f*x]]) + (10*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 +
f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*f)
```

Rubi [A] time = 0.077818, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2641}

$$-\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{10\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^4,x]
```

```
[Out] (-2*d^3*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*d*Cos[e + f*x])/(2
1*f*Sqrt[d*Csc[e + f*x]]) + (10*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 +
f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx = d^4\int \frac{1}{(d\csc(e+fx))^{7/2}}\,dx$$
$$= -\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} + \frac{1}{7}\left(5d^2\right)\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx$$
$$= -\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{5}{21}\int \sqrt{d\csc(e+fx)}\,dx$$
$$= -\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{1}{21}\left(5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int$$
$$= -\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{10\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right.}{21f}$$

Mathematica [A] time = 0.176135, size = 67, normalized size = 0.67

$$-\frac{\sqrt{d\csc(e+fx)}\left(26\sin(2(e+fx))-3\sin(4(e+fx))+40\sqrt{\sin(e+fx)}F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{84f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^4,x]
```

```
[Out] -(Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e +
 f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/(84*f)
```

Maple [C] time = 0.226, size = 214, normalized size = 2.1

$$\frac{\sqrt{2}\sin(fx+e)}{21f(-1+\cos(fx+e))}\sqrt{\frac{d}{\sin(fx+e)}}\left(-5i\sin(fx+e)\sqrt{\frac{i\cos(fx+e)+\sin(fx+e)-i}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-\sin(fx}{\sin(fx+e)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(d*csc(f*x+e))^(1/2),x)
```

```
[Out] 1/21/f*2^(1/2)*sin(f*x+e)*(d/sin(f*x+e))^(1/2)*(-5*I*sin(f*x+e)*((I*cos(f*x
+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e
))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/
2))*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)+3*2^(1/2)*cos(f*x+e)^4-3*2^(1/2)*
cos(f*x+e)^3-8*2^(1/2)*cos(f*x+e)^2+8*2^(1/2)*cos(f*x+e))/(-1+cos(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\csc(fx+e)}\sin(fx+e)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}+1\right)\sqrt{d\csc\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(d*csc(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.508 $\int \sqrt{d\csc(e+fx)}\sin^3(e+fx)\,dx$

Optimal. Leaf size=75

$$\frac{6dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d^2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

```
[Out] (-2*d^2*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*d*EllipticE[(e - Pi
/2 + f*x)/2, 2])/(5*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rubi [A] time = 0.0542168, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2639}

$$\frac{6dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d^2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^3,x]
```

```
[Out] (-2*d^2*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*d*EllipticE[(e - Pi
/2 + f*x)/2, 2])/(5*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\csc(e+fx)}\sin^3(e+fx)\,dx &= d^3\int \frac{1}{(d\csc(e+fx))^{5/2}}\,dx\\ &= -\frac{2d^2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{1}{5}(3d)\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\ &= -\frac{2d^2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{(3d)\int \sqrt{\sin(e+fx)}\,dx}{5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\ &= -\frac{2d^2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{6dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.115072, size = 62, normalized size = 0.83

$$-\frac{2\sqrt{d\csc(e+fx)}\left(\sin^2(e+fx)\cos(e+fx)+3\sqrt{\sin(e+fx)}E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^3,x]
```

```
[Out] (-2*Sqrt[d*Csc[e + f*x]]*(3*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e
+ f*x]] + Cos[e + f*x]*Sin[e + f*x]^2))/(5*f)
```

Maple [C] time = 0.198, size = 538, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(d*csc(f*x+e))^(1/2),x)
```

```
[Out] 1/5/f*2^(1/2)*(d/sin(f*x+e))^(1/2)*(-6*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)
-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin
(f*x+e))^(1/2),1/2*2^(1/2))+3*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+
e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^
(1/2),1/2*2^(1/2))+2^(1/2)*cos(f*x+e)^3-6*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x
+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))
^(1/2),1/2*2^(1/2))+3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+
sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(
1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-
4*2^(1/2)*cos(f*x+e)+3*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^{2}-1\right)\sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(d*csc(f*x + e))*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(d*csc(f*x+e))**(1/2),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="giac")

[Out] Timed out
```

3.509 $\int \sqrt{d \csc(e+fx)} \sin^2(e+fx)\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2d\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}$$

```
[Out] (-2*d*Cos[e + f*x])/(3*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*El
lipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```

Rubi [A] time = 0.0522891, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2d\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^2,x]
```

```
[Out] (-2*d*Cos[e + f*x])/(3*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*El
lipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = d^2\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx$$
$$= -\frac{2d\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{1}{3}\int \sqrt{d\csc(e+fx)}\,dx$$
$$= -\frac{2d\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{1}{3}\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx$$
$$= -\frac{2d\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{2\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{3f}$$

Mathematica [A] time = 0.0742789, size = 55, normalized size = 0.76

$$-\frac{\sqrt{d\csc(e+fx)}\left(\sin(2(e+fx))+2\sqrt{\sin(e+fx)}F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^2,x]
```

```
[Out] -(Sqrt[d*Csc[e + f*x]]*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e +
f*x]] + Sin[2*(e + f*x)]))/(3*f)
```

Maple [C] time = 0.142, size = 187, normalized size = 2.6

$$-\frac{\sqrt{2}\sin\left(fx+e\right)}{3f\left(-1+\cos\left(fx+e\right)\right)}\sqrt{\frac{d}{\sin\left(fx+e\right)}}\left(i\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sin\left(fx+e\right)\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{\frac{i\cdots}{\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*2^(1/2)*sin(f*x+e)*(d/sin(f*x+e))^(1/2)*(I*(-I*(-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*sin(f*x+e)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I
*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*
x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(f*x+e)^2-2^(1/2)*cos(f*x
+e))/(-1+cos(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^2-1\right)\sqrt{d\csc\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(d*csc(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\csc\left(e+fx\right)}\sin^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*csc(e + f*x))*sin(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^2, x)
```

3.510 $\int \sqrt{d \csc(e+fx)} \sin(e+fx)\,dx$

Optimal. Leaf size=44

$$\frac{2dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] (2*d*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e +
 f*x]])
```

Rubi [A] time = 0.0284428, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3771, 2639}

$$\frac{2dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x],x]
```

```
[Out] (2*d*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e +
 f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\csc(e+fx)}\sin(e+fx)\,dx &= d\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\ &= \frac{d\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\ &= \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0463764, size = 43, normalized size = 0.98

$$-\frac{2dE\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x],x]
```

```
[Out] (-2*d*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin
[e + f*x]])
```

Maple [C] time = 0.13, size = 525, normalized size = 11.9

$$-\frac{\sqrt{2}}{f}\left(2\cos\left(fx+e\right)\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -1/f*2^(1/2)*(2*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*
x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+
e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1
/2))-cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*
x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*E
llipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^
(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x
+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-
sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(
f*x+e))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(f*x+e)-2^(1/2))*(d/sin(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*sin(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \csc \left(e + fx\right)} \sin \left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*csc(e + f*x))*sin(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \csc \left(fx + e\right)} \sin \left(fx + e\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e), x)
```

3.511 $\int \sqrt{d \csc(e + fx)}\, dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{f}$$

```
[Out] (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]]
)/f
```

Rubi [A] time = 0.0193115, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]]
)/f
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\csc(e+fx)}\,dx &= \left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx\\ &= \frac{2\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{f}\end{aligned}$$

Mathematica [A] time = 0.0346121, size = 42, normalized size = 0.98

$$-\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\sqrt{d\csc(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f
*x]])/f
```

Maple [C] time = 0.093, size = 165, normalized size = 3.8

$$\frac{-i\sqrt{2}\left(-1+\cos\left(fx+e\right)\right)\left(\cos\left(fx+e\right)+1\right)^{2}}{f\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{d}{\sin\left(fx+e\right)}}\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(f\right.}{\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^(1/2),x)
```

```
[Out] -I/f*2^(1/2)*(d/sin(f*x+e))^(1/2)*(-1+cos(f*x+e))*((I*cos(f*x+e)+sin(f*x+e)
-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-I*(
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin
(f*x+e))^(1/2),1/2*2^(1/2))/sin(f*x+e)^2*(cos(f*x+e)+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\csc\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\csc\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\csc\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*csc(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \csc(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(f*x + e)), x)
```

3.512 $\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx$

Optimal. Leaf size=68

$$-\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}-\frac{2dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] (-2*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/f - (2*d*EllipticE[(e - Pi/2 + f*x)/
2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rubi [A] time = 0.0391602, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 3768, 3771, 2639}

$$-\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}-\frac{2dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/f - (2*d*EllipticE[(e - Pi/2 + f*x)/
2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx &= \frac{\int (d\csc(e+fx))^{3/2}\,dx}{d} \\
&= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - d\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx \\
&= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - \frac{d\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}} \\
&= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}
$$

Mathematica [A] time = 0.128092, size = 57, normalized size = 0.84

$$
\frac{(d\csc(e+fx))^{3/2}\left(2\sin^{\frac{3}{2}}(e+fx)E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)-\sin(2(e+fx))\right)}{df}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] ((d*Csc[e + f*x])^(3/2)*(2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f*x]
^(3/2) - Sin[2*(e + f*x)]))/(d*f)
```

Maple [C] time = 0.121, size = 514, normalized size = 7.6

$$
\frac{\sqrt{2}}{f}\left(2\cos\left(fx+e\right)\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\mathrm{E}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x)
```

```
[Out] 1/f*2^(1/2)*(2*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x
+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e
))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/
2))-cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x
+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*El
lipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*(
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(
1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+
e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-s
in(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f
*x+e))^(1/2),1/2*2^(1/2))-2^(1/2))*(d/sin(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\csc\left(e+fx\right)}\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*csc(e + f*x))*csc(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e), x)
```

3.513 $\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx$

Optimal. Leaf size=74

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df}$$

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*d*f) + (2*Sqrt[d*Csc[e + f*x]]*
EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```

Rubi [A] time = 0.0404922, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*d*f) + (2*Sqrt[d*Csc[e + f*x]]*
EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx = \frac{\int (d\csc(e+fx))^{5/2}\,dx}{d^2}$$
$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df} + \frac{1}{3}\int \sqrt{d\csc(e+fx)}\,dx$$
$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df} + \frac{1}{3}\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}$$
$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df} + \frac{2\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+f}}{3f}$$

Mathematica [A] time = 0.0896675, size = 55, normalized size = 0.74

$$-\frac{2(d\csc(e+fx))^{3/2}\left(\cos(e+fx)+\sin^{\frac{3}{2}}(e+fx)F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*(d*Csc[e + f*x])^(3/2)*(Cos[e + f*x] + EllipticF[(-2*e + Pi - 2*f*x)/4,
 2]*Sin[e + f*x]^(3/2)))/(3*d*f)
```

Maple [C] time = 0.127, size = 319, normalized size = 4.3

$$\frac{\sqrt{2}\left(\cos\left(fx+e\right)+1\right)^2\left(-1+\cos\left(fx+e\right)\right)^2}{3f\left(\sin\left(fx+e\right)\right)^5}\sqrt{\frac{d}{\sin\left(fx+e\right)}}\left(i\cos\left(fx+e\right)\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{\frac{-i(-1+}{\sin}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x)
```

```
[Out] 1/3/f*2^(1/2)*(d/sin(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(I*co
s(f*x+e)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-I*(-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*sin(f*x+
e)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+I*
(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*((I*cos(f*x+e)+sin(f*x+e)-
I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*Ellipt
icF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-2^(1/2)*cos
(f*x+e))/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\csc\left(e+fx\right)}\csc^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*csc(e + f*x))*csc(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e)^2, x)
```

3.514 $\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx$

Optimal. Leaf size=100

$$-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f}-\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}-\frac{6dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] (-6*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(5*f) - (2*Cos[e + f*x]*(d*Csc[e + f
*x])^(5/2))/(5*d^2*f) - (6*d*EllipticE[(e - Pi/2 + f*x)/2, 2])/(5*f*Sqrt[d*
Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rubi [A] time = 0.0584073, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2639}

$$-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f}-\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}-\frac{6dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-6*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(5*f) - (2*Cos[e + f*x]*(d*Csc[e + f
*x])^(5/2))/(5*d^2*f) - (6*d*EllipticE[(e - Pi/2 + f*x)/2, 2])/(5*f*Sqrt[d*
Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx = \frac{\int (d\csc(e+fx))^{7/2}\,dx}{d^3}$$
$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f} + \frac{3\int (d\csc(e+fx))^{3/2}\,dx}{5d}$$
$$= -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f} - \frac{1}{5}(3d)\int \frac{}{\sqrt{d\,\mathrm{cs}}}$$
$$= -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f} - \frac{(3d)\int \sqrt{\sin}}{5\sqrt{d\csc(e+f}}$$
$$= -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f} - \frac{6dE\left(\frac{1}{2}(e-\right.}{5f\sqrt{d\csc(e+}}$$

Mathematica [A] time = 0.139827, size = 68, normalized size = 0.68

$$-\frac{2\sqrt{d\csc(e+fx)}\left(3\cos(e+fx)+\cot(e+fx)\csc(e+fx)-3\sqrt{\sin(e+fx)}E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Sqrt[d*Csc[e + f*x]]*(3*Cos[e + f*x] + Cot[e + f*x]*Csc[e + f*x] - 3*El
lipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f*x]]))/(5*f)
```

Maple [C] time = 0.158, size = 1054, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -1/5/f*2^(1/2)*(6*cos(f*x+e)^3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(I*c
os(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+
e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e
))^(1/2)-3*cos(f*x+e)^3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+
e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))
^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2)
)+6*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e
)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^2*(-I*(-1+cos(f*x
+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)-3*(-(
I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f
*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^2*(-I*(-1+cos(f*x+e))/si
n(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)-6*cos(f*x+e)
*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x
+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*co
s(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+3*cos(f*x+e)*(-I*(-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2
)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+
sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-6*(-I*(-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-si
```

```
n(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*
x+e))^(1/2),1/2*2^(1/2))+3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*
x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+
e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1
/2))-3*2^(1/2)*cos(f*x+e)^2+2^(1/2)*cos(f*x+e)+3*2^(1/2))*(d/sin(f*x+e))^(1
/2)/sin(f*x+e)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\csc\left(e+fx\right)}\csc^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*csc(e + f*x))*csc(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e)^3, x)
```

3.515 $\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx$

Optimal. Leaf size=103

$$-\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{10d\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21f}$$

```
[Out] (-2*d^4*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*d^2*Cos[e + f*x])/
(21*f*Sqrt[d*Csc[e + f*x]]) + (10*d*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/
2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*f)
```

Rubi [A] time = 0.0758318, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2641}

$$-\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{10d\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] (-2*d^4*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*d^2*Cos[e + f*x])/
(21*f*Sqrt[d*Csc[e + f*x]]) + (10*d*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/
2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx = d^5\int \frac{1}{(d\csc(e+fx))^{7/2}}\,dx$$
$$= -\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} + \frac{1}{7}\left(5d^3\right)\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx$$
$$= -\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{1}{21}(5d)\int \sqrt{d\csc(e+fx)}\,dx$$
$$= -\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{1}{21}\left(5d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)$$
$$= -\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{10d\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right.\right.}{21f}$$

Mathematica [A] time = 0.123485, size = 68, normalized size = 0.66

$$-\frac{d\sqrt{d\csc(e+fx)}\left(26\sin(2(e+fx))-3\sin(4(e+fx))+40\sqrt{\sin(e+fx)}F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{84f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] -(d*Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e
 + f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/(84*f)
```

Maple [C] time = 0.113, size = 216, normalized size = 2.1

$$-\frac{\sqrt{2}\left(\sin\left(fx+e\right)\right)^2}{21f\left(-1+\cos\left(fx+e\right)\right)}\left(5i\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sin\left(fx+e\right)\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\mathrm{EllipticF}\left(\sqrt{\frac{i\,c}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x)
```

```
[Out] -1/21/f*2^(1/2)*(5*I*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*(-(I*
cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x
+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+
e))^(1/2)-3*2^(1/2)*cos(f*x+e)^4+3*2^(1/2)*cos(f*x+e)^3+8*2^(1/2)*cos(f*x+e
)^2-8*2^(1/2)*cos(f*x+e))*(d/sin(f*x+e))^(3/2)*sin(f*x+e)^2/(-1+cos(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\cos\left(fx+e\right)^4-2\,d\cos\left(fx+e\right)^2+d\right)\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e)^4 - 2*d*cos(f*x + e)^2 + d)*sqrt(d*csc(f*x + e))*c
sc(f*x + e)*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

3.516 $\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx$

Optimal. Leaf size=77

$$\frac{6d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

```
[Out] (-2*d^3*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*d^2*EllipticE[(e -
Pi/2 + f*x)/2, 2])/(5*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rubi [A] time = 0.0556424, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2639}

$$\frac{6d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (-2*d^3*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*d^2*EllipticE[(e -
Pi/2 + f*x)/2, 2])/(5*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx &= d^4\int \frac{1}{(d\csc(e+fx))^{5/2}}\,dx\\ &= -\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{1}{5}\left(3d^2\right)\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\ &= -\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{\left(3d^2\right)\int \sqrt{\sin(e+fx)}\,dx}{5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\ &= -\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{6d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.188771, size = 62, normalized size = 0.81

$$-\frac{2(d\csc(e+fx))^{3/2}\left(\sin^3(e+fx)\cos(e+fx)+3\sin^{\frac{3}{2}}(e+fx)E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (-2*(d*Csc[e + f*x])^(3/2)*(3*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f
*x]^(3/2) + Cos[e + f*x]*Sin[e + f*x]^3))/(5*f)
```

Maple [C] time = 0.151, size = 545, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x)
```

```
[Out] -1/5/f*2^(1/2)*(6*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(
f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*
x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^
(1/2))-3*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+si
n(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/
2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+6*
(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+
e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos
(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-3*(-I*(-1+cos(f*x+e))/
sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f
*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I
)/sin(f*x+e))^(1/2),1/2*2^(1/2))-2^(1/2)*cos(f*x+e)^3+4*2^(1/2)*cos(f*x+e)-
3*2^(1/2))*(d/sin(f*x+e))^(3/2)*sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\cos\left(fx+e\right)^4 - 2\,d\cos\left(fx+e\right)^2 + d\right)\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e)^4 - 2*d*cos(f*x + e)^2 + d)*sqrt(d*csc(f*x + e))*c
sc(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.517 $\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx$

Optimal. Leaf size=75

$$\frac{2d\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}$$

```
[Out] (-2*d^2*Cos[e + f*x])/(3*f*Sqrt[d*Csc[e + f*x]]) + (2*d*Sqrt[d*Csc[e + f*x]
]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```

Rubi [A] time = 0.0531429, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2641}

$$\frac{2d\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^3,x]
```


```
[Out] (-2*d^2*Cos[e + f*x])/(3*f*Sqrt[d*Csc[e + f*x]]) + (2*d*Sqrt[d*Csc[e + f*x]
]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx = d^3\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx$$
$$= -\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{1}{3}d\int \sqrt{d\csc(e+fx)}\,dx$$
$$= -\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{1}{3}\left(d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx$$
$$= -\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{2d\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{3f}$$

Mathematica [A] time = 0.0687143, size = 56, normalized size = 0.75

$$-\frac{d\sqrt{d\csc(e+fx)}\left(\sin(2(e+fx))+2\sqrt{\sin(e+fx)}F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^3,x]
```

```
[Out] -(d*Sqrt[d*Csc[e + f*x]]*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e
+ f*x]] + Sin[2*(e + f*x)]))/(3*f)
```

Maple [C] time = 0.118, size = 189, normalized size = 2.5

$$-\frac{\sqrt{2}\left(\sin\left(fx+e\right)\right)^2}{3f\left(-1+\cos\left(fx+e\right)\right)}\left(i\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sin\left(fx+e\right)\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-s}{\sin\left(fx\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x)
```

```
[Out] -1/3/f*2^(1/2)*(I*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*((I*cos(
f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*
x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^
(1/2))+2^(1/2)*cos(f*x+e)^2-2^(1/2)*cos(f*x+e))*(d/sin(f*x+e))^(3/2)*sin(f*
x+e)^2/(-1+cos(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(d\cos\left(fx+e\right)^{2}-d\right)\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral(-(d*cos(f*x + e)^2 - d)*sqrt(d*csc(f*x + e))*csc(f*x + e)*sin(f*x
+ e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^3, x)
```

3.518 $\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx$

Optimal. Leaf size=46

$$\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] (2*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e
 + f*x]])
```

Rubi [A] time = 0.0379558, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3771, 2639}

$$\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (2*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e
 + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx &= d^2\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\
&= \frac{d^2\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\
&= \frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.0267357, size = 45, normalized size = 0.98

$$-\frac{2d^2E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (-2*d^2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[S
in[e + f*x]])
```

Maple [C] time = 0.131, size = 531, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x)
```

```
[Out] -1/f*2^(1/2)*(2*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*
x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+
e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1
/2))-cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*
x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*E
llipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^
(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x
+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-
sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(
f*x+e))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(f*x+e)-2^(1/2))*(d/sin(f*x+e))^(3/2)
*sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(d\cos\left(fx+e\right)^2-d\right)\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*cos(f*x + e)^2 - d)*sqrt(d*csc(f*x + e))*csc(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d \csc \left(fx+e\right)\right)^{\frac{3}{2}} \sin \left(fx+e\right)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^2, x)
```

3.519 $\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx$

Optimal. Leaf size=44

$$\frac{2d\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{f}$$

```
[Out] (2*d*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x
]])/f
```

Rubi [A] time = 0.0280744, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3771, 2641}

$$\frac{2d\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x],x]
```

```
[Out] (2*d*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x
]])/f
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx &= d\int \sqrt{d\csc(e+fx)}\,dx\\ &= \left(d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx\\ &= \frac{2d\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{f}\end{aligned}$$

Mathematica [A] time = 0.0159633, size = 43, normalized size = 0.98

$$-\frac{2d\sqrt{\sin(e+fx)}F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\sqrt{d\csc(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x],x]
```

```
[Out] (-2*d*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e +
 f*x]])/f
```

Maple [C] time = 0.125, size = 165, normalized size = 3.8

$$\frac{-i\sqrt{2}\left(\cos\left(fx+e\right)+1\right)^{2}\left(-1+\cos\left(fx+e\right)\right)}{f\sin\left(fx+e\right)}\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+\right.}{\mathrm{si}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^(3/2)*sin(f*x+e),x)
```

```
[Out] -I/f*2^(1/2)*(cos(f*x+e)+1)^2*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*
x+e))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x
+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e)
)^(1/2)*(-1+cos(f*x+e))*(d/sin(f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\csc\left(fx+e\right)}d\csc\left(fx+e\right)\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*d*csc(f*x + e)*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}} \sin\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e), x)
```

3.520 $\int (d\csc(e+fx))^{3/2}\,dx$

Optimal. Leaf size=71

$$-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}$$

```
[Out] (-2*d*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/f - (2*d^2*EllipticE[(e - Pi/2 + f
*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rubi [A] time = 0.0334052, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3771, 2639}

$$-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^(3/2),x]
```



```
[Out] (-2*d*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/f - (2*d^2*EllipticE[(e - Pi/2 + f
*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```


Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (d\csc(e+fx))^{3/2}\,dx &= -\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}-d^2\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\
&= -\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}-\frac{d^2\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\
&= -\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}-\frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.078176, size = 54, normalized size = 0.76

$$\frac{(d\csc(e+fx))^{3/2}\left(2\sin^{\frac{3}{2}}(e+fx)E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)-\sin(2(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] ((d*Csc[e + f*x])^(3/2)*(2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f*x]
^(3/2) - Sin[2*(e + f*x)]))/f
```

Maple [C] time = 0.105, size = 520, normalized size = 7.3

$$\frac{\sqrt{2}\sin\left(fx+e\right)}{f}\left(2\cos\left(fx+e\right)\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(\right.}{\sin\left(fx+e\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^(3/2),x)
```

```
[Out] 1/f*2^(1/2)*(2*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x
+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e
))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/
2))-cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x
+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*El
lipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*(
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(
1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+
e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-s
in(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f
*x+e))^(1/2),1/2*2^(1/2))-2^(1/2))*(d/sin(f*x+e))^(3/2)*sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\csc\left(fx+e\right)}d\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*d*csc(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*csc(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2), x)
```

3.521 $\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx$

Optimal. Leaf size=72

$$\frac{2d\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}$$

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*f) + (2*d*Sqrt[d*Csc[e + f*x]]*
EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```

Rubi [A] time = 0.0388401, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 3768, 3771, 2641}

$$\frac{2d\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*f) + (2*d*Sqrt[d*Csc[e + f*x]]*
EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx = \frac{\int (d\csc(e+fx))^{5/2}\,dx}{d}$$
$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f} + \frac{1}{3}d\int \sqrt{d\csc(e+fx)}\,dx$$
$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f} + \frac{1}{3}\left(d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+f}}$$
$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f} + \frac{2d\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+}}{3f}$$

Mathematica [A] time = 0.17595, size = 58, normalized size = 0.81

$$-\frac{(d\csc(e+fx))^{5/2}\left(\sin(2(e+fx))+2\sin^{\frac{5}{2}}(e+fx)F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] -((d*Csc[e + f*x])^(5/2)*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f*x
]^(5/2) + Sin[2*(e + f*x)]))/(3*d*f)
```

Maple [C] time = 0.118, size = 319, normalized size = 4.4

$$\frac{\sqrt{2}\left(-1+\cos\left(fx+e\right)\right)^{2}\left(\cos\left(fx+e\right)+1\right)^{2}}{3f\left(\sin\left(fx+e\right)\right)^{4}}\left(i\cos\left(fx+e\right)\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x)
```

```
[Out] 1/3/f*2^(1/2)*(-1+cos(f*x+e))^2*(I*cos(f*x+e)*((I*cos(f*x+e)+sin(f*x+e)-I)/
sin(f*x+e))^(1/2)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin
(f*x+e)-I)/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)
-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+I*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*
sin(f*x+e)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-s
in(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f
*x+e))^(1/2),1/2*2^(1/2))-2^(1/2)*cos(f*x+e))*(cos(f*x+e)+1)^2*(d/sin(f*x+e
))^(3/2)/sin(f*x+e)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*csc(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d \csc\left(fx+e\right)} d \csc\left(fx+e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*d*csc(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d \csc\left(e+fx\right)\right)^{\frac{3}{2}} \csc\left(e+fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*csc(e + f*x))**(3/2)*csc(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d \csc\left(fx+e\right)\right)^{\frac{3}{2}} \csc\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*csc(f*x + e), x)
```

3.522 $\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx$

Optimal. Leaf size=103

$$-\frac{6d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df}-\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}$$

```
[Out] (-6*d*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(5*f) - (2*Cos[e + f*x]*(d*Csc[e +
 f*x])^(5/2))/(5*d*f) - (6*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(5*f*Sqrt[
d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rubi [A] time = 0.0580863, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2639}

$$-\frac{6d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df}-\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-6*d*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(5*f) - (2*Cos[e + f*x]*(d*Csc[e +
 f*x])^(5/2))/(5*d*f) - (6*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(5*f*Sqrt[
d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx = \frac{\int (d\csc(e+fx))^{7/2}\,dx}{d^2}$$
$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df} + \frac{3}{5}\int (d\csc(e+fx))^{3/2}\,dx$$
$$= -\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df} - \frac{1}{5}\left(3d^2\right)\int$$
$$= -\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df} - \frac{\left(3d^2\right)\int}{5\sqrt{d\csc(e}}$$
$$= -\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df} - \frac{6d^2E\left(\frac{1}{2}\right.}{5f\sqrt{d\csc(e}}$$

Mathematica [A] time = 0.243958, size = 68, normalized size = 0.66

$$\frac{(d\csc(e+fx))^{5/2}\left(-7\cos(e+fx)+3\cos(3(e+fx))+12\sin^{\frac{5}{2}}(e+fx)E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{10df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] ((d*Csc[e + f*x])^(5/2)*(-7*Cos[e + f*x] + 3*Cos[3*(e + f*x)] + 12*Elliptic
E[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f*x]^(5/2)))/(10*d*f)
```

Maple [C] time = 0.135, size = 1054, normalized size = 10.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x)
```

```
[Out] -1/5/f*2^(1/2)*(d/sin(f*x+e))^(3/2)*(6*cos(f*x+e)^3*(-I*(-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((
I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)+si
n(f*x+e)-I)/sin(f*x+e))^(1/2)-3*cos(f*x+e)^3*(-I*(-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(
f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+
e))^(1/2),1/2*2^(1/2))+6*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*El
lipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x
+e)^2*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/si
n(f*x+e))^(1/2)-3*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF
(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^2*(
-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e
))^(1/2)-6*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+
sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(
1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+
3*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e
)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*Elli
pticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-6*(-I*(-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/
```

```
2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)
+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+3*(-I*(-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-s
in(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f
*x+e))^(1/2),1/2*2^(1/2))-3*2^(1/2)*cos(f*x+e)^2+2^(1/2)*cos(f*x+e)+3*2^(1/
2))/sin(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\csc\left(fx+e\right)}d\csc\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*d*csc(f*x + e)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}\csc^{2}\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*csc(e + f*x))**(3/2)*csc(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*csc(f*x + e)^2, x)
```

3.523 $\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal. Leaf size=102

$$-\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{10\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21df}$$

```
[Out] (-2*d^2*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*Cos[e + f*x])/(21*
f*Sqrt[d*Csc[e + f*x]]) + (10*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*
x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*d*f)
```

Rubi [A] time = 0.0711687, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2641}

$$-\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{10\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21df}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*d^2*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*Cos[e + f*x])/(21*
f*Sqrt[d*Csc[e + f*x]]) + (10*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*
x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*d*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = d^3\int \frac{1}{(d\csc(e+fx))^{7/2}}\,dx$$
$$= -\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} + \frac{1}{7}(5d)\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx$$
$$= -\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{5\int \sqrt{d\csc(e+fx)}\,dx}{21d}$$
$$= -\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{\left(5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{21d}$$
$$= -\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{10\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+f}}{21df}$$

Mathematica [A] time = 0.136695, size = 70, normalized size = 0.69

$$-\frac{\sqrt{d\csc(e+fx)}\left(26\sin(2(e+fx)) - 3\sin(4(e+fx)) + 40\sqrt{\sin(e+fx)}F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{84df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] -(Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e +
 f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/(84*d*f)
```

Maple [C] time = 0.154, size = 208, normalized size = 2.

$$-\frac{\sqrt{2}}{21f\left(-1+\cos\left(fx+e\right)\right)}\left(5i\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sin\left(fx+e\right)\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\mathrm{EllipticF}\left(\sqrt{\frac{ic}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -1/21/f*2^(1/2)*(5*I*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*(-(I*
cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x
+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+
e))^(1/2)-3*2^(1/2)*cos(f*x+e)^4+3*2^(1/2)*cos(f*x+e)^3+8*2^(1/2)*cos(f*x+e
)^2-8*2^(1/2)*cos(f*x+e))/(-1+cos(f*x+e))/(d/sin(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^3}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^{2}-1\right)\sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)}{d\csc\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(d*csc(f*x + e))*sin(f*x + e)/(d*csc(f*x
 + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{3}}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

3.524 $\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal. Leaf size=72

$$\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

```
[Out] (-2*d*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*EllipticE[(e - Pi/2 +
 f*x)/2, 2])/(5*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rubi [A] time = 0.0514034, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2639}

$$\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*d*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*EllipticE[(e - Pi/2 +
 f*x)/2, 2])/(5*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = d^2\int \frac{1}{(d\csc(e+fx))^{5/2}}\,dx$$
$$= -\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{3}{5}\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx$$
$$= -\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{3\int \sqrt{\sin(e+fx)}\,dx}{5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$
$$= -\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{6E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

Mathematica [A] time = 0.169243, size = 57, normalized size = 0.79

$$\frac{-2\sin(2(e+fx)) - \frac{12E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{\sqrt{\sin(e+fx)}}}{10f\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] ((-12*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*x]] - 2*Sin[2*(e
+ f*x)])/(10*f*Sqrt[d*Csc[e + f*x]])
```

Maple [C] time = 0.148, size = 547, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -1/5/f*2^(1/2)*(6*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(
f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*
x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^
(1/2))-3*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+si
n(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/
2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+6*
(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+
e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos
(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-3*(-I*(-1+cos(f*x+e))/
sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f
*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I
)/sin(f*x+e))^(1/2),1/2*2^(1/2))-2^(1/2)*cos(f*x+e)^3+4*2^(1/2)*cos(f*x+e)-
3*2^(1/2))/(d/sin(f*x+e))^(1/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^{2}-1\right)\sqrt{d\csc\left(fx+e\right)}}{d\csc\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(d*csc(f*x + e))/(d*csc(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^{2}\left(e+fx\right)}{\sqrt{d\csc\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/sqrt(d*csc(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{2}}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

3.525 $\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal. Leaf size=74

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3df}-\frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}$$

```
[Out] (-2*Cos[e + f*x])/(3*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*Elli
pticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d*f)
```

Rubi [A] time = 0.044945, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 3769, 3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3df}-\frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x])/(3*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*Elli
pticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = d\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx$$
$$= -\frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{\int \sqrt{d\csc(e+fx)}\,dx}{3d}$$
$$= -\frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d}$$
$$= -\frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{2\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{3df}$$

Mathematica [A] time = 0.0799577, size = 64, normalized size = 0.86

$$-\frac{d\csc^2(e+fx)\left(\sin(2(e+fx))+2\sqrt{\sin(e+fx)}F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] -(d*Csc[e + f*x]^2*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f*x]
] + Sin[2*(e + f*x)]))/(3*f*(d*Csc[e + f*x])^(3/2))
```

Maple [C] time = 0.128, size = 181, normalized size = 2.5

$$-\frac{\sqrt{2}}{3f\left(-1+\cos\left(fx+e\right)\right)}\left(i\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sin\left(fx+e\right)\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-s}{\sin\left(fx\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*2^(1/2)*(I*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*((I*cos(
f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*
x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^
(1/2))+2^(1/2)*cos(f*x+e)^2-2^(1/2)*cos(f*x+e))/(-1+cos(f*x+e))/(d/sin(f*x+
e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)/sqrt(d*csc(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)}{d\csc\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*sin(f*x + e)/(d*csc(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(e+fx\right)}{\sqrt{d\csc\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)/sqrt(d*csc(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)/sqrt(d*csc(f*x + e)), x)
```

3.526 $\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal. Leaf size=43

$$\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f
*x]])
```

Rubi [A] time = 0.0185217, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2639}

$$\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f
*x]])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx &= \frac{\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\ &= \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0152657, size = 42, normalized size = 0.98

$$-\frac{2E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e
 + f*x]])
```

Maple [C] time = 0.124, size = 521, normalized size = 12.1

$$-\frac{\sqrt{2}}{f\sin\left(fx+e\right)}\left(2\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\cos\left(fx+e\right)\sqrt{\frac{-i\cos\left(fx+e\right)+\sin\left(fx+e\right)+i}{\sin\left(fx+e\right)}}\text{EllipticE}\left(\sqrt{\frac{i\cos\left(fx\right.}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -1/f*2^(1/2)*(2*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*((-I*cos(f
*x+e)+sin(f*x+e)+I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I
)/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(
1/2)-(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*((-I*cos(f*x+e)+sin(f
*x+e)+I)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*E
llipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((-I*cos(f*x+e)+sin(f*x+e)+I)/sin(f*x+e))
^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2)
)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)-(-I*(-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*((-I*cos(f*x+e)+sin(f*x+e)+I)/sin(f*x+e))^(1/2)*EllipticF(((I*c
os(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)+sin(f
*x+e)-I)/sin(f*x+e))^(1/2)+2^(1/2)*cos(f*x+e)-2^(1/2))/(d/sin(f*x+e))^(1/2)
/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(d*csc(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}}{d\csc\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))/(d*csc(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d \csc\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(d*csc(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d \csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(d*csc(f*x + e)), x)
```

3.527 $\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{df}$$

```
[Out] (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]]
)/(d*f)
```

Rubi [A] time = 0.0214527, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{df}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]]
)/(d*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx &= \frac{\int \sqrt{d\csc(e+fx)}\,dx}{d}\\ &= \frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{d}\\ &= \frac{2\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{df}\end{aligned}$$

Mathematica [A] time = 0.0166597, size = 45, normalized size = 0.98

$$-\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\sqrt{d\csc(e+fx)}}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f
*x]])/(d*f)
```

Maple [C] time = 0.109, size = 165, normalized size = 3.6

$$\frac{-i\left(-1+\cos\left(fx+e\right)\right)\left(\cos\left(fx+e\right)+1\right)^{2}\sqrt{2}}{f\left(\sin\left(fx+e\right)\right)^{3}}\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+\right.}{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -I/f*(-1+cos(f*x+e))*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*co
s(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*(
cos(f*x+e)+1)^2*2^(1/2)/(d/sin(f*x+e))^(1/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc\left(fx+e\right)}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/sqrt(d*csc(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}}{d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))/d, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(e+fx\right)}{\sqrt{d\csc\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(d*csc(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/sqrt(d*csc(f*x + e)), x)
```

3.528 $\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal. Leaf size=70

$$-\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df}-\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] (-2*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(d*f) - (2*EllipticE[(e - Pi/2 + f*x
)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rubi [A] time = 0.0385336, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2639}

$$-\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df}-\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(d*f) - (2*EllipticE[(e - Pi/2 + f*x
)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{\int (d\csc(e+fx))^{3/2}\,dx}{d^2}$$
$$= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df} - \int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx$$
$$= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df} - \frac{\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$
$$= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df} - \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

Mathematica [A] time = 0.0841349, size = 52, normalized size = 0.74

$$\frac{\frac{2E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{\sqrt{\sin(e+fx)}} - 2\cot(e+fx)}{f\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cot[e + f*x] + (2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*x
]])/(f*Sqrt[d*Csc[e + f*x]])
```

Maple [C] time = 0.117, size = 522, normalized size = 7.5

$$\frac{\sqrt{2}}{f\sin(fx+e)}\left(2\cos(fx+e)\sqrt{\frac{-i(-1+\cos(fx+e))}{\sin(fx+e)}}\sqrt{\frac{i\cos(fx+e)+\sin(fx+e)-i}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x)
```

```
[Out] 1/f*2^(1/2)*(2*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x
+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e
))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/
2))-cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x
+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*El
lipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*(
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(
1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+
e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-s
in(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f
*x+e))^(1/2),1/2*2^(1/2))-2^(1/2))/(d/sin(f*x+e))^(1/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)^2}{\sqrt{d\csc(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)}{d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e)/d, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2\left(e+fx\right)}{\sqrt{d\csc\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(d*csc(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

3.529 $\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3df}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f}$$

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*d^2*f) + (2*Sqrt[d*Csc[e + f*x]
]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d*f)
```

Rubi [A] time = 0.0393681, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3df}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*d^2*f) + (2*Sqrt[d*Csc[e + f*x]
]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{\int (d\csc(e+fx))^{5/2}\,dx}{d^3}$$

$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f} + \frac{\int \sqrt{d\csc(e+fx)}\,dx}{3d}$$

$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f} + \frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d}$$

$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f} + \frac{2\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{3df}$$

Mathematica [A] time = 0.0708461, size = 60, normalized size = 0.78

$$-\frac{2\csc^2(e+fx)\left(\cos(e+fx)+\sin^{\frac{3}{2}}(e+fx)F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3f\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Csc[e + f*x]^2*(Cos[e + f*x] + EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sin[
e + f*x]^(3/2)))/(3*f*Sqrt[d*Csc[e + f*x]])
```

Maple [C] time = 0.129, size = 319, normalized size = 4.1

$$\frac{\sqrt{2}\left(-1+\cos\left(fx+e\right)\right)^2\left(\cos\left(fx+e\right)+1\right)^2}{3f\left(\sin\left(fx+e\right)\right)^6}\left(i\cos\left(fx+e\right)\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x)
```

```
[Out] 1/3/f*2^(1/2)*(-1+cos(f*x+e))^2*(I*cos(f*x+e)*((I*cos(f*x+e)+sin(f*x+e)-I)/
sin(f*x+e))^(1/2)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin
(f*x+e)-I)/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)
-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+I*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*
sin(f*x+e)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-s
in(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f
*x+e))^(1/2),1/2*2^(1/2))-2^(1/2)*cos(f*x+e))*(cos(f*x+e)+1)^2/sin(f*x+e)^6
/(d/sin(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^3}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^{2}}{d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e)^2/d, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^{3}\left(e+fx\right)}{\sqrt{d\csc\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/sqrt(d*csc(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{3}}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

3.530 $\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{10\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21d^2f}-\frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}$$

```
[Out] (-2*d*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*Cos[e + f*x])/(21*d*
f*Sqrt[d*Csc[e + f*x]]) + (10*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*
x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*d^2*f)
```

Rubi [A] time = 0.0737739, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2641}

$$\frac{10\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21d^2f}-\frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*d*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*Cos[e + f*x])/(21*d*
f*Sqrt[d*Csc[e + f*x]]) + (10*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*
x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*d^2*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = d^2\int \frac{1}{(d\csc(e+fx))^{7/2}}\,dx$$
$$= -\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} + \frac{5}{7}\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx$$
$$= -\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}} + \frac{5\int \sqrt{d\csc(e+fx)}\,dx}{21d^2}$$
$$= -\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}} + \frac{\left(5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx}}}{21d^2}$$
$$= -\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}} + \frac{10\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin}}{21d^2f}$$

Mathematica [A] time = 0.113968, size = 70, normalized size = 0.68

$$-\frac{\sqrt{d\csc(e+fx)}\left(26\sin(2(e+fx))-3\sin(4(e+fx))+40\sqrt{\sin(e+fx)}F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{84d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] -(Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e +
 f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/(84*d^2*f)
```

Maple [C] time = 0.137, size = 216, normalized size = 2.1

$$-\frac{\sqrt{2}}{21f\left(-1+\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}\left(5i\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sin\left(fx+e\right)\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\mathrm{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] -1/21/f*2^(1/2)*(5*I*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*(-(I*
cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x
+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+
e))^(1/2)-3*2^(1/2)*cos(f*x+e)^4+3*2^(1/2)*cos(f*x+e)^3+8*2^(1/2)*cos(f*x+e
)^2-8*2^(1/2)*cos(f*x+e))/(-1+cos(f*x+e))/(d/sin(f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^{2}-1\right)\sqrt{d\csc\left(fx+e\right)}}{d^{2}\csc\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(d*csc(f*x + e))/(d^2*csc(f*x + e)^2), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^{2}\left(e+fx\right)}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/(d*csc(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{2}}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

3.531 $\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

```
[Out] (-2*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*EllipticE[(e - Pi/2 + f
*x)/2, 2])/(5*d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rubi [A] time = 0.0456729, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 3769, 3771, 2639}

$$\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
```


```
[Out] (-2*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*EllipticE[(e - Pi/2 + f
*x)/2, 2])/(5*d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = d\int \frac{1}{(d\csc(e+fx))^{5/2}}\,dx$$
$$= -\frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx}{5d}$$
$$= -\frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{3\int \sqrt{\sin(e+fx)}\,dx}{5d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$
$$= -\frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{6E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

Mathematica [A] time = 0.0324978, size = 60, normalized size = 0.81

$$\frac{-2\sin(2(e+fx)) - \frac{12E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{\sqrt{\sin(e+fx)}}}{10df\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] ((-12*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*x]] - 2*Sin[2*(e
+ f*x)])/(10*d*f*Sqrt[d*Csc[e + f*x]])
```

Maple [C] time = 0.128, size = 546, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] 1/5/f*2^(1/2)*(-6*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(
f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*
x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^
(1/2))+3*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+si
n(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/
2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2^
(1/2)*cos(f*x+e)^3-6*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+s
in(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1
/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+3
*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x
+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*co
s(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-4*2^(1/2)*cos(f*x+e)+
3*2^(1/2))/(d/sin(f*x+e))^(3/2)/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)}{d^2\csc\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*sin(f*x + e)/(d^2*csc(f*x + e)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(e+fx\right)}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)/(d*csc(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(fx+e\right)}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

3.532 $\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}$$

```
[Out] (-2*Cos[e + f*x])/(3*d*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*El
lipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f)
```

Rubi [A] time = 0.033067, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3769, 3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^(-3/2),x]
```

```
[Out] (-2*Cos[e + f*x])/(3*d*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*El
lipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f)
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx &= -\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}+\frac{\int \sqrt{d\csc(e+fx)}\,dx}{3d^2}\\
&= -\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}+\frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d^2}\\
&= -\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}+\frac{2\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{3d^2f}
\end{aligned}$$

Mathematica [A] time = 0.0177733, size = 63, normalized size = 0.82

$$-\frac{\csc^2(e+fx)\left(\sin(2(e+fx))+2\sqrt{\sin(e+fx)}F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^(-3/2),x]
```

```
[Out] -(Csc[e + f*x]^2*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f*x]]
+ Sin[2*(e + f*x)]))/(3*f*(d*Csc[e + f*x])^(3/2))
```

Maple [C] time = 0.108, size = 189, normalized size = 2.5

$$-\frac{\sqrt{2}}{3f\left(-1+\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}\left(i\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sin\left(fx+e\right)\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] -1/3/f*2^(1/2)*(I*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*((I*cos(
f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*
x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^
(1/2))+2^(1/2)*cos(f*x+e)^2-2^(1/2)*cos(f*x+e))/(-1+cos(f*x+e))/(d/sin(f*x+
e))^(3/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}}{d^2\csc\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))/(d^2*csc(f*x + e)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*csc(e + f*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^(-3/2), x)
```

3.533 $\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e +
 f*x]])
```

Rubi [A] time = 0.021443, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3771, 2639}

$$\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e +
 f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx &= \frac{\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx}{d}\\
&= \frac{\int \sqrt{\sin(e+fx)}\,dx}{d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\
&= \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.0311639, size = 45, normalized size = 0.98

$$-\frac{2E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin
[e + f*x]])
```

Maple [C] time = 0.106, size = 533, normalized size = 11.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] -1/f*2^(1/2)*(2*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*
x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+
e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1
/2))-cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*
x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*E
llipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^
(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x
+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-
sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(
f*x+e))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(f*x+e)-2^(1/2))/(d/sin(f*x+e))^(3/2)
/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}}{d^2\csc\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))/(d^2*csc(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(e+fx\right)}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)/(d*csc(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

3.534 $\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{d^2f}$$

```
[Out] (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]]
)/(d^2*f)
```

Rubi [A] time = 0.0219113, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]]
)/(d^2*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx &= \frac{\int \sqrt{d\csc(e+fx)}\,dx}{d^2}\\
&= \frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{d^2}\\
&= \frac{2\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{d^2f}
\end{aligned}$$

Mathematica [A] time = 0.0165215, size = 45, normalized size = 0.98

$$-\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\sqrt{d\csc(e+fx)}}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f
*x]])/(d^2*f)
```

Maple [C] time = 0.107, size = 165, normalized size = 3.6

$$\frac{-i\sqrt{2}\left(\cos\left(fx+e\right)+1\right)^2\left(-1+\cos\left(fx+e\right)\right)}{f\left(\sin\left(fx+e\right)\right)^4}\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(f\right.}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] -I/f*2^(1/2)*(cos(f*x+e)+1)^2*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*
x+e))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x
+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e)
)^(1/2)*(-1+cos(f*x+e))/(d/sin(f*x+e))^(3/2)/sin(f*x+e)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}}{d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))/d^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2\left(e+fx\right)}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(d*csc(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

3.535 $\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f}-\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] (-2*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(d^2*f) - (2*EllipticE[(e - Pi/2 + f
*x)/2, 2])/(d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rubi [A] time = 0.0388136, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2639}

$$-\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f}-\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(d^2*f) - (2*EllipticE[(e - Pi/2 + f
*x)/2, 2])/(d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\int (d\csc(e+fx))^{3/2}\,dx}{d^3}$$
$$= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f} - \frac{\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx}{d}$$
$$= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f} - \frac{\int \sqrt{\sin(e+fx)}\,dx}{d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$
$$= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f} - \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

Mathematica [A] time = 0.0513571, size = 55, normalized size = 0.75

$$\frac{\frac{2E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{\sqrt{\sin(e+fx)}} - 2\cot(e+fx)}{df\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cot[e + f*x] + (2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*x
]])/(d*f*Sqrt[d*Csc[e + f*x]])
```

Maple [C] time = 0.115, size = 522, normalized size = 7.2

$$\frac{\sqrt{2}}{f\left(\sin\left(fx+e\right)\right)^2}\left(2\cos\left(fx+e\right)\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx\right.}{\sin\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] 1/f*2^(1/2)*(2*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x
+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e
))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/
2))-cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x
+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*El
lipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*(
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(
1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+
e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-s
in(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f
*x+e))^(1/2),1/2*2^(1/2))-2^(1/2))/(d/sin(f*x+e))^(3/2)/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^3}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^3/(d*csc(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)}{d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e)/d^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3\left(e+fx\right)}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(d*csc(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^3}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/(d*csc(f*x + e))^(3/2), x)
```

3.536 $\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f}$$

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*d^3*f) + (2*Sqrt[d*Csc[e + f*x]
]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f)
```

Rubi [A] time = 0.0388047, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*d^3*f) + (2*Sqrt[d*Csc[e + f*x]
]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\int (d\csc(e+fx))^{5/2}\,dx}{d^4}$$
$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f} + \frac{\int \sqrt{d\csc(e+fx)}\,dx}{3d^2}$$
$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f} + \frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d^2}$$
$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f} + \frac{2\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{3d^2f}$$

Mathematica [A] time = 0.0667853, size = 60, normalized size = 0.78

$$-\frac{2\csc^3(e+fx)\left(\cos(e+fx)+\sin^{\frac{3}{2}}(e+fx)F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Csc[e + f*x]^3*(Cos[e + f*x] + EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sin[
e + f*x]^(3/2)))/(3*f*(d*Csc[e + f*x])^(3/2))
```

Maple [C] time = 0.138, size = 319, normalized size = 4.1

$$\frac{\sqrt{2}\left(-1+\cos\left(fx+e\right)\right)^2\left(\cos\left(fx+e\right)+1\right)^2}{3\,f\left(\sin\left(fx+e\right)\right)^7}\left(i\cos\left(fx+e\right)\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{\frac{-i\left(-1+\cos\left(fx+e\right.\right.}{\sin\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] 1/3/f*2^(1/2)*(-1+cos(f*x+e))^2*(I*cos(f*x+e)*((I*cos(f*x+e)+sin(f*x+e)-I)/
sin(f*x+e))^(1/2)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin
(f*x+e)-I)/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)
-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+I*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*
sin(f*x+e)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-s
in(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f
*x+e))^(1/2),1/2*2^(1/2))-2^(1/2)*cos(f*x+e))*(cos(f*x+e)+1)^2/sin(f*x+e)^7
/(d/sin(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^4}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^4/(d*csc(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^{2}}{d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e)^2/d^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc^{4}\left(e+fx\right)}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(d*csc(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc\left(fx+e\right)^{4}}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(d*csc(f*x + e))^(3/2), x)
```

3.537 $\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f}-\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f}-\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] (-6*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(5*d^2*f) - (2*Cos[e + f*x]*(d*Csc[e
 + f*x])^(5/2))/(5*d^4*f) - (6*EllipticE[(e - Pi/2 + f*x)/2, 2])/(5*d*f*Sqr
t[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rubi [A] time = 0.0590597, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2639}

$$-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f}-\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f}-\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-6*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(5*d^2*f) - (2*Cos[e + f*x]*(d*Csc[e
 + f*x])^(5/2))/(5*d^4*f) - (6*EllipticE[(e - Pi/2 + f*x)/2, 2])/(5*d*f*Sqr
t[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\int (d\csc(e+fx))^{7/2}\,dx}{d^5}$$

$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f} + \frac{3\int (d\csc(e+fx))^{3/2}\,dx}{5d^3}$$

$$= -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f} - \frac{3\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx}{5d}$$

$$= -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f} - \frac{3\int \sqrt{\sin(e+fx)}\,dx}{5d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+}}$$

$$= -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f} - \frac{6E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+}}$$

Mathematica [A] time = 0.113364, size = 73, normalized size = 0.7

$$\frac{\csc^4(e+fx)\left(-7\cos(e+fx)+3\cos(3(e+fx))+12\sin^{\frac{5}{2}}(e+fx)E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{10f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (Csc[e + f*x]^4*(-7*Cos[e + f*x] + 3*Cos[3*(e + f*x)] + 12*EllipticE[(-2*e
+ Pi - 2*f*x)/4, 2]*Sin[e + f*x]^(5/2)))/(10*f*(d*Csc[e + f*x])^(3/2))
```

Maple [C] time = 0.148, size = 1054, normalized size = 10.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] -1/5/f*2^(1/2)*(6*cos(f*x+e)^3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(I*c
os(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+
e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e
))^(1/2)-3*cos(f*x+e)^3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+
e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))
^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2)
)+6*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e
)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^2*(-I*(-1+cos(f*x
+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)-3*(-(
I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f
*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^2*(-I*(-1+cos(f*x+e))/si
n(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)-6*cos(f*x+e)
*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x
+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*co
s(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+3*cos(f*x+e)*(-I*(-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2
)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+
sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-6*(-I*(-1+cos(f*x+e))/sin(f*x+
```

```
e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-si
n(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*
x+e))^(1/2),1/2*2^(1/2))+3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*
x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+
e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1
/2))-3*2^(1/2)*cos(f*x+e)^2+2^(1/2)*cos(f*x+e)+3*2^(1/2))/sin(f*x+e)^4/(d/s
in(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^5}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^5/(d*csc(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^3}{d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e)^3/d^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^5\left(e+fx\right)}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**5/(d*csc(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^5}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^5/(d*csc(f*x + e))^(3/2), x)
```

3.538 $\int (b\csc(e+fx))^n (a\sin(e+fx))^m \, dx$

Optimal. Leaf size=87

$$\frac{\cos(e+fx)(a\sin(e+fx))^{m+1}(b\csc(e+fx))^n \, {}_2F_1\left(\frac{1}{2},\frac{1}{2}(m-n+1);\frac{1}{2}(m-n+3);\sin^2(e+fx)\right)}{af(m-n+1)\sqrt{\cos^2(e+fx)}}$$

```
[Out] (Cos[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[1/2, (1 + m - n)/2, (3 +
 m - n)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m - n)*Sqrt[
Cos[e + f*x]^2])
```

Rubi [A] time = 0.0636076, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2588, 2643}

$$\frac{\cos(e+fx)(a\sin(e+fx))^{m+1}(b\csc(e+fx))^n \, {}_2F_1\left(\frac{1}{2},\frac{1}{2}(m-n+1);\frac{1}{2}(m-n+3);\sin^2(e+fx)\right)}{af(m-n+1)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[1/2, (1 + m - n)/2, (3 +
 m - n)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m - n)*Sqrt[
Cos[e + f*x]^2])
```

Rule 2588

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[(a*b)^IntPart[n]*(a*Sin[e + f*x])^FracPart[n]*(b*Csc
[e + f*x])^FracPart[n], Int[(a*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b
, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\csc(e+fx))^n (a\sin(e+fx))^m \, dx = \left((b\csc(e+fx))^n (a\sin(e+fx))^n\right) \int (a\sin(e+fx))^{m-n} \, dx$$

$$= \frac{\cos(e+fx)(b\csc(e+fx))^n \, {}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+m-n);\frac{1}{2}(3+m-n);\sin^2(e+fx)\right)}{af(1+m-n)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 9.79536, size = 102, normalized size = 1.17

$$\frac{2\tan\left(\frac{1}{2}(e+fx)\right)(a\sin(e+fx))^m(b\csc(e+fx))^n\sec^2\left(\frac{1}{2}(e+fx)\right)^{m-n} \, {}_2F_1\left(\frac{1}{2}(m-n+1),m-n+1;\frac{1}{2}(m-n+3);-t\right.}{f(m-n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] (2*(b*Csc[e + f*x])^n*Hypergeometric2F1[(1 + m - n)/2, 1 + m - n, (3 + m -
n)/2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(m - n)*(a*Sin[e + f*x])^m*
Tan[(e + f*x)/2])/(f*(1 + m - n))
```

Maple [F] time = 0.781, size = 0, normalized size = 0.

$$\int \left(b \csc \left(fx+e\right)\right)^{n} \left(a \sin \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

```
[Out] int((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \csc \left(fx+e\right)\right)^{n} \left(a \sin \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \csc \left(fx+e\right)\right)^{n} \left(a \sin \left(fx+e\right)\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(e+fx\right)\right)^{m} \left(b \csc \left(e+fx\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*(a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*(b*csc(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n} \left(a\sin\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do
    not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
   else #ExpnType(result) > ExpnType(optimal)
       if debug then
          print("ExpnType(result) > ExpnType(optimal)");
       fi;
       return "C";
   end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
   (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```